CHAPTER V

CLASSICAL SEMI-INFINITE DIATOMIC CHAIN

In the last chapter we have reviewed the semi-infinite monatomic chain[1] shortly by the response function method. In this chapter we have proposed how to extend Martinez's work for the semi-infinite diatomic chain problem with some applications as follows.

The Response Function of the Semi-Infinite Diatomic Chain

The semi-infinite diatomic chain was performed by the sinusoidally varying force f(t) is shown in Fig 5.1,

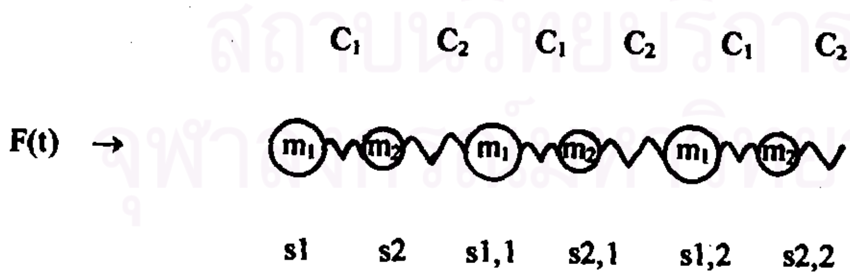

Fig 5.1 The semi-infinite diatomic chain was performed by the force.

where C_1 and C_2 are the spring constant and $F(t) = f_{01}\exp(i\omega t)$.

When the force is applied , the masses oscillate with the same frequency. Then $u_{sn}(t) = u_{sn0}(\omega)\exp(i\omega t)$, where $u_{sn0}(\omega)$ is the Fourier transform of $u_{sn}(t)$.

5.1 <u>The Recursion Relations</u>

The equation of motion of the first mass (s1), when the force is applied, is

$$m_1 \ddot{u}_{s1}(t) = C_1[u_{s2}(t) - u_{s1}(t)] + F(t) \quad ,$$

where $u_{sn}(t) = u_{sn0}(\omega)\exp(i\omega t)$, $F(t) = f_{01}\exp(i\omega t)$ and $\ddot{u}_{sn}(t) = \ddot{u}_{sn0}(\omega)\exp(i\omega t)$; $\ddot{u}_{sn}(t)$ is the acceleration of the sn th mass and $\ddot{u}_{sn}(\omega)$ is the Fourier transform of $\ddot{u}_{sn}(t)$.

Therefore

$$m_1 u_{s1}(t) = C_1(u_{s20}(\omega)\exp(i\omega t) - u_{s10}(\omega)\exp(i\omega t)) + f_{01}\exp(i\omega t) \quad .$$

Hence,
$$m_1 \ddot{u}_{s1}(\omega) = C_1(u_{s20}(\omega) - u_{s10}(\omega)) + f_{01} \quad , \quad (5.1.1)$$

and the definition of the Fourier transform of the response function is

$$\ddot{u}_{s10}(\omega) = \chi_1(\omega) f_{01} \quad , \quad (5.1.2.a)$$

where $\chi_1(\omega)$ is the first response function.

Considering the s1,n th mass (n > 0), the acting force is

$$C_1(u_{s20,n}(\omega) - u_{s10,n}(\omega)) + C_2(u_{s20,n-1}(\omega) - u_{s10,n}(\omega)) \quad .$$

But the acting force from the left is $C_2(u_{s20,n-1}(\omega)-u_{s10,n}(\omega))$, therefore ,

$$\ddot{u}_{s10,n}(\omega) = \chi_1(\omega)C_2(u_{s20,n-1}(\omega) - u_{s10,n}(\omega)) \quad . \tag{5.1.2.b}$$

In the case of Harmonic motion $\ddot{u}_{sn}(\omega) = -\omega^2 u_{sn}$ [, the Eq.(5.1.2.b) is written

$$-\omega^2 u_{s10,n}(\omega) = \chi_1(\omega)C_2(u_{s20,n-1}(\omega) - u_{s10,n}(\omega)) \quad .$$

Therefore, we can rewrite that

$$\frac{u_{s10,n}(\omega)}{u_{s20,n-1}(\omega)} = \frac{\chi_1(\omega)C_2}{\chi_1(\omega)C_2 - \omega^2} \quad . \tag{5.1.3}$$

This equation is called as the first recursion relation.

Similarly, the equation of motion of the s2 th mass is

$$m_2\ddot{u}_{s2}(t) = C_1(u_{s1}(t) - u_{s2}(t)) + C_2(u_{s1,1}(t) - u_{s2}(t)) \quad ,$$

where $u_{sn}(t) = u_{sn0}(\omega)\exp(i\omega t)$ and $\ddot{u}_{sn}(t) = \ddot{u}_{sn0}(\omega)\exp(i\omega t)$. Therefore

$$m_2\ddot{u}_{s20}(\omega) = C_1(u_{s10}(\omega) - u_{s20}(\omega)) + C_2(u_{s10,1}(\omega) - u_{s20}(\omega)) \quad ,$$

and let $C_1(u_{s10}(\omega) - u_{s20}(\omega)) = f_{02}$. Therefore, we can rewrite that

$$m_2\ddot{u}_{s20}(\omega) = f_{02} + C_2(u_{s10,1}(\omega) - u_{s20}(\omega)) \quad . \tag{5.1.4}$$

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From the definition of the Fourier transform of the response function, the acceleration is equal to the acting force from the left multiplying the response function. Therefore,we can write

$$\ddot{u}_{s20}(\omega) = \chi_2(\omega) f_{02} \quad , \qquad (5.1.5.a)$$

where $\chi_2(\omega)$ is the second response function; and $\chi_1(\omega)$ and $\chi_2(\omega)$ are the two coupled response functions.

Consider the s2,n th mass $n \geq 0$, the acting force is

$$C_1(u_{s10,n}(\omega) - u_{s20,n}(\omega)) + C_2(u_{s10,n+1}(\omega) - u_{s20,n}(\omega)).$$

The acting force from the left is $C_1(u_{s10,n}(\omega) - u_{s20,n}(\omega))$, therefore,

$$\ddot{u}_{s20,n}(\omega) = \chi_2(\omega)\, C_1(u_{s10,n}(\omega) - u_{s20,n}(\omega)) \quad . \qquad (5.1.5.b)$$

In the case of Harmonic motion $\ddot{u}_m(\omega) = -\omega^2 u_m$, the Eq.(5.1.5.b) is written as

$$-\omega^2 u_{s20,n}(\omega) = \chi_2(\omega) C_1(u_{s10,n}(\omega) - u_{s20,n}(\omega)) \quad .$$

Therefore, we can rewrite that

$$\frac{u_{s20,n}(\omega)}{u_{s10,n}(\omega)} = \frac{\chi_2(\omega) C_1}{\chi_2(\omega) C_1 - \omega^2} \quad . \qquad (5.1.6)$$

This equation is called as the second recursion relation.

5.2 The Response Functions

The equation of motion of the first mass (s1) is

$$m_1 \ddot{u}_{s10}(\omega) = C_1(u_{s20}(\omega) - u_{s10}(\omega)) + f_0(\omega) \quad . \tag{5.2.1}$$

Inserting Eq.(5.1.6) into Eq.(5.2.1) yields

$$m_1 \ddot{u}_{s10}(\omega) = C_1 u_{s10}(\omega) \left[\frac{\chi_2(\omega)C_1}{\chi_2(\omega)C_1 - \omega^2} - 1 \right] + f_{01} \ ,$$

$$= C_1 u_{s10}(\omega) \left[\frac{\omega^2}{\chi_2(\omega)C_1 - \omega^2} \right] + f_{01} \ ,$$

From the harmonic motion $\ddot{u}(\omega) = -\omega^2 u(\omega)$(, we can rewrite that

$$\ddot{u}_{s10}(\omega) = \left[\frac{C_1}{\chi_2(\omega)C_1 - \omega^2} + m_1 \right]^{-1} f_{01} \quad . \tag{5.2.2}$$

Comparing between Eq.(5.2.2) and Eq.(5.1.2.a) gives

$$\chi_1(\omega) = \left[\frac{C_1}{\chi_2(\omega)C_1 - \omega^2} + m_1 \right]^{-1} \quad .$$

Therefore $$\chi_1(\omega) = \frac{\chi_2(\omega)C_1 - \omega^2}{C_1 + m_1\chi_2(\omega)C_1 - m_1\omega^2} \quad . \tag{5.2.3}$$

Similarly, we consider the equation of motion of the s2 th mass and insert Eq.(5.1.3) into Eq.(5.1.4); we will get

$$m_2 \ddot{u}_{s20}(\omega) = C_2 u_{s20}(\omega) \left[\frac{\chi_1(\omega) C_2}{\chi_1(\omega) C_2 - \omega^2} - 1 \right] + f_{02} ,$$

$$= C_2 u_{s20}(\omega) \left[\frac{\omega^2}{\chi_1(\omega) C_2 - \omega^2} \right] + f_{02} .$$

From the harmonic motion $\ddot{u}(\omega) = -\omega^2 u(\omega)$[, we can rewrite that

$$\ddot{u}_{s20}(\omega) = \left[\frac{C_2}{\chi_1(\omega) C_2 - \omega^2} + m_2 \right]^{-1} f_{02} . \qquad (5.2.4)$$

Comparing between Eq.(5.2.4) and Eq.(5.1.5.a) yields

$$\chi_2(\omega) = \left[\frac{C_2}{\chi_1(\omega) C_2 - \omega^2} + m_2 \right]^{-1} .$$

Therefore,

$$\chi_2(\omega) = \frac{\chi_1(\omega) C_2 - \omega^2}{C_2 + m_2 \chi_1(\omega) C_2 - m_2 \omega^2} . \qquad \textbf{(5.2.5)}$$

Inserting Eq.(5.2.5) into Eq.(5.2.3) (self-consistent solutions), and then simplifying by MCAD, yield

$$\chi_1(\omega) = \frac{\omega^2}{2C_2}\left[1 - \frac{(m_1 - m_2)C_2}{m_1 m_2 \omega^2 - (m_1 + m_2)C_1}\right]$$
$$\left[1 - \sqrt{1 + \frac{4C_2[(C_1 + C_2) - m_2\omega^2][m_1 m_2 \omega^2 - (m_1 + m_2)C_1]}{\omega^2[m_1 m_2 \omega^2 - (m_1 + m_2)C_1 - (m_1 - m_2)C_2]^2}}\right] \quad . (5.2.6a)$$

Similarly, inserting Eq.(5.2.3) into Eq.(5.2.5) (self-consistent solutions), and then simplifying by MCAD, yield

$$\chi_2(\omega) = \frac{\omega^2}{2C_1}\left[1 + \frac{(m_1 - m_2)C_1}{m_1 m_2 \omega^2 - (m_1 + m_2)C_2}\right]$$
$$\left[1 - \sqrt{1 + \frac{4C_1[(C_1 + C_2) - m_2\omega^2][m_1 m_2 \omega^2 - (m_1 + m_2)C_2]}{\omega^2[m_1 m_2 \omega^2 - (m_1 + m_2)C_2 - (m_1 - m_2)C_1]^2}}\right] \quad . (5.2.6b)$$

Eq.(5.2.6a) and Eq.(5.2.6b) are one of two possible solutions of $\chi_1(\omega)$ and $\chi_2(\omega)$ respectively. The others solutions are divergent.

From the Fig. 2.2.1, we can overview that the diatomic molecules are connected with together without the end.

One of the example of the model is NaCl crystal, where the structure of the NaCl salt crystal is called face-centered cubic. If we obtain M.Born's model[11] for the analogue of the NaCl lattice, this implies that we limit the problem which the distances(d_1 and d_2) are equal(d_1=d_2=d) and the spring constants(C_1 and C_2) are equal (C_1=C_2=C). This is shown[12] in Fig. 5.2.1

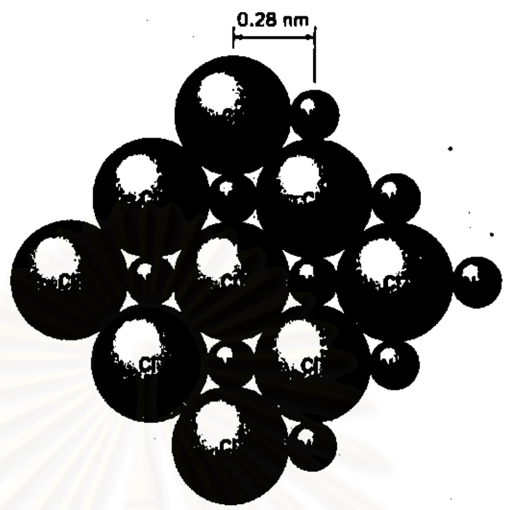

Fig. 5.2.1 The NaCl Structure.

Fig.5.2.1 is the NaCl structure in two dimensions. If we consider it in one dimension, it will be Fig. 5.2.2

Fig. 5.2.2 The NaCl Structure in One Dimension.

Therefore, the response function in Eq.(5.2.6a) and Eq.(5.2.6b) have been applied for the model of the semi-infinite diatomic molecules in Fig. 5.2.2. Hence, m_1 is the mass of Chlorine(Cl) and m_2 is the mass of Sodium(Na). C is the spring constant between the masses, m_1 and m_2.

From the periodic table of the elements we have the atomic mass of Na is equal to 22.98977 amu and the atomic mass of Cl is equal to 35.453 amu. Next step, we must find the values of the spring constant C by considering the molecular vibrations,

$$C \approx \frac{h^2}{m\, d^4_{atom}} ,$$

where Planck's constant $h = 6.62 \times 10^{-34}$ J-s , m is the electron mass and d_{atom} is the distance that confines the atom in a molecule, here it is 0.28 nm. Therefore,

$$C \approx \frac{(6.62 \times 10^{-34}\ \text{J - s})^2}{(9.109 \times 10^{-31}\,\text{kg})(0.28\text{nm})^4} ,$$

$$= 78.4945\ \text{N/m} . \tag{5.2.7}$$

We can evaluate the values of the $\chi_1(\omega)$ and $\chi_2(\omega)$ as shown in table 1 by MCAD.

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Table 1 .The Values of the $\chi_1(\omega)$ and $\chi_2(\omega)$.

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$	$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-9	0.535	0.902	-8.79	0.556	0.936
-8.995	0.535	0.902	-8.785	0.557	0.937
-8.99	0.536	0.903	-8.78	0.558	0.938
-8.985	0.536	0.904	-8.775	0.558	0.939
-8.98	0.537	0.905	-8.77	0.559	0.94
-8.975	0.537	0.905	-8.765	0.56	0.941
-8.97	0.537	0.906	-8.76	0.56	0.942
-8.965	0.538	0.907	-8.755	0.561	0.943
-8.96	0.538	0.908	-8.75	0.561	0.944
-8.955	0.539	0.908	-8.745	0.562	0.945
-8.95	0.539	0.909	-8.74	0.563	0.946
-8.945	0.54	0.91	-8.735	0.563	0.947
-8.94	0.54	0.911	-8.73	0.564	0.948
-8.935	0.541	0.911	-8.725	0.565	0.949
-8.93	0.541	0.912	-8.72	0.566	0.95
-8.925	0.542	0.913	-8.715	0.566	0.951
-8.92	0.542	0.914	-8.71	0.567	0.952
-8.915	0.543	0.915	-8.705	0.568	0.954
-8.91	0.543	0.915	-8.7	0.568	0.955
-8.905	0.544	0.916	-8.695	0.569	0.956
-8.9	0.544	0.917	-8.69	0.57	0.957
-8.895	0.545	0.918	-8.685	0.571	0.958
-8.89	0.545	0.919	-8.68	0.571	0.959
-8.885	0.546	0.92	-8.675	0.572	0.96
-8.88	0.546	0.92	-8.67	0.573	0.961
-8.875	0.547	0.921	-8.665	0.574	0.962
-8.87	0.547	0.922	-8.66	0.574	0.964
-8.865	0.548	0.923	-8.655	0.575	0.965
-8.86	0.548	0.924	-8.65	0.576	0.966
-8.855	0.549	0.925	-8.645	0.577	0.967
-8.85	0.549	0.926	-8.64	0.578	0.968
-8.845	0.55	0.926	-8.635	0.578	0.969
-8.84	0.551	0.927	-8.63	0.579	0.971
-8.835	0.551	0.928	-8.625	0.58	0.972
-8.83	0.552	0.929	-8.62	0.581	0.973
-8.825	0.552	0.93	-8.615	0.582	0.974
-8.82	0.553	0.931	-8.61	0.583	0.976
-8.815	0.553	0.932	-8.605	0.584	0.977
-8.81	0.554	0.933	-8.6	0.584	0.978
-8.805	0.555	0.934	-8.595	0.585	0.98
-8.8	0.555	0.935			
-8.795	0.556	0.935			

$w\cdot 10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$	$w\cdot 10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$
-8.59	0.586	0.981	-8.39	0.638	1.049
-8.585	0.587	0.982	-8.385	0.639	1.051
-8.58	0.588	0.983	-8.38	0.641	1.054
-8.575	0.589	0.985	-8.375	0.643	1.056
-8.57	0.59	0.986	-8.37	0.645	1.058
-8.565	0.591	0.988	-8.365	0.647	1.061
-8.56	0.592	0.989	-8.36	0.649	1.064
-8.555	0.593	0.99	-8.355	0.652	1.066
-8.55	0.594	0.992	-8.35	0.654	1.069
-8.545	0.595	0.993	-8.345	0.656	1.072
-8.54	0.596	0.995	-8.34	0.659	1.075
-8.535	0.597	0.996	-8.335	0.661	1.078
-8.53	0.598	0.998	-8.33	0.664	1.081
-8.525	0.599	0.999	-8.325	0.666	1.084
-8.52	0.601	1.001	-8.32	0.669	1.087
-8.515	0.602	1.002	-8.315	0.672	1.091
-8.51	0.603	1.004	-8.31	0.675	1.094
-8.505	0.604	1.005	-8.305	0.678	1.098
-8.5	0.605	1.007	-8.3	0.682	1.101
-8.495	0.606	1.009	-8.295	0.685	1.105
-8.49	0.608	1.01	-8.29	0.689	1.11
-8.485	0.609	1.012	-8.285	0.693	1.114
-8.48	0.61	1.014	-8.28	0.697	1.119
-8.475	0.611	1.015	-8.275	0.701	1.124
-8.47	0.613	1.017	-8.27	0.706	1.129
-8.465	0.614	1.019	-8.265	0.711	1.135
-8.46	0.615	1.021	-8.26	0.717	1.141
-8.455	0.617	1.022	-8.255	0.724	1.148
-8.45	0.618	1.024	-8.25	0.732	1.157
-8.445	0.62	1.026	-8.245	0.741	1.166
-8.44	0.621	1.028	-8.24	0.753	1.179
-8.435	0.623	1.03	-8.235	0.78	1.206
-8.43	0.624	1.032	-8.23	0.785 – 0.033i	1.212 – 0.033i
-8.425	0.626	1.034	-8.225	0.784 – 0.048i	1.211 – 0.048i
-8.42	0.627	1.036	-8.22	0.782 – 0.058i	1.21 – 0.058i
-8.415	0.629	1.038	-8.215	0.781 – 0.067i	1.209 – 0.067i
-8.41	0.631	1.04	-8.21	0.779 – 0.075i	1.209 – 0.075i
-8.405	0.632	1.042	-8.205	0.778 – 0.083i	1.208 – 0.083i
-8.4	0.634	1.044	-8.2	0.776 – 0.089i	1.207 – 0.089i
-8.395	0.636	1.047	-8.195	0.775 – 0.095i	1.206 – 0.095i

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$	$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-8.19	0.774 – 0.101i	1.205 – 0.101i	-7.99	0.714 – 0.228i	1.169 – 0.228i
-8.185	0.772 – 0.106i	1.204 – 0.106i	-7.985	0.713 – 0.23i	1.168 – 0.23i
-8.18	0.771 – 0.112i	1.203 – 0.112i	-7.98	0.711 – 0.232i	1.167 – 0.232i
-8.175	0.769 – 0.116i	1.202 – 0.116i	-7.975	0.71 – 0.234i	1.166 – 0.234i
-8.17	0.768 – 0.121i	1.201 – 0.121i	-7.97	0.708 – 0.236i	1.165 – 0.236i
-8.165	0.766 – 0.126i	1.2 – 0.126i	-7.965	0.706 – 0.238i	1.165 – 0.238i
-8.16	0.765 – 0.13i	1.199 – 0.13i	-7.96	0.705 – 0.24i	1.164 – 0.24i
-8.155	0.763 – 0.134i	1.198 – 0.134i	-7.955	0.703 – 0.242i	1.163 – 0.242i
-8.15	0.762 – 0.138i	1.197 – 0.138i	-7.95	0.702 – 0.244i	1.162 – 0.244i
-8.145	0.76 – 0.142i	1.196 – 0.142i	-7.945	0.7 – 0.246i	1.161 – 0.246i
-8.14	0.759 – 0.146i	1.195 – 0.146i	-7.94	0.699 – 0.248i	1.16 – 0.248i
-8.135	0.757 – 0.149i	1.195 – 0.149i	-7.935	0.697 – 0.25i	1.16 – 0.25i
-8.13	0.756 – 0.153i	1.194 – 0.153i	-7.93	0.696 – 0.251i	1.159 – 0.251i
-8.125	0.754 – 0.156i	1.193 – 0.156i	-7.925	0.694 – 0.253i	1.158 – 0.253i
-8.12	0.753 – 0.16i	1.192 – 0.16i	-7.92	0.693 – 0.255i	1.157 – 0.255i
-8.115	0.751 – 0.163i	1.191 – 0.163i	-7.915	0.691 – 0.257i	1.156 – 0.257i
-8.11	0.75 – 0.166i	1.19 – 0.166i	-7.91	0.69 – 0.258i	1.155 – 0.258i
-8.105	0.748 – 0.169i	1.189 – 0.169i	-7.905	0.688 – 0.26i	1.155 – 0.26i
-8.1	0.747 – 0.173i	1.188 – 0.173i	-7.9	0.687 – 0.262i	1.154 – 0.262i
-8.095	0.745 – 0.176i	1.187 – 0.176i	-7.895	0.685 – 0.264i	1.153 – 0.264i
-8.09	0.744 – 0.179i	1.186 – 0.179i	-7.89	0.684 – 0.265i	1.152 – 0.265i
-8.085	0.742 – 0.181i	1.185 – 0.181i	-7.885	0.682 – 0.267i	1.151 – 0.267i
-8.08	0.741 – 0.184i	1.185 – 0.184i	-7.88	0.681 – 0.269i	1.151 – 0.269i
-8.075	0.739 – 0.187i	1.184 – 0.187i	-7.875	0.679 – 0.27i	1.15 – 0.27i
-8.07	0.738 – 0.19i	1.183 – 0.19i	-7.87	0.678 – 0.272i	1.149 – 0.272i
-8.065	0.736 – 0.192i	1.182 – 0.192i	-7.865	0.676 – 0.273i	1.148 – 0.273i
-8.06	0.735 – 0.195i	1.181 – 0.195i	-7.86	0.675 – 0.275i	1.147 – 0.275i
-8.055	0.733 – 0.198i	1.18 – 0.198i	-7.855	0.673 – 0.276i	1.147 – 0.276i
-8.05	0.732 – 0.2i	1.179 – 0.2i	-7.85	0.672 – 0.278i	1.146 – 0.278i
-8.045	0.73 – 0.203i	1.178 – 0.203i	-7.845	0.67 – 0.28i	1.145 – 0.28i
-8.04	0.729 – 0.205i	1.177 – 0.205i	-7.84	0.669 – 0.281i	1.144 – 0.281i
-8.035	0.727 – 0.208i	1.177 – 0.208i	-7.835	0.667 – 0.283i	1.143 – 0.283i
-8.03	0.726 – 0.21i	1.176 – 0.21i	-7.83	0.666 – 0.284i	1.143 – 0.284i
-8.025	0.724 – 0.212i	1.175 – 0.212i	-7.825	0.664 – 0.285i	1.142 – 0.285i
-8.02	0.723 – 0.215i	1.174 – 0.215i	-7.82	0.662 – 0.287i	1.141 – 0.287i
-8.015	0.722 – 0.217i	1.173 – 0.217i	-7.815	0.661 – 0.288i	1.14 – 0.288i
-8.01	0.72 – 0.219i	1.172 – 0.219i	-7.81	0.659 – 0.29i	1.14 – 0.29i
-8.005	0.719 – 0.221i	1.171 – 0.221i	-7.805	0.658 – 0.291i	1.139 – 0.291i
-8	0.717 – 0.224i	1.171 – 0.224i	-7.8	0.656 – 0.293i	1.138 – 0.293i
-7.995	0.716 – 0.226i	1.17 – 0.226i	-7.795	0.655 – 0.294i	1.137 – 0.294i

$w\cdot 10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$
-7.79	0.653 – 0.295i	1.136 – 0.295i
-7.785	0.652 – 0.297i	1.136 – 0.297i
-7.78	0.65 – 0.298i	1.135 – 0.298i
-7.775	0.649 – 0.299i	1.134 – 0.299i
-7.77	0.647 – 0.301i	1.133 – 0.301i
-7.765	0.646 – 0.302i	1.133 – 0.302i
-7.76	0.644 – 0.303i	1.132 – 0.303i
-7.755	0.642 – 0.304i	1.131 – 0.304i
-7.75	0.641 – 0.306i	1.131 – 0.306i
-7.745	0.639 – 0.307i	1.13 – 0.307i
-7.74	0.638 – 0.308i	1.129 – 0.308i
-7.735	0.636 – 0.309i	1.128 – 0.309i
-7.73	0.635 – 0.311i	1.128 – 0.311i
-7.725	0.633 – 0.312i	1.127 – 0.312i
-7.72	0.632 – 0.313i	1.126 – 0.313i
-7.715	0.63 – 0.314i	1.125 – 0.314i
-7.71	0.628 – 0.315i	1.125 – 0.315i
-7.705	0.627 – 0.317i	1.124 – 0.317i
-7.7	0.625 – 0.318i	1.123 – 0.318i
-7.695	0.624 – 0.319i	1.123 – 0.319i
-7.69	0.622 – 0.32i	1.122 – 0.32i
-7.685	0.621 – 0.321i	1.121 – 0.321i
-7.68	0.619 – 0.322i	1.121 – 0.322i
-7.675	0.617 – 0.323i	1.12 – 0.323i
-7.67	0.616 – 0.324i	1.119 – 0.324i
-7.665	0.614 – 0.325i	1.119 – 0.325i
-7.66	0.613 – 0.327i	1.118 – 0.327i
-7.655	0.611 – 0.328i	1.117 – 0.328i
-7.65	0.61 – 0.329i	1.116 – 0.329i
-7.645	0.608 – 0.33i	1.116 – 0.33i
-7.64	0.606 – 0.331i	1.115 – 0.331i
-7.635	0.605 – 0.332i	1.114 – 0.332i
-7.63	0.603 – 0.333i	1.114 – 0.333i
-7.625	0.602 – 0.334i	1.113 – 0.334i
-7.62	0.6 – 0.335i	1.113 – 0.335i
-7.615	0.598 – 0.336i	1.112 – 0.336i
-7.61	0.597 – 0.337i	1.111 – 0.337i
-7.605	0.595 – 0.338i	1.111 – 0.338i
-7.6	0.594 – 0.339i	1.11 – 0.339i
-7.595	0.592 – 0.34i	1.109 – 0.34i

$w\cdot 10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$
-7.59	0.59 – 0.34i	1.109 – 0.34i
-7.585	0.589 – 0.341i	1.108 – 0.341i
-7.58	0.587 – 0.342i	1.107 – 0.342i
-7.575	0.586 – 0.343i	1.107 – 0.343i
-7.57	0.584 – 0.344i	1.106 – 0.344i
-7.565	0.582 – 0.345i	1.106 – 0.345i
-7.56	0.581 – 0.346i	1.105 – 0.346i
-7.555	0.579 – 0.347i	1.104 – 0.347i
-7.55	0.577 – 0.348i	1.104 – 0.348i
-7.545	0.576 – 0.348i	1.103 – 0.348i
-7.54	0.574 – 0.349i	1.103 – 0.349i
-7.535	0.573 – 0.35i	1.102 – 0.35i
-7.53	0.571 – 0.351i	1.101 – 0.351i
-7.525	0.569 – 0.352i	1.101 – 0.352i
-7.52	0.568 – 0.353i	1.1 – 0.353i
-7.515	0.566 – 0.353i	1.1 – 0.353i
-7.51	0.564 – 0.354i	1.099 – 0.354i
-7.505	0.563 – 0.355i	1.098 – 0.355i
-7.5	0.561 – 0.356i	1.098 – 0.356i
-7.495	0.559 – 0.357i	1.097 – 0.357i
-7.49	0.558 – 0.357i	1.097 – 0.357i
-7.485	0.556 – 0.358i	1.096 – 0.358i
-7.48	0.554 – 0.359i	1.096 – 0.359i
-7.475	0.553 – 0.36i	1.095 – 0.36i
-7.47	0.551 – 0.36i	1.095 – 0.36i
-7.465	0.549 – 0.361i	1.094 – 0.361i
-7.46	0.548 – 0.362i	1.094 – 0.362i
-7.455	0.546 – 0.362i	1.093 – 0.362i
-7.45	0.544 – 0.363i	1.092 – 0.363i
-7.445	0.543 – 0.364i	1.092 – 0.364i
-7.44	0.541 – 0.365i	1.091 – 0.365i
-7.435	0.539 – 0.365i	1.091 – 0.365i
-7.43	0.538 – 0.366i	1.09 – 0.366i
-7.425	0.536 – 0.367i	1.09 – 0.367i
-7.42	0.534 – 0.367i	1.089 – 0.367i
-7.415	0.533 – 0.368i	1.089 – 0.368i
-7.41	0.531 – 0.369i	1.088 – 0.369i
-7.405	0.529 – 0.369i	1.088 – 0.369i
-7.4	0.528 – 0.37i	1.087 – 0.37i
-7.395	0.526 – 0.37i	1.087 – 0.37i

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-7.39	0.524 – 0.371i	1.087 – 0.371i
-7.385	0.522 – 0.372i	1.086 – 0.372i
-7.38	0.521 – 0.372i	1.086 – 0.372i
-7.375	0.519 – 0.373i	1.085 – 0.373i
-7.37	0.517 – 0.373i	1.085 – 0.373i
-7.365	0.515 – 0.374i	1.084 – 0.374i
-7.36	0.514 – 0.374i	1.084 – 0.374i
-7.355	0.512 – 0.375i	1.083 – 0.375i
-7.35	0.51 – 0.376i	1.083 – 0.376i
-7.345	0.509 – 0.376i	1.083 – 0.376i
-7.34	0.507 – 0.377i	1.082 – 0.377i
-7.335	0.505 – 0.377i	1.082 – 0.377i
-7.33	0.503 – 0.378i	1.081 – 0.378i
-7.325	0.502 – 0.378i	1.081 – 0.378i
-7.32	0.5 – 0.379i	1.08 – 0.379i
-7.315	0.498 – 0.379i	1.08 – 0.379i
-7.31	0.496 – 0.38i	1.08 – 0.38i
-7.305	0.494 – 0.38i	1.079 – 0.38i
-7.3	0.493 – 0.381i	1.079 – 0.381i
-7.295	0.491 – 0.381i	1.079 – 0.381i
-7.29	0.489 – 0.382i	1.078 – 0.382i
-7.285	0.487 – 0.382i	1.078 – 0.382i
-7.28	0.486 – 0.382i	1.078 – 0.382i
-7.275	0.484 – 0.383i	1.077 – 0.383i
-7.27	0.482 – 0.383i	1.077 – 0.383i
-7.265	0.48 – 0.384i	1.076 – 0.384i
-7.26	0.478 – 0.384i	1.076 – 0.384i
-7.255	0.477 – 0.384i	1.076 – 0.384i
-7.25	0.475 – 0.385i	1.075 – 0.385i
-7.245	0.473 – 0.385i	1.075 – 0.385i
-7.24	0.471 – 0.386i	1.075 – 0.386i
-7.235	0.469 – 0.386i	1.075 – 0.386i
-7.23	0.467 – 0.386i	1.074 – 0.386i
-7.225	0.466 – 0.387i	1.074 – 0.387i
-7.22	0.464 – 0.387i	1.074 – 0.387i
-7.215	0.462 – 0.387i	1.073 – 0.387i
-7.21	0.46 – 0.388i	1.073 – 0.388i
-7.205	0.458 – 0.388i	1.073 – 0.388i
-7.2	0.456 – 0.388i	1.073 – 0.388i
-7.195	0.454 – 0.389i	1.072 – 0.389i
-7.19	0.452 – 0.389i	1.072 – 0.389i
-7.185	0.451 – 0.389i	1.072 – 0.389i
-7.18	0.449 – 0.389i	1.072 – 0.389i
-7.175	0.447 – 0.39i	1.071 – 0.39i
-7.17	0.445 – 0.39i	1.071 – 0.39i
-7.165	0.443 – 0.39i	1.071 – 0.39i
-7.16	0.441 – 0.39i	1.071 – 0.39i
-7.155	0.439 – 0.391i	1.071 – 0.391i
-7.15	0.437 – 0.391i	1.07 – 0.391i
-7.145	0.435 – 0.391i	1.07 – 0.391i
-7.14	0.433 – 0.391i	1.07 – 0.391i
-7.135	0.431 – 0.392i	1.07 – 0.392i
-7.13	0.43 – 0.392i	1.07 – 0.392i
-7.125	0.428 – 0.392i	1.07 – 0.392i
-7.12	0.426 – 0.392i	1.07 – 0.392i
-7.115	0.424 – 0.392i	1.069 – 0.392i
-7.11	0.422 – 0.392i	1.069 – 0.392i
-7.105	0.42 – 0.392i	1.069 – 0.392i
-7.1	0.418 – 0.393i	1.069 – 0.393i
-7.095	0.416 – 0.393i	1.069 – 0.393i
-7.09	0.414 – 0.393i	1.069 – 0.393i
-7.085	0.412 – 0.393i	1.069 – 0.393i
-7.08	0.41 – 0.393i	1.069 – 0.393i
-7.075	0.408 – 0.393i	1.069 – 0.393i
-7.07	0.406 – 0.393i	1.069 – 0.393i
-7.065	0.404 – 0.393i	1.069 – 0.393i
-7.06	0.402 – 0.393i	1.068 – 0.393i
-7.055	0.4 – 0.393i	1.068 – 0.393i
-7.05	0.397 – 0.393i	1.068 – 0.393i
-7.045	0.395 – 0.393i	1.068 – 0.393i
-7.04	0.393 – 0.393i	1.068 – 0.393i
-7.035	0.391 – 0.393i	1.068 – 0.393i
-7.03	0.389 – 0.393i	1.068 – 0.393i
-7.025	0.387 – 0.393i	1.068 – 0.393i
-7.02	0.385 – 0.393i	1.068 – 0.393i
-7.015	0.383 – 0.393i	1.069 – 0.393i
-7.01	0.381 – 0.393i	1.069 – 0.393i
-7.005	0.379 – 0.393i	1.069 – 0.393i
-7	0.376 – 0.393i	1.069 – 0.393i
-6.995	0.374 – 0.393i	1.069 – 0.393i

$w\cdot10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$
-6.99	0.372 – 0.393i	1.069 – 0.393i
-6.985	0.37 – 0.393i	1.069 – 0.393i
-6.98	0.368 – 0.393i	1.069 – 0.393i
-6.975	0.366 – 0.392i	1.069 – 0.392i
-6.97	0.363 – 0.392i	1.069 – 0.392i
-6.965	0.361 – 0.392i	1.07 – 0.392i
-6.96	0.359 – 0.392i	1.07 – 0.392i
-6.955	0.357 – 0.392i	1.07 – 0.392i
-6.95	0.355 – 0.391i	1.07 – 0.391i
-6.945	0.352 – 0.391i	1.07 – 0.391i
-6.94	0.35 – 0.391i	1.07 – 0.391i
-6.935	0.348 – 0.391i	1.071 – 0.391i
-6.93	0.346 – 0.39i	1.071 – 0.39i
-6.925	0.343 – 0.39i	1.071 – 0.39i
-6.92	0.341 – 0.39i	1.071 – 0.39i
-6.915	0.339 – 0.39i	1.072 – 0.39i
-6.91	0.336 – 0.389i	1.072 – 0.389i
-6.905	0.334 – 0.389i	1.072 – 0.389i
-6.9	0.332 – 0.388i	1.072 – 0.388i
-6.895	0.329 – 0.388i	1.073 – 0.388i
-6.89	0.327 – 0.388i	1.073 – 0.388i
-6.885	0.325 – 0.387i	1.073 – 0.387i
-6.88	0.322 – 0.387i	1.074 – 0.387i
-6.875	0.32 – 0.386i	1.074 – 0.386i
-6.87	0.317 – 0.386i	1.075 – 0.386i
-6.865	0.315 – 0.385i	1.075 – 0.385i
-6.86	0.312 – 0.385i	1.075 – 0.385i
-6.855	0.31 – 0.384i	1.076 – 0.384i
-6.85	0.308 – 0.384i	1.076 – 0.384i
-6.845	0.305 – 0.383i	1.077 – 0.383i
-6.84	0.303 – 0.383i	1.077 – 0.383i
-6.835	0.3 – 0.382i	1.078 – 0.382i
-6.83	0.298 – 0.382i	1.078 – 0.382i
-6.825	0.295 – 0.381i	1.079 – 0.381i
-6.82	0.292 – 0.38i	1.079 – 0.38i
-6.815	0.29 – 0.38i	1.08 – 0.38i
-6.81	0.287 – 0.379i	1.08 – 0.379i
-6.805	0.285 – 0.378i	1.081 – 0.378i
-6.8	0.282 – 0.377i	1.082 – 0.377i
-6.795	0.28 – 0.377i	1.082 – 0.377i

$w\cdot10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$
-6.79	0.277 – 0.376i	1.083 – 0.376i
-6.785	0.274 – 0.375i	1.084 – 0.375i
-6.78	0.272 – 0.374i	1.084 – 0.374i
-6.775	0.269 – 0.373i	1.085 – 0.373i
-6.77	0.266 – 0.372i	1.086 – 0.372i
-6.765	0.263 – 0.371i	1.086 – 0.371i
-6.76	0.261 – 0.37i	1.087 – 0.37i
-6.755	0.258 – 0.369i	1.088 – 0.369i
-6.75	0.255 – 0.368i	1.089 – 0.368i
-6.745	0.252 – 0.367i	1.089 – 0.367i
-6.74	0.249 – 0.366i	1.09 – 0.366i
-6.735	0.247 – 0.365i	1.091 – 0.365i
-6.73	0.244 – 0.364i	1.092 – 0.364i
-6.725	0.241 – 0.363i	1.093 – 0.363i
-6.72	0.238 – 0.361i	1.094 – 0.361i
-6.715	0.235 – 0.36i	1.095 – 0.36i
-6.71	0.232 – 0.359i	1.096 – 0.359i
-6.705	0.229 – 0.357i	1.097 – 0.357i
-6.7	0.226 – 0.356i	1.098 – 0.356i
-6.695	0.223 – 0.355i	1.099 – 0.355i
-6.69	0.22 – 0.353i	1.1 – 0.353i
-6.685	0.217 – 0.352i	1.101 – 0.352i
-6.68	0.214 – 0.35i	1.102 – 0.35i
-6.675	0.211 – 0.349i	1.103 – 0.349i
-6.67	0.208 – 0.347i	1.104 – 0.347i
-6.665	0.205 – 0.345i	1.105 – 0.345i
-6.66	0.201 – 0.343i	1.107 – 0.343i
-6.655	0.198 – 0.342i	1.108 – 0.342i
-6.65	0.195 – 0.34i	1.109 – 0.34i
-6.645	0.192 – 0.338i	1.11 – 0.338i
-6.64	0.189 – 0.336i	1.112 – 0.336i
-6.635	0.185 – 0.334i	1.113 – 0.334i
-6.63	0.182 – 0.332i	1.115 – 0.332i
-6.625	0.179 – 0.33i	1.116 – 0.33i
-6.62	0.175 – 0.327i	1.117 – 0.327i
-6.615	0.172 – 0.325i	1.119 – 0.325i
-6.61	0.168 – 0.323i	1.12 – 0.323i
-6.605	0.165 – 0.32i	1.122 – 0.32i
-6.6	0.161 – 0.318i	1.123 – 0.318i
-6.595	0.158 – 0.315i	1.125 – 0.315i

$w\cdot10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$
-6.59	0.154 – 0.312i	1.127 – 0.312i
-6.585	0.151 – 0.309i	1.128 – 0.309i
-6.58	0.147 – 0.307i	1.13 – 0.307i
-6.575	0.143 – 0.304i	1.132 – 0.304i
-6.57	0.139 – 0.3i	1.134 – 0.3i
-6.565	0.136 – 0.297i	1.135 – 0.297i
-6.56	0.132 – 0.294i	1.137 – 0.294i
-6.555	0.128 – 0.29i	1.139 – 0.29i
-6.55	0.124 – 0.287i	1.141 – 0.287i
-6.545	0.12 – 0.283i	1.143 – 0.283i
-6.54	0.116 – 0.279i	1.145 – 0.279i
-6.535	0.112 – 0.275i	1.147 – 0.275i
-6.53	0.108 – 0.271i	1.149 – 0.271i
-6.525	0.104 – 0.267i	1.151 – 0.267i
-6.52	0.1 – 0.262i	1.154 – 0.262i
-6.515	0.096 – 0.257i	1.156 – 0.257i
-6.51	0.092 – 0.252i	1.158 – 0.252i
-6.505	0.087 – 0.247i	1.161 – 0.247i
-6.5	0.083 – 0.242i	1.163 – 0.242i
-6.495	0.079 – 0.236i	1.165 – 0.236i
-6.49	0.074 – 0.23i	1.168 – 0.23i
-6.485	0.07 – 0.224i	1.17 – 0.224i
-6.48	0.065 – 0.217i	1.173 – 0.217i
-6.475	0.061 – 0.21i	1.176 – 0.21i
-6.47	0.056 – 0.203i	1.178 – 0.203i
-6.465	0.052 – 0.195i	1.181 – 0.195i
-6.46	0.047 – 0.186i	1.184 – 0.186i
-6.455	0.042 – 0.177i	1.187 – 0.177i
-6.45	0.037 – 0.167i	1.19 – 0.167i
-6.445	0.032 – 0.156i	1.193 – 0.156i
-6.44	0.027 – 0.144i	1.196 – 0.144i
-6.435	0.022 – 0.13i	1.199 – 0.13i
-6.43	0.017 – 0.115i	1.202 – 0.115i
-6.425	0.012 – 0.096i	1.206 – 0.096i
-6.42	0.007 – 0.072i	1.209 – 0.072i
-6.415	0.001 – 0.032i	1.212 – 0.032i
-6.41	0.053	1.159
-6.405	0.078	1.132
-6.4	0.095	1.113
-6.395	0.109	1.097

$w\cdot10^{-13}$	$\chi_1(w)\cdot M$	$(\chi_2(w))\cdot M$
-6.39	0.12	1.084
-6.385	0.13	1.072
-6.38	0.139	1.061
-6.375	0.147	1.051
-6.37	0.155	1.042
-6.365	0.162	1.033
-6.36	0.168	1.025
-6.355	0.174	1.017
-6.35	0.18	1.01
-6.345	0.185	1.002
-6.34	0.19	0.996
-6.335	0.195	0.989
-6.33	0.199	0.982
-6.325	0.204	0.976
-6.32	0.208	0.97
-6.315	0.212	0.964
-6.31	0.216	0.958
-6.305	0.22	0.953
-6.3	0.223	0.947
-6.295	0.227	0.942
-6.29	0.23	0.937
-6.285	0.233	0.932
-6.28	0.237	0.927
-6.275	0.24	0.922
-6.27	0.243	0.917
-6.265	0.246	0.912
-6.26	0.249	0.907
-6.255	0.251	0.903
-6.25	0.254	0.898
-6.245	0.257	0.893
-6.24	0.259	0.889
-6.235	0.262	0.885
-6.23	0.264	0.88
-6.225	0.267	0.876
-6.22	0.269	0.872
-6.215	0.272	0.868
-6.21	0.274	0.863
-6.205	0.276	0.859
-6.2	0.278	0.855
-6.195	0.281	0.851

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$	$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-6.19	0.283	0.847	-5.99	0.35	0.708
-6.185	0.285	0.843	-5.985	0.351	0.705
-6.18	0.287	0.839	-5.98	0.353	0.702
-6.175	0.289	0.836	-5.975	0.354	0.699
-6.17	0.291	0.832	-5.97	0.355	0.696
-6.165	0.293	0.828	-5.965	0.357	0.693
-6.16	0.295	0.824	-5.96	0.358	0.689
-6.155	0.297	0.82	-5.955	0.359	0.686
-6.15	0.299	0.817	-5.95	0.361	0.683
-6.145	0.301	0.813	-5.945	0.362	0.68
-6.14	0.303	0.809	-5.94	0.363	0.677
-6.135	0.304	0.806	-5.935	0.365	0.674
-6.13	0.306	0.802	-5.93	0.366	0.671
-6.125	0.308	0.799	-5.925	0.367	0.668
-6.12	0.31	0.795	-5.92	0.369	0.665
-6.115	0.311	0.791	-5.915	0.37	0.662
-6.11	0.313	0.788	-5.91	0.371	0.659
-6.105	0.315	0.784	-5.905	0.373	0.656
-6.1	0.317	0.781	-5.9	0.374	0.653
-6.095	0.318	0.777	-5.895	0.375	0.65
-6.09	0.32	0.774	-5.89	0.376	0.647
-6.085	0.321	0.771	-5.885	0.378	0.644
-6.08	0.323	0.767	-5.88	0.379	0.641
-6.075	0.325	0.764	-5.875	0.38	0.638
-6.07	0.326	0.76	-5.87	0.382	0.635
-6.065	0.328	0.757	-5.865	0.383	0.632
-6.06	0.329	0.754	-5.86	0.384	0.629
-6.055	0.331	0.75	-5.855	0.385	0.626
-6.05	0.332	0.747	-5.85	0.387	0.623
-6.045	0.334	0.744	-5.845	0.388	0.62
-6.04	0.335	0.74	-5.84	0.389	0.617
-6.035	0.337	0.737	-5.835	0.39	0.614
-6.03	0.338	0.734	-5.83	0.392	0.611
-6.025	0.34	0.731	-5.825	0.393	0.608
-6.02	0.341	0.727	-5.82	0.394	0.605
-6.015	0.343	0.724	-5.815	0.395	0.602
-6.01	0.344	0.721	-5.81	0.397	0.599
-6.005	0.346	0.718	-5.805	0.398	0.596
-6	0.347	0.715	-5.8	0.399	0.593
-5.995	0.349	0.711	-5.795	0.4	0.59

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$	$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-5.79	0.402	0.587	-5.59	0.453	0.468
-5.785	0.403	0.584	-5.585	0.455	0.465
-5.78	0.404	0.581	-5.58	0.456	0.462
-5.775	0.405	0.578	-5.575	0.458	0.459
-5.77	0.407	0.575	-5.57	0.459	0.456
-5.765	0.408	0.572	-5.565	0.46	0.453
-5.76	0.409	0.569	-5.56	0.462	0.45
-5.755	0.41	0.567	-5.555	0.463	0.447
-5.75	0.411	0.564	-5.55	0.465	0.444
-5.745	0.413	0.561	-5.545	0.466	0.441
-5.74	0.414	0.558	-5.54	0.468	0.437
-5.735	0.415	0.555	-5.535	0.469	0.434
-5.73	0.417	0.552	-5.53	0.471	0.431
-5.725	0.418	0.549	-5.525	0.472	0.428
-5.72	0.419	0.546	-5.52	0.474	0.425
-5.715	0.42	0.543	-5.515	0.475	0.422
-5.71	0.422	0.54	-5.51	0.477	0.418
-5.705	0.423	0.537	-5.505	0.479	0.415
-5.7	0.424	0.534	-5.5	0.48	0.412
-5.695	0.425	0.531	-5.495	0.482	0.409
-5.69	0.427	0.528	-5.49	0.483	0.406
-5.685	0.428	0.525	-5.485	0.485	0.402
-5.68	0.429	0.522	-5.48	0.487	0.399
-5.675	0.431	0.519	-5.475	0.488	0.396
-5.67	0.432	0.516	-5.47	0.49	0.392
-5.665	0.433	0.513	-5.465	0.492	0.389
-5.66	0.434	0.51	-5.46	0.494	0.386
-5.655	0.436	0.507	-5.455	0.495	0.382
-5.65	0.437	0.504	-5.45	0.497	0.379
-5.645	0.438	0.501	-5.445	0.499	0.376
-5.64	0.44	0.498	-5.44	0.501	0.372
-5.635	0.441	0.495	-5.435	0.502	0.369
-5.63	0.442	0.492	-5.43	0.504	0.365
-5.625	0.444	0.489	-5.425	0.506	0.362
-5.62	0.445	0.486	-5.42	0.508	0.358
-5.615	0.446	0.483	-5.415	0.51	0.355
-5.61	0.448	0.48	-5.41	0.512	0.351
-5.605	0.449	0.477	-5.405	0.514	0.348
-5.6	0.451	0.474	-5.4	0.516	0.344
-5.595	0.452	0.471	-5.395	0.518	0.34

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-5.39	0.52	0.337
-5.385	0.522	0.333
-5.38	0.525	0.329
-5.375	0.527	0.325
-5.37	0.529	0.322
-5.365	0.531	0.318
-5.36	0.534	0.314
-5.355	0.536	0.31
-5.35	0.538	0.306
-5.345	0.541	0.302
-5.34	0.543	0.298
-5.335	0.546	0.294
-5.33	0.548	0.289
-5.325	0.551	0.285
-5.32	0.554	0.281
-5.315	0.557	0.277
-5.31	0.559	0.272
-5.305	0.562	0.268
-5.3	0.565	0.263
-5.295	0.568	0.258
-5.29	0.572	0.254
-5.285	0.575	0.249
-5.28	0.578	0.244
-5.275	0.582	0.239
-5.27	0.585	0.234
-5.265	0.589	0.229
-5.26	0.593	0.223
-5.255	0.597	0.218
-5.25	0.601	0.212
-5.245	0.605	0.206
-5.24	0.61	0.2
-5.235	0.615	0.194
-5.23	0.62	0.187
-5.225	0.625	0.18
-5.22	0.63	0.173
-5.215	0.636	0.166
-5.21	0.643	0.158
-5.205	0.65	0.149
-5.2	0.657	0.141
-5.195	0.665	0.131

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-5.19	0.674	0.12
-5.185	0.684	0.108
-5.18	0.696	0.095
-5.175	0.711	0.079
-5.17	0.731	0.057
-5.165	0.775	0.011
-5.16	0.783 – 0.057i	0.003 – 0.057i
-5.155	0.778 – 0.081i	0.005 – 0.081i
-5.15	0.774 – 0.099i	0.008 – 0.099i
-5.145	0.77 – 0.114i	0.011 – 0.114i
-5.14	0.766 – 0.126i	0.013 – 0.126i
-5.135	0.762 – 0.138i	0.016 – 0.138i
-5.13	0.758 – 0.148i	0.018 – 0.148i
-5.125	0.754 – 0.157i	0.021 – 0.157i
-5.12	0.75 – 0.166i	0.023 – 0.166i
-5.115	0.746 – 0.174i	0.026 – 0.174i
-5.11	0.742 – 0.182i	0.028 – 0.182i
-5.105	0.739 – 0.189i	0.03 – 0.189i
-5.1	0.735 – 0.195i	0.032 – 0.195i
-5.095	0.731 – 0.202i	0.034 – 0.202i
-5.09	0.727 – 0.208i	0.037 – 0.208i
-5.085	0.724 – 0.213i	0.039 – 0.213i
-5.08	0.72 – 0.219i	0.041 – 0.219i
-5.075	0.717 – 0.224i	0.043 – 0.224i
-5.07	0.713 – 0.229i	0.045 – 0.229i
-5.065	0.71 – 0.234i	0.047 – 0.234i
-5.06	0.706 – 0.238i	0.049 – 0.238i
-5.055	0.703 – 0.243i	0.051 – 0.243i
-5.05	0.7 – 0.247i	0.052 – 0.247i
-5.045	0.696 – 0.251i	0.054 – 0.251i
-5.04	0.693 – 0.255i	0.056 – 0.255i
-5.035	0.69 – 0.259i	0.058 – 0.259i
-5.03	0.687 – 0.262i	0.06 – 0.262i
-5.025	0.683 – 0.266i	0.061 – 0.266i
-5.02	0.68 – 0.269i	0.063 – 0.269i
-5.015	0.677 – 0.272i	0.065 – 0.272i
-5.01	0.674 – 0.276i	0.066 – 0.276i
-5.005	0.671 – 0.279i	0.068 – 0.279i
-5	0.668 – 0.282i	0.069 – 0.282i
-4.995	0.665 – 0.285i	0.071 – 0.285i

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-4.99	0.662 – 0.287i	0.072 – 0.287i
-4.985	0.659 – 0.29i	0.074 – 0.29i
-4.98	0.656 – 0.293i	0.075 – 0.293i
-4.975	0.653 – 0.295i	0.077 – 0.295i
-4.97	0.65 – 0.298i	0.078 – 0.298i
-4.965	0.648 – 0.3i	0.08 – 0.3i
-4.96	0.645 – 0.303i	0.081 – 0.303i
-4.955	0.642 – 0.305i	0.082 – 0.305i
-4.95	0.639 – 0.307i	0.083 – 0.307i
-4.945	0.636 – 0.309i	0.085 – 0.309i
-4.94	0.634 – 0.311i	0.086 – 0.311i
-4.935	0.631 – 0.313i	0.087 – 0.313i
-4.93	0.628 – 0.315i	0.088 – 0.315i
-4.925	0.626 – 0.317i	0.09 – 0.317i
-4.92	0.623 – 0.319i	0.091 – 0.319i
-4.915	0.62 – 0.321i	0.092 – 0.321i
-4.91	0.618 – 0.323i	0.093 – 0.323i
-4.905	0.615 – 0.325i	0.094 – 0.325i
-4.9	0.613 – 0.326i	0.095 – 0.326i
-4.895	0.61 – 0.328i	0.096 – 0.328i
-4.89	0.608 – 0.33i	0.097 – 0.33i
-4.885	0.605 – 0.331i	0.099 – 0.331i
-4.88	0.603 – 0.333i	0.1 – 0.333i
-4.875	0.6 – 0.334i	0.101 – 0.334i
-4.87	0.598 – 0.336i	0.102 – 0.336i
-4.865	0.596 – 0.337i	0.103 – 0.337i
-4.86	0.593 – 0.339i	0.103 – 0.339i
-4.855	0.591 – 0.34i	0.104 – 0.34i
-4.85	0.588 – 0.342i	0.105 – 0.342i
-4.845	0.586 – 0.343i	0.106 – 0.343i
-4.84	0.584 – 0.344i	0.107 – 0.344i
-4.835	0.581 – 0.345i	0.108 – 0.345i
-4.83	0.579 – 0.347i	0.109 – 0.347i
-4.825	0.577 – 0.348i	0.11 – 0.348i
-4.82	0.575 – 0.349i	0.111 – 0.349i
-4.815	0.572 – 0.35i	0.111 – 0.35i
-4.81	0.57 – 0.351i	0.112 – 0.351i
-4.805	0.568 – 0.352i	0.113 – 0.352i
-4.8	0.566 – 0.354i	0.114 – 0.354i
-4.795	0.564 – 0.355i	0.114 – 0.355i
⋮	⋮	⋮

$w \cdot 10^{-13}$	$\chi_1(w) \cdot M$	$(\chi_2(w)) \cdot M$
-4.79	0.562 – 0.356i	0.115 – 0.356i
-4.785	0.559 – 0.357i	0.116 – 0.357i
-4.78	0.557 – 0.358i	0.117 – 0.358i
-4.775	0.555 – 0.359i	0.117 – 0.359i
-4.77	0.553 – 0.359i	0.118 – 0.359i
-4.765	0.551 – 0.36i	0.119 – 0.36i
-4.76	0.549 – 0.361i	0.119 – 0.361i
-4.755	0.547 – 0.362i	0.12 – 0.362i
-4.75	0.545 – 0.363i	0.121 – 0.363i
-4.745	0.543 – 0.364i	0.121 – 0.364i
-4.74	0.541 – 0.365i	0.122 – 0.365i
-4.735	0.539 – 0.366i	0.122 – 0.366i
-4.73	0.537 – 0.366i	0.123 – 0.366i
-4.725	0.535 – 0.367i	0.124 – 0.367i
-4.72	0.533 – 0.368i	0.124 – 0.368i
-4.715	0.531 – 0.369i	0.125 – 0.369i
-4.71	0.529 – 0.369i	0.125 – 0.369i
-4.705	0.527 – 0.37i	0.126 – 0.37i
-4.7	0.525 – 0.371i	0.126 – 0.371i
-4.695	0.523 – 0.371i	0.127 – 0.371i
-4.69	0.521 – 0.372i	0.127 – 0.372i
-4.685	0.519 – 0.373i	0.128 – 0.373i
-4.68	0.518 – 0.373i	0.128 – 0.373i
-4.675	0.516 – 0.374i	0.129 – 0.374i
-4.67	0.514 – 0.374i	0.129 – 0.374i
-4.665	0.512 – 0.375i	0.13 – 0.375i
-4.66	0.51 – 0.376i	0.13 – 0.376i
-4.655	0.508 – 0.376i	0.131 – 0.376i
-4.65	0.507 – 0.377i	0.131 – 0.377i
-4.645	0.505 – 0.377i	0.132 – 0.377i
-4.64	0.503 – 0.378i	0.132 – 0.378i
-4.635	0.501 – 0.378i	0.132 – 0.378i
-4.63	0.499 – 0.379i	0.133 – 0.379i
-4.625	0.498 – 0.379i	0.133 – 0.379i
-4.62	0.496 – 0.38i	0.134 – 0.38i
-4.615	0.494 – 0.38i	0.134 – 0.38i
-4.61	0.492 – 0.381i	0.134 – 0.381i
-4.605	0.491 – 0.381i	0.135 – 0.381i
-4.6	0.489 – 0.382i	0.135 – 0.382i
-4.595	0.487 – 0.382i	0.135 – 0.382i
⋮	⋮	⋮

When we have plotted the graphs(by MCAD) for the real and the imaginary part of the response functions ($\chi_1(\omega)$ and $\chi_2(\omega)$) vs. the frequency ω ,as shown in Fig. 5.2.3, 5.2.4*.

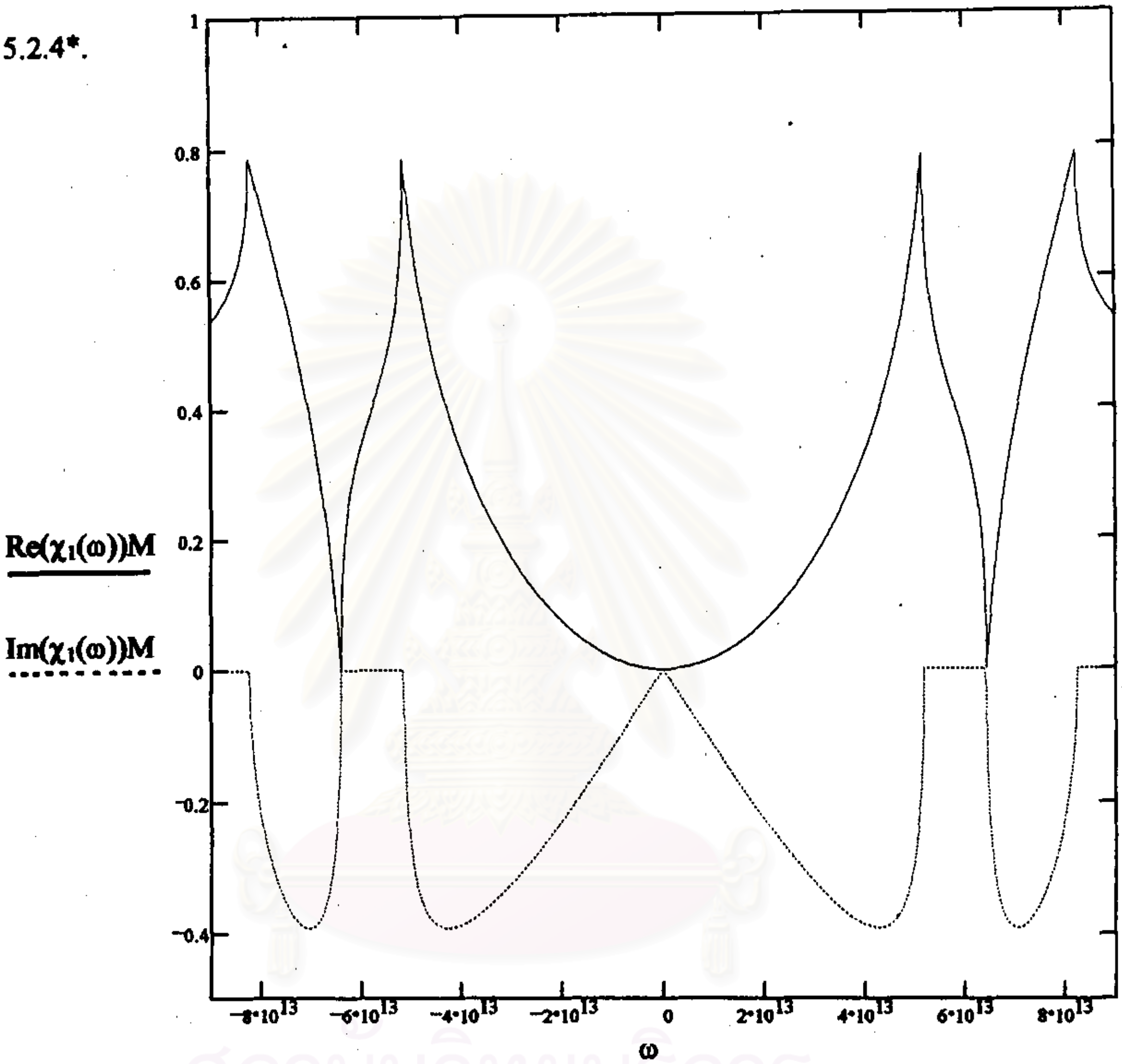

Fig. 5.2.3 The Response Function of $\chi_1(\omega)$.

*See APPENDIX F.

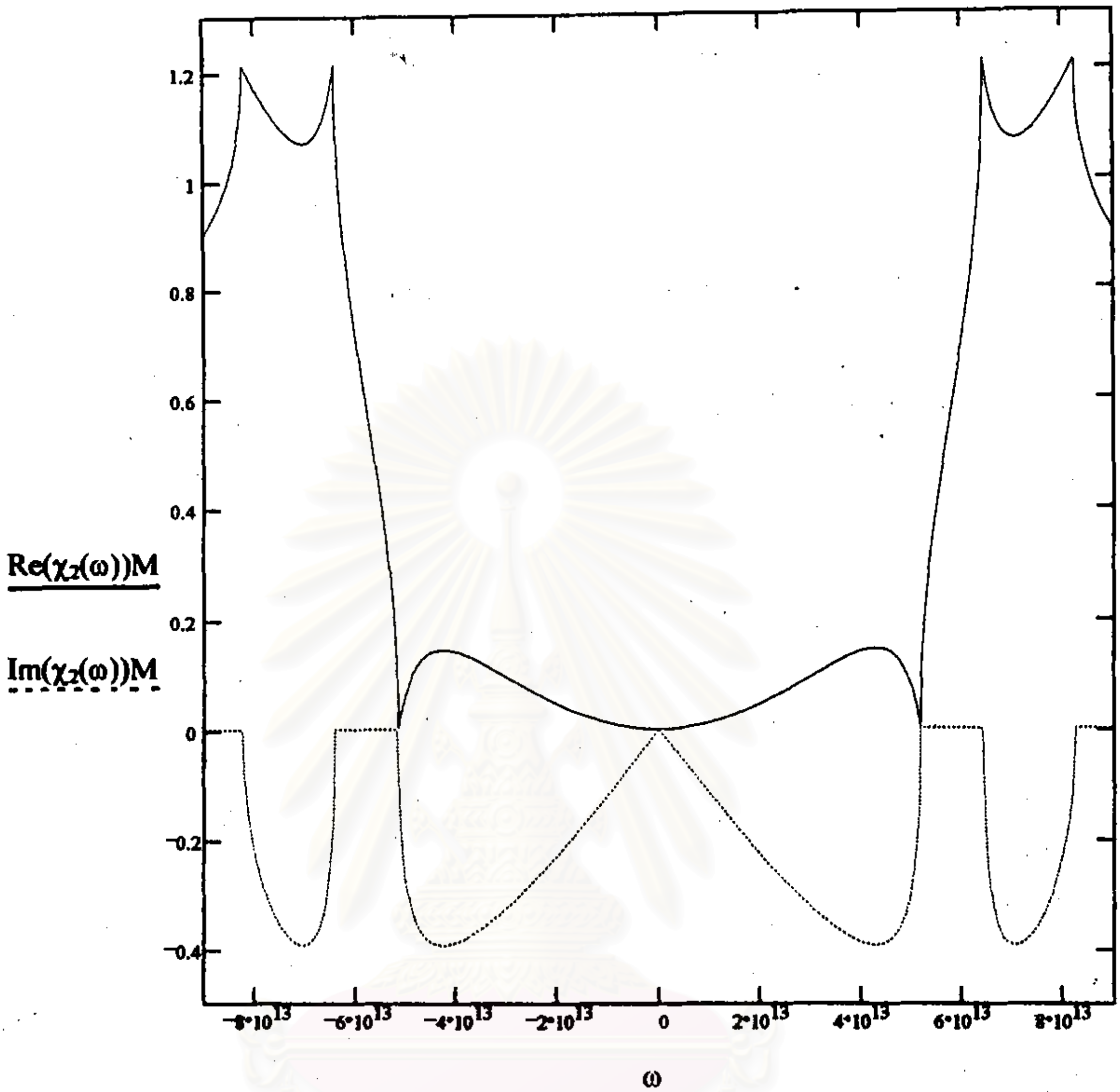

Fig. 5.2.4 The Response Function of $\chi_2(\omega)$.

These response functions are complex, and its imaginary part means the acceleration is not in phase with the applied force.

The Application for the Response Functions

The application which is presented here has been used to find the motions of the chain and the energy transport in the chain from the response functions as follows.

5.3 <u>The Motions of the Chain</u>

From the first recursion relations, Eq.(5.1.3),

$$\frac{u_{s1Q,n}(\omega)}{u_{s2Q,n-1}(\omega)} = \frac{\chi_1(\omega)C_2}{\chi_1(\omega)C_2-\omega^2} , \tag{5.1.3}$$

and the second recursion relation, Eq.(5.1.6)

$$\frac{u_{s2Q,n}(\omega)}{u_{s1Q,n}(\omega)} = \frac{\chi_2(\omega)C_1}{\chi_2(\omega)C_1-\omega^2} . \tag{5.1.6}$$

We can hence evaluate the numerical values of the recursion relations [Eq.(5.1.3) and Eq.(5.1.6)] by MCAD, whereas the response functions $\chi_1(\omega)$ and $\chi_2(\omega)$[Eq.(5.2.6a) and Eq.(5.2.6b)] and the spring constant C from Eq.(5.2.7) have been used for NaCl crystal as shown in table 2.

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Table 2 The values of $\dfrac{C\chi_1(\omega)}{C\chi_1(\omega)-\omega^2}$ **and** $\dfrac{C\chi_2(\omega)}{C\chi_2(\omega)-\omega^2}$.

$w \cdot 10^{-13}$	$\dfrac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\dfrac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-9	-0.288	-0.606
-8.995	-0.289	-0.608
-8.99	-0.29	-0.61
-8.985	-0.291	-0.612
-8.98	-0.291	-0.614
-8.975	-0.292	-0.616
-8.97	-0.293	-0.618
-8.965	-0.294	-0.62
-8.96	-0.294	-0.622
-8.955	-0.295	-0.624
-8.95	-0.296	-0.626
-8.945	-0.297	-0.628
-8.94	-0.297	-0.63
-8.935	-0.298	-0.632
-8.93	-0.299	-0.634
-8.925	-0.3	-0.636
-8.92	-0.301	-0.638
-8.915	-0.301	-0.64
-8.91	-0.302	-0.642
-8.905	-0.303	-0.644
-8.9	-0.304	-0.646
-8.895	-0.305	-0.648
-8.89	-0.305	-0.651
-8.885	-0.306	-0.653
-8.88	-0.307	-0.655
-8.875	-0.308	-0.657
-8.87	-0.309	-0.659
-8.865	-0.31	-0.662
-8.86	-0.31	-0.664
-8.855	-0.311	-0.666
-8.85	-0.312	-0.669
-8.845	-0.313	-0.671
-8.84	-0.314	-0.673
-8.835	-0.315	-0.676
-8.83	-0.316	-0.678
-8.825	-0.317	-0.68
-8.82	-0.317	-0.683
-8.815	-0.318	-0.685
-8.81	-0.319	-0.688
-8.805	-0.32	-0.69
-8.8	-0.321	-0.693
-8.795	-0.322	-0.695

$w \cdot 10^{-13}$	$\dfrac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\dfrac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-8.79	-0.323	-0.698
-8.785	-0.324	-0.7
-8.78	-0.325	-0.703
-8.775	-0.326	-0.705
-8.77	-0.327	-0.708
-8.765	-0.328	-0.711
-8.76	-0.329	-0.713
-8.755	-0.33	-0.716
-8.75	-0.331	-0.719
-8.745	-0.332	-0.721
-8.74	-0.333	-0.724
-8.735	-0.334	-0.727
-8.73	-0.335	-0.73
-8.725	-0.336	-0.733
-8.72	-0.337	-0.735
-8.715	-0.338	-0.738
-8.71	-0.339	-0.741
-8.705	-0.34	-0.744
-8.7	-0.342	-0.747
-8.695	-0.343	-0.75
-8.69	-0.344	-0.753
-8.685	-0.345	-0.756
-8.68	-0.346	-0.759
-8.675	-0.347	-0.762
-8.67	-0.348	-0.766
-8.665	-0.35	-0.769
-8.66	-0.351	-0.772
-8.655	-0.352	-0.775
-8.65	-0.353	-0.778
-8.645	-0.354	-0.782
-8.64	-0.356	-0.785
-8.635	-0.357	-0.788
-8.63	-0.358	-0.792
-8.625	-0.359	-0.795
-8.62	-0.361	-0.799
-8.615	-0.362	-0.802
-8.61	-0.363	-0.806
-8.605	-0.365	-0.809
-8.6	-0.366	-0.813
-8.595	-0.367	-0.817

$w \cdot 10^{-13}$	$\dfrac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\dfrac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-8.59	-0.369	-0.821
-8.585	-0.37	-0.824
-8.58	-0.372	-0.828
-8.575	-0.373	-0.832
-8.57	-0.374	-0.836
-8.565	-0.376	-0.84
-8.56	-0.377	-0.844
-8.555	-0.379	-0.848
-8.55	-0.38	-0.852
-8.545	-0.382	-0.856
-8.54	-0.383	-0.86
-8.535	-0.385	-0.865
-8.53	-0.387	-0.869
-8.525	-0.388	-0.873
-8.52	-0.39	-0.878
-8.515	-0.392	-0.882
-8.51	-0.393	-0.887
-8.505	-0.395	-0.891
-8.5	-0.397	-0.896
-8.495	-0.398	-0.901
-8.49	-0.4	-0.906
-8.485	-0.402	-0.911
-8.48	-0.404	-0.915
-8.475	-0.406	-0.921
-8.47	-0.408	-0.926
-8.465	-0.41	-0.931
-8.46	-0.412	-0.936
-8.455	-0.414	-0.942
-8.45	-0.416	-0.947
-8.445	-0.418	-0.953
-8.44	-0.42	-0.958
-8.435	-0.422	-0.964
-8.43	-0.424	-0.97
-8.425	-0.426	-0.976
-8.42	-0.429	-0.982
-8.415	-0.431	-0.988
-8.41	-0.433	-0.995
-8.405	-0.436	-1.001
-8.4	-0.438	-1.008
-8.395	-0.441	-1.014

$w \cdot 10^{-13}$	$\dfrac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\dfrac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-8.39	-0.443	-1.021
-8.385	-0.446	-1.028
-8.38	-0.449	-1.036
-8.375	-0.451	-1.043
-8.37	-0.454	-1.05
-8.365	-0.457	-1.058
-8.36	-0.46	-1.066
-8.355	-0.463	-1.074
-8.35	-0.466	-1.083
-8.345	-0.47	-1.091
-8.34	-0.473	-1.1
-8.335	-0.476	-1.11
-8.33	-0.48	-1.119
-8.325	-0.484	-1.129
-8.32	-0.488	-1.139
-8.315	-0.492	-1.15
-8.31	-0.496	-1.161
-8.305	-0.5	-1.172
-8.3	-0.505	-1.184
-8.295	-0.51	-1.197
-8.29	-0.515	-1.21
-8.285	-0.52	-1.224
-8.28	-0.526	-1.239
-8.275	-0.532	-1.254
-8.27	-0.539	-1.271
-8.265	-0.546	-1.29
-8.26	-0.554	-1.31
-8.255	-0.563	-1.333
-8.25	-0.574	-1.359
-8.245	-0.586	-1.391
-8.24	-0.603	-1.433
-8.235	-0.639	-1.519
-8.23	-0.647 + 0.045i	-1.539 + 0.108i
-8.225	-0.645 + 0.065i	-1.536 + 0.154i
-8.22	-0.643 + 0.079i	-1.533 + 0.189i
-8.215	-0.641 + 0.092i	-1.53 + 0.219i
-8.21	-0.639 + 0.102i	-1.527 + 0.245i
-8.205	-0.637 + 0.112i	-1.524 + 0.268i
-8.2	-0.635 + 0.121i	-1.521 + 0.29i
-8.195	-0.633 + 0.129i	-1.518 + 0.31i

$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-8.19	-0.631 + 0.137i	-1.515 + 0.329i
-8.185	-0.629 + 0.144i	-1.511 + 0.346i
-8.18	-0.627 + 0.151i	-1.508 + 0.363i
-8.175	-0.625 + 0.157i	-1.505 + 0.379i
-8.17	-0.623 + 0.164i	-1.502 + 0.395i
-8.165	-0.621 + 0.17i	-1.499 + 0.41i
-8.16	-0.619 + 0.175i	-1.496 + 0.424i
-8.155	-0.617 + 0.181i	-1.493 + 0.438i
-8.15	-0.615 + 0.186i	-1.49 + 0.451i
-8.145	-0.613 + 0.191i	-1.487 + 0.465i
-8.14	-0.611 + 0.196i	-1.484 + 0.477i
-8.135	-0.609 + 0.201i	-1.481 + 0.49i
-8.13	-0.607 + 0.206i	-1.478 + 0.502i
-8.125	-0.605 + 0.211i	-1.475 + 0.513i
-8.12	-0.603 + 0.215i	-1.472 + 0.525i
-8.115	-0.601 + 0.219i	-1.469 + 0.536i
-8.11	-0.599 + 0.223i	-1.466 + 0.547i
-8.105	-0.597 + 0.228i	-1.463 + 0.558i
-8.1	-0.595 + 0.232i	-1.46 + 0.568i
-8.095	-0.593 + 0.236i	-1.457 + 0.579i
-8.09	-0.591 + 0.239i	-1.454 + 0.589i
-8.085	-0.589 + 0.243i	-1.45 + 0.599i
-8.08	-0.587 + 0.247i	-1.447 + 0.609i
-8.075	-0.585 + 0.25i	-1.444 + 0.618i
-8.07	-0.583 + 0.254i	-1.441 + 0.628i
-8.065	-0.581 + 0.257i	-1.438 + 0.637i
-8.06	-0.579 + 0.261i	-1.435 + 0.646i
-8.055	-0.577 + 0.264i	-1.432 + 0.655i
-8.05	-0.575 + 0.267i	-1.429 + 0.664i
-8.045	-0.573 + 0.271i	-1.426 + 0.673i
-8.04	-0.571 + 0.274i	-1.423 + 0.682i
-8.035	-0.569 + 0.277i	-1.42 + 0.691i
-8.03	-0.567 + 0.28i	-1.417 + 0.699i
-8.025	-0.566 + 0.283i	-1.414 + 0.708i
-8.02	-0.564 + 0.286i	-1.411 + 0.716i
-8.015	-0.562 + 0.289i	-1.408 + 0.724i
-8.01	-0.56 + 0.292i	-1.405 + 0.732i
-8.005	-0.558 + 0.294i	-1.402 + 0.74i
-8	-0.556 + 0.297i	-1.399 + 0.748i
-7.995	-0.554 + 0.3i	-1.396 + 0.756i

$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-7.99	-0.552 + 0.303i	-1.393 + 0.764i
-7.985	-0.55 + 0.305i	-1.39 + 0.771i
-7.98	-0.548 + 0.308i	-1.387 + 0.779i
-7.975	-0.546 + 0.31i	-1.384 + 0.787i
-7.97	-0.544 + 0.313i	-1.381 + 0.794i
-7.965	-0.542 + 0.315i	-1.378 + 0.801i
-7.96	-0.54 + 0.318i	-1.375 + 0.809i
-7.955	-0.538 + 0.32i	-1.372 + 0.816i
-7.95	-0.536 + 0.322i	-1.369 + 0.823i
-7.945	-0.534 + 0.325i	-1.366 + 0.83i
-7.94	-0.533 + 0.327i	-1.363 + 0.837i
-7.935	-0.531 + 0.329i	-1.36 + 0.844i
-7.93	-0.529 + 0.332i	-1.357 + 0.851i
-7.925	-0.527 + 0.334i	-1.354 + 0.858i
-7.92	-0.525 + 0.336i	-1.351 + 0.865i
-7.915	-0.523 + 0.338i	-1.349 + 0.872i
-7.91	-0.521 + 0.34i	-1.346 + 0.879i
-7.905	-0.519 + 0.342i	-1.343 + 0.885i
-7.9	-0.517 + 0.344i	-1.34 + 0.892i
-7.895	-0.515 + 0.346i	-1.337 + 0.899i
-7.89	-0.513 + 0.348i	-1.334 + 0.905i
-7.885	-0.511 + 0.35i	-1.331 + 0.912i
-7.88	-0.509 + 0.352i	-1.328 + 0.918i
-7.875	-0.508 + 0.354i	-1.325 + 0.925i
-7.87	-0.506 + 0.356i	-1.322 + 0.931i
-7.865	-0.504 + 0.358i	-1.319 + 0.937i
-7.86	-0.502 + 0.36i	-1.316 + 0.944i
-7.855	-0.5 + 0.362i	-1.313 + 0.95i
-7.85	-0.498 + 0.363i	-1.31 + 0.956i
-7.845	-0.496 + 0.365i	-1.307 + 0.962i
-7.84	-0.494 + 0.367i	-1.304 + 0.969i
-7.835	-0.492 + 0.369i	-1.301 + 0.975i
-7.83	-0.49 + 0.37i	-1.298 + 0.981i
-7.825	-0.488 + 0.372i	-1.295 + 0.987i
-7.82	-0.487 + 0.374i	-1.292 + 0.993i
-7.815	-0.485 + 0.375i	-1.29 + 0.999i
-7.81	-0.483 + 0.377i	-1.287 + 1.005i
-7.805	-0.481 + 0.379i	-1.284 + 1.011i
-7.8	-0.479 + 0.38i	-1.281 + 1.017i
-7.795	-0.477 + 0.382i	-1.278 + 1.023i

$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$	$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-7.79	-0.475 + 0.383i	-1.275 + 1.028i	-7.59	-0.4 + 0.43i	-1.16 + 1.246i
-7.785	-0.473 + 0.385i	-1.272 + 1.034i	-7.585	-0.399 + 0.431i	-1.157 + 1.251i
-7.78	-0.471 + 0.386i	-1.269 + 1.04i	-7.58	-0.397 + 0.432i	-1.154 + 1.256i
-7.775	-0.47 + 0.388i	-1.266 + 1.046i	-7.575	-0.395 + 0.433i	-1.151 + 1.261i
-7.77	-0.468 + 0.389i	-1.263 + 1.051i	-7.57	-0.393 + 0.433i	-1.148 + 1.266i
-7.765	-0.466 + 0.391i	-1.26 + 1.057i	-7.565	-0.391 + 0.434i	-1.145 + 1.271i
-7.76	-0.464 + 0.392i	-1.257 + 1.063i	-7.56	-0.389 + 0.435i	-1.143 + 1.276i
-7.755	-0.462 + 0.393i	-1.255 + 1.069i	-7.555	-0.388 + 0.436i	-1.14 + 1.281i
-7.75	-0.46 + 0.395i	-1.252 + 1.074i	-7.55	-0.386 + 0.436i	-1.137 + 1.287i
-7.745	-0.458 + 0.396i	-1.249 + 1.08i	-7.545	-0.384 + 0.437i	-1.134 + 1.292i
-7.74	-0.456 + 0.398i	-1.246 + 1.085i	-7.54	-0.382 + 0.438i	-1.131 + 1.297i
-7.735	-0.454 + 0.399i	-1.243 + 1.091i	-7.535	-0.38 + 0.439i	-1.128 + 1.302i
-7.73	-0.453 + 0.4i	-1.24 + 1.097i	-7.53	-0.378 + 0.439i	-1.126 + 1.307i
-7.725	-0.451 + 0.401i	-1.237 + 1.102i	-7.525	-0.377 + 0.44i	-1.123 + 1.312i
-7.72	-0.449 + 0.403i	-1.234 + 1.108i	-7.52	-0.375 + 0.441i	-1.12 + 1.317i
-7.715	-0.447 + 0.404i	-1.231 + 1.113i	-7.515	-0.373 + 0.441i	-1.117 + 1.322i
-7.71	-0.445 + 0.405i	-1.228 + 1.119i	-7.51	-0.371 + 0.442i	-1.114 + 1.327i
-7.705	-0.443 + 0.406i	-1.226 + 1.124i	-7.505	-0.369 + 0.443i	-1.112 + 1.332i
-7.7	-0.441 + 0.408i	-1.223 + 1.129i	-7.5	-0.367 + 0.443i	-1.109 + 1.337i
-7.695	-0.439 + 0.409i	-1.22 + 1.135i	-7.495	-0.366 + 0.444i	-1.106 + 1.342i
-7.69	-0.438 + 0.41i	-1.217 + 1.14i	-7.49	-0.364 + 0.444i	-1.103 + 1.347i
-7.685	-0.436 + 0.411i	-1.214 + 1.146i	-7.485	-0.362 + 0.445i	-1.1 + 1.353i
-7.68	-0.434 + 0.412i	-1.211 + 1.151i	-7.48	-0.36 + 0.445i	-1.097 + 1.358i
-7.675	-0.432 + 0.413i	-1.208 + 1.156i	-7.475	-0.358 + 0.446i	-1.095 + 1.363i
-7.67	-0.43 + 0.415i	-1.205 + 1.162i	-7.47	-0.356 + 0.447i	-1.092 + 1.368i
-7.665	-0.428 + 0.416i	-1.203 + 1.167i	-7.465	-0.355 + 0.447i	-1.089 + 1.373i
-7.66	-0.426 + 0.417i	-1.2 + 1.172i	-7.46	-0.353 + 0.448i	-1.086 + 1.378i
-7.655	-0.425 + 0.418i	-1.197 + 1.178i	-7.455	-0.351 + 0.448i	-1.083 + 1.383i
-7.65	-0.423 + 0.419i	-1.194 + 1.183i	-7.45	-0.349 + 0.449i	-1.081 + 1.388i
-7.645	-0.421 + 0.42i	-1.191 + 1.188i	-7.445	-0.347 + 0.449i	-1.078 + 1.393i
-7.64	-0.419 + 0.421i	-1.188 + 1.194i	-7.44	-0.346 + 0.449i	-1.075 + 1.398i
-7.635	-0.417 + 0.422i	-1.185 + 1.199i	-7.435	-0.344 + 0.45i	-1.072 + 1.403i
-7.63	-0.415 + 0.423i	-1.182 + 1.204i	-7.43	-0.342 + 0.45i	-1.07 + 1.408i
-7.625	-0.413 + 0.424i	-1.18 + 1.209i	-7.425	-0.34 + 0.451i	-1.067 + 1.413i
-7.62	-0.412 + 0.425i	-1.177 + 1.214i	-7.42	-0.338 + 0.451i	-1.064 + 1.419i
-7.615	-0.41 + 0.426i	-1.174 + 1.22i	-7.415	-0.337 + 0.452i	-1.061 + 1.424i
-7.61	-0.408 + 0.427i	-1.171 + 1.225i	-7.41	-0.335 + 0.452i	-1.058 + 1.429i
-7.605	-0.406 + 0.427i	-1.168 + 1.23i	-7.405	-0.333 + 0.452i	-1.056 + 1.434i
-7.6	-0.404 + 0.428i	-1.165 + 1.235i	-7.4	-0.331 + 0.453i	-1.053 + 1.439i
-7.595	-0.402 + 0.429i	-1.162 + 1.24i	-7.395	-0.329 + 0.453i	-1.05 + 1.444i

$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$	$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-7.39	-0.328 + 0.453i	-1.047 + 1.449i	-7.19	-0.257 + 0.456i	-0.938 + 1.666i
-7.385	-0.326 + 0.454i	-1.045 + 1.454i	-7.185	-0.255 + 0.456i	-0.935 + 1.671i
-7.38	-0.324 + 0.454i	-1.042 + 1.459i	-7.18	-0.253 + 0.455i	-0.933 + 1.677i
-7.375	-0.322 + 0.454i	-1.039 + 1.465i	-7.175	-0.251 + 0.455i	-0.93 + 1.683i
-7.37	-0.32 + 0.454i	-1.036 + 1.47i	-7.17	-0.25 + 0.455i	-0.927 + 1.689i
-7.365	-0.319 + 0.455i	-1.033 + 1.475i	-7.165	-0.248 + 0.455i	-0.925 + 1.695i
-7.36	-0.317 + 0.455i	-1.031 + 1.48i	-7.16	-0.246 + 0.454i	-0.922 + 1.701i
-7.355	-0.315 + 0.455i	-1.028 + 1.485i	-7.155	-0.244 + 0.454i	-0.919 + 1.707i
-7.35	-0.313 + 0.455i	-1.025 + 1.491i	-7.15	-0.243 + 0.454i	-0.916 + 1.713i
-7.345	-0.311 + 0.456i	-1.022 + 1.496i	-7.145	-0.241 + 0.454i	-0.914 + 1.719i
-7.34	-0.31 + 0.456i	-1.02 + 1.501i	-7.14	-0.239 + 0.453i	-0.911 + 1.726i
-7.335	-0.308 + 0.456i	-1.017 + 1.506i	-7.135	-0.238 + 0.453i	-0.908 + 1.732i
-7.33	-0.306 + 0.456i	-1.014 + 1.511i	-7.13	-0.236 + 0.452i	-0.906 + 1.738i
-7.325	-0.304 + 0.456i	-1.011 + 1.517i	-7.125	-0.234 + 0.452i	-0.903 + 1.744i
-7.32	-0.303 + 0.457i	-1.009 + 1.522i	-7.12	-0.232 + 0.452i	-0.9 + 1.751i
-7.315	-0.301 + 0.457i	-1.006 + 1.527i	-7.115	-0.231 + 0.451i	-0.898 + 1.757i
-7.31	-0.299 + 0.457i	-1.003 + 1.533i	-7.11	-0.229 + 0.451i	-0.895 + 1.763i
-7.305	-0.297 + 0.457i	-1 + 1.538i	-7.105	-0.227 + 0.45i	-0.892 + 1.77i
-7.3	-0.295 + 0.457i	-0.998 + 1.543i	-7.1	-0.225 + 0.45i	-0.89 + 1.776i
-7.295	-0.294 + 0.457i	-0.995 + 1.549i	-7.095	-0.224 + 0.45i	-0.887 + 1.783i
-7.29	-0.292 + 0.457i	-0.992 + 1.554i	-7.09	-0.222 + 0.449i	-0.884 + 1.789i
-7.285	-0.29 + 0.457i	-0.99 + 1.559i	-7.085	-0.22 + 0.449i	-0.882 + 1.796i
-7.28	-0.288 + 0.457i	-0.987 + 1.565i	-7.08	-0.219 + 0.448i	-0.879 + 1.803i
-7.275	-0.287 + 0.457i	-0.984 + 1.57i	-7.075	-0.217 + 0.448i	-0.876 + 1.809i
-7.27	-0.285 + 0.457i	-0.981 + 1.576i	-7.07	-0.215 + 0.447i	-0.874 + 1.816i
-7.265	-0.283 + 0.457i	-0.979 + 1.581i	-7.065	-0.213 + 0.447i	-0.871 + 1.823i
-7.26	-0.281 + 0.457i	-0.976 + 1.587i	-7.06	-0.212 + 0.446i	-0.869 + 1.83i
-7.255	-0.28 + 0.457i	-0.973 + 1.592i	-7.055	-0.21 + 0.445i	-0.866 + 1.837i
-7.25	-0.278 + 0.457i	-0.97 + 1.598i	-7.05	-0.208 + 0.445i	-0.863 + 1.844i
-7.245	-0.276 + 0.457i	-0.968 + 1.603i	-7.045	-0.207 + 0.444i	-0.861 + 1.851i
-7.24	-0.274 + 0.457i	-0.965 + 1.609i	-7.04	-0.205 + 0.444i	-0.858 + 1.858i
-7.235	-0.272 + 0.457i	-0.962 + 1.614i	-7.035	-0.203 + 0.443i	-0.855 + 1.865i
-7.23	-0.271 + 0.457i	-0.96 + 1.62i	-7.03	-0.201 + 0.442i	-0.853 + 1.873i
-7.225	-0.269 + 0.457i	-0.957 + 1.625i	-7.025	-0.2 + 0.442i	-0.85 + 1.88i
-7.22	-0.267 + 0.457i	-0.954 + 1.631i	-7.02	-0.198 + 0.441i	-0.847 + 1.887i
-7.215	-0.265 + 0.457i	-0.951 + 1.637i	-7.015	-0.196 + 0.44i	-0.845 + 1.895i
-7.21	-0.264 + 0.457i	-0.949 + 1.642i	-7.01	-0.195 + 0.44i	-0.842 + 1.902i
-7.205	-0.262 + 0.456i	-0.946 + 1.648i	-7.005	-0.193 + 0.439i	-0.84 + 1.91i
-7.2	-0.26 + 0.456i	-0.943 + 1.654i	-7	-0.191 + 0.438i	-0.837 + 1.918i
-7.195	-0.258 + 0.456i	-0.941 + 1.66i	-6.995	-0.189 + 0.437i	-0.834 + 1.925i

$w\cdot 10^{-13}$	$\frac{C\cdot\chi_1(w)}{C\cdot\chi_1(w)-w^2}$	$\frac{C\cdot\chi_2(w)}{C\cdot\chi_2(w)-w^2}$	$w\cdot 10^{-13}$	$\frac{C\cdot\chi_1(w)}{C\cdot\chi_1(w)-w^2}$	$\frac{C\cdot\chi_2(w)}{C\cdot\chi_2(w)-w^2}$
-6.99	-0.188 + 0.436i	-0.832 + 1.933i	-6.79	-0.121 + 0.389i	-0.728 + 2.345i
-6.985	-0.186 + 0.436i	-0.829 + 1.941i	-6.785	-0.119 + 0.387i	-0.726 + 2.359i
-6.98	-0.184 + 0.435i	-0.826 + 1.949i	-6.78	-0.117 + 0.385i	-0.723 + 2.374i
-6.975	-0.183 + 0.434i	-0.824 + 1.957i	-6.775	-0.116 + 0.384i	-0.721 + 2.388i
-6.97	-0.181 + 0.433i	-0.821 + 1.966i	-6.77	-0.114 + 0.382i	-0.718 + 2.403i
-6.965	-0.179 + 0.432i	-0.819 + 1.974i	-6.765	-0.113 + 0.38i	-0.716 + 2.418i
-6.96	-0.178 + 0.431i	-0.816 + 1.982i	-6.76	-0.111 + 0.378i	-0.713 + 2.434i
-6.955	-0.176 + 0.43i	-0.813 + 1.991i	-6.755	-0.109 + 0.377i	-0.711 + 2.449i
-6.95	-0.174 + 0.43i	-0.811 + 1.999i	-6.75	-0.108 + 0.375i	-0.708 + 2.466i
-6.945	-0.173 + 0.429i	-0.808 + 2.008i	-6.745	-0.106 + 0.373i	-0.706 + 2.482i
-6.94	-0.171 + 0.428i	-0.806 + 2.017i	-6.74	-0.104 + 0.371i	-0.703 + 2.499i
-6.935	-0.169 + 0.427i	-0.803 + 2.025i	-6.735	-0.103 + 0.369i	-0.7 + 2.516i
-6.93	-0.167 + 0.426i	-0.8 + 2.034i	-6.73	-0.101 + 0.367i	-0.698 + 2.534i
-6.925	-0.166 + 0.425i	-0.798 + 2.044i	-6.725	-0.099 + 0.365i	-0.695 + 2.552i
-6.92	-0.164 + 0.424i	-0.795 + 2.053i	-6.72	-0.098 + 0.363i	-0.693 + 2.57i
-6.915	-0.162 + 0.423i	-0.793 + 2.062i	-6.715	-0.096 + 0.361i	-0.69 + 2.589i
-6.91	-0.161 + 0.421i	-0.79 + 2.071i	-6.71	-0.095 + 0.358i	-0.688 + 2.609i
-6.905	-0.159 + 0.42i	-0.787 + 2.081i	-6.705	-0.093 + 0.356i	-0.685 + 2.629i
-6.9	-0.157 + 0.419i	-0.785 + 2.091i	-6.7	-0.091 + 0.354i	-0.683 + 2.649i
-6.895	-0.156 + 0.418i	-0.782 + 2.101i	-6.695	-0.09 + 0.352i	-0.68 + 2.67i
-6.89	-0.154 + 0.417i	-0.78 + 2.11i	-6.69	-0.088 + 0.349i	-0.678 + 2.691i
-6.885	-0.152 + 0.416i	-0.777 + 2.121i	-6.685	-0.086 + 0.347i	-0.675 + 2.714i
-6.88	-0.151 + 0.415i	-0.774 + 2.131i	-6.68	-0.085 + 0.345i	-0.673 + 2.736i
-6.875	-0.149 + 0.413i	-0.772 + 2.141i	-6.675	-0.083 + 0.342i	-0.67 + 2.76i
-6.87	-0.147 + 0.412i	-0.769 + 2.152i	-6.67	-0.081 + 0.34i	-0.668 + 2.784i
-6.865	-0.146 + 0.411i	-0.767 + 2.162i	-6.665	-0.08 + 0.337i	-0.665 + 2.808i
-6.86	-0.144 + 0.41i	-0.764 + 2.173i	-6.66	-0.078 + 0.335i	-0.663 + 2.834i
-6.855	-0.142 + 0.408i	-0.762 + 2.184i	-6.655	-0.077 + 0.332i	-0.66 + 2.86i
-6.85	-0.141 + 0.407i	-0.759 + 2.196i	-6.65	-0.075 + 0.329i	-0.658 + 2.887i
-6.845	-0.139 + 0.405i	-0.756 + 2.207i	-6.645	-0.073 + 0.327i	-0.655 + 2.915i
-6.84	-0.137 + 0.404i	-0.754 + 2.218i	-6.64	-0.072 + 0.324i	-0.653 + 2.944i
-6.835	-0.136 + 0.403i	-0.751 + 2.23i	-6.635	-0.07 + 0.321i	-0.65 + 2.974i
-6.83	-0.134 + 0.401i	-0.749 + 2.242i	-6.63	-0.069 + 0.318i	-0.648 + 3.005i
-6.825	-0.132 + 0.4i	-0.746 + 2.254i	-6.625	-0.067 + 0.315i	-0.645 + 3.037i
-6.82	-0.131 + 0.398i	-0.744 + 2.267i	-6.62	-0.065 + 0.312i	-0.643 + 3.07i
-6.815	-0.129 + 0.397i	-0.741 + 2.279i	-6.615	-0.064 + 0.309i	-0.64 + 3.105i
-6.81	-0.127 + 0.395i	-0.739 + 2.292i	-6.61	-0.062 + 0.306i	-0.638 + 3.14i
-6.805	-0.126 + 0.394i	-0.736 + 2.305i	-6.605	-0.061 + 0.303i	-0.635 + 3.177i
-6.8	-0.124 + 0.392i	-0.733 + 2.318i	-6.6	-0.059 + 0.299i	-0.633 + 3.216i
-6.795	-0.122 + 0.391i	-0.731 + 2.332i	-6.595	-0.057 + 0.296i	-0.631 + 3.256i

$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$	$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-6.59	-0.056 + 0.293i	-0.628 + 3.298i	-6.39	-0.111	-9.017
-6.585	-0.054 + 0.289i	-0.626 + 3.342i	-6.385	-0.122	-8.229
-6.58	-0.053 + 0.286i	-0.623 + 3.388i	-6.38	-0.131	-7.622
-6.575	-0.051 + 0.282i	-0.621 + 3.436i	-6.375	-0.14	-7.135
-6.57	-0.049 + 0.278i	-0.618 + 3.486i	-6.37	-0.149	-6.733
-6.565	-0.048 + 0.274i	-0.616 + 3.539i	-6.365	-0.156	-6.393
-6.56	-0.046 + 0.27i	-0.613 + 3.595i	-6.36	-0.164	-6.101
-6.555	-0.045 + 0.266i	-0.611 + 3.653i	-6.355	-0.171	-5.847
-6.55	-0.043 + 0.262i	-0.608 + 3.715i	-6.35	-0.178	-5.622
-6.545	-0.041 + 0.258i	-0.606 + 3.78i	-6.345	-0.184	-5.422
-6.54	-0.04 + 0.254i	-0.603 + 3.849i	-6.34	-0.191	-5.242
-6.535	-0.038 + 0.249i	-0.601 + 3.923i	-6.335	-0.197	-5.078
-6.53	-0.037 + 0.244i	-0.599 + 4.001i	-6.33	-0.203	-4.929
-6.525	-0.035 + 0.24i	-0.596 + 4.084i	-6.325	-0.209	-4.793
-6.52	-0.033 + 0.235i	-0.594 + 4.174i	-6.32	-0.214	-4.667
-6.515	-0.032 + 0.23i	-0.591 + 4.27i	-6.315	-0.22	-4.55
-6.51	-0.03 + 0.225i	-0.589 + 4.373i	-6.31	-0.225	-4.441
-6.505	-0.029 + 0.219i	-0.586 + 4.486i	-6.305	-0.23	-4.34
-6.5	-0.027 + 0.214i	-0.584 + 4.607i	-6.3	-0.236	-4.245
-6.495	-0.025 + 0.208i	-0.581 + 4.74i	-6.295	-0.241	-4.155
-6.49	-0.024 + 0.202i	-0.579 + 4.887i	-6.29	-0.246	-4.071
-6.485	-0.022 + 0.196i	-0.577 + 5.048i	-6.285	-0.251	-3.992
-6.48	-0.021 + 0.189i	-0.574 + 5.228i	-6.28	-0.255	-3.917
-6.475	-0.019 + 0.182i	-0.572 + 5.429i	-6.275	-0.26	-3.845
-6.47	-0.018 + 0.175i	-0.569 + 5.657i	-6.27	-0.265	-3.777
-6.465	-0.016 + 0.167i	-0.567 + 5.918i	-6.265	-0.269	-3.712
-6.46	-0.014 + 0.159i	-0.564 + 6.22i	-6.26	-0.274	-3.65
-6.455	-0.013 + 0.151i	-0.562 + 6.577i	-6.255	-0.278	-3.591
-6.45	-0.011 + 0.142i	-0.56 + 7.006i	-6.25	-0.283	-3.535
-6.445	-0.01 + 0.132i	-0.557 + 7.534i	-6.245	-0.287	-3.48
-6.44	-0.008 + 0.121i	-0.555 + 8.208i	-6.24	-0.292	-3.428
-6.435	-0.007 + 0.109i	-0.552 + 9.11i	-6.235	-0.296	-3.378
-6.43	-0.005 + 0.096i	-0.55 + 10.403i	-6.23	-0.3	-3.33
-6.425	-0.004 + 0.08i	-0.548 + 12.486i	-6.225	-0.305	-3.283
-6.42	-0.002 + 0.06i	-0.545 + 16.738i	-6.22	-0.309	-3.238
-6.415	$-3.837 \cdot 10^{-4} + 0.027i$	-0.543 + 37.605i	-6.215	-0.313	-3.194
-6.41	-0.045	-21.99	-6.21	-0.317	-3.152
-6.405	-0.069	-14.578	-6.205	-0.321	-3.112
-6.4	-0.085	-11.722	-6.2	-0.326	-3.072
-6.395	-0.099	-10.099	-6.195	-0.33	-3.034

$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$	$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-6.19	-0.334	-2.997	-5.99	-0.494	-2.024
-6.185	-0.338	-2.961	-5.985	-0.498	-2.007
-6.18	-0.342	-2.926	-5.98	-0.502	-1.99
-6.175	-0.346	-2.891	-5.975	-0.507	-1.974
-6.17	-0.35	-2.858	-5.97	-0.511	-1.957
-6.165	-0.354	-2.826	-5.965	-0.515	-1.941
-6.16	-0.358	-2.794	-5.96	-0.519	-1.925
-6.155	-0.362	-2.764	-5.955	-0.524	-1.909
-6.15	-0.366	-2.733	-5.95	-0.528	-1.894
-6.145	-0.37	-2.704	-5.945	-0.532	-1.878
-6.14	-0.374	-2.675	-5.94	-0.537	-1.863
-6.135	-0.378	-2.647	-5.935	-0.541	-1.848
-6.13	-0.382	-2.62	-5.93	-0.546	-1.833
-6.125	-0.386	-2.593	-5.925	-0.55	-1.818
-6.12	-0.39	-2.567	-5.92	-0.555	-1.803
-6.115	-0.394	-2.541	-5.915	-0.559	-1.789
-6.11	-0.397	-2.516	-5.91	-0.564	-1.774
-6.105	-0.401	-2.491	-5.905	-0.568	-1.76
-6.1	-0.405	-2.467	-5.9	-0.573	-1.746
-6.095	-0.409	-2.443	-5.895	-0.577	-1.732
-6.09	-0.413	-2.42	-5.89	-0.582	-1.718
-6.085	-0.417	-2.397	-5.885	-0.587	-1.704
-6.08	-0.421	-2.374	-5.88	-0.592	-1.69
-6.075	-0.425	-2.352	-5.875	-0.596	-1.677
-6.07	-0.429	-2.331	-5.87	-0.601	-1.663
-6.065	-0.433	-2.309	-5.865	-0.606	-1.65
-6.06	-0.437	-2.288	-5.86	-0.611	-1.637
-6.055	-0.441	-2.267	-5.855	-0.616	-1.624
-6.05	-0.445	-2.247	-5.85	-0.621	-1.611
-6.045	-0.449	-2.227	-5.845	-0.626	-1.598
-6.04	-0.453	-2.207	-5.84	-0.631	-1.585
-6.035	-0.457	-2.188	-5.835	-0.636	-1.573
-6.03	-0.461	-2.169	-5.83	-0.641	-1.56
-6.025	-0.465	-2.15	-5.825	-0.646	-1.547
-6.02	-0.469	-2.131	-5.82	-0.651	-1.535
-6.015	-0.473	-2.113	-5.815	-0.657	-1.523
-6.01	-0.477	-2.094	-5.81	-0.662	-1.51
-6.005	-0.482	-2.077	-5.805	-0.667	-1.498
-6	-0.486	-2.059	-5.8	-0.673	-1.486
-5.995	-0.49	-2.041	-5.795	-0.678	-1.474

$w\cdot 10^{-13}$	$\frac{C\cdot\chi_1(w)}{C\cdot\chi_1(w)-w^2}$	$\frac{C\cdot\chi_2(w)}{C\cdot\chi_2(w)-w^2}$
-5.79	-0.684	-1.462
-5.785	-0.689	-1.45
-5.78	-0.695	-1.439
-5.775	-0.701	-1.427
-5.77	-0.707	-1.415
-5.765	-0.712	-1.404
-5.76	-0.718	-1.392
-5.755	-0.724	-1.381
-5.75	-0.73	-1.37
-5.745	-0.736	-1.358
-5.74	-0.742	-1.347
-5.735	-0.749	-1.336
-5.73	-0.755	-1.325
-5.725	-0.761	-1.314
-5.72	-0.768	-1.303
-5.715	-0.774	-1.292
-5.71	-0.781	-1.281
-5.705	-0.787	-1.27
-5.7	-0.794	-1.259
-5.695	-0.801	-1.249
-5.69	-0.808	-1.238
-5.685	-0.815	-1.227
-5.68	-0.822	-1.217
-5.675	-0.829	-1.206
-5.67	-0.836	-1.196
-5.665	-0.844	-1.185
-5.66	-0.851	-1.175
-5.655	-0.859	-1.165
-5.65	-0.866	-1.154
-5.645	-0.874	-1.144
-5.64	-0.882	-1.134
-5.635	-0.89	-1.124
-5.63	-0.898	-1.113
-5.625	-0.906	-1.103
-5.62	-0.915	-1.093
-5.615	-0.923	-1.083
-5.61	-0.932	-1.073
-5.605	-0.941	-1.063
-5.6	-0.95	-1.053
-5.595	-0.959	-1.043

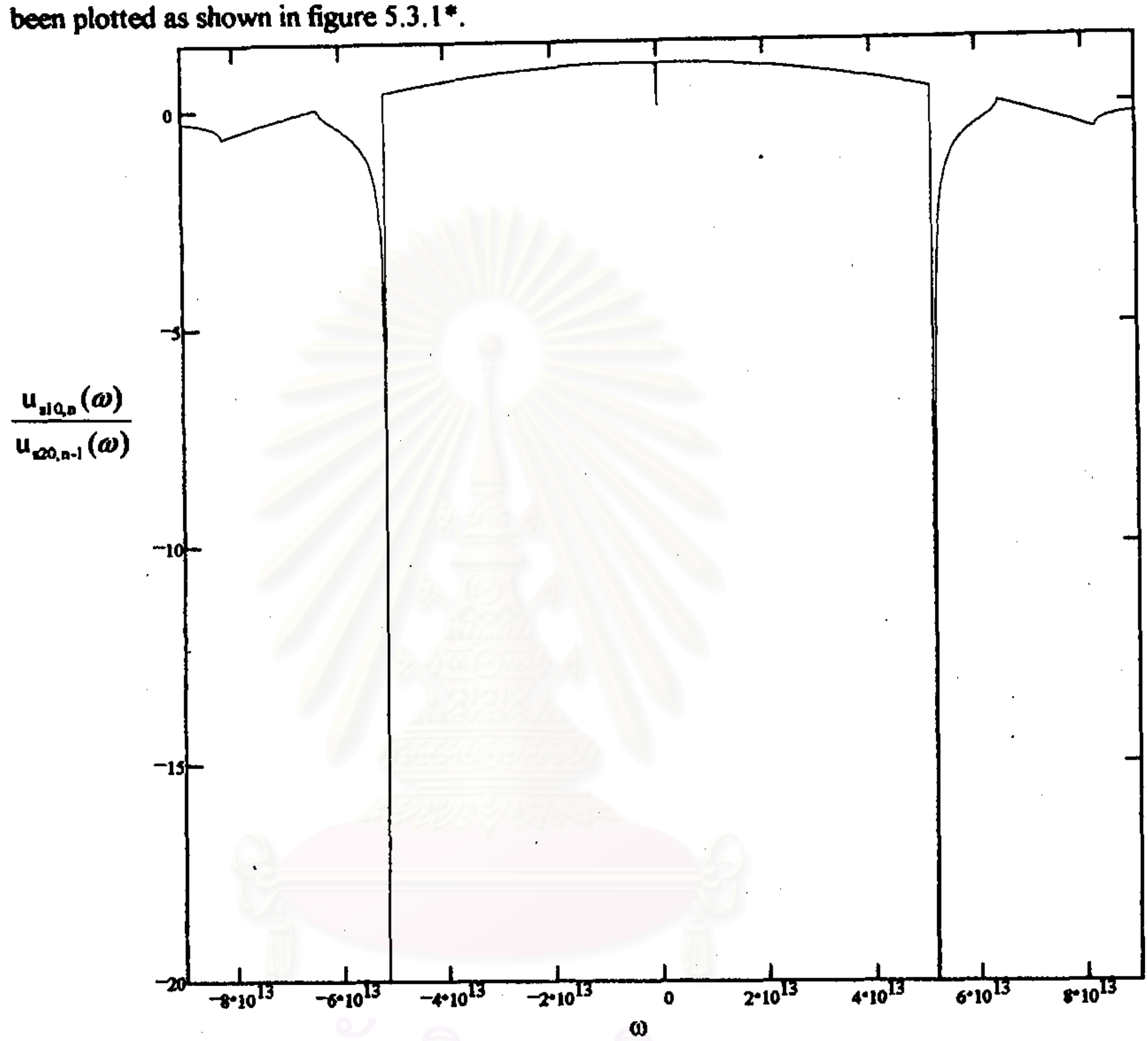
$w\cdot 10^{-13}$	$\frac{C\cdot\chi_1(w)}{C\cdot\chi_1(w)-w^2}$	$\frac{C\cdot\chi_2(w)}{C\cdot\chi_2(w)-w^2}$
-5.59	-0.968	-1.033
-5.585	-0.977	-1.023
-5.58	-0.987	-1.013
-5.575	-0.996	-1.004
-5.57	-1.006	-0.994
-5.565	-1.016	-0.984
-5.56	-1.027	-0.974
-5.555	-1.037	-0.964
-5.55	-1.048	-0.955
-5.545	-1.058	-0.945
-5.54	-1.069	-0.935
-5.535	-1.08	-0.926
-5.53	-1.092	-0.916
-5.525	-1.104	-0.906
-5.52	-1.115	-0.897
-5.515	-1.127	-0.887
-5.51	-1.14	-0.877
-5.505	-1.152	-0.868
-5.5	-1.165	-0.858
-5.495	-1.178	-0.849
-5.49	-1.192	-0.839
-5.485	-1.206	-0.829
-5.48	-1.22	-0.82
-5.475	-1.234	-0.81
-5.47	-1.249	-0.801
-5.465	-1.264	-0.791
-5.46	-1.279	-0.782
-5.455	-1.295	-0.772
-5.45	-1.312	-0.762
-5.445	-1.328	-0.753
-5.44	-1.345	-0.743
-5.435	-1.363	-0.734
-5.43	-1.381	-0.724
-5.425	-1.399	-0.715
-5.42	-1.418	-0.705
-5.415	-1.438	-0.695
-5.41	-1.458	-0.686
-5.405	-1.479	-0.676
-5.4	-1.501	-0.666
-5.395	-1.523	-0.657

$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$	$w \cdot 10^{-13}$	$\frac{C \cdot \chi_1(w)}{C \cdot \chi_1(w) - w^2}$	$\frac{C \cdot \chi_2(w)}{C \cdot \chi_2(w) - w^2}$
-5.39	-1.546	-0.647	-5.19	-5.606	-0.178
-5.385	-1.569	-0.637	-5.185	-6.312	-0.158
-5.38	-1.594	-0.628	-5.18	-7.342	-0.136
-5.375	-1.619	-0.618	-5.175	-9.055	-0.11
-5.37	-1.645	-0.608	-5.17	-12.847	-0.078
-5.365	-1.672	-0.598	-5.165	-68.02	-0.015
-5.36	-1.7	-0.588	-5.16	0.353 + 13.766i	0.002 + 0.073i
-5.355	-1.729	-0.578	-5.155	0.354 + 9.654i	0.004 + 0.103i
-5.35	-1.76	-0.568	-5.15	0.355 + 7.869i	0.006 + 0.127i
-5.345	-1.791	-0.558	-5.145	0.357 + 6.815i	0.008 + 0.146i
-5.34	-1.824	-0.548	-5.14	0.358 + 6.1i	0.01 + 0.163i
-5.335	-1.858	-0.538	-5.135	0.359 + 5.574i	0.012 + 0.179i
-5.33	-1.894	-0.528	-5.13	0.36 + 5.166i	0.013 + 0.193i
-5.325	-1.932	-0.518	-5.125	0.361 + 4.839i	0.015 + 0.206i
-5.32	-1.971	-0.507	-5.12	0.363 + 4.568i	0.017 + 0.218i
-5.315	-2.012	-0.497	-5.115	0.364 + 4.34i	0.019 + 0.229i
-5.31	-2.055	-0.487	-5.11	0.365 + 4.143i	0.021 + 0.239i
-5.305	-2.101	-0.476	-5.105	0.366 + 3.973i	0.023 + 0.25i
-5.3	-2.149	-0.465	-5.1	0.368 + 3.822i	0.025 + 0.259i
-5.295	-2.199	-0.455	-5.095	0.369 + 3.689i	0.027 + 0.268i
-5.29	-2.253	-0.444	-5.09	0.37 + 3.569i	0.029 + 0.277i
-5.285	-2.31	-0.433	-5.085	0.371 + 3.46i	0.031 + 0.286i
-5.28	-2.37	-0.422	-5.08	0.373 + 3.362i	0.033 + 0.294i
-5.275	-2.435	-0.411	-5.075	0.374 + 3.272i	0.034 + 0.302i
-5.27	-2.504	-0.399	-5.07	0.375 + 3.189i	0.036 + 0.309i
-5.265	-2.578	-0.388	-5.065	0.376 + 3.113i	0.038 + 0.317i
-5.26	-2.657	-0.376	-5.06	0.378 + 3.042i	0.04 + 0.324i
-5.255	-2.743	-0.365	-5.055	0.379 + 2.977i	0.042 + 0.331i
-5.25	-2.837	-0.352	-5.05	0.38 + 2.916i	0.044 + 0.337i
-5.245	-2.939	-0.34	-5.045	0.381 + 2.858i	0.046 + 0.344i
-5.24	-3.051	-0.328	-5.04	0.383 + 2.804i	0.048 + 0.35i
-5.235	-3.175	-0.315	-5.035	0.384 + 2.754i	0.05 + 0.356i
-5.23	-3.312	-0.302	-5.03	0.385 + 2.706i	0.052 + 0.362i
-5.225	-3.466	-0.289	-5.025	0.386 + 2.661i	0.053 + 0.368i
-5.22	-3.641	-0.275	-5.02	0.387 + 2.619i	0.055 + 0.374i
-5.215	-3.84	-0.26	-5.015	0.389 + 2.578i	0.057 + 0.379i
-5.21	-4.072	-0.246	-5.01	0.39 + 2.54i	0.059 + 0.385i
-5.205	-4.345	-0.23	-5.005	0.391 + 2.503i	0.061 + 0.39i
-5.2	-4.675	-0.214	-5	0.392 + 2.468i	0.063 + 0.395i
-5.195	-5.083	-0.197	-4.995	0.393 + 2.435i	0.065 + 0.4i

$w\cdot 10^{-13}$	$\dfrac{C\cdot\chi_1(w)}{C\cdot\chi_1(w)-w^2}$	$\dfrac{C\cdot\chi_2(w)}{C\cdot\chi_2(w)-w^2}$
-4.99	0.395 + 2.403i	0.067 + 0.405i
-4.985	0.396 + 2.373i	0.068 + 0.41i
-4.98	0.397 + 2.344i	0.07 + 0.415i
-4.975	0.398 + 2.316i	0.072 + 0.419i
-4.97	0.4 + 2.289i	0.074 + 0.424i
-4.965	0.401 + 2.263i	0.076 + 0.428i
-4.96	0.402 + 2.238i	0.078 + 0.433i
-4.955	0.403 + 2.214i	0.08 + 0.437i
-4.95	0.404 + 2.191i	0.081 + 0.441i
-4.945	0.406 + 2.169i	0.083 + 0.445i
-4.94	0.407 + 2.147i	0.085 + 0.45i
-4.935	0.408 + 2.127i	0.087 + 0.454i
-4.93	0.409 + 2.106i	0.089 + 0.457i
-4.925	0.41 + 2.087i	0.091 + 0.461i
-4.92	0.412 + 2.068i	0.093 + 0.465i
-4.915	0.413 + 2.05i	0.094 + 0.469i
-4.91	0.414 + 2.032i	0.096 + 0.472i
-4.905	0.415 + 2.015i	0.098 + 0.476i
-4.9	0.416 + 1.998i	0.1 + 0.48i
-4.895	0.418 + 1.982i	0.102 + 0.483i
-4.89	0.419 + 1.966i	0.104 + 0.486i
-4.885	0.42 + 1.951i	0.105 + 0.49i
-4.88	0.421 + 1.936i	0.107 + 0.493i
-4.875	0.422 + 1.922i	0.109 + 0.496i
-4.87	0.423 + 1.908i	0.111 + 0.5i
-4.865	0.425 + 1.894i	0.113 + 0.503i
-4.86	0.426 + 1.88i	0.115 + 0.506i
-4.855	0.427 + 1.867i	0.116 + 0.509i
-4.85	0.428 + 1.855i	0.118 + 0.512i
-4.845	0.429 + 1.842i	0.12 + 0.515i
-4.84	0.431 + 1.83i	0.122 + 0.518i
-4.835	0.432 + 1.818i	0.124 + 0.521i
-4.83	0.433 + 1.806i	0.125 + 0.524i
-4.825	0.434 + 1.795i	0.127 + 0.526i
-4.82	0.435 + 1.784i	0.129 + 0.529i
-4.815	0.436 + 1.773i	0.131 + 0.532i
-4.81	0.438 + 1.763i	0.133 + 0.534i
-4.805	0.439 + 1.752i	0.134 + 0.537i
-4.8	0.44 + 1.742i	0.136 + 0.54i
-4.795	0.441 + 1.732i	0.138 + 0.542i

$w\cdot 10^{-13}$	$\dfrac{C\cdot\chi_1(w)}{C\cdot\chi_1(w)-w^2}$	$\dfrac{C\cdot\chi_2(w)}{C\cdot\chi_2(w)-w^2}$
-4.79	0.442 + 1.722i	0.14 + 0.545i
-4.785	0.443 + 1.713i	0.142 + 0.547i
-4.78	0.445 + 1.703i	0.143 + 0.55i
-4.775	0.446 + 1.694i	0.145 + 0.552i
-4.77	0.447 + 1.685i	0.147 + 0.554i
-4.765	0.448 + 1.676i	0.149 + 0.557i
-4.76	0.449 + 1.668i	0.151 + 0.559i
-4.755	0.45 + 1.659i	0.152 + 0.561i
-4.75	0.452 + 1.651i	0.154 + 0.564i
-4.745	0.453 + 1.642i	0.156 + 0.566i
-4.74	0.454 + 1.634i	0.158 + 0.568i
-4.735	0.455 + 1.626i	0.16 + 0.57i
-4.73	0.456 + 1.619i	0.161 + 0.572i
-4.725	0.457 + 1.611i	0.163 + 0.574i
-4.72	0.458 + 1.603i	0.165 + 0.577i
-4.715	0.46 + 1.596i	0.167 + 0.579i
-4.71	0.461 + 1.589i	0.168 + 0.581i
-4.705	0.462 + 1.582i	0.17 + 0.583i
-4.7	0.463 + 1.575i	0.172 + 0.585i
-4.695	0.464 + 1.568i	0.174 + 0.586i
-4.69	0.465 + 1.561i	0.175 + 0.588i
-4.685	0.466 + 1.554i	0.177 + 0.59i
-4.68	0.468 + 1.547i	0.179 + 0.592i
-4.675	0.469 + 1.541i	0.181 + 0.594i
-4.67	0.47 + 1.535i	0.182 + 0.596i
-4.665	0.471 + 1.528i	0.184 + 0.598i
-4.66	0.472 + 1.522i	0.186 + 0.599i
-4.655	0.473 + 1.516i	0.188 + 0.601i
-4.65	0.474 + 1.51i	0.189 + 0.603i
-4.645	0.476 + 1.504i	0.191 + 0.605i
-4.64	0.477 + 1.498i	0.193 + 0.606i
-4.635	0.478 + 1.492i	0.195 + 0.608i
-4.63	0.479 + 1.486i	0.196 + 0.61i
-4.625	0.48 + 1.481i	0.198 + 0.611i
-4.62	0.481 + 1.475i	0.2 + 0.613i
-4.615	0.482 + 1.47i	0.202 + 0.614i
-4.61	0.483 + 1.464i	0.203 + 0.616i
-4.605	0.484 + 1.459i	0.205 + 0.617i
-4.6	0.486 + 1.454i	0.207 + 0.619i
-4.595	0.487 + 1.448i	0.208 + 0.62i

The recursion relations which depends on the frequency(look for only real part) have been plotted as shown in figure 5.3.1*.



(a)

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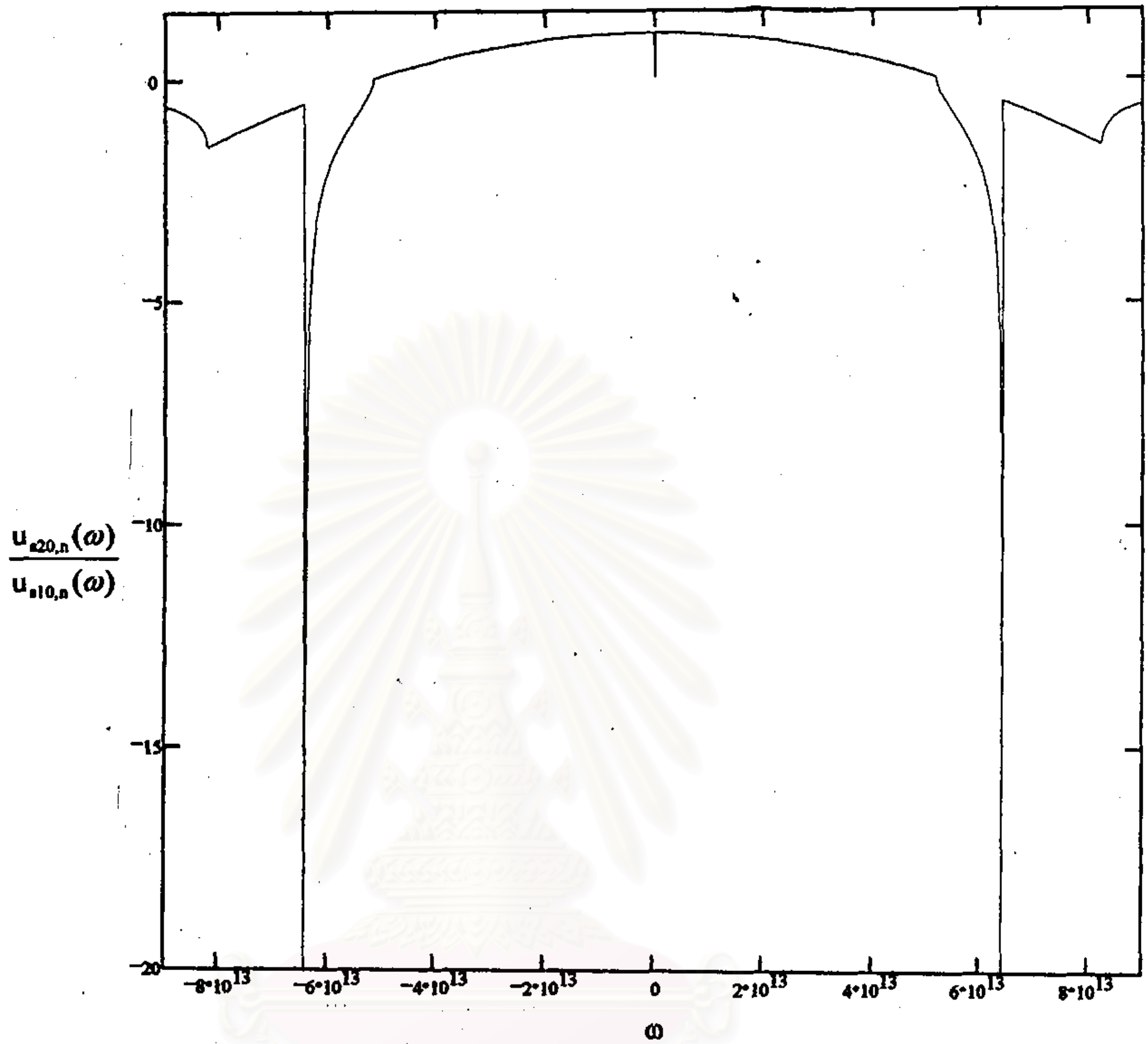
*See APPENDIX F.



(b)

Fig. 5.3.1 The Recursion Relations.

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We have observed as follows

$0 < |u_{s10,n}(w)/u_{s20,n-1}(w)| < 1$, means $|u_{s10,n}(w)| < |u_{s20,n-1}(w)|$, and $0 < |u_{s20,n}(w)/u_{s10,n}(w)| < 1$, means $|u_{s20,n}(w)| < |u_{s10,n}(w)|$. $|u_{s10,n}(w)/u_{s20,n-1}(w)|$ and $|u_{s20,n}(w)/u_{s10,n}(w)|$ are equal to one mean the masses move with the same amplitude. $|u_{s10,n}(w)/u_{s20,n-1}(w)| > 1$, means $|u_{s10,n}(w)| > |u_{s20,n-1}(w)|$, and $|u_{s20,n}(w)/u_{s10,n}(w)| > 1$, means $|u_{s20,n}(w)| > |u_{s10,n}(w)|$.

The minus sign means the direction of vibrations are opposite and the plus sign means the direction of vibrations are the same.

5.4 The Energy Transport in the Chain

We have considered the energy transport of the masses in two cases; the energy transport in the s1,n th mass and the energy transport in the s2,n th mass.

5.4.1 The energy transport in the s1,n th mass

The energy in the s1,n th mass at time t is

$$E_{s1,n} = \frac{P_{s1,n}^{\;2}}{2m_1} + \frac{C\,[u_{s1,n} - u_{s2,n}]^2}{2} \qquad (5.4.1.1)$$

Then its change with time is given by its Poisson bracket with the Hamiltonian, is

$$\dot{E}_{s1,n} = [E_{s1,n}\,,\, H] = \sum_i \left[\frac{\partial E_{s1,n}}{\partial u_{s1,i}} \frac{\partial H}{\partial P_{s1,i}} - \frac{\partial H}{\partial u_{s1,i}} \frac{\partial E_{s1,n}}{\partial P_{s1,i}} \right] + \sum_i \left[\frac{\partial E_{s1,n}}{\partial u_{s2,i}} \frac{\partial H}{\partial P_{s2,i}} - \frac{\partial H}{\partial u_{s2,i}} \frac{\partial E_{s1,n}}{\partial P_{s2,i}} \right] \quad [13]\,, \qquad (5.4.1.2)$$

where

$$H = (1/2m_1)\sum_{n=0}^{\infty} P_{s1,n}^2 + (C/2)\sum_{n=0}^{\infty} (u_{s2,n} - u_{s1,n})^2$$
$$+ (1/2m_2)\sum_{n=0}^{\infty} P_{s2,n}^2 + (C/2)\sum_{n=0}^{\infty} \ldots (u_{s1,n+1} - u_{s2,n})^2$$

We then get

$$\frac{\partial E_{s1,n}}{\partial u_{s1,i}} = \frac{1}{2m_1}\frac{\partial P_{s1,n}{}^2}{\partial u_{s1,i}} + \frac{C}{2}\frac{\partial [u_{s1,n} - u_{s2,n}]^2}{\partial u_{s1,i}},$$

$$= \frac{C}{2}\frac{\partial [u_{s1,n} - u_{s2,n}]^2}{\partial u_{s1,i}},$$

$$\frac{\partial H}{\partial P_{s1,i}} = \frac{P_{s1,i}}{m_1},$$

$$\frac{\partial H}{\partial u_{s1,i}} = -C(u_{s2,i} - u_{s1,i}) + C(u_{s1,i} - u_{s2,i-1}),$$

$$\frac{\partial E_{s1,n}}{\partial P_{s1,i}} = \frac{1}{2m_1}\frac{\partial P_{s1,n}{}^2}{\partial P_{s1,i}},$$
$$\frac{\partial H}{\partial P_{s2,i}} = \frac{P_{s2,i}}{m_2}$$

$$\frac{\partial E_{s1,n}}{\partial u_{s2,i}} = \frac{C}{2}\frac{\partial [u_{s1,n} - u_{s2,n}]}{\partial u_{s2,i}},$$

Inserting these equations into Eq.(5.15) yields

$$\dot{E}_{s1,n} = \sum_i \frac{C}{2}\frac{\partial[u_{s1,n}-u_{s2,n}]^2}{\partial u_{s1,i}}\frac{P_{s1,i}}{m_1} - \left[\frac{-C(u_{s2,i}-u_{s1,i})+C(u_{s1,i}-u_{s2,i-1})}{2m_1}\right]\frac{\partial P_{s1,n}{}^2}{\partial P_{s1,i}}$$
$$+\sum_i \frac{C}{2}\frac{\partial[u_{s1,n}-u_{s2,n}]^2}{\partial u_{s2,i}}\frac{P_{s2,i}}{m_2} \quad ,$$

$$= \frac{C[u_{s1,n}-u_{s2,n}]P_{s1,i}\delta_{in}+C[u_{s2,i}-u_{s1,i}]P_{s1,n}\delta_{in}-C[u_{s1,i}-u_{s2,i-1}]P_{s1,n}\delta_{in}}{m_1}$$
$$-\frac{C[u_{s1,n}-u_{s2,n}]P_{s2,i}\delta_{in}}{m_2} \quad ,$$

$$= \frac{C[u_{s2,n-1}-u_{s1,n}]P_{s1,n}}{m_1}+\frac{C[u_{s2,n}-u_{s1,n}]P_{s2,n}}{m_2} \, ,$$

$$= C\dot{u}_{s1,n}[u_{s2,n-1}-u_{s1,n}]+C\dot{u}_{s2,n}[u_{s2,n}-u_{s1,n}] \quad . \tag{5.4.1.3}$$

<u>Energy flux</u> ($\dot{E}_f$)

The energy flux has thus been defined as $\dot{E}_{f(s1,n)}(t)$. Substitute $u_{s1,n}$ = Aexp i ($\omega t+n\Delta\Phi'$) and $u_{s2,n}$ = Bexp i($\omega t+n\Delta\Phi'$) into Eq.(5.4.1.3), and consider only real part, we have obtained

$$\dot{E}_{f(s1,n)}(t) = C\,Re\{\dot{u}_{s1,n}[u_{s2,n-1}-u_{s1,n}]+\dot{u}_{s2,n}[u_{s2,n}-u_{s1,n}]\}$$
$$= C[Re(\dot{u}_{s1,n})Re(u_{s2,n-1}-u_{s1,n})-Im(\dot{u}_{s1,n})Im(u_{s2,n-1}-u_{s1,n})$$
$$+Re(\dot{u}_{s2,n})Re(u_{s2,n}-u_{s1,n})-Im(\dot{u}_{s2,n})Im(u_{s2,n}-u_{s1,n})]$$

$$E_{f(s1,n)}^{\bullet}(t) = C[Re(\dot{u}_{s1,n})Re(u_{s1,n}[\frac{u_{s2,n-1}}{u_{s1,n}} - 1]) - Im(\dot{u}_{s1,n})Im(u_{s1,n}[\frac{u_{s2,n-1}}{u_{s1,n}} - 1]) + Re(\dot{u}_{s2,n})Re(u_{s2,n}[1 - \frac{u_{s1,n}}{u_{s2,n}}]) - Im(\dot{u}_{s2,n})Im(u_{s2,n}[1 - \frac{u_{s1,n}}{u_{s2,n}}])]$$

Inserting the recursion relations Eq.(5.1.3) and Eq.(5.1.6) into the last equation yield,

$$E_{f(s1,n)}^{\bullet}(t) = \frac{\omega^2}{|\chi_1|^2}[Re(\dot{u}_{s1,n})Re(u_{s1,n}\dot{\chi}_1) - Im(\dot{u}_{s1,n})Im(u_{s1,n}\dot{\chi}_1)] + \frac{\omega^2}{|\chi_2|^2}[Re(\dot{u}_{s2,n})Re(u_{s2,n}\dot{\chi}_2) - Im(\dot{u}_{s2,n})Im(u_{s2,n}\dot{\chi}_2)] \quad , \quad (5.4.1.4)$$

$$Re(\dot{u}_{s1,n}) = -A\,\omega\sin[\omega t + n\Delta\phi]\exp(-n\gamma) \quad ,$$

$$Im(\dot{u}_{s1,n}) = A\,\omega\cos[\omega t + n\Delta\phi]\exp(-n\gamma) \quad ,$$

$$Re(u_{s1,n}\dot{\chi}_1) = A\exp(-n\gamma)[\cos(\omega t + n\Delta\phi)Re(\chi_1) + \sin(\omega t + n\Delta\phi)Im(\chi_1)] \quad ,$$

$$Im(u_{s1,n}\dot{\chi}_1) = A\exp(-n\gamma)[-\cos(\omega t + n\Delta\phi)Im(\chi_1) + \sin(\omega t + n\Delta\phi)Re(\chi_1)] \quad ,$$

$$Re(\dot{u}_{s2,n}) = -B\,\omega\sin[\omega t + n\Delta\phi]\exp(-n\gamma) \quad ,$$

$$Im(\dot{u}_{s2,n}) = B\,\omega\cos[\omega t + n\Delta\phi]\exp(-n\gamma) \quad ,$$

$$Re(u_{s2,n}\dot{\chi}_2) = B\exp(-n\gamma)[\cos(\omega t + n\Delta\phi)Re(\chi_2) + \sin(\omega t + n\Delta\phi)Im(\chi_2)] \quad ,$$

$$Im(u_{s2,n}\dot{\chi}_2) = B\exp(-n\gamma)[-\cos(\omega t + n\Delta\phi)Im(\chi_2) + \sin(\omega t + n\Delta\phi)Re(\chi_2)] \quad ,$$

where $\Delta\Phi' = \Delta\phi + i\gamma$, $\Delta\phi$ is phase difference and γ is attenuation factor and let

$\omega t + n\Delta\phi = \beta$, therefore,

$$Re(\dot{u}_{s1,n})Re(u_{s1,n}\dot{\chi}_1) = \frac{-A^2\omega\, exp(-2n\gamma)}{2}[\sin 2\beta\ \mathrm{Re}(\chi_1) + \mathrm{Im}(\chi_1) - \cos 2\beta\ \mathrm{Im}(\chi_1)]$$

$$Im(\dot{u}_{s1,n})Im(u_{s1,n}\dot{\chi}_1) = \frac{A^2\omega\, exp(-2n\gamma)}{2}[\sin 2\beta\ \mathrm{Re}(\chi_1) - \mathrm{Im}(\chi_1) - \cos 2\beta\ \mathrm{Im}(\chi_1)]$$

$$Re(\dot{u}_{s2,n})Re(u_{s2,n}\dot{\chi}_2) = \frac{-B^2\omega\, exp(-2n\gamma)}{2}[\sin 2\beta\ \mathrm{Re}(\chi_2) + \mathrm{Im}(\chi_2) - \cos 2\beta\ \mathrm{Im}(\chi_2)]$$

$$Im(\dot{u}_{s2,n})Im(u_{s2,n}\dot{\chi}_2) = \frac{B^2\omega\, exp(-2n\gamma)}{2}[\sin 2\beta\ \mathrm{Re}(\chi_2) - \mathrm{Im}(\chi_2) - \cos 2\beta\ \mathrm{Im}(\chi_2)]$$

Inserting these equations into Eq.(5.4.1.4), yields

$$\begin{aligned}\dot{E}_{f(s1,n)}(t) &= -\frac{A^2\omega^3\ exp(-2n\gamma)}{2}[\sin 2\beta\ \mathrm{Re}(\chi_1) - \cos 2\beta\ \mathrm{Im}(\chi_1)] \\ &-\frac{B^2\omega^3\ exp(-2n\gamma)}{2}[\sin 2\beta\ \mathrm{Re}(\chi_2) - \cos 2\beta\ \mathrm{Im}(\chi_2)]\end{aligned} \quad . \quad (5.4.1.5)$$

The terms in Eq.(5.4.1.5) are expressed as the energy flow in the chain which is oscillating in time. They are proportional to the real and the imaginary parts of χ_1 and χ_2.

5.4.2 <u>The energy transport in the s2,n th mass</u>

The energy in the s2,n th mass at time t is

$$E_{s2,n} = \frac{P_{s2,n}{}^{2}}{2m_2} + \frac{C\,[u_{s2,n} - u_{s1,n+1}]^2}{2} \qquad (5.4.2.1)$$

Then its change with time is given by its Poisson bracket with the Hamiltonian, is

$$\dot{E}_{s2,n} = [E_{s2,n}, H] = \sum_{l}\left[\frac{\partial E_{s2,n}}{\partial u_{s2,l}}\frac{\partial H}{\partial P_{s2,l}} - \frac{\partial H}{\partial u_{s2,l}}\frac{\partial E_{s2,n}}{\partial P_{s2,l}}\right] + \sum_{l}\left[\frac{\partial E_{s2,n}}{\partial u_{s1,l}}\frac{\partial H}{\partial P_{s1,l}} - \frac{\partial H}{\partial u_{s1,l}}\frac{\partial E_{s2,n}}{\partial P_{s1,l}}\right] \quad [13] \quad , \qquad (5.4.2.2)$$

where

$$H = (1/2m_1)\sum_{n=0}^{\infty} P_{s1,n}^2 + (C/2)\sum_{n=0}^{\infty}(u_{s2,n} - u_{s1,n})^2 + (1/2m_2)\sum_{n=0}^{\infty} P_{s2,n}^2 + (C/2)\sum_{n=0}^{\infty}(u_{s1,n+1} - u_{s2,n})^2$$

and thus

$$\frac{\partial E_{s2,n}}{\partial u_{s2,l}} = \frac{C}{2}\,\frac{\partial [u_{s2,n} - u_{s1,n+1}]^2}{\partial u_{s2,l}} ,$$

$$\frac{\partial H}{\partial P_{s2,l}} = \frac{P_{s2,l}}{m2} ,$$

$$\frac{\partial H}{\partial u_{s2,l}} = C\,(u_{s2,l} - u_{s1,l}) - C\,(u_{s1,l+1} - u_{s2,l}) ,$$

$$\frac{\partial E_{s2,n}}{\partial P_{s2,i}} = \frac{1}{2m_2}\frac{\partial {P_{s2,n}}^2}{\partial P_{s2,i}} \quad ,$$

$$\frac{\partial E_{s2,n}}{\partial u_{s1,i}} = \frac{C}{2}\frac{\partial [u_{s2,n} - u_{s1,n+1}]^2}{\partial u_{s1,i}} \text{, and}$$

$$\frac{\partial H}{\partial P_{s1,i}} = \frac{P_{s1,i}}{m_1}.$$

Inserting these equations into Eq.(5.4.2.2) yields

$$\dot{E}_{s2,n} = \frac{C[u_{s2,n} - u_{s1,n+1}]P_{s2,i}\delta_{in} - C[u_{s2,i} - u_{s1,i}]P_{s2,n}\delta_{in} + C[u_{s1,i+1} - u_{s2,i}]P_{s2,n}\delta_{in}}{m_2} - \frac{C[u_{s2,n} - u_{s1,n+1}]P_{s1,i}\delta_{i(n+1)}}{m_1} ,$$

$$= C\dot{u}_{s2,n}[u_{s1,n} - u_{s2,n}] - C\dot{u}_{s1,n+1}[u_{s2,n} - u_{s1,n+1}] \qquad (5.4.2.3)$$

Energy flux ($\dot{E}_f$)

Similarly, substitute $u_{s1,n}$ = Aexp i(ωt+n$\Delta\Phi'$) and $u_{s2,n}$ = Bexp i(ωt+n$\Delta\Phi'$) into Eq. (5.4.2.3) and consider only real part; we have thus obtained

$$\dot{E}_{f(s2,n)}(t) = C[Re(\dot{u}_{s2,n})Re(u_{s2,n}[\frac{u_{s1,n}}{u_{s2,n}} - 1]) - Im(\dot{u}_{s2,n})Im(u_{s2,n}[\frac{u_{s1,n}}{u_{s2,n}} - 1]) + Re(\dot{u}_{s1,n+1})Re(u_{s1,n+1}[1 - \frac{u_{s2,n}}{u_{s1,n+1}}]) - Im(\dot{u}_{s1,n+1})Im(u_{s1,n+1}[1 - \frac{u_{s2,n}}{u_{s1,n+1}}])]$$

Inserting the recursion relations Eq.(5.1.3) and Eq.(5.1.6) into the last equation yield,

$$\dot{E}_{f(s2,n)}(t) = \frac{\omega^2}{|\chi_2|^2}[Re(\dot{u}_{s2,n})Re(u_{s1,n}\dot{\chi}_2) - Im(\dot{u}_{s2,n})Im(u_{s2,n}\dot{\chi}_2)] + \frac{\omega^2}{|\chi_1|^2}[Re(\dot{u}_{s1,n+1})Re(u_{s1,n+1}\dot{\chi}_1) - Im(\dot{u}_{s1,n+1})Im(u_{s1,n+1}\dot{\chi}_1)] \quad , (5.4.2.4)$$

$$Re(\dot{u}_{s1,n+1}) = -A\,\omega sin[\omega t + n\Delta\phi]\exp(-[n+1]\gamma) \quad ,$$

$$Im(\dot{u}_{s1,n+1}) = A\,\omega cos[\omega t + n\Delta\phi]\exp(-[n+1]\gamma) \quad ,$$

$$Re(u_{s1,n+1}\dot{\chi}_1) = Aexp(-[n+1]\gamma)[cos(\omega t + n\Delta\phi)\mathrm{Re}(\chi_1) + sin(\omega t + n\Delta\phi)\mathrm{Im}(\chi_1)] \quad ,$$

$$Im(u_{s1,n+1}\dot{\chi}_1) = Aexp(-[n+1]\gamma)[-cos(\omega t + n\Delta\phi)\mathrm{Im}(\chi_1) + sin(\omega t + n\Delta\phi)\mathrm{Re}(\chi_1)] \quad ,$$

$$Re(\dot{u}_{s2,n}) = -B\,\omega sin[\omega t + n\Delta\phi]\,exp(-n\gamma) \quad ,$$

$$Im(\dot{u}_{s2,n}) = B\,\omega cos[\omega t + n\Delta\phi]\,exp(-n\gamma) \quad ,$$

$$Re(u_{s2,n}\dot{\chi}_2) = Bexp(-n\gamma)[cos(\omega t + n\Delta\phi)Re(\chi_2) + sin(\omega t + n\Delta\phi)Im(\chi_2)] \quad ,$$

$$Im(u_{s2,n}\dot{\chi}_2) = Bexp(-n\gamma)[-cos(\omega t + n\Delta\phi)Im(\chi_2) + sin(\omega t + n\Delta\phi)Re(\chi_2)] \quad ,$$

where $\Delta\Phi' = \Delta\phi + i\gamma$, $\Delta\phi$ is phase difference and γ is attenuation factor and let

$\omega t + n\Delta\phi = \beta$, therefore,

$$Re(\dot{u}_{s1,n+1})Re(u_{s1,n+1}\dot{\chi}_1) = \frac{-A^2\omega\,exp(-2[n+1]\gamma)}{2}[\sin 2\beta\,\mathrm{Re}(\chi_1) + \mathrm{Im}(\chi_1) - \cos 2\beta\,\mathrm{Im}(\chi_1)]$$

$$Im(\dot{u}_{s1,n+1})Im(u_{s1,n+1}\dot{\chi}_1) = \frac{A^2\omega\, exp(-2[n+1]\gamma)}{2}[\sin 2\beta\, \mathrm{Re}(\chi_1) - \mathrm{Im}(\chi_1) - \cos 2\beta\, \mathrm{Im}(\chi_1)]$$

$$Re(\dot{u}_{s2,n})Re(u_{s2,n}\dot{\chi}_2) = \frac{-B^2\omega\, exp(-2n\gamma)}{2}[\sin 2\beta\, \mathrm{Re}(\chi_2) + \mathrm{Im}(\chi_2) - \cos 2\beta\, \mathrm{Im}(\chi_2)],$$

$$Im(\dot{u}_{s2,n})Im(u_{s2,n}\dot{\chi}_2) = \frac{B^2\omega\, exp(-2n\gamma)}{2}[\sin 2\beta\, \mathrm{Re}(\chi_2) - \mathrm{Im}(\chi_2) - \cos 2\beta\, \mathrm{Im}(\chi_2)]$$

Inserting these equations into Eq.(5.4.2.4), yields

$$\begin{aligned}\dot{E}_{f(s2,n)}(t) &= -\frac{A^2\omega^3\, exp(-2[n+1]\gamma)}{2}[\sin 2\beta\, \mathrm{Re}(\chi_1) - \cos 2\beta\, \mathrm{Im}(\chi_1)] \\ &+ \frac{B^2\omega^3\, exp(-2n\gamma)}{2}[\sin 2\beta\, \mathrm{Re}(\chi_2) - \cos 2\beta\, \mathrm{Im}(\chi_2)]\end{aligned} \quad .(5.4.2.5)$$

The terms in Eq.(5.4.2.5) are expressed as the energy flow in the chain which is oscillating in time. They are proportional to the real and the imaginary parts of χ_1 and χ_2.1