การวิเคราะห์ความอ่อนไหวของแบบจำลองการตั้งราคา CDO

นางสาว รมณี บุญดีเจริญ

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ลิขสิทธิ์ของจุฬาลงกรณ์มหาวิทยาลัย

Sensitivity Analysis of CDO Pricing Models

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เราเปรียบเทียบตัวแบบที่เป็นที่นิยมใช้ในการตั้งราคา CDO และความอ่อนไหวต่อค่าความน่าจะเป็นในการผิดสัญญาและค่าพารามิเตอร์สหสัมพันธ์ของตัวแบบเหล่านั้น ในที่นี่การขึ้นแก่กันระหว่าง ระยะเวลาผิดสัญญาถูกจำลองผ่านตัวแบบ Gaussian, stochastic correlation, Student t, and Clayton copulas เราเปรียบเทียบความอ่อนไหวของตัวแบบเหล่านี้โดยพิจารณาค่าส่วนต่างในด้านเครดิตของผู้ให้กู้ยืมในกลุ่มอ้างอิง การวิเคราะห์ความอ่อนไหวนี้ให้ข้อสรุปสำคัญในการประกันความเสี่ยง เราจะศึกษาว่าตัวแบบที่เหมาะสมกับราคาตลาดได้ดีนั้นมีข้อสรุปเกี่ยวกับความอ่อนไหวของความน่าจะเป็นที่จะผิดสัญญาเหมือนกันหรือไม่ นอกจากนั้นเราศึกษาความอ่อนไหวที่มีต่อพารามิเตอร์สหสัมพันธ์ โดยเฉพาะอย่างยิ่งเราศึกษาว่าตัวแบบที่เหมาะสมกับตลาดนั้นมีรูปแบบความอ่อนไหวของสหสัมพันธ์เหมือนกันหรือไม่ จากนั้นเราสรุปลักษณะจำเพาะของตัวแบบต่างๆ ที่สามารถเลียนกับราคาตลาดได้ดี

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We compare some popular CDO pricing models and their sensitivity to default probabilities and correlation parameters. Here, the dependence between default times is modeled through Gaussian, stochastic correlation, Student t, Double t, and Clayton copulas. First, we compare these models' sensitivity with respect to the credit spreads of obligors in the underlying portfolio. This sensitivity analysis has an important implication on hedging. We will investigate whether the models that fit market quotes well agree on the sensitivity to default probability. Moreover, we investigate the sensitivity of tranche premiums with respect to correlation parameters. More specifically, we investigate whether the models that fit market well share a correlation sensitivity pattern. We then conclude about the characteristics of models that are able to fit market quotes.

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ศูนย์วิทยทรัพยากร
จุฬาลงกรณ์มหาวิทยาลัย

Contents

List of Table Contents

ศูนย์วิทยทรัพยากร
จุฬาลงกรณ์มหาวิทยาลัย

List of Figure Content

ศูนย์วิทยทรัพยากร
จุฬาลงกรณ์มหาวิทยาลัย

Graph Content

ศูนย์วิทยทรัพยากร
จุฬาลงกรณ์มหาวิทยาลัย

CHAPTER I

INTRODUCTION

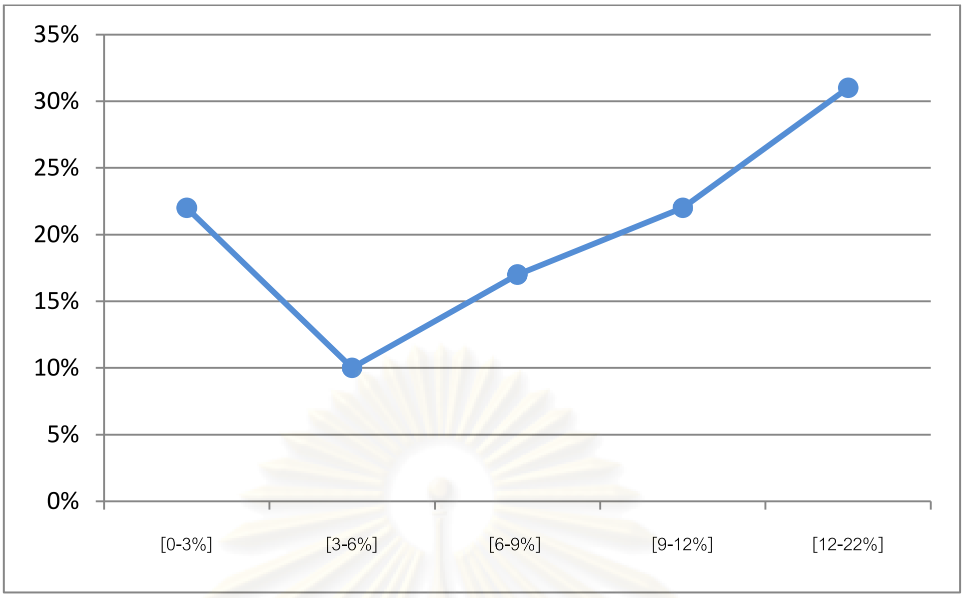
In this work, we consider collateralized debt obligation (CDO) and present a comparison of some popular CDO pricing models.

1. The Gaussian copula, the model, widely used by the financial industry, was introduced to the credit field by Li (2000).
2. A stochastic correlation introduced by Andersen and Sidenius (2005)
3. The Student t copula which has been considered by a number of authors.(for example Andersen et al (2003)
4. A Double t one factor model
5. The Clayton copula model

We focus on "copula models" since they are the most widely used model in the credit derivatives markets, though the factor approach also applies to various intensity models (see Mortensen (2006)). The pricing of synthetic CDOs involves the computation of aggregate loss distributions over different time horizons. In our "bottom-up" approach, CDO tranche premiums depend upon the individual credit risk of names in the underlying portfolio and the dependence structure between default times.

Burtschell, Gregory, and Laurent (2008), compared some popular CDO pricing models. Dependence between default times is modeled through Gaussian, stochastic correlation, Student t, Double t, Clayton and Marshall-Olkin copulas. They detailed the model properties and compared the semi-analytic pricing approach with large portfolio approximation techniques. They mentioned that base correlation is monotonic with respect to the model's dependence parameter (Burtschell, Gregory, and Laurent (2008), p. 2). However, they did not compare Mezzanine, Senior, and relationship among tranches. They concluded that Clayton tends to give similar tranche premium as Gaussian and did not produce implied correlation smile. Student t also did not produce correlation smile. On the other hand, Double t and stochastic correlation appeared to fit the skew better. However, they did not link this to sensitivity analysis.

Graph 1: Correlation Smile in Market Quote



In practice, CDO prices are quoted using the so-called "implied correlation," which is the flat correlation parameter in the Gaussian copula that makes the tranche spread match the market quote. For the market quote, the implied correlation is lower in the mezzanine tranche which results in the "correlation smile". Gaussian model does not create correlation smile and fails to price of the iTraxx tranches. According to Burtschell, Gregory, and Laurent (2008), they observed that for both Clayton and Student t copula model don't produce any correlation smile as well. Double t and stochastic correlation model can fit the market quote reasonably well.

Andersen and Sidenius (2005), Gregory and Laurent (2004) and Hull and White (2004) also studied about sensitivity. Nonetheless, they did not compare between each model, and they did not offer a link between sensitivity and the ability to fit market quotes.

In this work, we investigate these models' sensitivity to both

→ Default probability, which is the probability that the obligor cannot return debt and it is related to credit ratings.

→ Correlation, or the dependence, among the default of obligor.

1.1 Literature review

Burtschell, Gregory, and Laurent (2008) compare several copula CDO pricing models which are uses to model the dependence between default times. They explain the model properties and compare the semi-analytic pricing approach with large portfolio approximation techniques. In addition, they also discuss the ability of each model to fit the market quote.

The Gaussian copula which applied to CDOs was first introduced by Li (2000) became a tool for financial institutions be used to specify the joint distribution of survival times after marginal distributions of survival times. The copula functions and theirs basic properties are introduced to provide a way of joint distribution which can be called marginal. Also, the popular credit model is used in the calibration of the correlation parameter.

Gregory and Laurent (2004) focus on the correction of correlation for structure within a Gaussian copula framework. They come up with a more flexible two-factor model to incorporate a realistic correlation structure. In addition, they integrate the dependence between recovery rates and default times and found that it can match the correlation smile better.

Andersen and Sidenius (2005) introduced two new models which are extensions of the standard Gaussian copula model. The first extension is the randomized recovery which produces a heavy upper tail. The second extension is the randomized factor loadings which can produce the correlation smile similar to those observed in the CDO market.

Andersen et al (2003) show methods to improve the way to calculate the prices and hedge parameters for credit basket derivatives. The new technique focus on single-tranche CDO sensitivity and hedge ratio calculations.

Demarta and McNeil (2005) construct two new copulas which are the skewed t copula and the grouped t copula. Two new copulas derive by the considerations of extreme value which are the t extreme value copula and the t lower tail. The objective of their studies is to combined what is known about the t copula with its extremely properties and to present some extensions of the t copula.

Hull and White (2004) develop two quick procedures for valuing tranches of CDO and n^{th} to default swaps. Copula model of times to default and Fourier transforms are the procedures which involve calculating the probability distribution of the number of defaults and using a “probability bucketing” numerical procedure to build up the loss distribution. Many new copula models can be created by the different distribution assumption.

Schönbucher and Schubert (2001) illustrate a new method to find the default dependency in risk models which is specified by the Copula of the default times. The Gumbel and the Clayton Copula are the two models they focus on. Gregory and Laurent (2003), Rogge and Schönbucher (2003), Madan *et al*. (2004), Laurent and Gregory (2005), Schloegl and O'Kane (2005), and Friend and Rogge (2005)) are the other studies that mention Clayton copular.

Schönbucher (2002) focuses on formula for the distribution of a loan portfolio loss that easy to be compute. He shows three things which are modeling joint distributions, investigating the effect of the implicit assumption of a Gaussian dependency structure on the risk measures and the portfolio returns distribution, and providing an application for the model.

1.2 Objective

- Investigate tranche sensitivity to the credit rating of obligors in the underlying portfolio. This issue finds practical implication when practitioners wish to hedge CDO tranche with other credit derivatives such as CDS. We will investigate whether different models give different sensitivity to probability default sensitivities.
- Investigate sensitivity of tranche premium with respect to the correlation parameter. This issue has important practical implication, since practitioners often require calibrating the correlation parameter to fit the market quotes. While it is well known that the equity tranche sensitivity is usually monotonic with correlation parameter, no one has provided summary across different models as to the correlation affects on mezzanine and senior tranche as suggested by Burtschell, Gregory, and Laurent (2008). We also want to see if all models agree on relationship between tranche premium and correlation.
- Investigate whether models that fit market well share a sensitivity pattern. We hypothesize that the ability to fit market quotes can be determined by the pattern of tranche sensitivity to correlation parameter. Indeed, we shall see that Double t and stochastic correlations which have been observed to fit market well (Burtschell, Gregory, and Laurent (2008)) share a correlation sensitivity pattern, while Clayton and Student-t do not have this pattern.

The work is organized as follows: The second chapter recalls the factor or conditional independence approach and provides the basic understanding of each model. We consider Gaussian, stochastic correlation, Student T, Double t, and Clayton Copula. In addition, we also explain about the payoff description of CDO and summarize the MATLAB code. In the third and fourth chapter, we discuss about the result of sensitivity with respect to correlation and the sensitivity of CDO tranches with respect to default probability. The last chapter is the conclusion.

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CHAPTER II

METHODOLOGY

In this chapter, we introduce how the factor or conditional independence approach can be associated with computations and CDO tranches (see Laurent and Gregory (2005)). In this work, we consider n number of underlying obligors and $(\tau_1, \dots, \tau_n)$ characterized as the random vectors of default dates.

$$F(t_1, \dots, t_n) = Q(\tau_1 \le t_1, \dots, \tau_n \le t_n) \qquad 1.1$$

where Q stands for some pricing probability measure. F represents the joint distribution. $F_1, \dots, F_n$ indicate the marginal distribution functions.

The loss given default for name *i* denotes by:

$$M_i = E_i(1 - \delta_i) \qquad 1.2$$

where E_i for $i = 1, \dots, n$ is the tranche notional associated with n credits. Here, δ_i represents the corresponding recovery rates. Later, we will assume that recovery rates are deterministic and focus on the dependence of default times.

2.1. One factor Gaussian copula

Gaussian copula was introduced by Li (2000) and its default dates are given by:

$$\tau_i = F_i^{-1}\big(\Phi(V_i)\big) \text{ for } i = 1, \dots, n \qquad 1.1.1$$

where Φ is the cumulative distribution function of standard Gaussian. F_i^{-1} denotes the inverse of F_i which is the probability that obligors will default before time *i*.

Consider on one factor case of the Gaussian vector,

$$V_i = \rho V + \sqrt{1 - \rho^2}\bar{V}_i \qquad 1.1.2$$

where V, $\bar{V}_i$ are independent standard Gaussian random variables and $0 \le \rho \le 1$.

$\rho = 0$ corresponds to independent default times while $\rho = 1$ is correlated with the perfectly positively dependent case.

2.2. Stochastic correlation

The stochastic correlation (as discussed by Andersen and Sidenius (2005)) has been developed as an attempt to match "correlation smiles" in the CDO market. This model is the mixture of Gaussian with both low and high correlation parameter. The latent variables are given by:

$$V_i = B_i\left(\rho V + \sqrt{1-\rho^2}\bar{V}_i\right) + (1-B_i)\left(\beta V + \sqrt{1-\beta^2}\bar{V}_i\right) \text{ for } i = 1, \dots, n, \qquad 1.2.1$$

where B_i are Bernoulli random variables which either takes value 1 with success probability and value 0 with failure probability, V and V_i are standard Gaussian random variables, all these being jointly independent and ρ is some high correlation parameter , β is some low correlation parameters, $0 \leq \beta \leq \rho \leq 1$.

We define the default time as:

$$\tau_i = F_i^{-1}\left(\Phi(V_i)\right) \text{ for } i = 1, \dots, n. \qquad 1.2.2$$

The default times are independent conditionally on V

2.3. Student t copula

The Student t copula (see example Andersen et al (2003), Demarta and McNeil (2005)) is a modification of Gaussian copula. Student t copula exhibits higher tail dependence which Gaussian copula cannot capture. In the Student t approach, the underlying vector $(V_1, \dots, V_n)$ follows a Student t distribution with ν degrees of freedom. In the symmetric case which we are going to consider, we have

$$V_i = \sqrt{W}\left(\rho V + \sqrt{1-\rho^2}\bar{V}_i\right) \qquad 1.3.1$$

$V, \bar{V}_i$ are independent Gaussian random variables, W is an inverse Gamma distribution with parameters equal to $V/_2$ (or equivalently $v/_W$ follows a χ_v^2 distribution). Let t_ν be the distribution function of the standard univariate Student t. Default times of Student t is defined as:

$$\tau_i = F_i^{-1}\left(t_\nu(V_i)\right) \qquad 1.3.2$$

It can be seen that conditional on (V, W) default times are independent.

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2.4. Double t copula

The Double t copula, introduced by Hull and White (2004), is expanded from Gaussian copula to create heavier tails than the normal distribution. The latent variables which is used to model default time are

$$V_i = \rho\left(\frac{v-2}{v}\right)^{1/2} V + \sqrt{1-\rho^2}\left(\frac{v-2}{v}\right)^{1/2} \bar{V}_i \qquad 1.4.1$$

where V_i is an independent random variable following Student t distributions with v degrees of freedom and $\bar{V}_i$ is also an independent random variable following Double t distributions with $\bar{v}$ degrees of freedom and ρ is equal to or more than zero .

The default dates are then given by:

$$\tau_i = F_i^{-1}(H_i(V_i)) \text{ for } i = 1, \dots, n \qquad 1.4.2$$

where H_i is the distribution function of V_i.

2.5. Clayton copula

In the following, we present Clayton copula model which had been studied by many authors. See the literature review for the authors information.

We consider on Clayton copula because it has lower tail dependence. Let the positive random variable V , which is called a frailty, follow a standard Gamma distribution. Its shape parameter is $1/\theta$ and $\theta > 0$.

Its probability density can be expressed as

$$f(x) = \frac{1}{\Gamma(\frac{1}{\theta})} e^{-x} x^{(1-\theta)/\theta} \text{ for } x > 0. \qquad 1.5.1$$

The Laplace transform Ψ of probability density function is

$$\Psi(s) = \int_0^{\infty} f(x) e^{-sx} dx = (1+s)^{-1/\theta}. \qquad 1.5.2$$

Let define some latent variables V_i with:

$$V_i = \Psi\left(\frac{In\, U_i}{V}\right) \qquad 1.5.3$$

The independent uniform random variable U_i is independent from V. The default times are

$$\tau_i = F_i^{-1}(V_i), i = 1, \dots, n \qquad 1.5.4$$

ศูนย์วิทยทรัพยากร
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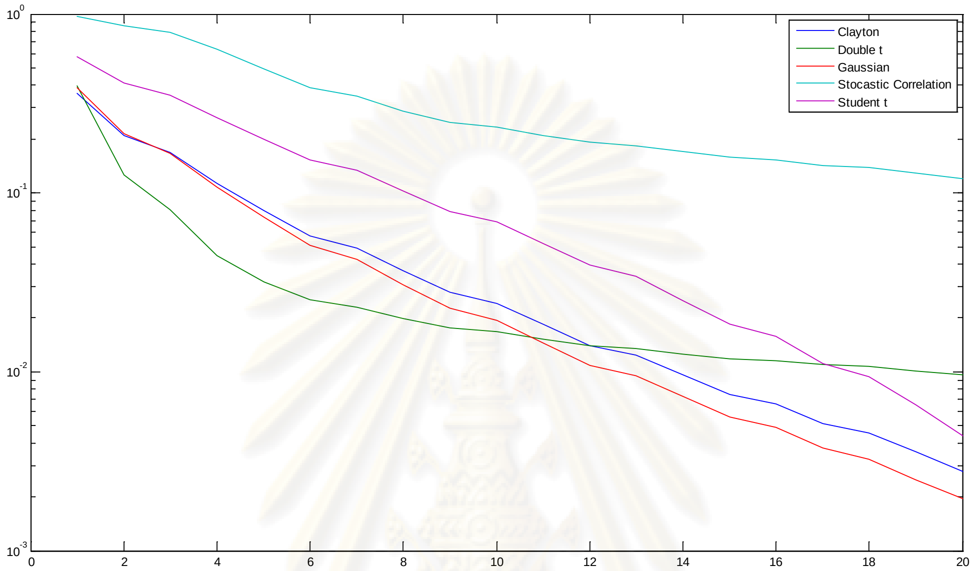
having V as the factor.

2.6. Distribution Plot

In this section, we present the loss distribution plot of each copula. Note the different tail behavior of each model.

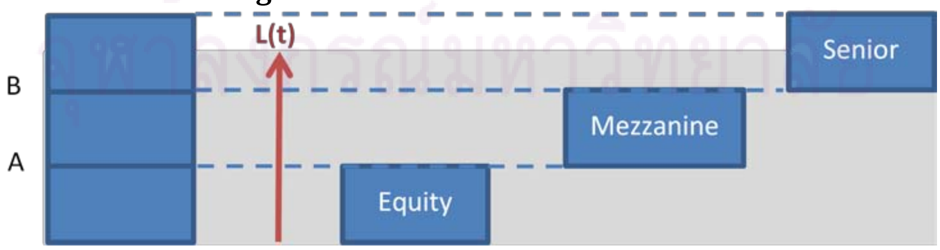
Figure 1 Plot Distribution



2.7. Payoff Description

In a CDO, default losses on the credit portfolio are divided along some thresholds or the attachment points and allocated to the different tranches. For example, consider a three tranches CDO, denoted as equity, mezzanine, and senior, as shown in the Figure 1. Each tranche is divided by the attachment points indicated by A and B.

Figure 2 Three Tranches of CDO



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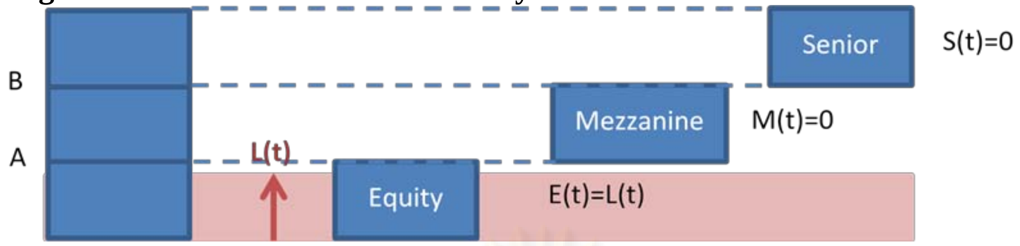
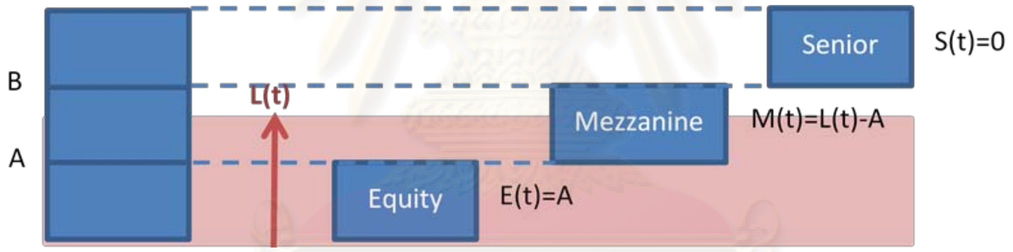
The cumulative loss by the time t, which denoted by L(t), can be separated into three scenarios. The first case is the case where cumulative default payment is less than attachment point A. In this scenario, equity tranche default payment, which can be referred to as E(t), will be equal to L(t) and for both the Mezzanine tranche default payment, represented by M(t) and the Senior tranche default payment, denoted as S(t), are equal to zero.

Figure 3 The Cumulative Default Payment is Less Than Attachment Point A



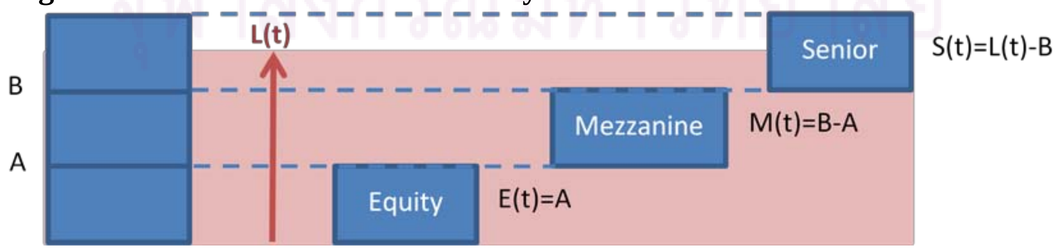
In the second scenario, the cumulative default payment is higher than attachment point A but less than attachment point B. The cumulative default payment for each tranche is shown in Figure 4. For the equity tranche, the default payment is denoted by E(t)=A. For the mezzanine tranche, default payments is denoted by M(t)=L(t)-A. In the senior tranche, the default payment is equal to zero.

Figure 4 The Cumulative Default Payment More Than A But Less Than Attachment Point B



In the last case, the cumulative default payment expands more than attachment point B as shown in Figure 5. In this scenario, tranche default payment is equal to A for the equity and equal to B-A for the mezzanine. For the senior tranche, default payment is equal to L(t)-B.

Figure 5 The Cumulative Default Payment More Than Attachment Point B



As mentioned that cumulative default payment can occur in three scenarios, we can summarize the cumulative default payment of each tranche by the equations below.

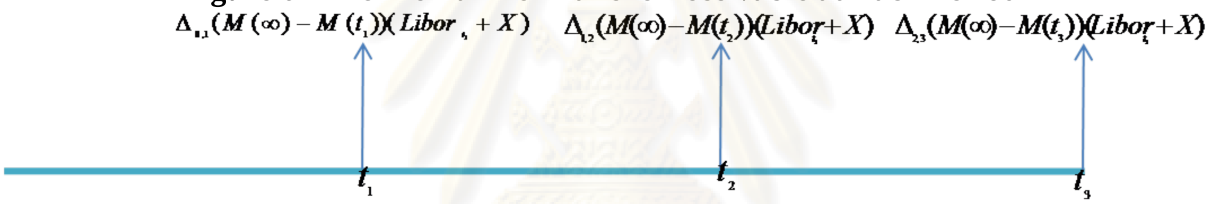
$$E(t) = L(t)1_{L(t)<A} + A\ 1_{L(t)>A} \quad 2.1$$

$$M(t) = (L(t) - A)1_{A<L(t)<B} + (B - A)\ 1_{L(t)>B} \quad 2.2$$

$$S(t) = (L(t) - B)1_{L(t)>B} \quad 2.3$$

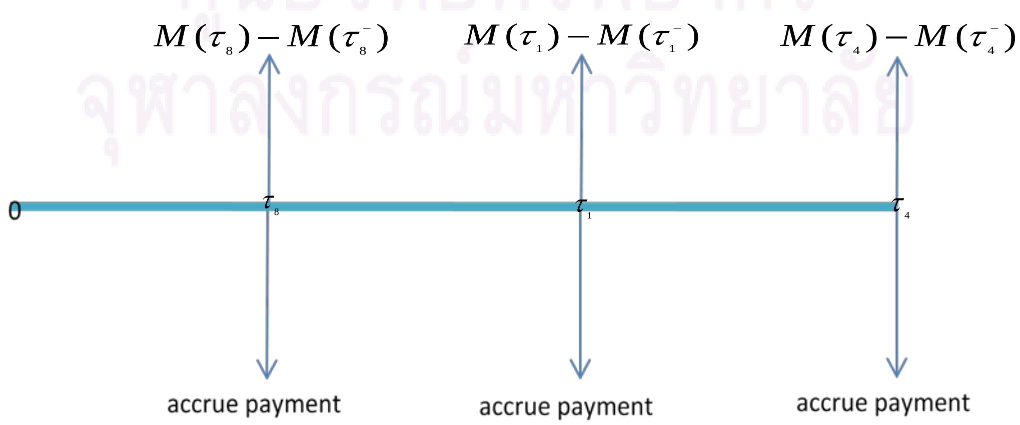
For more details on how to compute tranche premium, refer to Laurent and Gregory (2003). Here, we summarize the tranche premium of mezzanine tranches. Holders of synthetic CDO for the mezzanine tranche receive at time T a principal payment of $M(\infty) - M(T)$. $M(\infty)$ denotes the initial nominal of mezzanine tranche which is equal to $B - A$ and $M(\infty) - M(T)$ denotes the remaining nominal of the tranche. Figure 6 shows the cash flow that the mezzanine tranche holders receive at the end of each period.

Figure 6 The Mezzanine Tranche Receivable at Each Period



The payments are usually equal to a floating rate plus a fixed margin, which is specific to each tranche, and also based on the outstanding nominal on the tranche. For Figure 7, it shows the cash flow out of the mezzanine tranche at time of defaults.

Figure 7 The Mezzanine Tranche Cash flow In and Cash flow Out



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The interest payment at t_i is equivalent to $\Delta_{i-1,t_i}(M(\infty) - \mathrm{M}\ (t_i)\,)\,(Libor_{t_{i-1}} + X)$ where X determines the CDO margin, $\Delta_{i-1,i}$ represents the duration of period, and $Libor_{t_{i-1}}$ is the Libor rate for the period. There are some accrued interest payments since the interest is calculated at the end of the period. In practice, X is determined so that the present values of all cash flows are zero.

2.8. Summary of MATLAB Code

In order to conduct the CDO tranche premium study, we created MATLAB Code to simulate the outcome for each model using Monte-Carlo simulation. The MATLAB codes for each model are shown in the appendix. The MATLAB codes can be summarize as follow:

First, we start by generating the V and V_ which are the components of Vi, where Vi is the latent variable follow in different distribution. Then, we use Vi to generate default times. After that we compute E (t), M (t), and S (t) which are the accumulated loss absorbed by the equity, mezzanine, and senior tranches respectively. In the next step, we compute the regular coupon payment at each coupon date based on remaining notional payment at that date. At each default time, we will compute two things. The first thing that we compute is default payment based on the increase in E (t), M (t), and S (t). The other one is the accrued payments based on the default time from the nearest coupon date. Finally, we discount all the cash flow to time zero and find the premium that make present value of these cash flow equal zero.

ศูนย์วิทยทรัพยากร
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CHAPTER III
SENSITIVITY WITH RESPECT TO CORRELATION

We studied five models' sensitivities to the correlation parameter, which is the dependency among the defaults of obligor.

3.1. Gaussian

We use the numerical example from Burtschell, Gregory, and Laurent (2008) for comparability, we considered 100 names, all with a recovery rate of δ = 40% and equal unit nominal. The credit spreads are all equal to 100 bps. They are assumed to be constant until the maturity of the CDO. The attachment points of the tranches are A = 3% and B = 10%. The CDO maturity is equal to five years. The default free rates are provided in the appendix. We considered CDO margins of the equity, mezzanine and senior tranches using the different models. We begin first by the Gaussian model and compute the margins with respect to the correlation parameter ρ^2 .

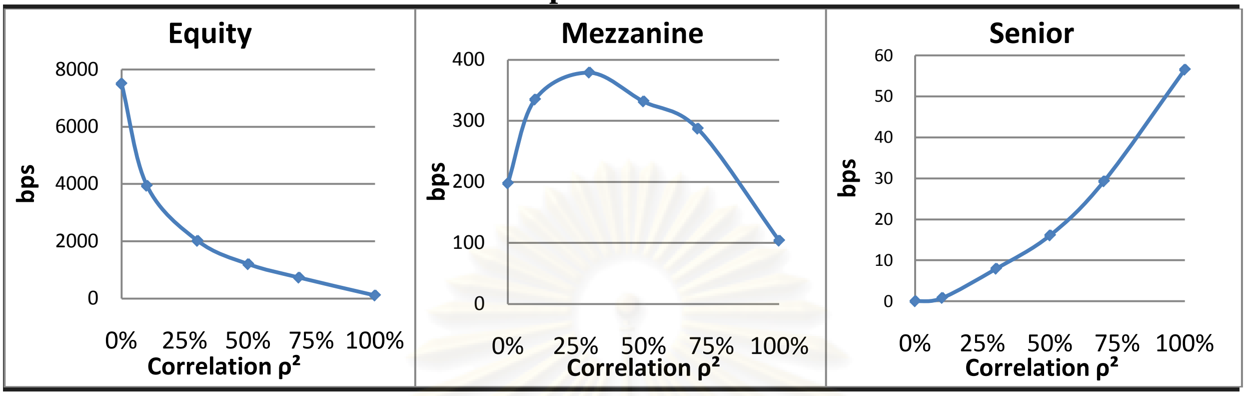
The result of the CDO margins from the Gaussian copula with respect to the correlation parameter is shown in the table 1. We will compute the following models' table by calibrating those correlation parameters and fit each model's equity tranche margin with Gaussian copula.

Table 1 CDO margins (bp pa) Gaussian copula with respect to the correlation parameter.

ρ 2	Equity	Mezzanine	Senior
0%	7505.26	197.55	0
10%	3940.96	334.75	0.82
30%	2015.83	378.92	7.94
50%	1200.64	331.67	16.08
70%	731.26	287.07	29.28
100%	106.86	103.72	56.54

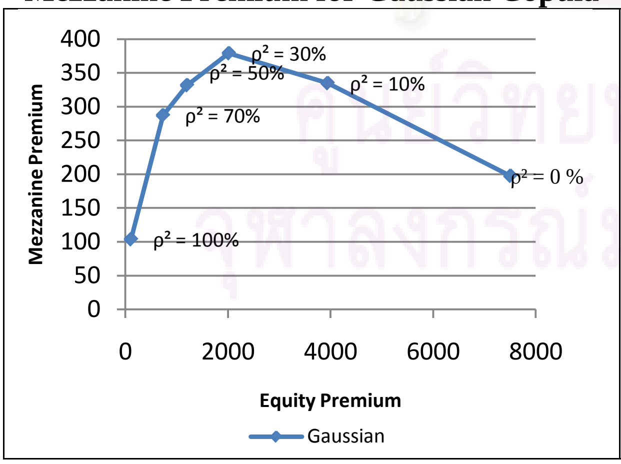
The graph beneath plots the CDO margins from the Gaussian copula with respect to the correlation parameter for each tranche. For the equity tranche of Gaussian copula, it shows a strong negative dependence with respect to the correlation parameter since the non-diversification defines that the unexpected loss on the tranche is smaller. When we consider the senior tranche, it shows a positive dependence come from the fact that the unexpected loss on the tranche is larger. For the mezzanine, the result produces a bumped curve.

Graph 2: CDO margins (bp pa) Gaussian copula with respect to the correlation parameter

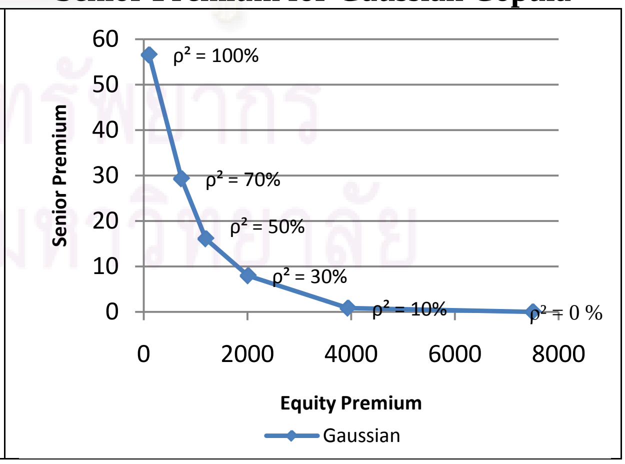


As the basis for comparison with other models, we construct two charts, one depicting relationship between the mezzanine and the equity premiums, the other between the senior and the equity, as a benchmark to compare with those of the other models. We believe that the shapes of these plots are indicative of the ability to fit market model. We will compare with other models in the next sections.

Graph 3: Comparing between Equity and Mezzanine Premium for Gaussian Copula



Graph 4: Comparing between Equity and Senior Premium for Gaussian Copula



To compare between different models, we set the dependence parameters to get the same equity tranche premiums by calibrating the other models. Then, we use outcome of the

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Gaussian copula as the benchmark. In order to get the same equity tranche premiums for different models, we calibrate the other models as already mention.

3.2. Clayton

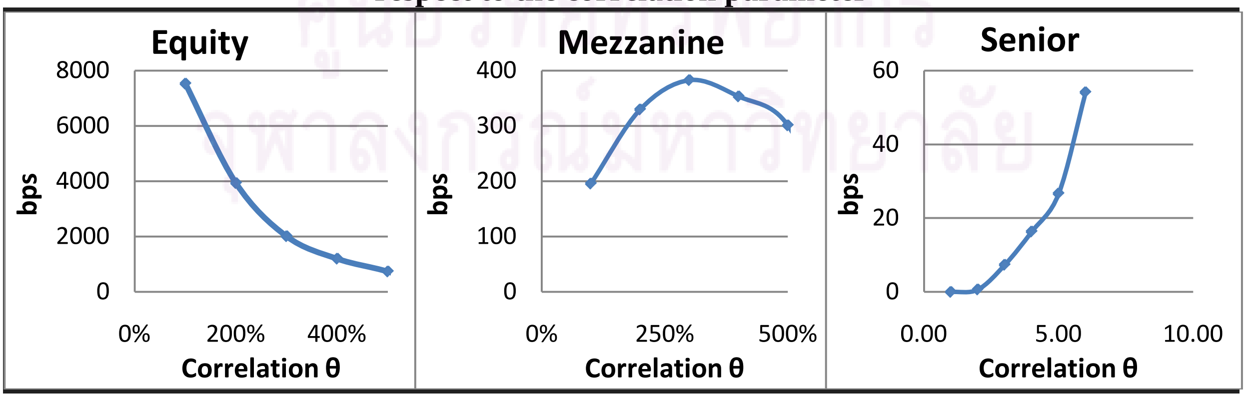
The result of the CDO margins for the Clayton copula with respect to the correlation parameter is shown in Table 2 below. As an example of the calibration notice that Clayton premium is approximately equal to Gaussian spread when we set its parameter, θ equal to 0.17 while Gaussian parameter ρ is equal to 30%. θ's in Table 2 produce equity premiums that are almost identical to those of the Gaussian.

Table 2 CDO margins (bp pa) Clayton copula with respect to the correlation parameter

θ	Equity	Mezzanine	Senior
0	7533.31	195.98	0.00
0.05	3935.93	329.83	0.67
0.1725	2011.95	382.81	7.36
0.34	1199.17	353.39	16.36
0.59	735.82	301.13	26.66
∞	107.27	102.84	54.10

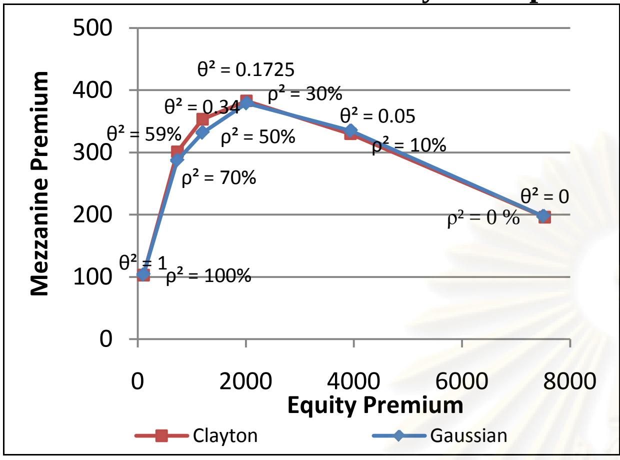
Graph 5 shows the CDO margins from Clayton copula with respect to its correlation parameter. However, it shows a similar result with Gaussian copula. For the equity tranche of Clayton copula, it also shows a strong negative dependence. For both mezzanine and senior tranches, it produces very similar outcomes to those of the Gaussian as well.

Graph 5: CDO margins (bp pa) Clayton Copula copula (6 degree of freedom) with respect to the correlation parameter

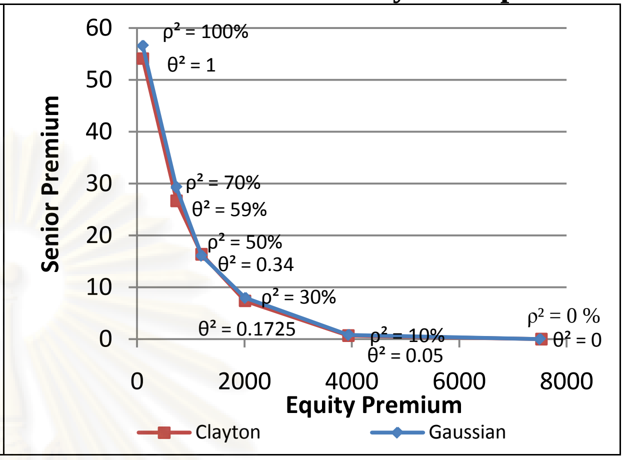


The next illustration compares the mezzanine-equity and the senior-equity premiums between that of the Gaussian model and the Clayton coupon spread. Gaussian and Clayton have almost the same sensitivity to correlation parameter. This might be the reason why Clayton cannot produce correlation smile as reported in Burtschell, Gregory, and Laurent (2008)

Graph 6: Comparing between Equity and Mezzanine Premium for Clayton Copula



Graph 7: Comparing between Equity and Senior Premium for Clayton Copula



3.3. Student T

Table 3 shows the resulting CDO margins with respect to the correlation parameter calibrated from the Student t copula.

Table 3 CDO margins (bp pa) Student t copula (4 degree of freedom) with respect to the correlation parameter

ρ 2	Equity	Mezzanine	Senior
0 %	2018.757	413.5097	6.02773
28%	1216.763	354.1069	16.44014
55%	742.0458	293.9838	27.96059
100%	96.57632	94.01169	51.3378

For Student t copula, we apply it with 4 degree of freedom. Table 3 shows the Student t copula with 6 degree of freedom with respect to the correlation parameter. In this model, both equity and senior tranches give similar results with Clayton copula.

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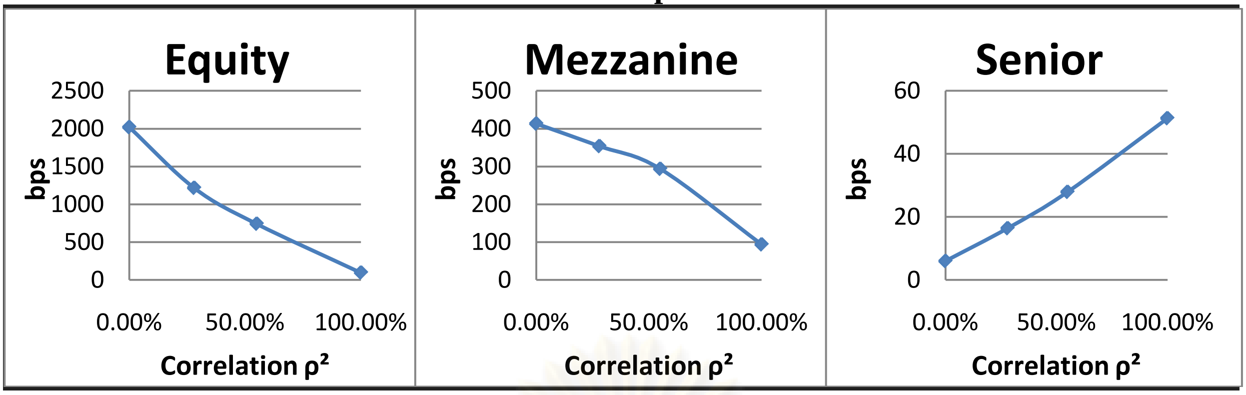
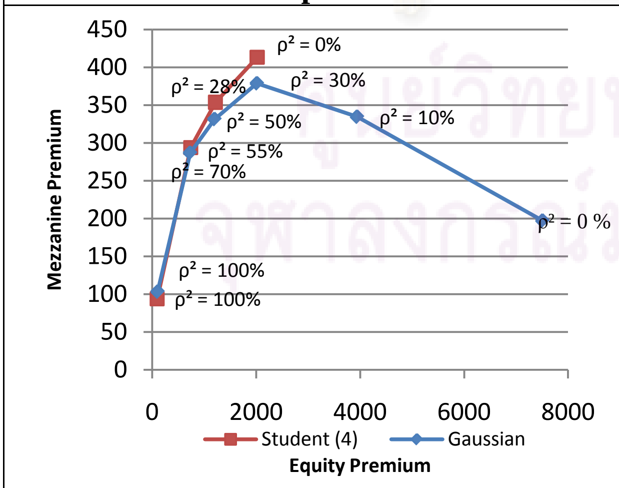
Graph 8: CDO margins (bp pa) Student t copula (4 degree of freedom) with respect to the correlation parameter

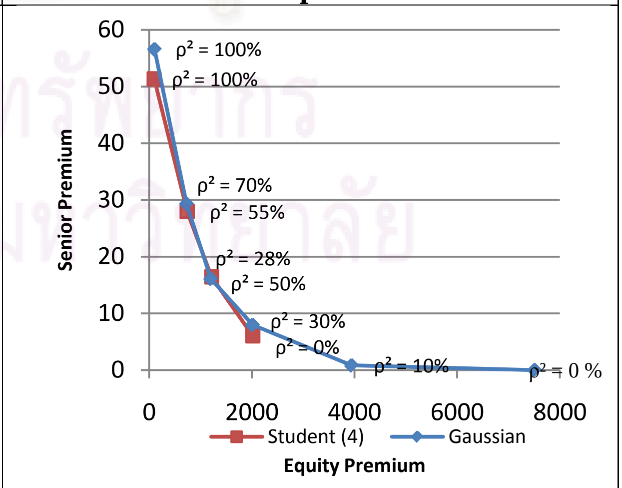


Now, we compare the mezzanine-equity and the senior-equity premiums relationships produced by the Student t with degree of freedom 4 to those by the Gaussian. Mezzanine premium from Student t is different from Gaussian. When we focus on comparing between equity and mezzanine tranches as we can see from Graph 9, we observe that as equity premium decreases as the result of increasing correlation, the mezzanine premium decreases faster than Gaussian. Then we compare equity and senior premium. We found that equity premium decreases when correlation increase and the senior premium increases just as fast as Gaussian. An interesting fact is that Student t is not only unable to create correlation smile, but also creating frown. It perhaps results from this characteristic.

Graph 9 : Comparing between Equity and Mezzanine Premium for Student t (4) Copula

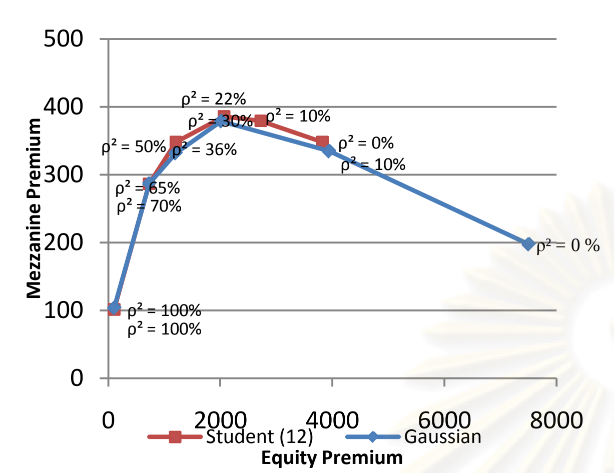


Graph 10 : Comparing between Equity and Senior Premium for Student t (4) Copula

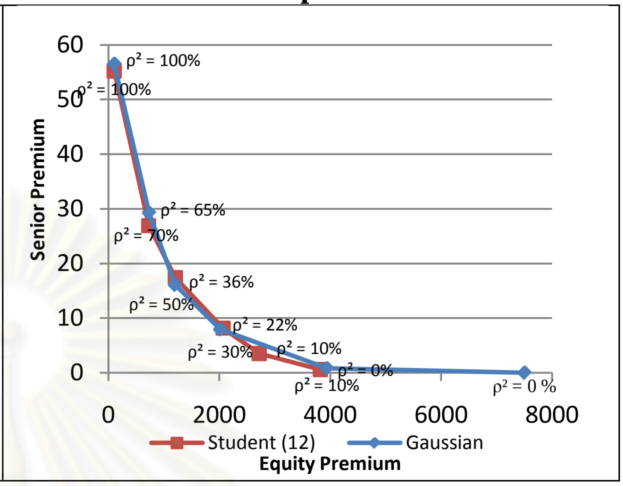


Next, we compare the mezzanine and the senior premiums relationships produced by the Student t with degree of freedom 12 to those by the Gaussian. It shows that the graphs look similar to Gaussian. We observe that when the degree of freedom increases, the graphs converge to Gaussian.

Graph 11 : Comparing between Equity and Mezzanine Premium for Student t (12) Copula



Graph 12 : Comparing between Equity and Senior Premium for Student t (12) Copula



3.4. Double t

Table 4 and Table 5 show the CDO margins for Double t copula 4-3 and Double t copula 3-3 with respect to the correlation parameter.

Table 4 CDO margins (bp pa) Double t Copula t(4)-t(3) with respect to the correlation parameter

ρ 2	Equity	Mezzanine	Senior
0%	7341.467	190.7516	0.000143
10%	3922.312	235.2195	3.839536
28%	2035.192	225.0991	13.85731
48%	1199.518	205.5088	25.16618
67%	728.687	181.7121	34.45001
100%	102.3232	99.45657	54.26102

Table 5 CDO margins (bp pa) Double t Copula t(3)-t(3) with respect to the correlation parameter

ρ 2	Equity	Mezzanine	Senior
0%	7493.363	198.5279	0
16%	3951.291	235.2241	5.878178
40%	2080.035	222.6465	16.25295
61%	1191.599	190.4813	26.00436
76%	753.8428	173.8419	33.51745
100%	109.0013	105.7383	57.61654

When we consider Double t copula with different degrees of freedom, we find that its result come out in the same way with Gaussian copula in all tranches.

ศูนย์วิทยทรัพยากร
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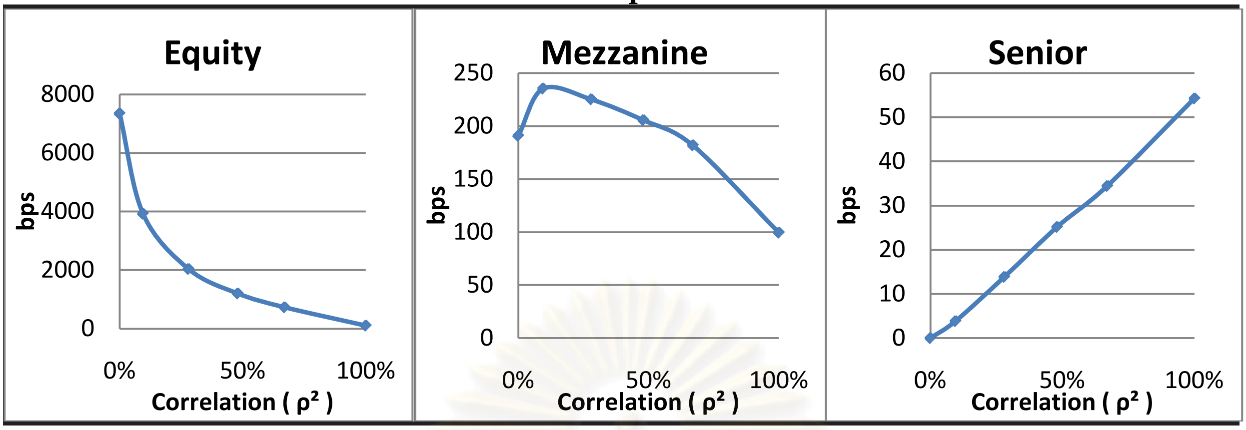
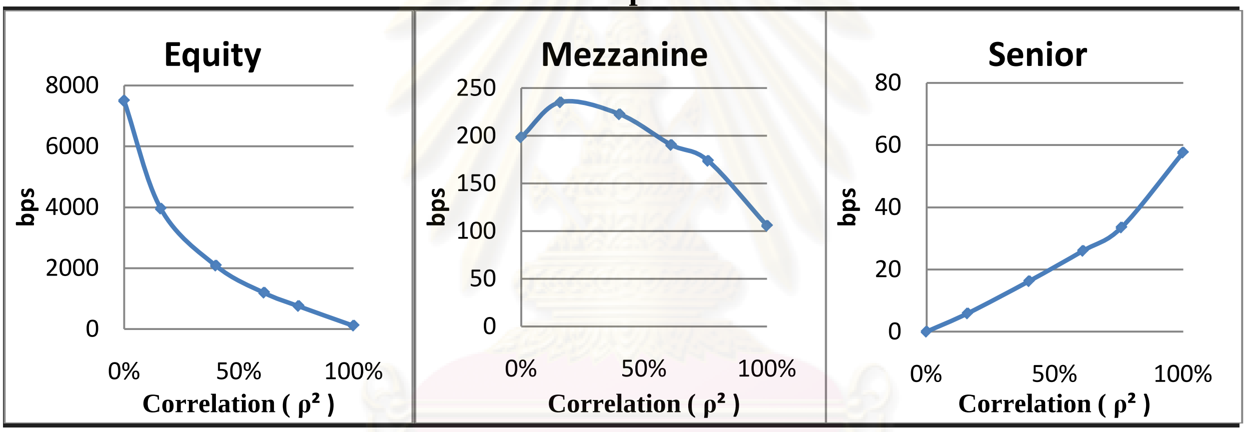
Graph 13 : CDO margins (bp pa) Double t Copula t(4)-t(3) with respect to the correlation parameter

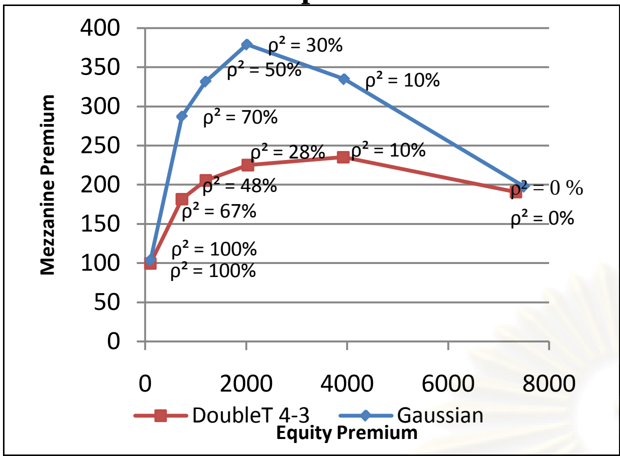


Graph 14 : CDO margins (bp pa) Double t Copula t(3)-t(3) with respect to the correlation parameter

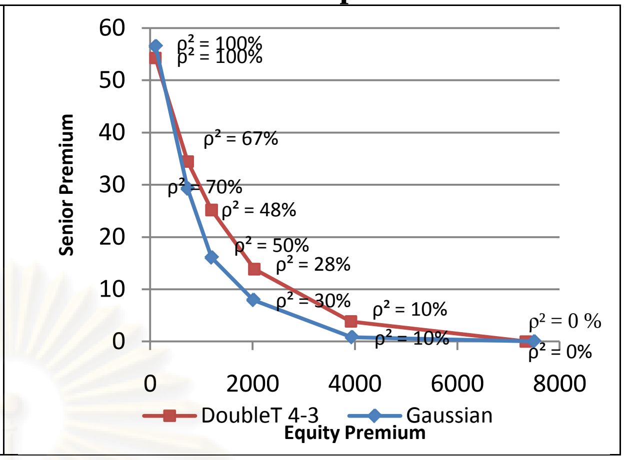


However, when we look at mezzanine and senior to equity premium plots, the shapes are very different. In mezzanine, the "bump" of Double t is much less pronounced. In senior, Double t lies above Gaussian. This may be the characteristic of smile-producing model. As equity premium decreases, Double t moves slowly than Gaussian in mezzanine premium.

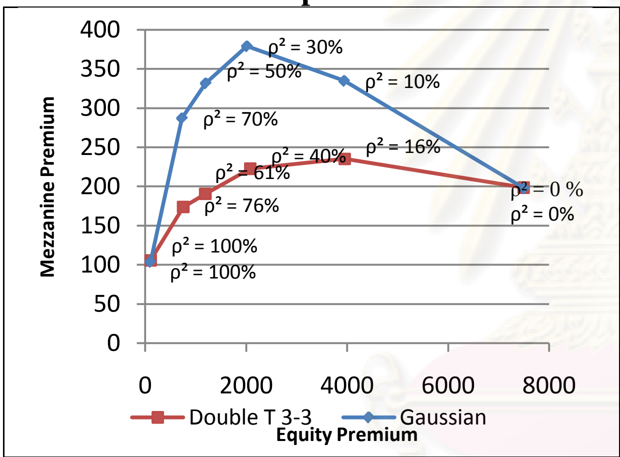
Graph 15 : Comparing between Equity and Mezzanine Premium for Double t 4-3 Copula



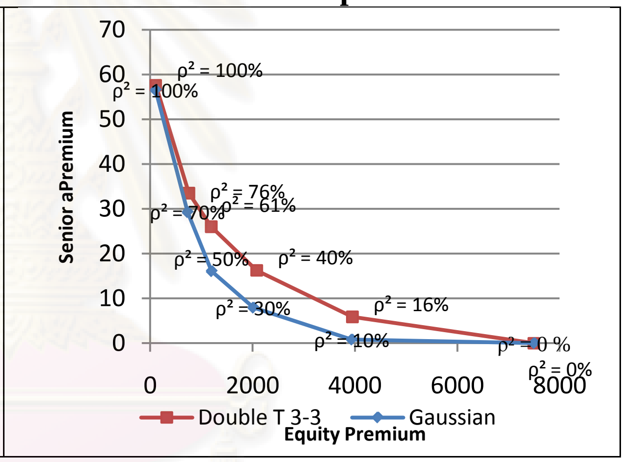
Graph 16 : Comparing between Equity and Senior Premium for Double t Double t 4-3 Copula



Graph 17 : Comparing between Equity and Mezzanine Premium for Double t 3-3 Copula



Graph 18 : Comparing between Equity and Senior Premium for Double t Double t 3-3 Copula



3.5. Stochastic correlation

Lastly, we focus on stochastic correlation. Table 3 presents the CDO margins of stochastic correlation copula with respect to the correlation parameter.

Table 6 CDO margins (bp pa) Stochastic correlation copula with respect to the correlation parameter

P	Equity	Mezzanine	Senior
0	3233.47	330.72	0.77
0.1	2881.38	322.25	3.43
0.35	2005.88	268.40	13.18
0.61	1218.33	231.97	24.93
0.8	749.62	219.36	33.51
1	339.91	202.78	42.79

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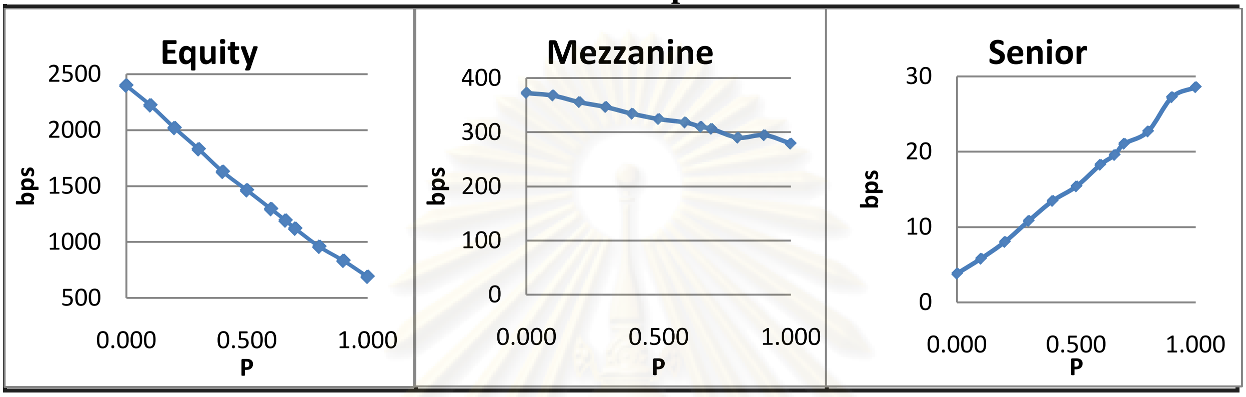
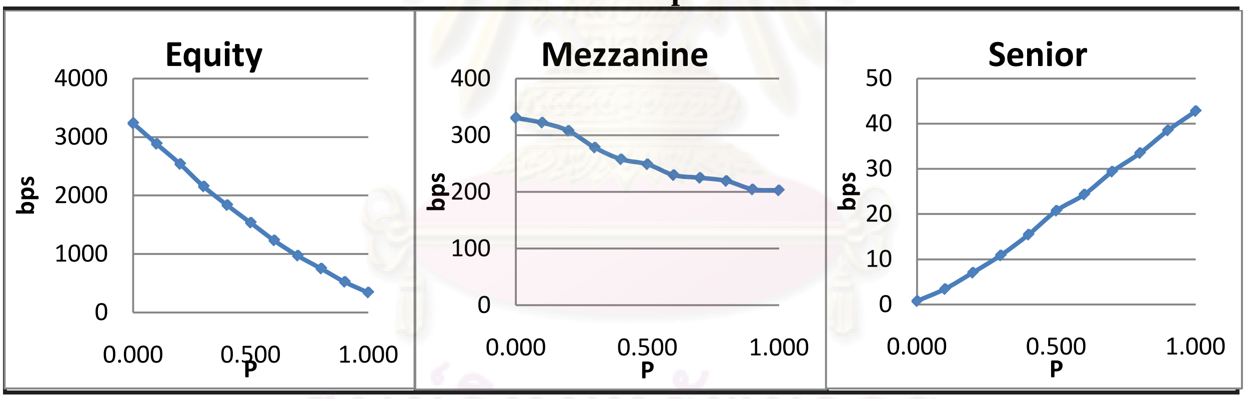
We find that stochastic correlation gives different result from other models. When we vary correlation, stochastic correlation results in linear downward sloping for both the equity and mezzanine tranches. However, the slope in mezzanine tranche is less steep compared to other models.

Graph 19: CDO margins (bp pa) Stochastic correlation Rho=0.7 Beta=0.2 with respect to the correlation parameter

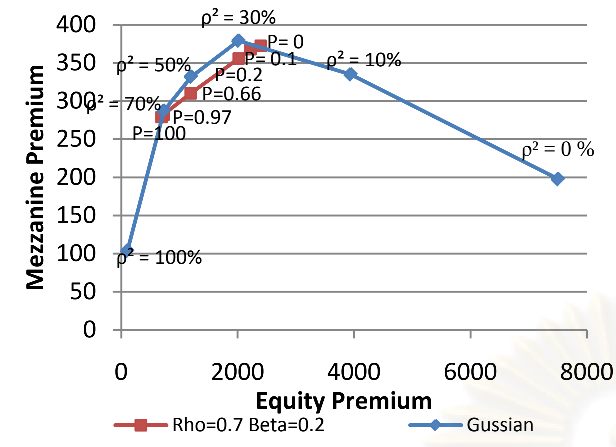


Graph 20: CDO margins (bp pa) Stochastic correlation Rho=0.9 Beta=0.1 with respect to the correlation parameter

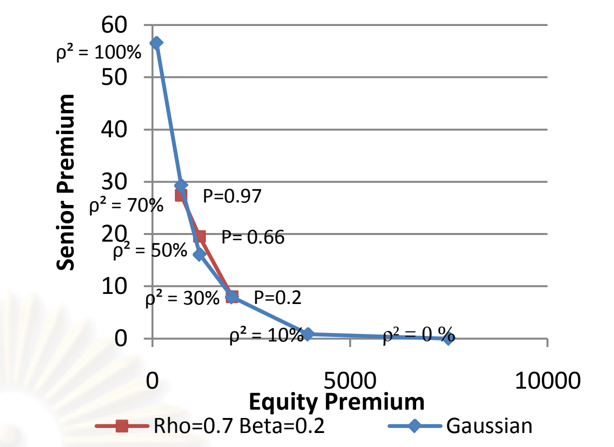


Let's look at the comparison of the mezzanine and the senior premiums. Again, note that stochastic correlation which is reported by Burtschell, Gregory, and Laurent (2008) to produce correlation smile, has the same property with Double t. For the mezzanine, the line of stochastic correlation lies below Gaussian. As equity premium decreases as the result of increasing correlation, the mezzanine premium also decreases but not as fast as Gaussian. In senior, stochastic correlation lies on top of Gaussian.

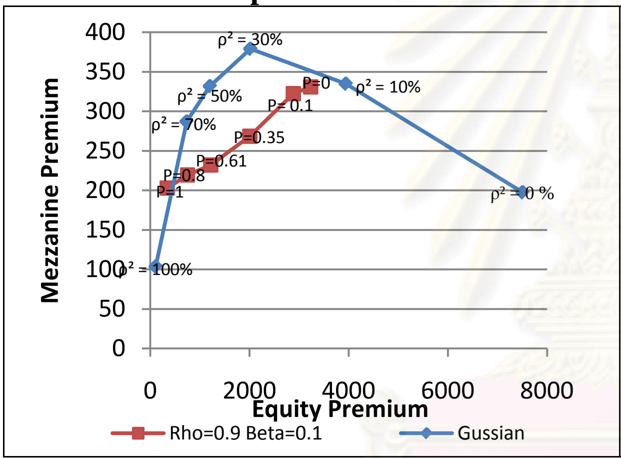
Graph 21 : Comparing between Equity and Mezzanine Premium for Stochastic correlation Rho=0.7 Beta=0.2



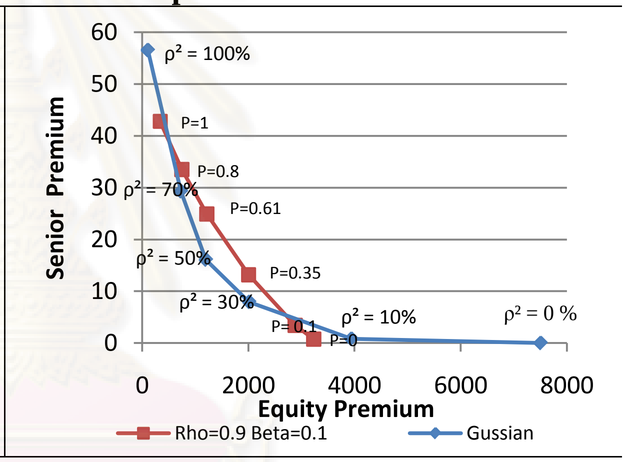
Graph 22 : Comparing between Equity and Senior Premium for Stochastic correlation Rho=0.7 Beta=0.2



Graph 23 : Comparing between Equity and Mezzanine Premium for Stochastic correlation copula Rho=0.9 Beta=0.1



Graph 24: Comparing between Equity and Senior Premium for Stochastic correlation copula Rho=0.9 Beta=0.1



<u>Summary</u>

- We find that all models agree in equity and senior tranches. Equity coupon spread decreases with correlation. Nonetheless, senior premium increases with correlation. Most models agree in mezzanine which creates "bump".

- Burtschell, Gregory, and Laurent (2008) find that Clayton and Student t cannot produce correlation smile, while Double t and stochastic correlation can produce the correlation smile. From plotting mezzanine premium and senior premium against equity coupon spread, we detect a pattern that might be characteristic of smile-producing models. We found that, for Double t and Stochastic correlation, the graph lies below Gaussian for mezzanine and above for senior. Clayton and Student t do not exhibit such pattern.

CHAPTER IV

SENSITIVITY OF CDO TRANCHES WITH RESPECT TO DEFAULT PROBABILITY

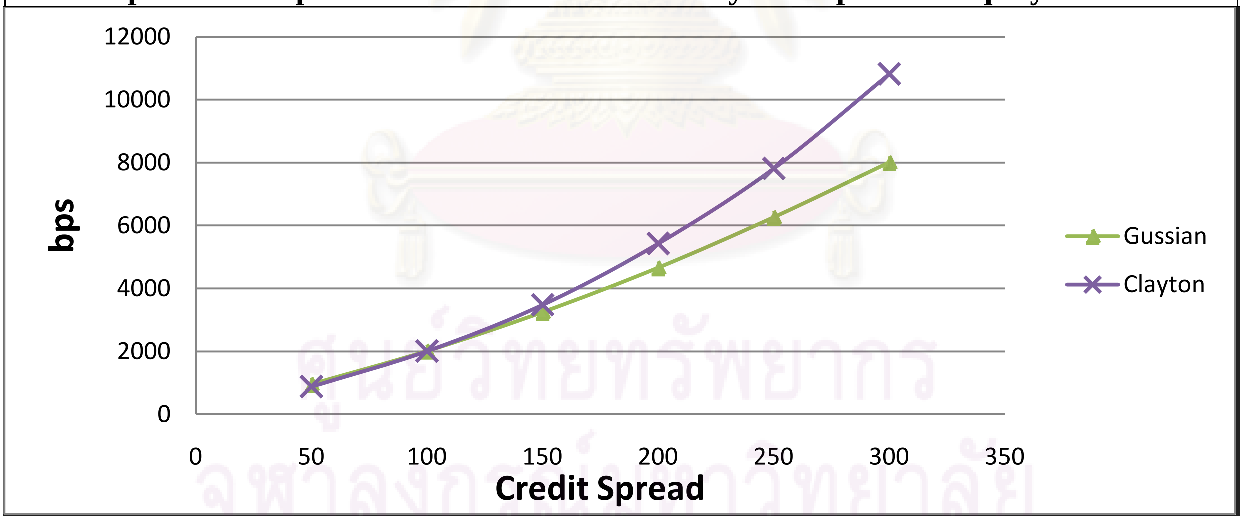
5.1 Sensitivity to Default Probability of Equity tranche

When we consider the sensitivity to default probability, we restrict all the variables and perturb credit spread which is a function of default probability:

$$F(t) = 1 - e^{t*credit\ spread} \qquad 4.1$$

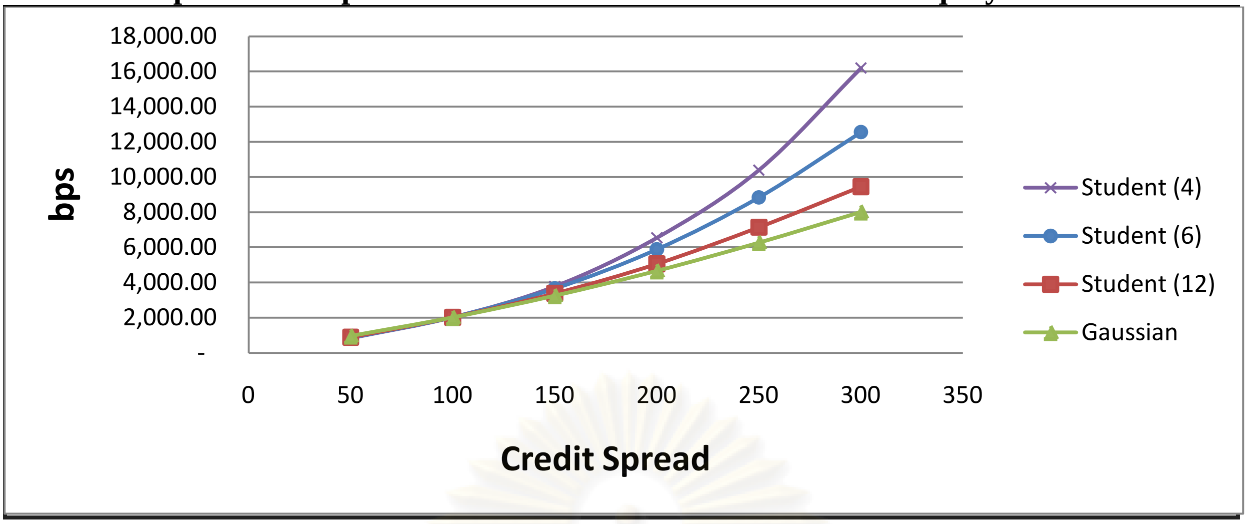
The first study is a comparison between Gaussian copula and Clayton copula shown in the graph below. It can be explained from the graph that Gaussian copula is less sensitive to the change of credit spread comparing to Clayton copula. We observed that Clayton rise more than Gaussian as the credit spread increase.

Graph 25 : Compare between Gaussian and Clayton Copula for Equity Tranche



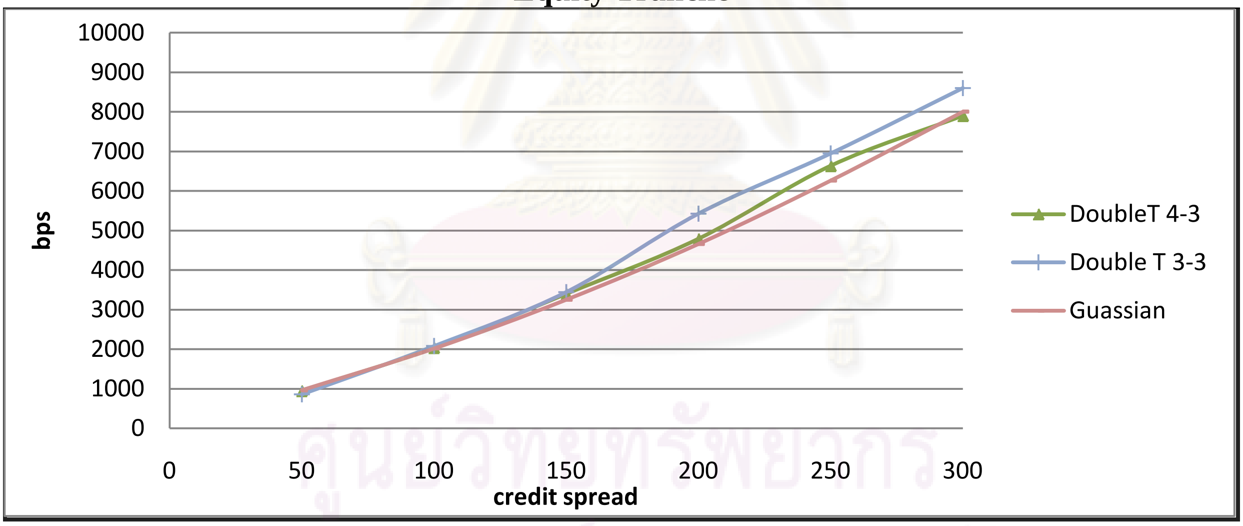
Now, we move on to Student t copula. We observe that Student t copula with 4 degree of freedom is more sensitive to the default probability than the Student t copula with 6 and 12 degree of freedom as examined from the graph below.

Graph 26 : Compare between Gaussian and Student t for Equity Tranche



We observe that Student t credit spread with high degree of freedom will approach Gaussian premium.

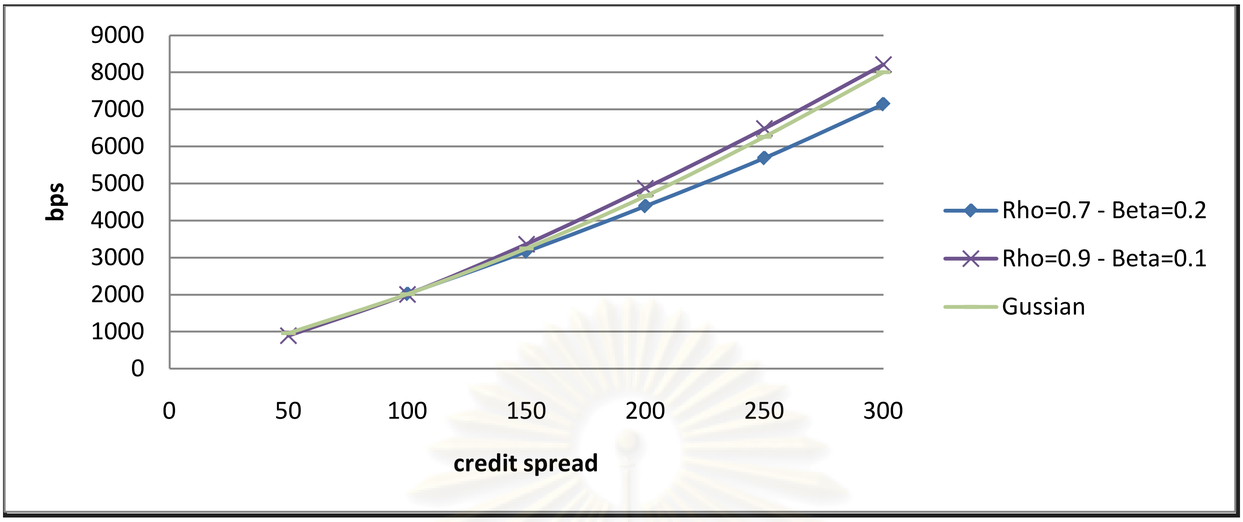
Graph 27 : Compare between Gaussian ,Double t 4-3 and Double t 3-3 Copula for Equity Tranche



The Graph reports that both Double t 4-3 and Double t 3-3 produce result that are not much different from Gaussian. As observed, we can conclude that as the degree freedom increases, Double t copula converses to Gaussian.

The graph underneath illustrates the comparison between Gaussian and stochastic correlation copula. Stochastic correlation pattern doesn't differ much from Gaussian.

Graph 28 : Compare between Gaussian and Stochastic correlation copula for Equity Tranche



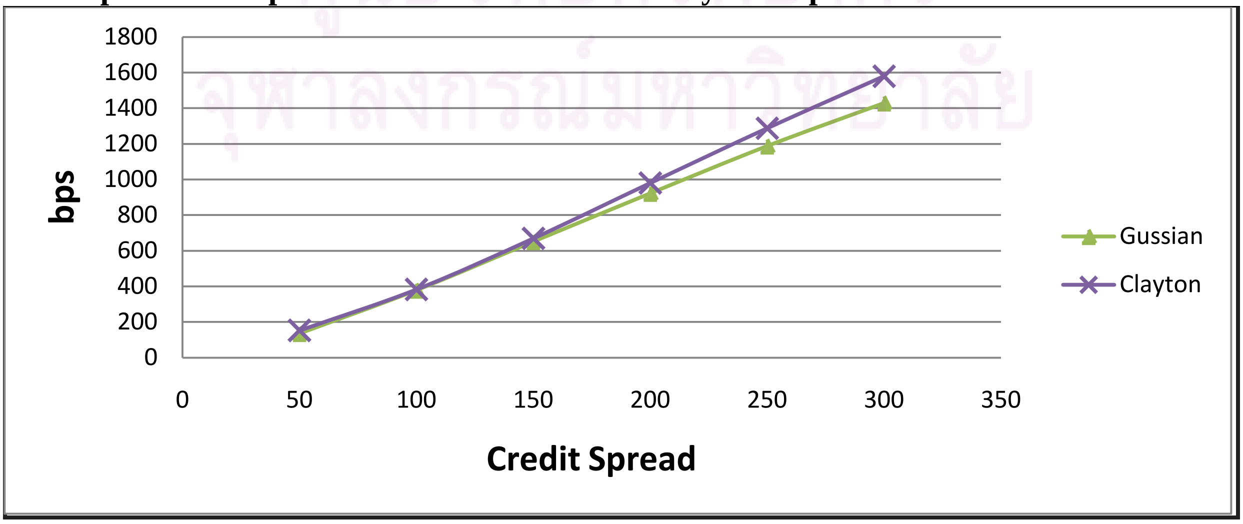
Summary for Equity Tranche

- We observe that Clayton copula is more sensitivity to default probability than the Gaussian copula. Furthermore, Gaussian copula is less sensitive to default probability than Student t copula. For Student t copula, we observed that as the degree of t increase, it converse to Gaussian copula.
- For Gaussian, Double t, and stochastic correlation copula, the results are not much different.

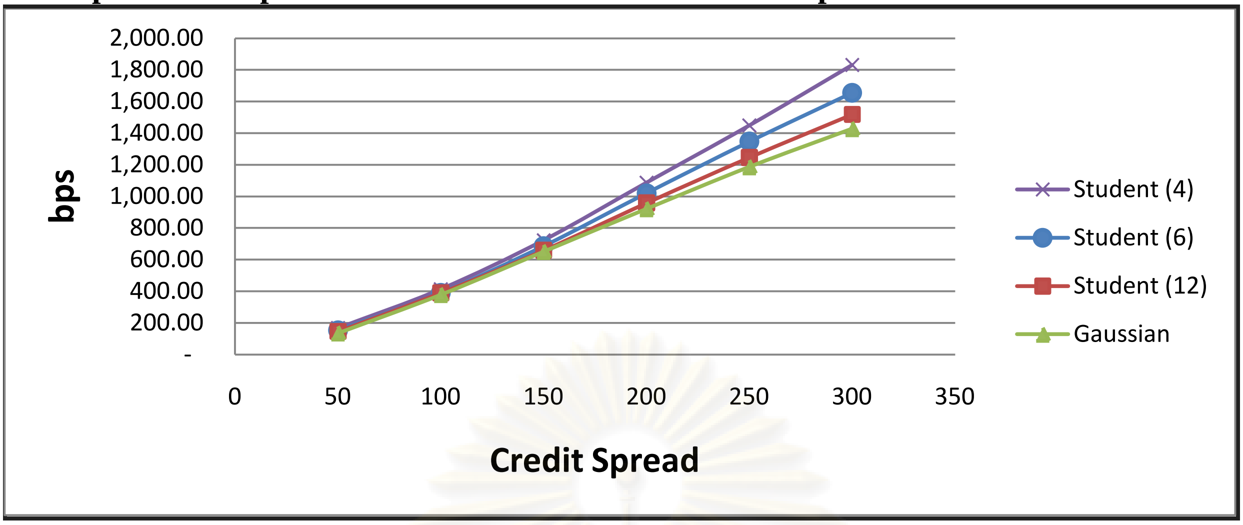
5.2 Sensitivity to Default Probability of Mezzanine tranche

In this section, we concentrated on Mezzanine tranche. Following graphs show comparison with Clayton, Student t, Double t, and stochastic correlation copula. The difference is so little that it can't be concluded which is more sensitive.

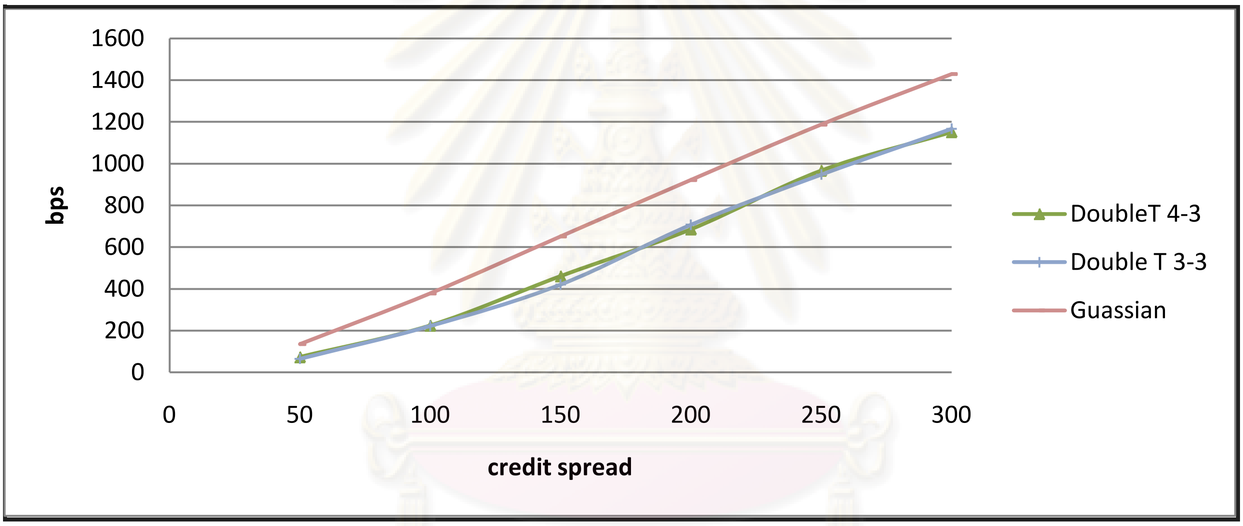
Graph 29 : Compare between Gaussian and Clayton Copula for Mezzanine Tranche



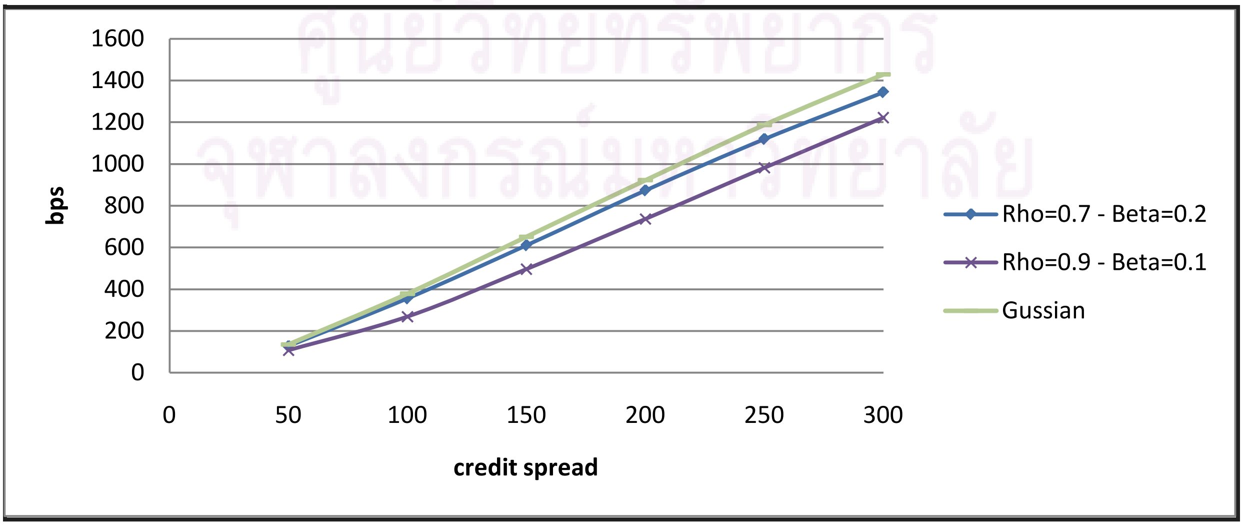
Graph 30 : Compare between Student t and Gaussian Copula for Mezzanine Tranche



Graph 31 : Compare between Gaussian and Double t 4-3 and 3-3 Copula for Mezzanine Tranche



Graph 32 : Compare between Gaussian and Stochastic correlation copula for Mezzanine Tranche



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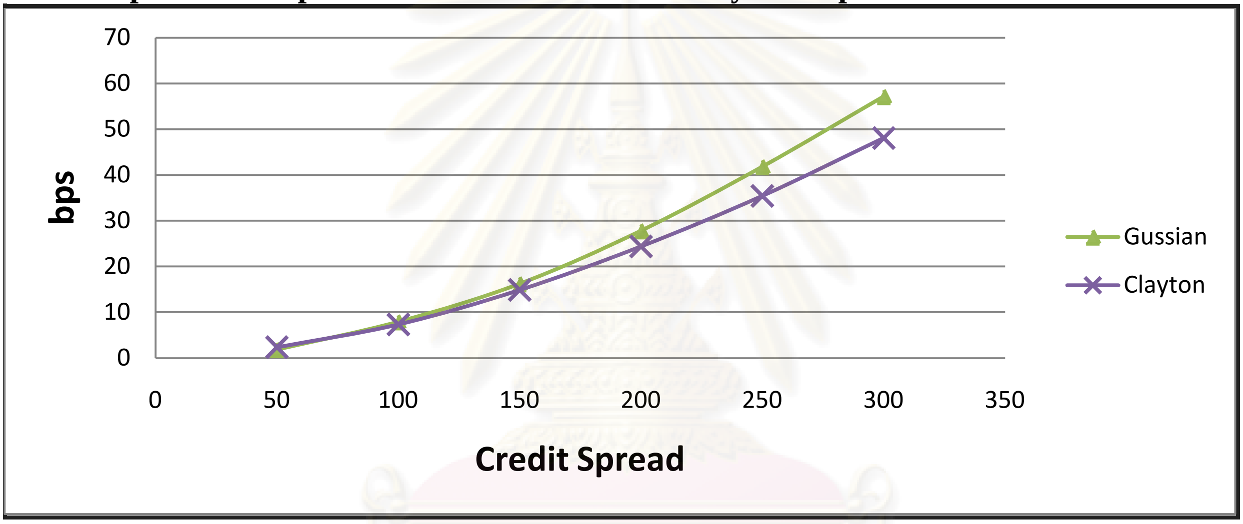
Summary for Mezzanine Tranche

- The percentage changes in all models are close.
- In other words, the difference among the models in terms of sensitivity to default probabilities is much less pronounced in mezzanine tranche.

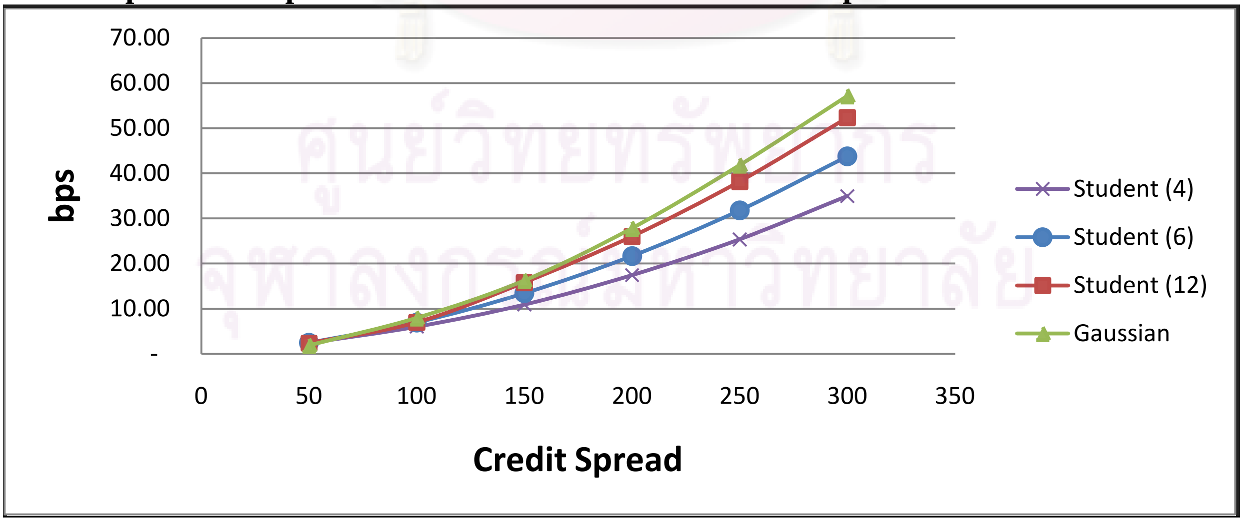
5.3 Sensitivity to Default Probability of Senior Tranche

Last, we concentrate on senior tranche. For all models of this tranche, we notice that the outcomes inverse equity tranche. We observe that the graph from Gaussian is more sensitive than Clayton copula. Next graph shows that Student t less sensitive than Gaussian copula.

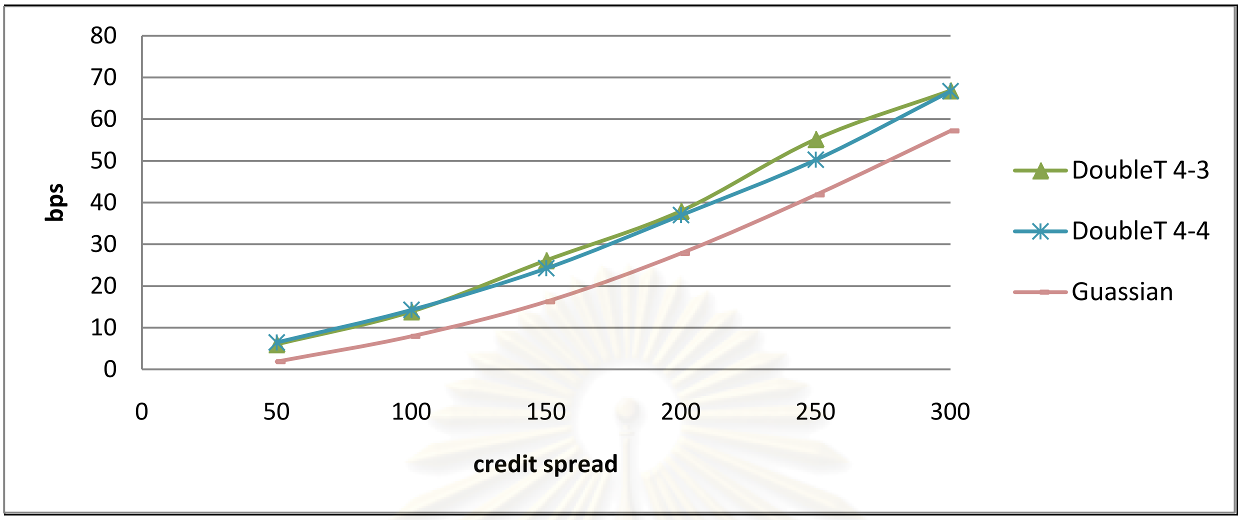
Graph 33 : Compare between Gaussian and Clayton Copula for Senior Tranche



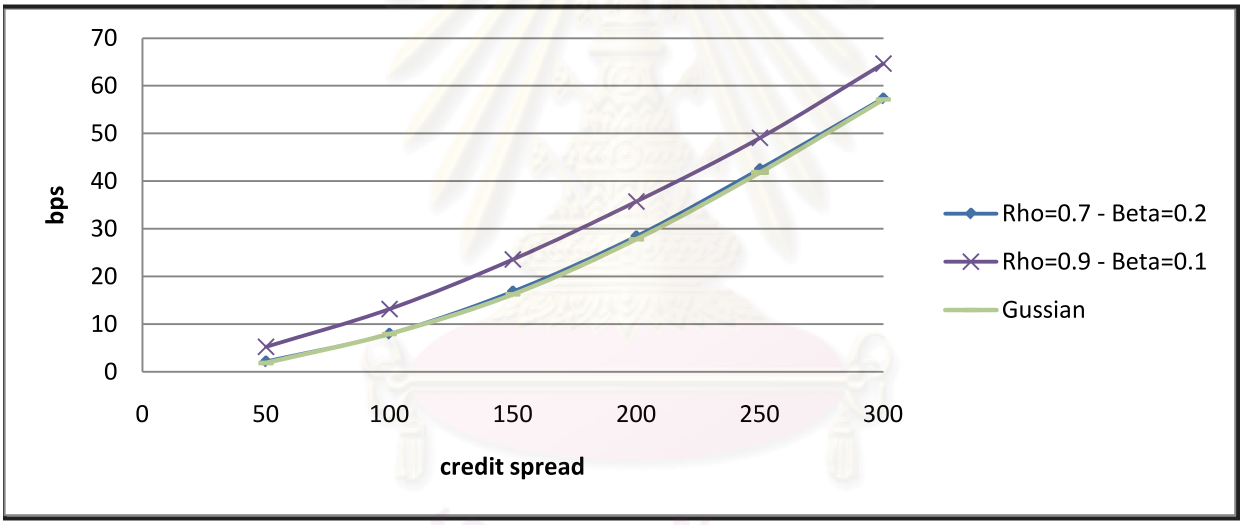
Graph 34: Compare between Gaussian and Student t Copula for Senior Tranche



Graph 35: Compare between Gaussian and Double t 4-3 and 4-4 Copula for Senior Tranche



Graph 36: Compare between Gaussian and Stochastic correlation copula for Mezzanine Tranche



Just like what we find in the equity tranche, we find that the sensitivity to default probability produced by Double t and Stochastic Correlations models are not much different from the Gaussian model.

<u>Summary for Senior Tranche</u>

- Gaussian is more sensitive than Clayton copula. Student t is less sensitive than Gaussian. Model that is more sensitive for equity tranche will be less sensitive for senior tranche.
- Double t and stochastic correlation are nearly indistinguishable from Gaussian in terms of sensitivity to default probability.

CHAPTER V
CONCLUSION

This paper is devoted to analysis of CDO tranches sensitivity to the default probability and to the degree of dependence between defaults in various models. The models under investigation are one factor Gaussian copula, stochastic correlation extension of the Gaussian copula, Clayton copula, Student t copula, and Double t copula.

- In term of the sensitivity to correlation, we found a characteristic of models that are able to fit market quotes. For Double t copula and stochastic correlation, both of which are known o fit market quotes well, the sensitivity to correlation parameters in the mezzanine tranche is much less pronounced compared to Gaussian copula. This can be seen in the plot of mezzanine tranche spread versus equity tranche spread, where the plots for Double t copula and Stochastic correlation are flatter then Gaussian copula. This characteristic is not seen in Clayton copula and Student t copula. Therefore, insensitivity to correlation in the mezzanine tranche seems to be a characteristic of smile-producing models. To offer an intuition behind this, the insensitivity of mezzanine implied correlation to remain level, while the sensitivity of equity and senior tranches causes the implied correlation of equity and senior tranches causes the implied correlation of equity and senior tranches to shoot up, producing a correlation smile as observed in the market.
- Compared to Gaussian, all models agree in both equity and senior tranche as to the effect of increasing correlation parameter. Equity premium decreases with correlation and senior premium increases with correlation. Every model agrees in mezzanine which creates “bump”.
- We found that for Double t and stochastic correlations, which fit market well, the sensitivity to default probability produced by these two models do not differ much. Interestingly, the one-factor Gaussian copula, which is known to be unable to fit market quote, produces sensitivity that is close to Double t and Stochastic correlation.
- Compared to Gaussian, Clayton and Student t produce very different sensitivity to default probability. Therefore, delta-hedging under different models will be different. Also a model that gives more sensitivity to default probability in equity tranche will give less sensitivity to senior tranche and vice versa.

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APPENDIX

APPENDIX A

Data for the basket default swaps and CDO examples

Basket default swaps and homogeneous CDO examples

Table A1: Default Free Yield Curve (Continuous Rates)

1D	1W	1M	2M	3M	6M	9M	1Y	2Y	3Y	4Y	5Y
2.02	2.05	2.06	2.07	2.08	2.14	2.2	2.37	2.8	3.17	3.47	3.71

The default free rates were obtained from the swap market in Euros on the 08/02/2005.

Table A3: Default Free Yield Curve (Continuous Rates)

1D	1W	1M	2M	3M	6M	9M	1Y	2Y	3Y	4Y	5Y
2.07	2.09	2.10	2.12	2.14	2.18	2.24	2.34	2.59	2.78	2.93	3.06

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APPENDIX B

1.) Matlab Code

1.1.) Gussian Copula

```
for i=1:n

    V         = randn();
    V_         = randn(100,1);
    Vi        = rho*V + sqrt(1-rho^2)*V_ ;
   tau        = -log(1-normcdf(Vi))./spread ;

 (tau_, num)    = sort(tau);
  tau_         = tau_(1:sum(tau_<5));
  prev_date    = repmat(0,length(tau_),1);

  for k=1:length(tau_)

        prev_date(k) = t(sum(tau_(k)>t));

  end

   num        = num (1:length(tau_));
    RF         = interp1(TERM,yield,tau_);
  default      = E(num).*(1-recover(num));
    Me         = min(A,cumsum(default));
    Mm          = max(0,min(cumsum(default)-A,B-A));
    Ms         = max(0,cumsum(default)-B);

    if length(tau_)==0
        DPLe(i)=0;
        DPLm(i)=0;
        DPLs(i)=0;
        AMPe   =0;
        AMPm   =0;
        AMPs   =0;

    else

        DPLe(i)    = sum(diff((0;Me))./(1+RF).^tau_);
        DPLm(i)    = sum(diff((0;Mm))./(1+RF).^tau_);
        DPLs(i)    = sum(diff((0;Ms))./(1+RF).^tau_);

        AMPe       = sum((tau_-prev_date).*diff((0;Me))./(1+RF).^tau_);
        AMPm        = sum((tau_-prev_date).*diff((0;Mm))./(1+RF).^tau_);
        AMPs       = sum((tau_-prev_date).*diff((0;Ms))./(1+RF).^tau_);

    end

   MPe     =0;
   MPm     =0;
   MPs     =0;

    for j=2:length(t)

        if length(tau_)==0

            AccuLosse =0;
            AccuLossm =0;
            AccuLosss =0;

        else
            AccuLosse      = Me(length(tau_<t(j)));
            AccuLossm       = Mm(length(tau_<t(j)));
            AccuLosss      = Ms(length(tau_<t(j)));

        end
```

```
     MPe     = MPe + (t(j)-t(j-1))*(A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
     MPm     = MPm +(t(j)-t(j-1))*(B-A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
     MPs     = MPs +(t(j)-t(j-1))*(100-B-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));

    end

  MarginE(i)    = AMPe+MPe;
  MarginM(i)    = AMPm+MPm;
  MarginS(i)    = AMPs+MPs;

end
      Xe     =mean(DPLe)/mean(MarginE);
      Xm     =mean(DPLm)/mean(MarginM);
      Xs     =mean(DPLs)/mean(MarginS);
```

1.2.) Stochastic correlation

```
for i=1:n

    Bi      =  (rand(100,1) < p);
     V      =  randn();
    V_      =  randn(100,1);
    Vi      =  Bi.*(rho*V + sqrt(1-rho^2)*V_) + (1-Bi).*(beta*V + sqrt(1-beta^2)*V_);
    tau     = -log(1-normcdf(Vi))./spread;

 (tau_, num)   =  sort(tau);
  tau_         =  tau_(1:sum(tau_<5));
  prev_date    =  repmat(0,length(tau_),1);

  for k=1:length(tau_)

       prev_date(k) = t(sum(tau_(k)>t));

  end

   num        = num (1:length(tau_));
    RF        = interp1(TERM,yield,tau_);
  default     = E(num).*(1-recover(num));
    Me        = min(A,cumsum(default));
    Mm        = max(0,min(cumsum(default)-A,B-A));
    Ms        = max(0,cumsum(default)-B);

    if  length(tau_)  ==  0

       DPLe(i)  =  0;
       DPLm(i)  =  0;
       DPLs(i)  =  0;
       AMPe     =  0;
       AMPm     =  0;
       AMPs     =  0;

    else

       DPLe(i)   =  sum(diff((0;Me))./(1+RF).^tau_);
       DPLm(i)   =  sum(diff((0;Mm))./(1+RF).^tau_);
       DPLs(i)   =  sum(diff((0;Ms))./(1+RF).^tau_);

       AMPe      =  sum((tau_-prev_date).*diff((0;Me))./(1+RF).^tau_);
       AMPm      =  sum((tau_-prev_date).*diff((0;Mm))./(1+RF).^tau_);
       AMPs      =  sum((tau_-prev_date).*diff((0;Ms))./(1+RF).^tau_);

    end

   MPe    =0;
   MPm    =0;
   MPs    =0;

    for j=2:length(t)

       if length(tau_)==0

         AccuLosse =0;
         AccuLossm =0;
```

```
          AccuLosss =0;

        else
          AccuLosse     = Me(length(tau_<t(j)));
          AccuLossm     = Mm(length(tau_<t(j)));
          AccuLosss     = Ms(length(tau_<t(j)));

        end

       MPe    = MPe + (t(j)-t(j-1))*(A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
       MPm     = MPm +(t(j)-t(j-1))*(B-A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
       MPs    = MPs +(t(j)-t(j-1))*(100-B-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));

    end

 MarginE(i)    = AMPe+MPe;
 MarginM(i)    = AMPm+MPm;
 MarginS(i)    = AMPs+MPs;

 if mod(i,100)==0
 fprintf('Replication %-6d, Equity %-5.0f, Mezz %-5.0f, Senior %-
5.0f\n',i,10000*mean(DPLe(1:i))/mean(MarginE(1:i)),10000*mean(DPLm(1:i))/mean(MarginM(1:i)),10000*mean(DPLs(1:i))/mean(Margi
nS(1:i)));
 end

end
    Xe     =mean(DPLe)/mean(MarginE);
    Xm     =mean(DPLm)/mean(MarginM);
    Xs     =mean(DPLs)/mean(MarginS);
```

1.3.) Student t Copula

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```
for i=1:n

    V    = randn();
    V_   = randn(100,1);
    W    = degree/chi2rnd(degree);
    X    = rho*V + sqrt(1-rho^2)*V_ ;
    Vi   = sqrt(W)*X;
 tau = -log(1-tcdf(Vi,degree))./spread ;

 (tau_, num)    = sort(tau);
   tau_         = tau_(1:sum(tau_<5));
   prev_date    = repmat(0,length(tau_),1);

   for k=1:length(tau_)

         prev_date(k) = t(sum(tau_(k)>t));

   end

    num         = num (1:length(tau_));
     RF         = interp1(TERM,yield,tau_);
   default      = E(num).*(1-recover(num));
     Me         = min(A,cumsum(default));
     Mm          = max(0,min(cumsum(default)-A,B-A));
     Ms         = max(0,cumsum(default)-B);

    if length(tau_)==0
        DPLe(i)=0;
        DPLm(i)=0;
        DPLs(i)=0;
        AMPe   =0;
        AMPm   =0;
        AMPs   =0;

    else
```

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```
    DPLe(i)   = sum(diff((0;Me))./(1+RF).^tau_);

    DPLm(i)   = sum(diff((0;Mm))./(1+RF).^tau_);

    DPLs(i)   = sum(diff((0;Ms))./(1+RF).^tau_);


    AMPe      = sum((tau_-prev_date).*diff((0;Me))./(1+RF).^tau_);

    AMPm      = sum((tau_-prev_date).*diff((0;Mm))./(1+RF).^tau_);

    AMPs      = sum((tau_-prev_date).*diff((0;Ms))./(1+RF).^tau_);


  end


MPe    =0;

MPm    =0;

MPs    =0;


 for j=2:length(t)


    if length(tau_)==0


      AccuLosse =0;

      AccuLossm =0;

      AccuLosss =0;


    else

      AccuLosse    = Me(length(tau_<t(j)));

      AccuLossm    = Mm(length(tau_<t(j)));

      AccuLosss    = Ms(length(tau_<t(j)));


    end


   MPe    = MPe + (t(j)-t(j-1))*(A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));

   MPm    = MPm +(t(j)-t(j-1))*(B-A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));

   MPs    = MPs +(t(j)-t(j-1))*(100-B-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));


 end
```

```
 MarginE(i)    = AMPe+MPe;

 MarginM(i)    = AMPm+MPm;

 MarginS(i)    = AMPs+MPs;


End


 if mod(i,100)==0

 fprintf('Replication %-6d, Equity %-5.0f, Mezz %-5.0f, Senior %-5.0f\n',i,10000*mean(DPLe(1:i))/mean(MarginE(1:i)),10000*mean(DPLm(1:i))/mean(MarginM(1:i)),10000*mean(DPLs(1:i))/mean(MarginS(1:i)));

 end

    Xe     =mean(DPLe)/mean(MarginE);

    Xm     =mean(DPLm)/mean(MarginM);

    Xs     =mean(DPLs)/mean(MarginS);
```

1.4.) Double t Copula

```
O     = rho*((degree-2)/degree)^1/2*trnd(degree,1,1000000)  +  sqrt(1-rho^2)*((degreev-2)/degreev)^1/2*trnd(degreev,1,1000000);
x     = -5:.2:5;
H     =();

for o=x;
   H=(H mean(O<=o));
end

for i=1:n


    V      = trnd(degree);
    V_     = trnd(degree,100,1);
    Vi     = rho*((degree-2)/degree)^1/2*V  +  sqrt(1-rho^2)*((degreev-2)/degreev)^1/2*V_ ;
     tau   = -log(1-interp1(x,H,Vi))./spread ;


 (tau_, num)    = sort(tau);
  tau_         = tau_(1:sum(tau_<5));
  prev_date    = repmat(0,length(tau_),1);

  for k=1:length(tau_)

       prev_date(k) = t(sum(tau_(k)>t));

  end

   num         = num (1:length(tau_));
    RF         = interp1(TERM,yield,tau_);
  default      = E(num).*(1-recover(num));
    Me         = min(A,cumsum(default));
    Mm          = max(0,min(cumsum(default)-A,B-A));
    Ms         = max(0,cumsum(default)-B);

    if length(tau_)==0
       DPLe(i)=0;
       DPLm(i)=0;
       DPLs(i)=0;
       AMPe   =0;
       AMPm   =0;
       AMPs   =0;

    else
```

```
        DPLe(i)    = sum(diff((0;Me))./(1+RF).^tau_);
        DPLm(i)    = sum(diff((0;Mm))./(1+RF).^tau_);
        DPLs(i)    = sum(diff((0;Ms))./(1+RF).^tau_);

        AMPe       = sum((tau_-prev_date).*diff((0;Me))./(1+RF).^tau_);
        AMPm       = sum((tau_-prev_date).*diff((0;Mm))./(1+RF).^tau_);
        AMPs       = sum((tau_-prev_date).*diff((0;Ms))./(1+RF).^tau_);

    end

   MPe     =0;
   MPm     =0;
   MPs     =0;

    for j=2:length(t)

        if length(tau_)==0

          AccuLosse =0;
          AccuLossm =0;
          AccuLosss =0;

        else
          AccuLosse      =  Me(length(tau_<t(j)));
          AccuLossm      =  Mm(length(tau_<t(j)));
          AccuLosss      =  Ms(length(tau_<t(j)));

        end

       MPe      =  MPe + (t(j)-t(j-1))*(A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
       MPm      =  MPm +(t(j)-t(j-1))*(B-A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
       MPs      =  MPs +(t(j)-t(j-1))*(100-B-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));

    end


  MarginE(i)    = AMPe+MPe;
  MarginM(i)    = AMPm+MPm;
  MarginS(i)    = AMPs+MPs;

End
  if mod(i,100)==0
  fprintf('Replication %-6d, Equity %-5.0f, Mezz %-5.0f, Senior %-
5.0f\n',i,10000*mean(DPLe(1:i))/mean(MarginE(1:i)),10000*mean(DPLm(1:i))/mean(MarginM(1:i)),10000*mean(DPLs(1:i))/mean(Margi
nS(1:i)));
  end
       Xe     =mean(DPLe)/mean(MarginE);
       Xm     =mean(DPLm)/mean(MarginM);
       Xs     =mean(DPLs)/mean(MarginS);
```

1.5.) Clayton Copula

```
for i=1:n

    V    =  randg(1/theta);
    U    =  rand(100,1);
    Vi   = (1-log(U)./V).^(-1/theta);
 tau = -log(1-unifinv(Vi,0,1))./spread;

  (tau_, num)    = sort(tau);
  tau_          = tau_(1:sum(tau_<5));
  prev_date     = repmat(0,length(tau_),1);

  for k=1:length(tau_)

       prev_date(k) = t(sum(tau_(k)>t));

  end

   num          = num (1:length(tau_));
```

```
  RF        = interp1(TERM,yield,tau_);
 default      = E(num).*(1-recover(num));
  Me         = min(A,cumsum(default));
  Mm         = max(0,min(cumsum(default)-A,B-A));
  Ms         = max(0,cumsum(default)-B);

  if length(tau_)==0
     DPLe(i)=0;
     DPLm(i)=0;
     DPLs(i)=0;
     AMPe   =0;
     AMPm   =0;
     AMPs   =0;

  else

     DPLe(i)    = sum(diff((0;Me))./(1+RF).^tau_);
     DPLm(i)    = sum(diff((0;Mm))./(1+RF).^tau_);
     DPLs(i)    = sum(diff((0;Ms))./(1+RF).^tau_);

     AMPe       = sum((tau_-prev_date).*diff((0;Me))./(1+RF).^tau_);
     AMPm       = sum((tau_-prev_date).*diff((0;Mm))./(1+RF).^tau_);
     AMPs       = sum((tau_-prev_date).*diff((0;Ms))./(1+RF).^tau_);

  end

 MPe     =0;
 MPm     =0;
 MPs     =0;

  for j=2:length(t)

     if length(tau_)==0

        AccuLosse =0;
        AccuLossm =0;
        AccuLosss =0;

     else
        AccuLosse      =  Me(length(tau_<t(j)));
        AccuLossm      =  Mm(length(tau_<t(j)));
        AccuLosss      =  Ms(length(tau_<t(j)));

     end

    MPe     =  MPe + (t(j)-t(j-1))*(A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
    MPm     =  MPm +(t(j)-t(j-1))*(B-A-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));
    MPs     =  MPs +(t(j)-t(j-1))*(100-B-AccuLosse)./((1+interp1(TERM,yield,t(j))).^t(j));

  end

 MarginE(i)     = AMPe+MPe;
 MarginM(i)     = AMPm+MPm;
 MarginS(i)     = AMPs+MPs;

End
 if mod(i,100)==0
 fprintf('Replication %-6d, Equity %-5.0f, Mezz %-5.0f, Senior %-
5.0f\n',i,10000*mean(DPLe(1:i))/mean(MarginE(1:i)),10000*mean(DPLm(1:i))/mean(MarginM(1:i)),10000*mean(DPLs(1:i))/mean(Margi
nS(1:i)));
 end
      Xe      =mean(DPLe)/mean(MarginE);
      Xm      =mean(DPLm)/mean(MarginM);
      Xs      =mean(DPLs)/mean(MarginS);
```

BIOGRAPHY

Miss Romani Boondicharern was born on July 16, 1986, in Bangkok. At the secondary school, she graduated from Cushing Academy. At the undergraduate level, she graduated from the Civil Engineering and Technology, Sirindhorn International Institute of Technology Thammasat University in May 2007 with a Bachelor of Engineering Civil Engineering. She joined the Master of Science in Finance program, Chulalongkorn University in June 2008.

ศูนย์วิทยทรัพยากร
จุฬาลงกรณ์มหาวิทยาลัย