CHAPTER 3

DEVELOPMENT OF MATHEMATICAL MODEL AND CALCULATION EXAMPLE

The mathematical model contain 2 parts of calculation cyclone efficiency and optimization calculation.

3.1 Cyclone efficiency calculation.

In this paper both calculation method, grade-efficiency η and pressure drop ΔP, relied in W., Licht and D., Leith (1972).

3.1.1 Basic design data

- Gas volume flow rate, m^3/s; Q
- Particle density, kg/m^3; ρ_p
- Fluid density, kg/m^3; ρ
- Dust loading, kg/m^3; DL
- absolute temperature, K^o; T_1
- fluid viscosity, kg/m.s; μ
- required cutsize particle diameter, m; d_{cpr}

The experiment cyclone, Shepherd and Lapple[c] type which is 0.96 m diameter use to collected fly ash from boiler combustion. There are 3 experiments for test the simulation program and measured data by SGS (THAILAND) Co.,Ltd. as the following Table

Table 3.1 The experimental cyclone dimension.

	The experiment cyclone : Shepherd and Lapple		
Term	Description	Value	Unit
D	Body diameter	960	mm
a	Inlet height:	480	mm
b	Inlet width:	240	mm
S	Outlet length:	600	mm
D_e	Outlet diameter:	480	mm
h	Cylinder height:	1,920	mm
H	Overall height:	3,840	mm
B	Dust outlet diameter:	240	mm
K	Configuration parameter	402.9	-
N_H	Inlet velocity heads	8.0	-
$Surf$	Surface parameter	3.78	-

Table 3.2 Details of fuel oil and exhaust gas from fuel (Buncker C) combustion

Fuel data	Value	Unit
Particle size MMD	10.0	μm
Boiler capacity	2,000	kg/hr
Oil consumption	245	kg/hr
Sulfur content in fuel	3.0	%
CO_2 in exhaust gas	13.5	%
Fuel gravity	20.0	API
Exhaust gas pressure	1.01×10^5	Pa
lb. of exhaust gas/lb. of fuel	17.2	-
lb. of exhaust gas	9,291.87	lb./hr
emission factor for TSP	0.033	lb. TSP/gal fuel
lb. TSP/hr	1.0372	kg TSP/hr

Table 3.3 Experiment cyclone result measurement No. 1

Parameter	Operating condition
	Experiment cyclone
K_a	0.5
K_b	0.25
N_H	8.0
d_{cpr} ; m	3.20×10^{-6}
Q ; m^3/s	1.501
ρ_p ; Kg/m^3	1500
ρ ; Kg/m^3	0.73625
μ ; Pa.s	25.97×10^{-6}
DL ; Kg/m^3	0.0001919
Temperature ; K^{o}	473
Cyclone Diameter ; m	0.96
Operating hours/year	6,000
Y ; year	5
c_e ; baht/unit	1.07
N	1

3.1.2 Efficiency Calculation

In this paper both calculation method, grade-efficiency η and pressure drop ΔP, relied in W., Licht and D., Leith (1972).

$$\eta = 1 - \exp - 2\left[M d_p^{\ N} \right] \qquad (2.20a)$$

in which

$$M = 2\left[\frac{KQ}{D^3} \frac{\rho_p^{n+1}}{18\mu} \right]^{N/2} \qquad and \qquad N = \frac{1}{n+1} \qquad (2.20b)$$

A 50% cut diameter may be found from

$$d_{cp_{50}} = \left(\frac{0.6931}{M}\right)^{n+1} \qquad (2.20c)$$

$$n = 1 - \left[1 - 0.67 D^{0.14}\right]\left(\frac{T_1}{283}\right)^{0.3} \qquad (2.20d)$$

where D = diameter of cyclone, m

T_1 = absolute temperature of gas, K

This is equivalent to starting that n depends on a Stokes number and a geometric configuration parameter K for the cyclone. The configuration parameter is calculated from the dimension ratios only, and is independent of the size of the clclone. Values of it for some standard desings are given in table 2.4. For other configurations, the calculation is explained by Licht (1972).

This model is satisfactory for cyclones of common sizes (say $D > 8$ in.) and over a fairly extended range of operating temperatures. It does not apply to very small cyclones, such as ones used in respirable dust sampling, nor to operation under high pressure. There is some indication that here η depends on a Reynolds number for the cyclones in addition to the Stokes number involved in M.

The model gives a conservative estimate of performance such as would be obtained on fairly dilute particle/gas streams, that is, at low "grain loading" say below 10 g/m^3. The limited data available indicate that efficiency improves as grain loading increases.

$$\Delta P = \frac{1}{2}\rho_g u_{T_2}^2 N_H \quad \text{or} \quad \frac{1}{2}\rho_g\left(\frac{Q}{K_a K_b N}\right)^2 N_H \qquad (3.1)$$

where

ρ_g = density of gas-particle stream, g/cm^3

u_{T_2} = inlet velocity, m/sec

N_H = number of inlet velocity heads

For grade efficiency curve calculation will calculate from d_{p_i} = 1 μm to 15 μm. by equation below

$$\eta_i = \left[1 - \exp\left(-M d_{p_i}^{\frac{1}{n+1}}\right)\right]$$

For overall grade efficiency will calculate form 0 μm to 100 μm by specified in 7 particles diameter ranges.

$$g_i = \frac{m_i}{Sum(m_i)}$$

$$\eta_i = \left[1 - \exp\left(-M d_{p_{avr_i}}^{\frac{1}{n+1}}\right)\right]$$

$$Overall\ grade\ Eff. = Sum(g_i \times \eta_i)$$

m_i	=	Mass of particle at specific diameter range ; g
g_i	=	Mass fraction at specific diameter range
$d_{p_{avr_i}}$	=	Average of particle at specific diameter range ; μm

3.2 Optimization calculation.

An optimization simulation account for two variables D and N then objective equation be :

Maximize objective equation and subject to six constraints.

Objective equation $\eta / \Delta P$ Maximize

Constraints

1) 15 m/s $\leq u_{T_2} \leq (u_{T_2 \max}$, 30 m/s, 1.35 $v_s)$

2) $N_{\min} \leq N_o \leq N_{\max}$;Integer

3) $0.5 \leq n \leq 0.9$

4) $\Delta P \leq 2500$ N/m^2

5) $d_{cp_{50}} \geq d_{cpr}$

6) Minimum total cost ; c_t

Objective equation be a function of two variables D and N. If proper D and N were used in which give maximum η and minimum ΔP in which subject to all constrains then maximized value of objective equation.

In this paper both calculation method, grade-efficiency η and pressure drop ΔP, relied in W., Licht and D., Leith (1972) as Equation 2.20a to 2.20d.

Find feasible solution in which subject to all constrains which maximized objective equation, $\eta \,/\, \Delta P$.

1st costrain

Find maximum inlet velocity, m/s; $u_{T_2\,\max}$ at $\Delta P \leq 2500$ N/m^2.

$$\Delta P = \frac{1}{2}\rho_g u_{T_2}^2 N_H \quad \text{or} \quad \frac{1}{2}\rho_g\left(\frac{Q}{K_a K_b N}\right)^2 N_H \tag{3.1}$$

$$u_{T_2\,\max} \leq \left[\frac{2\times 2{,}500}{(\rho + DL)N_H}\right]^{1/2}$$

if $u_{T_2\,\max} > 30$ m/s then $u_{T_2\,\max} = 30$ m/s

$$u_{T_2} = \left[\frac{Q}{K_a K_b N D^2}\right]$$

$$v_s = 4.91\left(\frac{4g\mu(\rho_p - \rho)}{3\rho^2}\right) \times \frac{K_b^{\,0.4}}{(1-K_b)^{1/3}} D^{0.067} u_{T_2}^{\;2/3}$$

2nd costrain

$$N_{\max} = \left(\frac{Q}{15 K_a K_b D^2}\right) + \tag{3.2a}$$

$$N_{\min} = \left(\frac{Q}{u_{T_2\,\max} K_a K_b D^2}\right) \quad \text{if } N_{\min} = 0 \text{ then set } N_{\min} \text{ to unity} \tag{3.2b}$$

3rd constrain

$$n = 1 - \left[1 - 0.67(D)^{0.14}\right]\left(\frac{T_1}{283}\right)^{0.3}$$

4th constrain

$$\Delta P = \frac{1}{2}\rho_g u_{T_2}^2 N_H \quad \text{or} \quad \frac{1}{2}\rho_g\left(\frac{Q}{K_a K_b N}\right)^2 N_H \qquad (3.1)$$

5th constrain

$$M = 2\left[\frac{KQ}{D^3}\frac{\rho_p^{\;n+1}}{18\mu}\right]^{N/2}$$

$$d_{cp_{50}} = \left(\frac{0.6931}{M}\right)^{n+1}$$

6th constrain

$$C_t = \left[M_{cy} + F_{cy} + R_{cy}\right] \times N + \frac{1}{2}\rho_g\left[\frac{Q}{K_a K_b N}\right]^2 N_H \times c_e \times yrs \times hpy \qquad (3.3)$$

Then comparing feasible solution that offered $C_{t_{\min}}$. If there were feasible solution more than 1 answer in which given the same $C_{t_{\min}}$ value but objective equation value difference the answer will be the maximum objective equation value.

3.2.1 Cost and thickness data

- power rate, baht/kw.hr; c_e
- amount operating years, year; *yrs*
- operating hours per year, hr/yr; *hpy*
- steel thickness, mm; *Thk*
- steel rolled cost, baht/m^2; *Roc*
- steel cost, baht/kg of steel; *Mac*
- steel weight, kg of steel/m^2; *Wt*

Cyclone expense emphasis the Thailand real expense both total fixed cost and total operating cost. Total fixed cost include material and fabrication cost for cyclone body only but exclude support structure and transportation cost. Total operating cost concern only electrical cost from the Electricity Generating Authority of Thailand (EGAT) but maintenance cost excluded.

The total cost which base on Thailand expense consist of total fixed cost and total operating cost .

The fixed cost can be expressed:

$$T_{fc} = \left[M_{cy} + F_{cy} + R_{cy} \right] \qquad (3.4)$$

$$C_{fixed} = T_{fc} \times N$$

where

M_{cy}	=	Each cyclone Metal sheet cost, Baht/ N
R_{cy}	=	Each cyclone metal sheet rolled cost, Baht/ N
F_{cy}	=	Each cyclone fabrication cost, Baht/ N
N	=	Amount of cyclones
M_{cy}	=	metal sheet cost × each cyclone metal sheet weight
	=	25 baht/Kg. × each cyclone metal sheet weight
R_{cy}	=	metal sheet rolled cost (up to D and thickness) × each cyclone total metal sheet area
	=	350 baht/m^2 at thickness 2 mm× each cyclone total metal sheet area
F_{cy}	=	fabrication cost × each cyclone metal sheet weight
	=	18 baht/Kg. × each cyclone metal sheet weight
A_{cy}	=	each cyclone total metal sheet area, m^2

$$const = \left[h + (1+B)(H-h) + D_e \times S\right]$$

$$A_{cy} = cont \times \pi \times D^2 \quad m^2$$

$$W_{cy} = A_{cy} \times Wt \quad kg$$

$$M_{cy} = W_{cy} \times Mac \quad baht$$

$$F_{cy} = W_{cy} \times Fac \quad baht$$

$$R_{cy} = Roc \times Rpa \times A_{cy} \quad baht$$

$$T_{fc} = M_{cy} + F_{cy} + R_{cy} \quad baht$$

The operating cost = power cost are:

$$c_{power} = Q \Delta P c_e \qquad (2.29)$$

$$\Delta P = \frac{1}{2}\rho_g u_{T_2}^2 N_H \quad \text{or} \quad \frac{1}{2}\rho_g\left(\frac{Q}{K_a K_b N}\right)^2 N_H \qquad (3.1)$$

The total cost will then be

$$C_t = C_{fixed} + C_{oper} \qquad (3.5)$$

The operating costs include only the cost of the power, since maintenance cost is negligible.

An equation will then be obtained which gives C_t in terms of N, combined Equation (2.29) with Equation (3.2).

$$C_t = \left[M_{cy} + F_{cy} + R_{cy}\right] \times N + \frac{1}{2}\rho_g\left[\frac{Q}{K_a K_b N}\right]^2 N_H \times c_e \times yrs \times hpy \qquad (3.3)$$

An objective eq^n value curve calculation will calculate from d_{p_i} = 1 μm to 15 μm. by equation below

$$Obj_i = \left[1 - \exp\left(-Md_{p_i}^{\frac{1}{n+1}}\right)\right] / \Delta P$$

3.3 Calculation steps

The calculation procedure will find the optimized solution with respect to 6 constrains, which will be detailed in section 3.4. The calculation routine of the program is composed of finding the maximum inlet velocity, the assume cyclone diameter, the minimum amount of cyclone, and checking the feasibility of the solution. Each calculation sub-routine is detailed as follow:

3.3.1. Find the maximum inlet velocity $u_{T_2 \max}$ in m/s at $\Delta P \leq 2500$ N/m^2.

$$\Delta P = \frac{1}{2}\rho_g u_{T_2}^2 N_H \quad \text{or} \quad \frac{1}{2}\rho_g\left(\frac{Q}{K_a K_b N}\right)^2 N_H \tag{3.1}$$

$$u_{T_2 \max} \leq \left[\frac{5000}{(\rho + DL)N_H}\right]^{1/2}$$

if $u_{T_2 \max} > 30$ m/s then $u_{T_2 \max} = 30$ m/s

if $u_{T_2 \max} < 15$ m/s then print "Please input new data cause $u_{T_2 \max} < 15$ m/s" and

End.

3.3.2. Find assume cyclone diameter, m; D_a by using the 5th constrain, $d_{cp} \leq d_{cpr}$. First let $D_a = 0.3\,m$ then

Repeat

$$D_a = D_a + 0.0001$$

$$n = 1 - \left[1 - 0.67(D_a)^{0.14}\right]\left(\frac{T_1}{283}\right)^{0.3}$$

$$M = 2\left[\frac{KQ}{D_a^{\;3}}\frac{\rho_p^{\;n+1}}{18\mu}\right]^{N/2}$$

$$d_{cp} = \left(\frac{0.6931}{M}\right)^{n+1}$$

until

$$d_{cpr} - d_{cp_{50}} < 1 \times 10^{-7}$$

then save D_a value to D

$$D = D_a$$

3.3.3. Find the minimum amount of cyclones $N_{\min}$ and the maximum amount of cyclones $N_{\max}$

$$N_{\max} = trunc\left(\frac{Q}{15K_a K_b D^2}\right) + 1 \quad (3.2a)$$

$$N_{\min} = trunc\left(\frac{Q}{u_{T_2\max} K_a K_b D^2}\right) + 1$$

if $N_{\min} = 0$ then set $N_{\min}$ to unity (3.2b)

if user type cyclone = True then $N_{\min} = N_{\max} = AC$.

3.3.4. Checked feasible solution respect to all 6 constrains.

2^{nd} constrain: $N_{\min} \le N_o \le N_{\max}$; This constrain is a loop constrain, $N_{\min}$ to $N_{\max}$, for the calculation iteration of the other constrains.

1^{st} constrain: 15 m/s $\le u_{T_2} \le$ ($u_{T_2\max}$, 30 m/s, 1.35 v_s)

Calculate cyclone entry velocity, m/s; v_{en} and v_s from Equation (2.35).

$$v_{en} = \left[\frac{Q}{K_a K_b N D^2}\right]$$

$$v_s = 4.91\left(\frac{4g\mu\left(\rho_p - \rho\right)}{3\rho^2}\right) \times \frac{K_b^{\ 0.4}}{\left(1 - K_b\right)^{1/3}} D^{0.067} v_{en}^{\ 2/3}$$

if $v_{en} > 1.35 v_s$ then print "**Now particle re-entainment**" and **End**.

if $v_{en} < v_i$ m/s and user type = False then

Repeat

$$D = D - 0.0001$$

$$v_{en} = \left(\frac{Q}{K_a K_b N D^2}\right)$$

Until

$$\left(v_{en} - 15\right) \ge 0.0001$$

3rd constrain: $0.5 \le n \le 0.9$

calculate n from Equation (2.20d).

$$n = 1 - \left[1 - 0.67 D^{0.14}\right]\left(\frac{T_1}{283}\right)^{0.3} \quad (2.20d)$$

if $n < 0.5\, or\, n > 0.9$ print "**Cyclone too small or large**" and **End**.

4th constrain: $\Delta P \le 2500$ N/m^2

calculate ΔP from Equation (3.1).

$$\Delta P = \frac{1}{2}(\rho + DL) v_{en}^{2} N_H \quad (3.1)$$

if user type = False and $\Delta P < 2500$ N/m^2 then print "**Volume flow rate too high**" and **End**.

5th constrain: $d_{cp_{50}} \le d_{cpr}$

calculate M and $d_{p_{50}}$ from Equation (2.20b) and (2.20c).

$$M = 2\left[\frac{KQ}{D^3}\frac{\rho_p^{n+1}}{18\mu}\right]^{N/2} \quad and \quad N = \frac{1}{n+1} \quad (2.20b)$$

$$d_{cp_{50}} = \left(\frac{0.6931}{M}\right)^{n+1} \quad (2.20c)$$

if user type = false and $d_{cp_{50}} > 0.85 d_{cpr}$ then next N.

calculate fixed cost, Baht; C_{fixed} from Equation (3.6)

calculate operating cost, Baht; C_{oper} from $c_{power} \times hpy \times yrs$.

calculate total cost, Baht; C_t from Equation (3.3)

$$C_{fixed} = T_{fc} \times N \quad (3.6)$$

$$C_{oper} = Q\Delta P c_e \times hpy \times yrs$$

$$C_t = C_{fixed} + C_{oper} \quad (3.5)$$

store feasible solution data

$amount\ of\ cyclone; N$

$cyclone\ diameter, m; D$

$vortex\ number, n$

$pressure\ drop, N / m^2; \Delta P$

$50\%\ cutsize\ particle\ diameter, m; d_{cp}$

$objective\ value; Obj$

$1.35 v_s$

$total\ fixed\ \cos t, baht; C_{fixed}$

$total\ operating\ \cos t, baht; C_{oper}$

$total\ \cos t, baht; C_t$

next N

3.3.5 Calculation 6^{th} constrain and Grade Objective eq^n value and Grade Efficiency curve

6^{th} constrain: Minimum total cost ; C_t

find the optimized solution of feasible solutions of the cyclone type and its minimum total cost.

$$C_t = C_{fixed} + C_{oper} \quad baht$$

$$C_{fixed} = T_{fc} \times N \quad baht$$

$$C_{oper} = Q \Delta P c_e \times hpy \times yrs \quad baht$$

$$const = \left[h + (1+B)(H-h) + D_e \times S \right]$$

$$A_{cy} = cont \times \pi \times D^2 \times N \quad m^2$$

$$W_{cy} = A_{cy} \times Wt \quad kg$$

$$M_{cy} = W_{cy} \times Mac \quad baht$$

$$F_{cy} = W_{cy} \times Fac \quad baht$$

$$R_{cy} = Roc \times Rpa \times A_{cy} \quad baht$$

$$T_{fc} = M_{cy} + F_{cy} + R_{cy} \quad baht$$

if there are more than 1 group of feasible solutions which have the same minimum total cost. Find the maximum Objective value among them.

Calculation Grade Objective eq^n value and Grade Efficiency curve at the optimized solution from particle diameter 1-15 μm.

$$\eta_i = \left[1 - \exp\left(-Md_{p_i}^{\frac{1}{n+1}}\right)\right]$$

$$Obj_i = \left[1 - \exp\left(-Md_{p_i}^{\frac{1}{n+1}}\right)\right] / \Delta P$$

calculation Overall Grade Efficiency from specific particle diameter range at the optimized solution.

$$g_i = \frac{m_i}{Sum(m_i)}$$

$$\eta_i = \left[1 - \exp\left(-Md_{p_{avr_i}}^{\frac{1}{n+1}}\right)\right]$$

$$Overall\ grade\ Eff. = Sum(g_i \times \eta_i)$$

finally print out the optimized solution and the following details of each cyclone type.

$cyclone\ types\ (user\ selected\ types\ only)$
$amount\ of\ cyclone;\ N$
$cyclone\ diameter, m;\ D$
$pressure\ drop, N/m^2;\ \Delta P$
$50\%\ cutsize\ particle\ diameter, m;\ d_{cp_{50}}$
$cutsize\ particle\ diameter, m;\ d_{cpr}$
$objective\ value;\ Obj$

$total\ fixed\ cost, baht;\ C_{fixed}$
$total\ operating\ cost, baht;\ C_{oper}$
$total\ cost, baht;\ C_t$

End.

3.4 Calculation example

This calculation example for standard cyclone type, Shepherd and Lapple type in which the same type that used for the experiment cyclone and the experiment cyclone at the experiment Batch No.1

Details of the test condition the experiment Batch No.1 and the experiment cyclone configuration show in the following Table and Table 3.4 to Table 3.5 below.

Table 3.4 Test condition measurement No.1 and cyclone configuration for both Shepherd and Lapple type standard cyclone and the experiment cyclone.

Parameter	Operating condition	
	Experiment cyclone	Shepherd and Lapple cyclone
K_a	0.5	0.5
K_b	0.25	0.25
N_H	8.0	8.0
d_{cpr} ; m	3.20×10^{-6}	3.20×10^{-6}
Q ; m^3/s	1.501	1.501
ρ_p ; Kg/m^3	1500	1500
ρ ; Kg/m^3	0.73625	0.73625
μ ; Pa.s	25.97×10^{-6}	25.97×10^{-6}
DL ; Kg/m^3	0.0001919	0.0001919
Temperature ; K°	473	473
Cyclone Diameter ; m	0.96	-
Operating hours/year	6,000	6,000
Y ; year	5	5
N	1	-
c_e ; Baht/unit	1.07	1.07
Thk ; mm	2.0	2.0
Roc ; $Baht/m^2$	350.0	350.0
Mac ; Baht/kg of steel	25.0	25.0
Wt ; kg of $steel/m^2$	19.62	19.62

Table 3.5 Particle analysis report operating condition measurement No.1

Sampling Date : June 30,1995 Date : July 26,1995

Sample designated as : Stack air emission/boiler Page 1/3

Parameter	Unit	Value	Standard
Stack diameter	m	0.48	-
Temperature	^{0}C	200	-
Air velocity	m/s	8.29	-
Emission rate**	m^3/s	1.501	-
Humidity	%	8.42	-
TSP_{in}	mg/Nm^3	457.30	-
TSP_{out}	mg/Nm^3	149.38	300*
NO_2	mg/Nm^3	82.50	470*
SO_2	mg/Nm^3	395.62	400***
ΔP	N/m^2	510.6	-
ρ_p	-	1.50	-
0 - 5 μm	g	35.8	
<5 -10 μm	g	22.3	
<10 - 15 μm	g	14.9	
<15 - 20 μm	g	6.4	
<20 -30 μm	g	7.1	
<30 -50 μm	g	7.0	
<50 - 100 μm	g	6.5	

Remark : * Notification of Ministry of Industrial Vol. 1993

** Flue condition

*** Recommended Value by Industrial Work Department Ministry of Industry

N Normal Temperature and Pressure ($25^{0}C$, 760 mmHg)

3.4.1. Find the maximum inlet velocity $u_{T_2 \max}$ in m/s at $\Delta P \le 2500$ N/m^2.

if $u_{T_2 \max} > 30$ m/s then $u_{T_2 \max} = 30$ m/s

if $u_{T_2 \max} < 15$ m/s then print "Please input new data cause $u_{T_2 \max} < 15$ m/s" and End.

$$\Delta P = \frac{1}{2}\rho_g u_{T_2}^2 N_H \quad \text{or} \quad \frac{1}{2}\rho_g\left(\frac{Q}{K_a K_b N}\right)^2 N_H \tag{3.1}$$

$$u_{T_2 \max} \le \left[\frac{2\times 2{,}500}{(\rho + DL)N_H}\right]^{1/2}$$

$$u_{T_2 \max} \le \left[\frac{2\times 2{,}500}{(1{,}500 + 0.0001919)8}\right]^{1/2}$$

$$u_{T_2 \max} \le 29.132 \quad m/s$$

3.4.2. Find assume cyclone diameter, m; D_a by using the 5th constrain, $d_{cp} \le d_{cpr}$. First let $D_a = 0.3\,m$ then repeat $D_a = D_a + 0.0001$ until $d_{cpr} - d_{cp_{50}} < 1\times 10^{-7}$ then save D_a value to D last calculation this procedure is

$$D_a = 0.971$$

$$n = 1 - \left[1 - 0.67(D_a)^{0.14}\right]\left(\frac{T_1}{283}\right)^{0.3}$$

$$n = 1 - \left[1 - 0.67(0.971)^{0.14}\right]\left(\frac{473}{283}\right)^{0.3}$$

$$n = 0.611808$$

$$M = 2\left[\frac{KQ}{D_a^{\,3}}\frac{\rho_p^{\,n+1}}{18\mu}\right]^{\frac{1}{2n+2}}$$

$$M = 2\left[\frac{402.9\times 1.501}{0.971^3}\frac{1{,}500^{0.611808+1}}{18\times 2.6\times 10^{-5}}\right]^{0.31021}$$

$$M = 1{,}813.211$$

$$d_{cp_{50}} = \left(\frac{0.6931}{M}\right)^{n+1}$$

$$d_{cp_{50}} = \left(\frac{0.6931}{1,813.211}\right)^{1.611808}$$

$$d_{cp_{50}} = 3.01 \times 10^{-6} \quad m$$

$$D = 0.971 \quad m$$

3.4.3. Find the minimum amount of cyclones $N_{\min}$ and the maximum amount of cyclones $N_{\max}$

if $N_{\min}$ = 0 then set $N_{\min}$ to unity (3.2b)

if user type cyclone = True then $N_{\min} = N_{\max} = AC$.

$$N_{\max} = trunc\left(\frac{Q}{15K_aK_bD^2}\right) + 1$$

$$N_{\max} = trunc\left(\frac{1.501}{15 \times 0.5 \times 0.25 \times 0.895^2}\right) + \qquad (3.2a)$$

$$N_{\max} = 1$$

$$N_{\min} = \left(\frac{Q}{u_{T_2\max}K_aK_bD^2}\right)$$

$$N_{\min} = trunc\left(\frac{1.501}{29.132 \times 0.5 \times 0.25 \times 0.895^2}\right) + 1$$

$$N_{\min} = 1$$

3.4.4 Checked feasible solution respect to all 6 constrains.

2nd constrain: $N_{\min} \le N_o \le N_{\max}$; This constrain is a loop constrain, $N_{\min}$ to $N_{\max}$, for the calculation iteration of the other constrains.

$$N_{\min} \le N_o \le N_{\max}$$

$$1 \le N_o \le 1$$

1st constrain: 15 m/s $\le u_{T_2} \le$ ($u_{T_2\max}$, 30 m/s, 1.35 v_s)

Calculate cyclone entry velocity, m/s; v_{en} and v_s from Equation (2.35).

$$v_{en}=\left[\frac{Q}{K_a K_b N D^2}\right]$$

$$v_{en}=\left[\frac{1.501}{0.5\times 0.25\times 1\times 0.971^2}\right]$$

$$v_{en}=12.736 \quad m/s$$

$$v_s=4.91\left(\frac{4g\mu\left(\rho_p-\rho\right)}{3\rho^2}\right)\times\frac{K_b^{\ 0.4}}{\left(1-K_b\right)^{1/3}}D^{0.067}v_{en}^{\ 2/3}$$

$$v_s=4.91\left(\frac{49.81\times 2.6\times 10^{-5}(1{,}500-0.73625)}{3\times 0.73625^2}\right)$$

$$\times\frac{0.25^{0.4}}{(1-0.25)^{1/3}}0.971^{0.067}12.736^{2/3}$$

$$v_s=16.547 \quad m/s$$

if $v_{en}>1.35v_s$ then print "**Now particle re-entainment**" and **End**.

Now $v_{en}<15\,m/s<1.35v_s$

if $v_{en}<15$ m/s and user type = False then adjust D till $\left(v_{en}-15\right)\geq 0.0001$

last calculation in this procedure is

$$D=0.895 \quad m$$

$$v_{en}=\left(\frac{Q}{K_a K_b N D^2}\right)$$

$$v_{en}=\left(\frac{1.501}{0.5\times 0.25\times 1\times 0.895^2}\right)$$

$$v_{en}=15.00 \quad m/s$$

3rd constrain: $0.5\leq n\leq 0.9$

calculate n from Equation (2.20d).

$$D_a = 0.895$$

$$n = 1 - \left[1 - 0.67\left(D_a\right)^{0.14}\right]\left(\frac{T_1}{283}\right)^{0.3}$$

$$n = 1 - \left[1 - 0.67(0.895)^{0.14}\right]\left(\frac{473}{283}\right)^{0.3}$$

$$n = 0.60294$$

if $n < 0.5$ *or* $n > 0.9$ print "**Cyclone too small or large**" and **End**.

Now $0.5 \le n \le 0.9$

4^{th} constrain: $\Delta P \le 2500$ N/m^2

calculate ΔP from Equation (3.1).

$$\Delta P = \frac{1}{2}\left(\rho + DL\right){v_{en}}^2 N_H$$

$$\Delta P = \frac{1}{2}(0.73625 + 0.0001919)15.00^2 \times 8$$

$$\Delta P = 662.7837 \quad N/m^2$$

Now $\Delta P \le 2500$ N/m^2

if user type = False and $\Delta P < 2500$ N/m^2 then print "**Volume flow rate too high**" and **End**.

5^{th} constrain: $d_{cp_{50}} \le d_{cpr}$

calculate M and $d_{p_{50}}$ from Equation (2.20b) and (2.20c).

if user type = false and $d_{cp_{50}} > 0.85 d_{cpr}$ then *increase* N.

$$M = 2\left[\frac{KQ}{{D_a}^3}\frac{{\rho_p}^{n+1}}{18\mu}\right]^{\frac{1}{2n+2}}$$

$$M = 2\left[\frac{402.9 \times 1.501}{0.895^3}\frac{1{,}500^{0.60294+1}}{18 \times 2.6 \times 10^{-5}}\right]^{0.31193}$$

$$M = 2{,}029.087$$

$$d_{cp_{50}} = \left(\frac{0.6931}{M}\right)^{n+1}$$

$$d_{cp_{50}} = \left(\frac{0.6931}{2{,}029.087}\right)^{1.60294}$$

$$d_{cp_{50}} = 2.78 \times 10^{-6} \quad m$$

Now $d_{cp_{50}} > 0.85 d_{cpr}$; $d_{cp_{50}} = 2.78 \times 10^{-6}\ m > 0.85 d_{cpr} = 2.72 \times 10^{-6}\ m$ then *increase* N the next calculation $N = 2$

calculate fixed cost, Baht; C_{fixed} from Equation (3.6)

calculate operating cost, Baht; C_{oper} from $c_{power} \times hpy \times yrs$.

calculate total cost, Baht; C_t from Equation (3.3) and store the feasible solution answer.

$$const = \left[h + (1+B)(H-h) + D_e \times S\right]$$
$$const = [2 + (1+0.25)(4-2) + 0.5 \times 0.625]$$
$$const = 4.8125$$
$$A_{cy} = cont \times \pi \times D^2 \times N$$
$$A_{cy} = 4.8125 \times \pi \times 0.895^2 \times 1$$
$$A_{cy} = 12.10819 \quad m^2$$
$$W_{cy} = A_{cy} \times Wt$$
$$W_{cy} = 12.10819 \times 19.62$$
$$W_{cy} = 237.56277 \quad kg$$
$$M_{cy} = W_{cy} \times Mac$$
$$M_{cy} = 237.56277 \times 25.0$$
$$M_{cy} = 5{,}935.07 \quad baht$$

$$F_{cy} = W_{cy} \times Fac$$
$$F_{cy} = 237.56277 \times 18.0$$
$$F_{cy} = 4{,}276.13 \quad baht$$
$$R_{cy} = Roc \times Rpa \times A_{cy}$$
$$R_{cy} = 350.0 \times 1.10 \times 12.10819$$
$$R_{cy} = 4{,}661.65 \quad baht$$
$$T_{fc} = M_{cy} + F_{cy} + R_{cy}$$
$$T_{fc} = 237.56277 + 4{,}276.13 + 4{,}661.65$$
$$T_{fc} = 14{,}876.85 \quad baht$$

$$C_{fixed} = T_{fc} \times N$$
$$C_{fixed} = 14{,}876.85 \times 1$$
$$C_{fixed} = 14{,}876.85 \quad baht$$

$$C_{oper} = Q\Delta Pc_e \times hpy \times yrs$$
$$C_{oper} = 1.501 \times 662.7837 \times 6{,}000 \times 5$$
$$C_{oper} = 31{,}934.31 \quad baht$$
$$C_t = C_{fixed} + C_{oper}$$
$$C_t = 14{,}876.85 + 31{,}934.3$$
$$C_t = 46{,}811.17 \quad baht$$

Table 3.6 Feasible solution at $N = 1$

Parameter	Unit	Value
amount of cyclone ; N	-	1
cyclone diameter ; D	m	0.895
vortex number ; n	-	0.60294
pressure drop ; ΔP	N/m^2	662.7837
50% cutsize particle diameter ; $d_{cp_{50}}$	m	2.78×10^{-6}
Objective Equation value	-	8.0134×10^{-4}
$1.35 v_s$	m/s	24.7758
total fixed cost ; C_{fixed}	baht	14,876.85
total operating cost ; C_{oper}	baht	31,934.31
total cost ; C_t	baht	46,811.17

next N

For next N at $N = 2$ repeat Calculation 3.3.1 to 3.3.4 in which check 1^{st} constrain to 5^{th} constrain and store feasible solution data shown in Table 3.7

Table 3.7 Feasible solution at $N = 2$

Parameter	Unit	Value
amount of cyclone ; N	-	2
cyclone diameter ; D	m	0.633
vortex number ; n	-	0.56650
pressure drop ; ΔP	N/m^2	662.7822
50% cutsize particle diameter ; $d_{cp_{50}}$	m	2.45×10^{-6}
Objective Equation value	-	8.4494×10^{-4}
$1.35v_s$	m/s	24.2071
total fixed cost ; C_{fixed}	baht	15,300.66
total operating cost ; C_{oper}	baht	31,934.24
total cost ; C_t	baht	47,234.90

3.4.5 Calculation 6^{th} constrain and grade objective eq^{n} value and grade efficiency curve

6^{th} constrain: Minimum total cost ; C_t

Find the optimized solution of feasible solutions of the cyclone type and its minimum total cost by comparing total cost ; C_t from $N = 1$ from Table 3.6 between $N = 2$ from Table 3.7 then the optimized solution is $N = 1$ Table 3.6 which give the lowest C_t.

calculation grade objective eq^{n} value and grade efficiency curve at the optimized solution from particle diameter 1-15 μm

$$\eta_i = \left[1 - \exp\left(-Md_{p_i}^{\frac{1}{n+1}}\right)\right]$$

$$Obj_i = \left[1 - \exp\left(-Md_{p_i}^{\frac{1}{n+1}}\right)\right] / \Delta P$$

$$\eta_{1\times10^{-6}} = \left[1 - \exp\left(-2{,}029.087 \times 1\times10^{-6\,\frac{1}{0.60294+1}}\right)\right]$$

$$\eta_{1\times10^{-6}} = 0.30691$$

$$\eta_{2\times10^{-6}} = 0.43159$$

$$\vdots$$

$$\eta_{15\times10^{-6}} = 0.86270$$

$$Obj_{1\times10^{-6}} = \left[1-\exp\left(-2{,}029.087\times 1\times 10^{-6\,\frac{1}{0.60294+1}}\right)\right]/\,662.7837$$

$$Obj_{1\times10^{-6}} = 4.6306\times10^{-4}$$

$$Obj_{2\times10^{-6}} = 6.5118\times10^{-4}$$

$$\vdots$$

$$Obj_{15\times10^{-6}} = 13.0164\times10^{-4}$$

calculation overall grade efficiency from specific particle diameter range at the optimized solution.

$$g_i = \frac{m_i}{Sum(m_i)}$$

$$\eta_i = \left[1-\exp\left(-Md_{p_{avr_i}}^{\frac{1}{n+1}}\right)\right]$$

$$\eta_i = \left[1-\exp\left(-2{,}028.357\times 2.5\times 10^{-6\,\frac{1}{0.60294+1}}\right)\right]$$

$$\eta_i = 0.4775$$

Calculation answer show in Table 3.8

Table 3.8 Overall grade efficiency of the optimized solution.

$d_p\times10^6$; m	$d_{p_{avr}}\times10^6$; m	M_i ; g	g_i	η_i (%)	$\eta_i\times g_i$ (%)
0.0 - 5.0	2.50	35.8	0.358	47.75	17.09
<5.0 - 10.0	7.50	22.3	0.223	72.42	16.15
<10.0 - 15.0	12.50	14.9	0.149	82.99	12.37
<15.0 - 20.0	17.50	6.4	0.064	88.76	5.68
<20.0 - 30.0	25.0	7.1	0.071	93.48	6.64
<30.0 - 50.0	40.0	7.0	0.070	97.43	6.82
<50.0 - 100.0	75.0	6.5	0.065	99.56	6.47
Sum	-	100	1.0	-	71.23

finally print out the optimized solution and the following details of each cyclone type.

$cyclone\ types\ (user\ selected\ types\ only)$

$amount\ of\ cyclone;\ N$

$cyclone\ diameter, m;\ D$

$pressure\ drop, N/m^2;\ \Delta P$

$50\%\ cutsize\ particle\ diameter, m;\ d_{cp_{50}}$

$cutsize\ particle\ diameter, m;\ d_{cpr}$

$objective\ value;\ Obj$

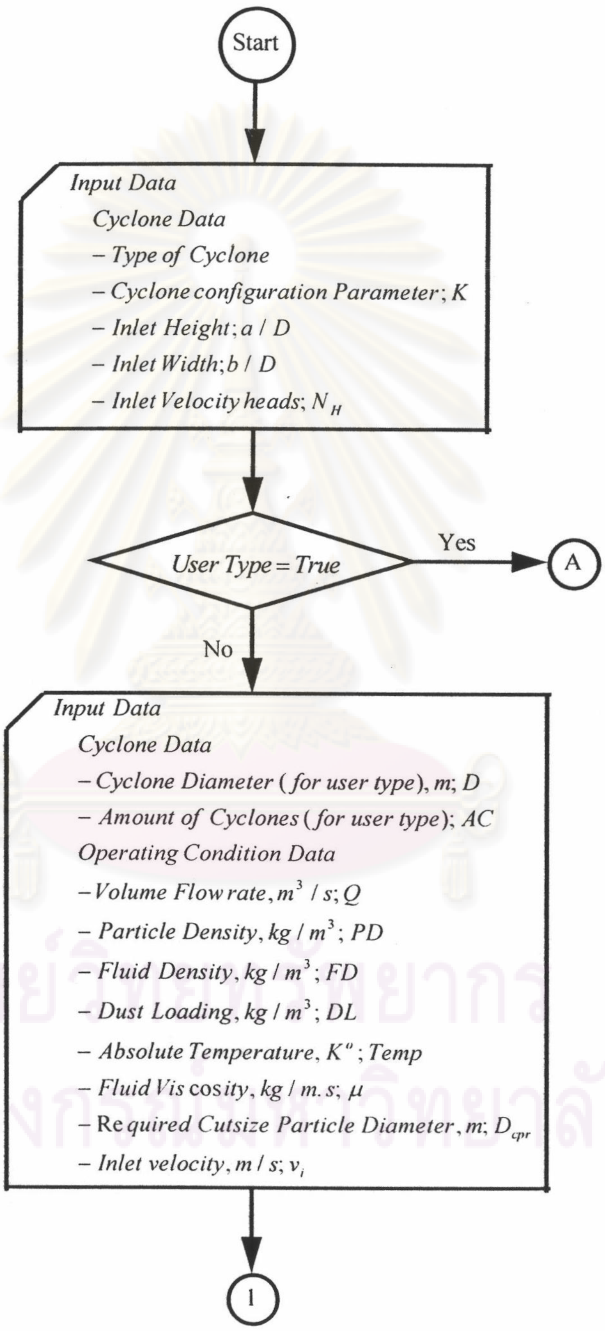
$total\ fixed\ \cos t, baht;\ C_{fixed}$

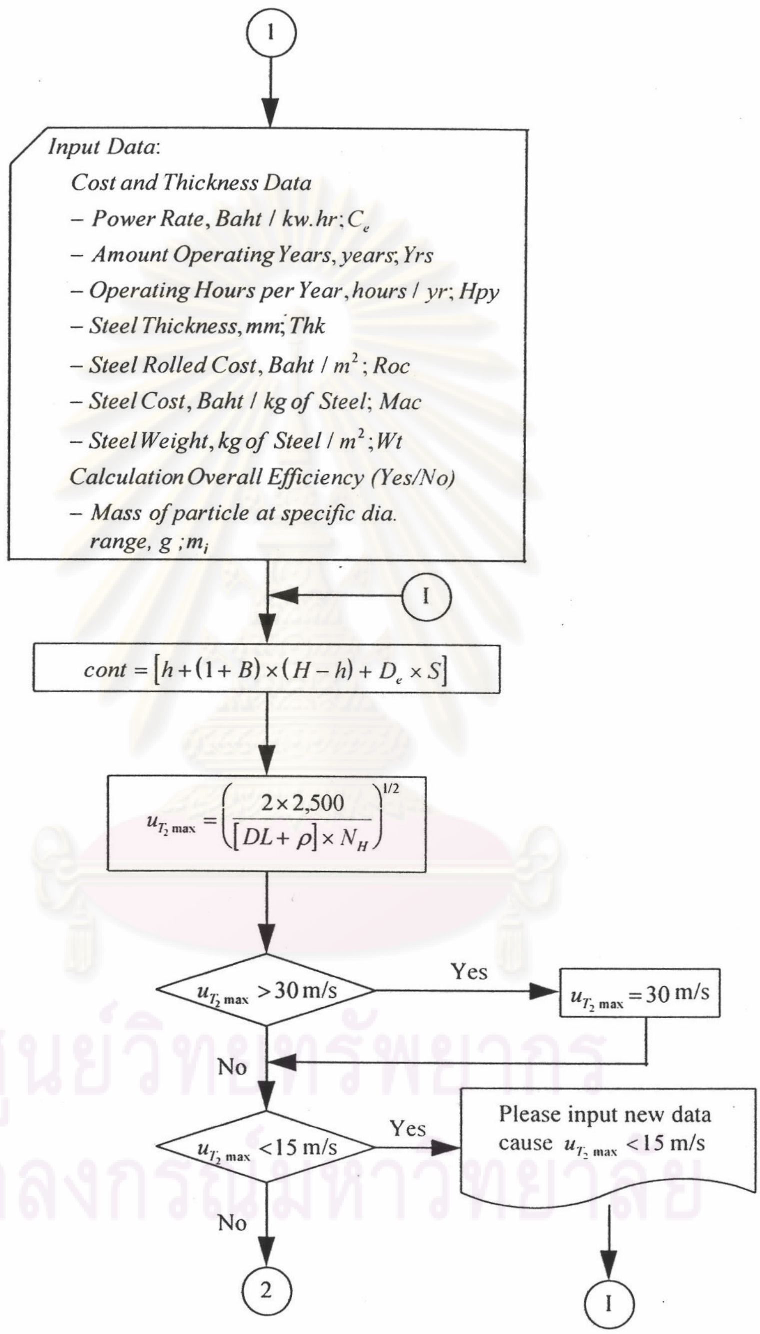
$total\ operating\ \cos t, baht;\ C_{oper}$

$total\ \cos t, baht;\ C_t$

End.

3.5 Program flowchart




1
Input Data:
Cost and Thickness Data
– Power Rate, Baht / kw.hr; C_e
– Amount Operating Years, years; Yrs
– Operating Hours per Year, hours / yr; Hpy
– Steel Thickness, mm; Thk
– Steel Rolled Cost, Baht / m^2; Roc
– Steel Cost, Baht / kg of Steel; Mac
– Steel Weight, kg of Steel / m^2; Wt
Calculation Overall Efficiency (Yes/No)
– Mass of particle at specific dia. range, g ; m_i
I
$cont = [h + (1 + B) \times (H - h) + D_e \times S]$
$u_{T_2 \max} = \left(\frac{2 \times 2{,}500}{[DL + \rho] \times N_H} \right)^{1/2}$
$u_{T_2 \max} > 30$ m/s
Yes
$u_{T_2 \max} = 30$ m/s
No
$u_{T_2 \max} < 15$ m/s
Yes
Please input new data cause $u_{T_2 \max} < 15$ m/s
No
2
I

(2)

$$D_a = 0.3$$

$\mathrm{Re}\,peat$

$$D_a = D_a + 0.001$$

$$n = 1 - \left[1 - 0.67\left(D_a\right)^{0.14}\right]\left(\frac{T}{283}\right)^{0.3}$$

$$M = 2\left[\frac{KQ}{D_a^{\,3}}\,\frac{\rho_p^{\;n+1}}{18\mu}\right]^{N/2}$$

$$D_{c\,p_{50}} = \left(\frac{0.6931}{M}\right)^{n+1}$$

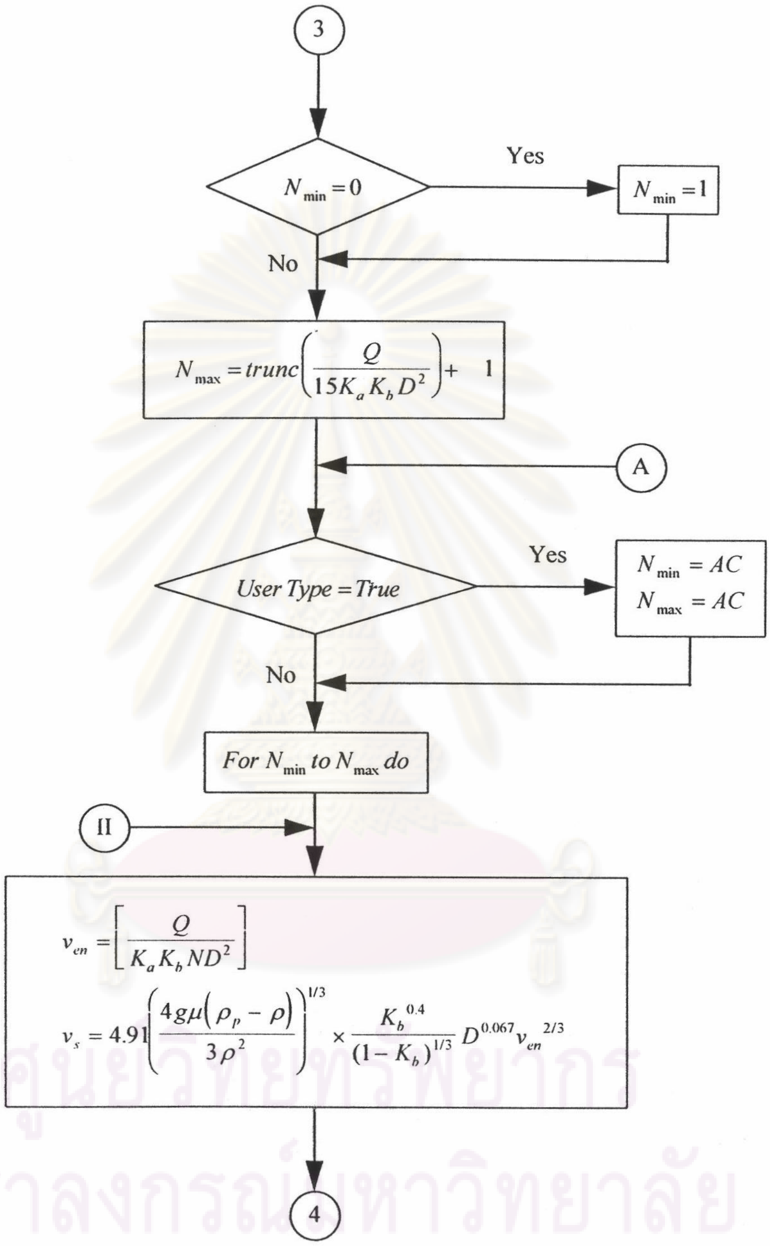
Until

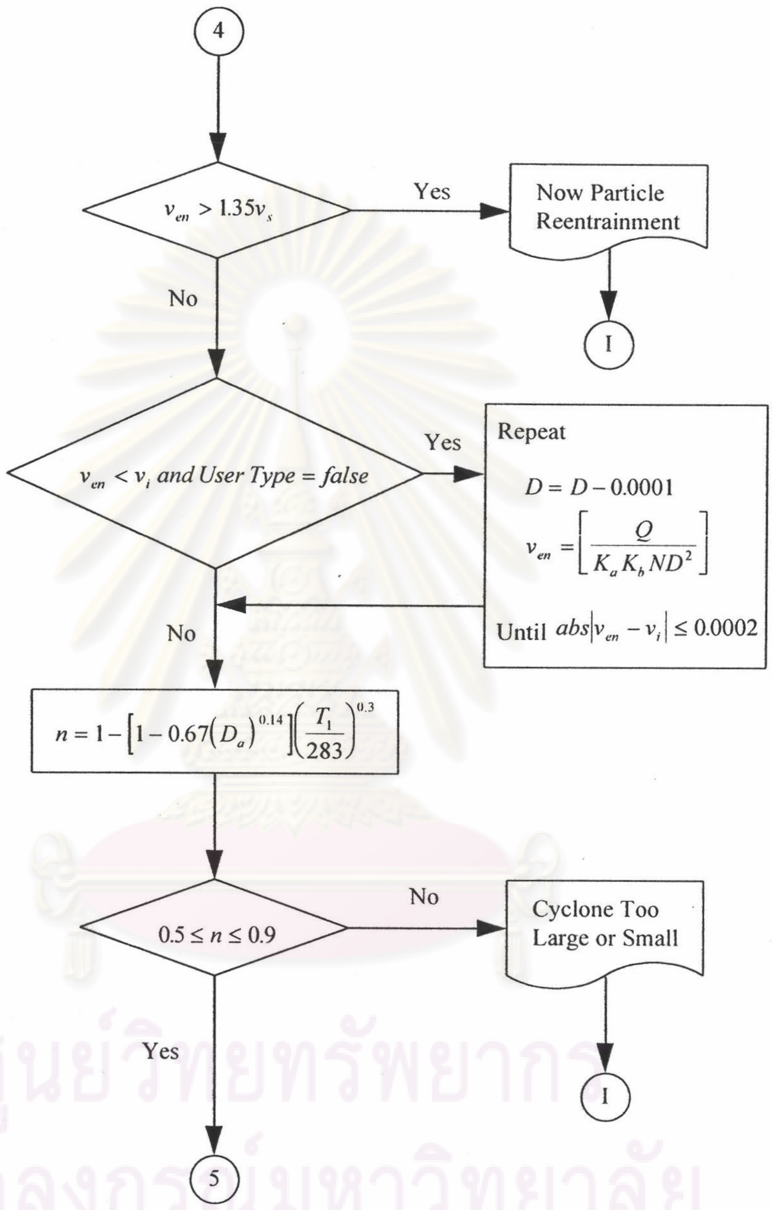
$$D_{c\,pr} - D_{c\,p_{50}} < 1\times 10^{-7}$$

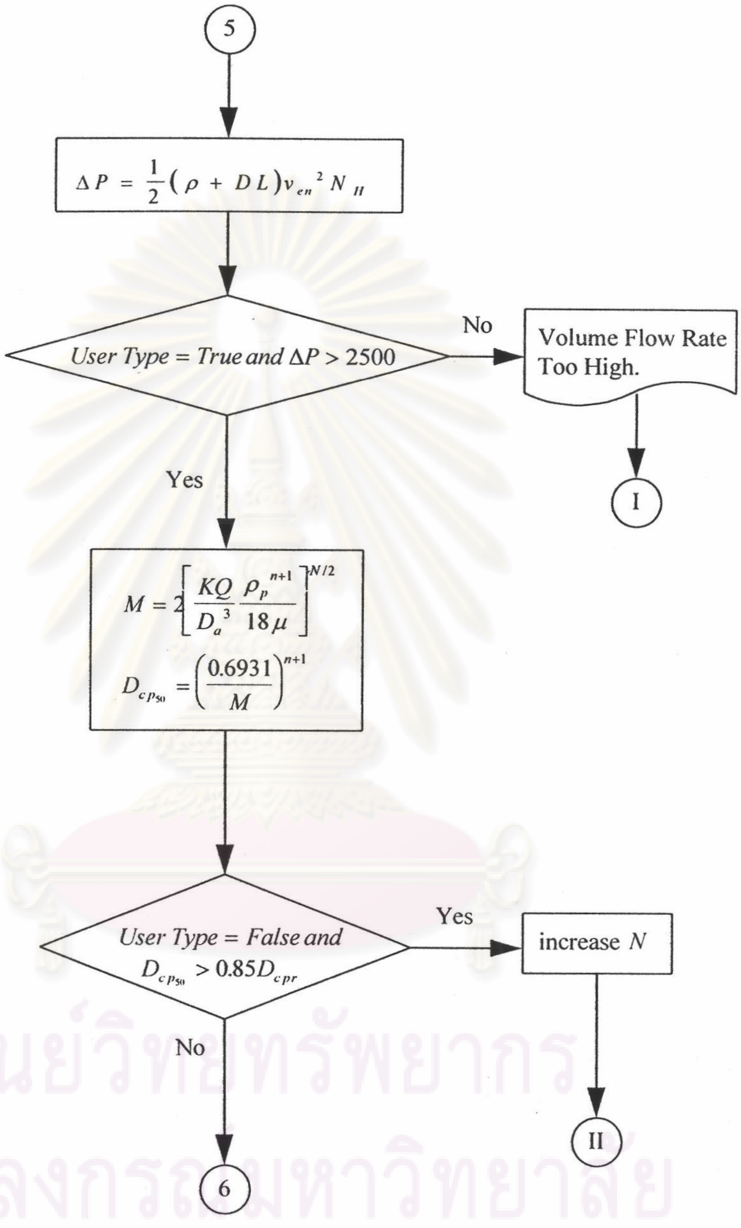
$$D = D_a$$

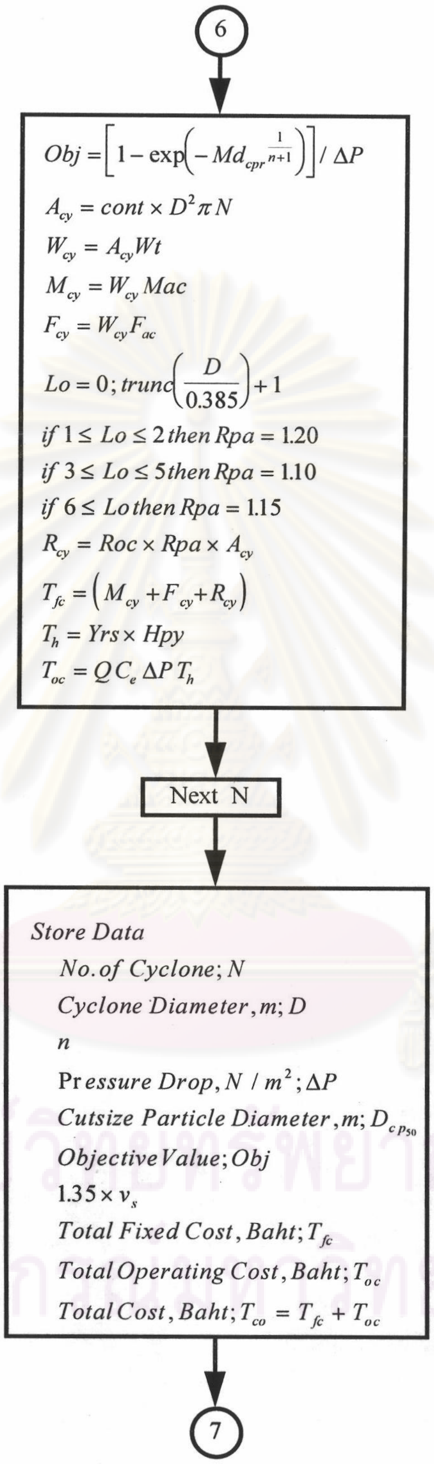
$$N_{\min} = trunc\left(\frac{Q}{K_a K_b u_{T_2\,\max} D^2}\right)$$

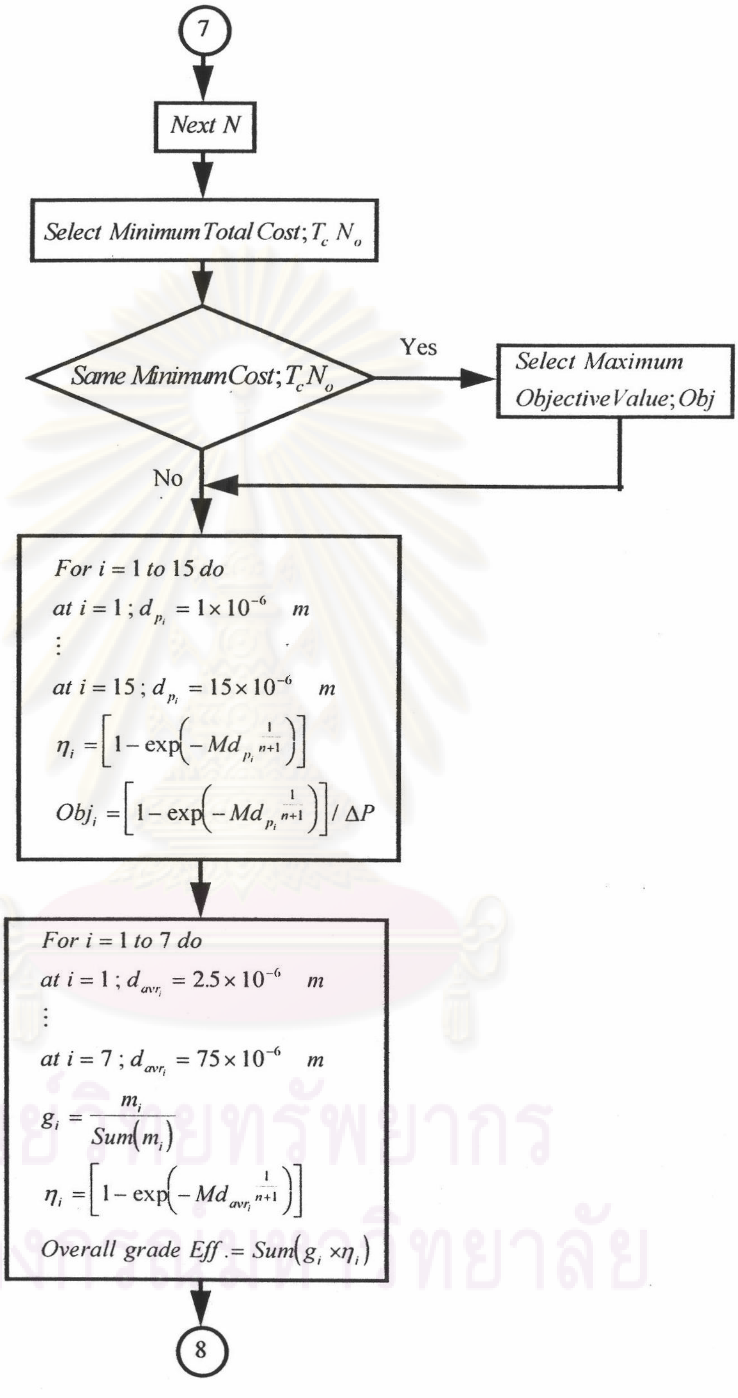
(3)

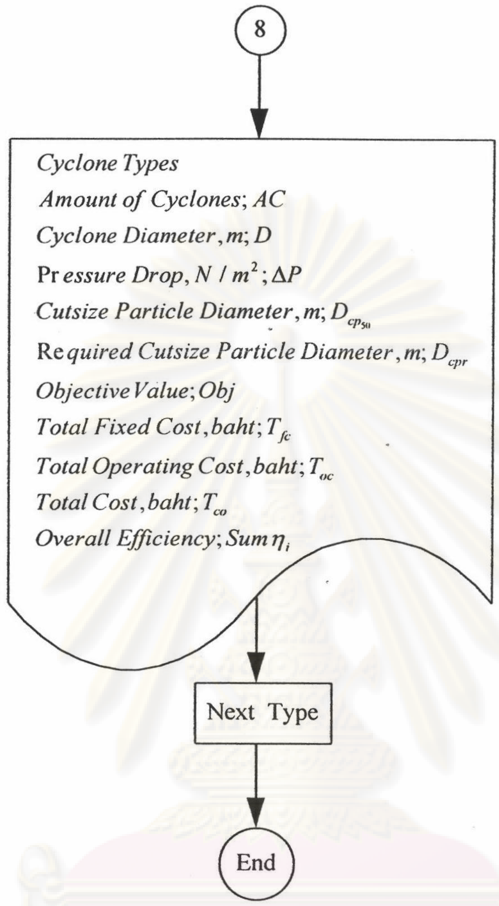

3
N min = 0
Yes
N min = 1
No
N max = trunc(Q / (15 Ka Kb D^2)) + 1
A
User Type = True
Yes
N min = AC
N max = AC
No
For N min to N max do
II
v_en = [Q / (Ka Kb N D^2)]
v_s = 4.91 (4gμ(ρ_p − ρ) / 3ρ^2)^(1/3) × Kb^0.4 / (1 − Kb)^(1/3) D^0.067 v_en^(2/3)
4


4
$v_{en} > 1.35v_s$
Yes
Now Particle Reentrainment
I
No
$v_{en} < v_i$ and User Type = false
Yes
Repeat
$D = D - 0.0001$
$v_{en} = \left[\frac{Q}{K_a K_b N D^2}\right]$
Until $abs|v_{en} - v_i| \le 0.0002$
No
$n = 1 - \left[1 - 0.67(D_a)^{0.14}\right]\left(\frac{T_1}{283}\right)^{0.3}$
$0.5 \le n \le 0.9$
No
Cyclone Too Large or Small
I
Yes
5


5
$\Delta P = \frac{1}{2}(\rho + DL)v_{en}^2 N_H$
User Type = True and $\Delta P > 2500$
No
Volume Flow Rate Too High.
I
Yes
$M = 2\left[\frac{KQ}{D_a^3}\frac{\rho_p^{n+1}}{18\mu}\right]^{N/2}$
$D_{cp_{50}} = \left(\frac{0.6931}{M}\right)^{n+1}$
User Type = False and $D_{cp_{50}} > 0.85D_{cpr}$
Yes
increase N
II
No
6


6
$Obj = \left[1 - \exp\left(-Md_{cpr}^{\frac{1}{n+1}}\right)\right] / \Delta P$
$A_{cy} = cont \times D^2 \pi N$
$W_{cy} = A_{cy} Wt$
$M_{cy} = W_{cy} Mac$
$F_{cy} = W_{cy} F_{ac}$
$Lo = 0; trunc\left(\frac{D}{0.385}\right) + 1$
$if\ 1 \le Lo \le 2\ then\ Rpa = 1.20$
$if\ 3 \le Lo \le 5\ then\ Rpa = 1.10$
$if\ 6 \le Lo\ then\ Rpa = 1.15$
$R_{cy} = Roc \times Rpa \times A_{cy}$
$T_{fc} = \left(M_{cy} + F_{cy} + R_{cy}\right)$
$T_h = Yrs \times Hpy$
$T_{oc} = Q C_e \Delta P T_h$
Next N
Store Data
No. of Cyclone; N
Cyclone Diameter, m; D
n
Pressure Drop, N / m^2; ΔP
Cutsize Particle Diameter, m; $D_{c p_{50}}$
Objective Value; Obj
$1.35 \times v_s$
Total Fixed Cost, Baht; T_{fc}
Total Operating Cost, Baht; T_{oc}
Total Cost, Baht; $T_{co} = T_{fc} + T_{oc}$
7


7
Next N
Select Minimum Total Cost; $T_c\ N_o$
Same Minimum Cost; $T_c N_o$
Yes
Select Maximum Objective Value; Obj
No
For i = 1 to 15 do
at i = 1 ; $d_{p_i} = 1\times10^{-6}$ m
⋮
at i = 15 ; $d_{p_i} = 15\times10^{-6}$ m
$\eta_i = \left[1-\exp\left(-Md_{p_i}^{\frac{1}{n+1}}\right)\right]$
$Obj_i = \left[1-\exp\left(-Md_{p_i}^{\frac{1}{n+1}}\right)\right]/\Delta P$
For i = 1 to 7 do
at i = 1 ; $d_{avr_i} = 2.5\times10^{-6}$ m
⋮
at i = 7 ; $d_{avr_i} = 75\times10^{-6}$ m
$g_i = \frac{m_i}{Sum(m_i)}$
$\eta_i = \left[1-\exp\left(-Md_{avr_i}^{\frac{1}{n+1}}\right)\right]$
Overall grade Eff. = $Sum(g_i \times \eta_i)$
8


8
Cyclone Types
Amount of Cyclones; AC
Cyclone Diameter, m; D
Pressure Drop, N / m²; ΔP
Cutsize Particle Diameter, m; D_cp50
Required Cutsize Particle Diameter, m; D_cpr
Objective Value; Obj
Total Fixed Cost, baht; T_fc
Total Operating Cost, baht; T_oc
Total Cost, baht; T_co
Overall Efficiency; Sum η_i
Next Type
End