Chapter 5

Concluding Remarks and Suggestions

5.1 Concluding Remarks

A large-sample approximation of the maximum likelihood estimator is proposed herein for joint estimation of nominal angles and angular spread due to mutiple spatially distributed sources. Asymptotic distribution assessment of AML estimate indicates that it holds the efficiency of exact ML performance in large sample. Rather than $(3N_s + 1)$ parameter spaces as required by the exact ML estimator [9], one having to concentrate on is that this enables us to only a $2N_s$-dimensional optimization. Numerical simulations of joint estimations are also investigated against most popular estimators existed. Under the same dimensional search, it is noteworthy that the AML is higher in performance than the *l*east *s*quare (LS) criterion [40] and the *a*symptotic *b*est *c*onsistent (ABC) estimator [9] in various aspects. It yields, in addition, satisfactory tradeoff with respect to the exact ML estimator at small number of samples.

A variational expression between the redundancy averaging [47] and weighted covariance-matching [48] is first explored. It is indicated that the RA methodology is equivalent to the unweighed version of the WCM loss function. According to the principle of best linear unbiased estimator, the WCM estimate with optimal weight outperforms that given from the RA method in any Gauss-Makov model. Such a statement is then verifiable when incorporating both Toeplitzifications into the spatially distributed source localization. Replacing the consistent weight matrix due to sample covariance estimate with one due to RA or WCM covariance estimate, numerical examples also illustrate that the RMSEs of estimated directions can be improved over the ordinary weighted least squares criteria in single source case [37]. As seen pictorially, the RMSEs of WLS invoked RA and WCM agree well with the relationship indicated herein.

An estimation of the matrix with linearly affine structure is proposed and then verified that it yields the same solution of the previous WLS approach. This enables us to a deep insight into the relationship between AML and WLS. For estimating the nominal direction, an improvement of spatially distributed source has been proposed. This contribution is due to the Toeplitz structure imposed in array covariance matrix. Replacing the ordinary sample covariance matrix with the Toeplitz-structured covariance estimate in the large-sample approximated maximum likelihood estimator, numerical examples emphasize that it yields better in non-asymptotic performance than one losing this additional knowledge.

5.2 Suggestions

- In our experience, the the effect of angular truncations in Gaussian and Laplacian cases still wait for an analysis of the unattainable gaps between exact ML and CRB, especially in high SNR.
- The AML estimator can be extended to the imperfect spial wavefront model.
- Note that one may substitute the Toeplitz-constrained matrix into its underlying loss function. This would more refine the Toeplitz structure, especially in small number of temporal snapshots.
- Replacing the RA covariance estimate with the WCM, it may improve the performance of RACM estimator.