Chapter 4

NUMERICAL EXAMPLES

In order to investigate the effectiveness of the proposed iterative technique, several example problems were solved using the new iterative scheme together with the others. The numbers of iterative cycles nacessary to obtain the desired equilibrium configuration from each iterative method were compared in order to assess the rate of convergence.

The rate of convergence can be studied by investigating the number of cycles to accomplish successfully the deformed shape of the structure or considering the computational time. Since there are some differences among the various schemes regarding the computation procedure employed, the comparison of the computation time is more meaningful. However, both of them are included in this report. All problems in this study were solved on the PRIME 9750 computer.

Due to lack of complete information on initial configuration of example problems given by some authors, it was necessary to assume some reasonable initial state which might have been different from the original paper in order to run the problems. Consequently, it is not possible to compare the results from different sources. Therefore, the NONSAP program (5) was also employed to solve the exmple problems using the same initial configuration in both analyses.

Example problem 1 An isolated cable hanging in vertical plane YZ subjected to a concentrated load as shown in figure 4.1 was first studied. As in Jayaraman & Knudson (13), this cable was modelled by 2-catenary elements. The vertical live load of 8 kips was applied to the cable under self-weight equilibrium configuration at node 2. Other relevant data are given in table 4.1. Three iterative schemes, namely, Newton-Raphson, Kar method, and the proposed iterative technique were used to solved the problem. In the proposed scheme the initial displacement was divided into 15 segments.

Figure 4.2 delineates the vertical displacement at node 2 to the numbers of iteration. Converged solution of the Newton-Raphson method, the Kar method, and the proposed iterative technique were obtained after 6, 4 and 3 cycles, respectively. It is interesting to note that very accurate estimate of the displacement was obtained even within the first cycle of iteration in the proposed technique. However, the computational time is not much different for all methods (see table 4.2). The reason could be explained by examining the algorithm of those schemes. In every cycle, while the Newton-Raphson method requires only 1 execution to generate the end forces of a catenary element by flexibility iteration, it takes twice in the Kar method in which the second operation is needed to evaluate the modifying factor. Depending on the flexibility iteration, the Kar method generally consumes 1 more operation in each cycle than the Newton-Raphson method. In the case of the proposed iterative technique which differs from the Newton-Raphson method only in the first cycle, the effort used in evaluating incremental trial equilibrium loads within the first cycle of iteration apparently leads to a somewhat faster convergence.

This problem was also solved by using the NONSAP program (5) with the cable modelled by 10 truss elements. The final solution is shown in table 4.3 together with those given by Knudson (ref.(15), using truss elements), Jayaraman et. al. (13) and the proposed method. The vertical displacement at the load point obtained by the proposed technique differs by only 0.007% from that by Jayaraman et.al.(13) and approximately 2.3% from the other two sources. It can be observed that the catenary model yields slightly larger displacement than the truss model. This is because a catenary element is a curved element which should be more flexible than a straight one.

<u>Example problem 2</u> A test of stability is of particular interest. The single cable of example problem 1 was presented again with some modification. The material is assumed to be nearly inextensible. The modulus of elasticity is therefore assumed to be very large ($1.368X10^{14}$ ksf.).

In this case, the Newton-Raphson method and the Kar method suffer numerical overflow right at the first cycle of iteration, whereas the proposed technique did converge to the correct solution. For the formers, the incremental load procedure was then applied with 4 load steps (0.5 k., 2.0 k., 3.5 k. and 8.0 k.). The final solutions obtained from the Newton-Raphson method, the Kar method and the proposed iterative technique were achieved after 45, 41 and 17 iterative cycles, respectively, as shown in figure 4.3. The computational time is tabulated in table 4.4.

The result shows clearly that the proposed iterative technique is much superior to the other procedures in view of stability. The utilization of the subdivision procedure in evaluating an appropriate configuration in the first cycle clearly overcomes numerical difficulty.

<u>Example problem 3</u> The system of cable net taken from ref. (13) and depicted in figure 4.4 is taken as the third example. This net structure was modelled as 12 catenary elements. Table 4.5 lists all the relevant properties of the structure. A set of vertical loads is applied downwards at nodes 4, 5, 8 and 9 with the same magnitude of 8 kips. An initial configuration was assumed in ref. (13) as given in table 4.5. This assumed state does not necessarily satisfy the equilibrium condition. Therefore, in this study, the self-weight equilibrium configuration (without external loading) was actually solved and subsequently used as the initial equilibrium state in both analyses by the proposed method and by NONSAP. However, the assumed configuration of ref.(13) was also considered.

Convergence of the Newton-Raphson method, the Kar method and the proposed iterative technique was achieved after 8, 7 and 2 cycles, respectively. As evident from figure 4.5 and table 4.6, the proposed method is much more superior to the other methods in terms of speed of convergence. The computation time consumed by the proposed scheme was only 43.4% and 31.6% of the Newton-Raphson and Kar methods, respectively.

As indicated in table 4.7 the final displacements (measured from the reference X-Y plane) obtained by various methods agree extremely well, with the maximum discrepancy being less than 0.04%. It should be mentioned that the solution by NONSAP was computed using the assumed initial configuration of ref.(13) rather than the actual one. Since the initial cable forces were very small under self weight, the resulting stiffness matrix at the (actual) initial configuration was almost singular and NONSAP failed to give a converged solution even though the loading was applied in 100 load incrementals.

It might be interesting to see the rate of convergence with respect to the number of subdivision of the first trial displacement. This problem was analyzed using coarse to very fine subdivisions ranging from 10 to 1000 as tabulated in table 4.8. The results of analyses are shown in table 4.8 and figure 4.6. It might be observed that with a small subdivision of the first trial displacement (eg. 10 in case A), the scaled down displacement might exceed the true equilibrium value, and extrapolation for the displacement of the first iterative cycle could lead to a much smaller value than the true value as depicted in figure 4.7. Consequently, a large number of iterations was required for convergence. On the other hand, when the number of subdivisions was increased to 300 or more, only 2 operations of the trial equilibrium load were required to bracket the range that contained the correct displacement, and interpolation by eqn. (3.8) yielded quite an accurate estimate of the true value (which is 4.139 ft.) even at the end of the first iterative cycle only.

<u>Example problem 4</u> The synclastic cable net from ref.(4) with spring supports having the stiffness of $1X10^{10}$ kip./ft. in all three

directions as shown in figure 4.8 is used in this example. A set of dead loads was applied vertically downwards at all free nodes plus a vertically unsymmetrical live load of 200 kips. applied at node 3. Since the unstretched length and the unit weight of the cable are not given in ref.(4), in this study the unit weight of cable element was assumed to be 8.17 kip./ft. based on the density of steel of 490 lb./ft.. Furthermore, the unstretched length was assumed in such a way that under the initial cable force of 24.999 kips. specified by ref.(4) the same initial chord length was obtained. The complete data are given in table 4.9. The configuration under self weight and concentrated dead load (deformation and internal stress) was determined using the proposed method and subsequently input as the initial state in the separate analysis by NONSAP. Twenty subdivisions of the first trial displacement was performed in the former solution.

This example again indicates the effectiveness of the proposed technique over the other two schemes as can be seen from table 4.10 and figure 4.9. Similar trends were observed as in the previous example.

Table 4.11 lists the final displacements at node 3 computed by the proposed technique and NONSAP. The solution of the present study differs by only 0.02% from that computed by NONSAP, and 1.2% from Kar's solution. The close agreement with Kar's result is due to the fact that the initial state computed in the present study is almost identical with that assumed by Kar, with only 0.4% difference in the initial cable forces and vertical displacements.

Example problem 5 A single cable stretched between a ground anchor point and a tower point (figure 4.10) was analysed. This problem is the same as that given in NONSAP (5). The unit weight of the cable was 0.106667 lb./in. and the unstretched length was assumed in the same way as in example 4. The initial state was solved by the present method in a similar way as in example 3, and then used as the initial state for computation by NONSAP. Table 4.12 gives the complete data.

The results of analyses are shown in tables 4.13 and 4.14, and figure 4.11. One may note that the effectiveness of the Kar method and the proposed technique (where ten number of subdivisions were prescribed) are nearly the same for this example in which initial prestress is moderate, and much better than the Newton-Raphson scheme. The solution in this study and that by NONSAP agree very well (to within 1.2%). However, the original result by ref. (5) differs by 1.4% from the present study due possibly to the use of slightly different initial configurations in the analyses.

Example problem 6 The saddle-shaped cable net considered by Baron and Venkatesan (8) was taken as the next example (figure 4.12). The unstretched length was assumed in the same way as before. Table 4.15 contains all the pertinent information of the problem. Note that the cable net is highly prestressed in this example.

From the values of the displacements resulting at the end of each iteration as tabulated in table 4.16, it can be observed that, for the case of a highly prestressed net, the displacement at the end of the first cycle of iteration is quite close to the final one, with only 2% discrepancy in this example. Therefore, the displacement at that stage can very well serve as a trial displacement for the proposed technique, and the number of subdivisions of the first trial displacement is taken as 1.

A converged solution was obtained after 2 cycles of iteration for the proposed technique as well as for the Newton-Raphson method, but the former consumed slightly more computation time due to one extra evaluation of the trial equilibrium state. However, the proposed scheme approaches the final solution with a more rapid speed of convergence as illustrated in figure 4.13.

Table 4.17 shows the three components of the displacements of various nodes resulting from various analyses. The results are in close agreement with very small difference with a maximum discrepancy of only 1%.