CHAPTER VI

DISCUSSION AND CONCLUSION

In the previous chapter, we have calculated the density of states of a two-dimensional electron gas with a random potential under the influence of a transverse magnetic field. The method of calculation we use here follows Sa-yakanit et al. [27, 33]. In this chapter, we would like to conclude the result that we obtained from the previous chapter. Finally some suggestions will be also introduced.

Discussion

In this thesis, the path integral method of Feynman is applied to the problem of the two-dimensional electron gas in a transverse magnetic field with a random potential with the aim of obtaining the density of states. The random system which we have presented corresponds to an electron interacting with a set of dense, weak and random scatterers.

The starting point of the investigation is a single electron confined to the xy plane, moving among a set of impurities. The magnetic field is perpendicular. After introducing the Hamiltonian (4.3)

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e\vec{A}}{c}\right)^2 + \sum_{i=1}^{N} v\left(\vec{r} - \vec{R}_i\right),$$

the propagator (4.5) representing the motion of an electron in this system is obtained

and is expressed in the Feynman path integral representation as

$$K\left(\vec{r}\,',\vec{r};t,\{\vec{R}\}\right) \;=\; \int \mathcal{D}(\vec{r}(\tau))\exp\left[\frac{i}{\hbar}\int_0^t d\tau\left\{\frac{m}{2}\left(\dot{x}^2(\tau)+\dot{y}^2(\tau)\right.\right.\right.$$
$$\left.\left.\left.+\,\omega\left(x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)\right)\right)-\sum_{i=1}^{N}v\left(\vec{r}-\vec{R}_i\right)\right\}\right],$$

we use the symmetric gauge here is $\vec{A} \;=\; (-By, 0, 0)$. As pointed out by Edwards

and Gulyaev [30], the configurational average of the propagator (4.5) with respect to

the probability (4.6) can be performed exactly (4.7)

$$\overline{K}\left(\vec{r}\,',\vec{r};t,\right) \;=\; \left\langle K\left(\vec{r}\,',\vec{r};t,\{\vec{R}\}\right)\right\rangle$$

$$=\; \int \mathcal{D}(\vec{r}(\tau))\exp\left[\frac{i}{\hbar}\int_0^t d\tau\frac{m}{2}\left\{\dot{x}^2(\tau)+\dot{y}^2(\tau)\right.\right.$$
$$\left.+\,\omega\left(x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)\right)\right\}$$
$$\left.+\,\rho\int d\vec{r}\left\{\exp\left(-\frac{i}{\hbar}\int_0^t d\tau\,v\left(\vec{r}(\tau)-\vec{R}\right)\right)-1\right\}\right].$$

In the limit of weak $v \to 0$ and dense $\rho \to \infty$ scatterers so that $\rho v^2 =$ finite, the average

propagator reduces to a simple expression (4.8)

$$\overline{K}\left(\vec{r}\,',\vec{r};t,\right) \;=\; \int \mathcal{D}(\vec{r}(\tau))\exp\left[\frac{i}{\hbar}\int_0^t d\tau\frac{m}{2}\left\{\dot{x}^2(\tau)+\dot{y}^2(\tau)\right.\right.$$
$$\left.+\,\omega\left(x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)\right)\right\}$$

$$-\frac{\rho\eta^2}{2\hbar^2}\int_0^t\int_0^t d\tau\, d\sigma\, W\left(\vec{r}(\tau)-\vec{r}(\sigma)\right)\Bigg].$$

In order to obtain approximate expressions for the average propagator we introduce a trial action S_0 (4.16)

$$S_0 = \frac{m}{2}\int_0^t\left\{\dot{x}^2(\tau)+\dot{y}^2(\tau)\right\}d\tau + \frac{m\omega}{2}\int_0^t\left\{x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)\right\}d\tau$$

and use a cumulant approximation which enables us to obtain the expressions (4.25)

$$K_1(0,0;t) = \frac{m}{2\pi i\hbar t}\left(\frac{\omega t}{2\sin\frac{\omega t}{2}}\right)\cdot\exp\left[-\frac{\xi_L t}{2\hbar^2}\int_0^t\left\{1+8i\frac{E_L}{E_\omega}g(t,y)\right\}^{-1}dy\right]$$

for the first-order approximation, where $g(t,y) = \sin\left(\frac{\omega}{2}(\tau-y)\right)\dfrac{\sin\frac{\omega y}{2}}{\sin\frac{\omega t}{2}}$,

and (5.30) is

$$K_1(0,0;t) = K_0(0,0;t)\exp\left[\frac{i\xi'_L t^2\sin t}{2x}\int_0^1 d\tau_1\int_0^1 d\sigma_1\,\frac{1}{B_{11}}\right.$$

$$-\frac{\xi'^2_L t^4\sin^2 t}{8x^2}\int_0^1 d\tau_1\int_0^1 d\sigma_1\int_0^1 d\tau_2\int_0^1 d\sigma_2$$

$$\left.\cdot\left(\frac{1}{B_{11}B_{22}-|B_{12}|^2}-\frac{1}{B_{11}B_{22}}\right)\right]$$

for the second-order approximation respectively. Here B_{ij} is given as

$$B_{ij} = \frac{x}{4i} \sin t \, \delta_{ij} - [\, g(\tau_i - \tau_j) - g(\tau_i - \sigma_j) - g(\sigma_i - \tau_j) + g(\sigma_i - \sigma_j)\,]$$

Having obtained the approximate average propagators, the density of states, n(E) can be calculated by using (5.5). In this thesis interest is given to considering the density of states for the second cumulant approximation average propagator. After obtaining the expression for the density of states (5.31)

$$n(E) = (A/\pi\hbar)\,\mathrm{Re} \int_0^\infty \exp\left[\frac{i}{\hbar}\{(E - E_n)t + f(t)\}\right] dt$$

where

$$f(t) = \frac{i\xi'_L t^2 \sin t}{2x} \int_0^1 d\tau_1 \int_0^1 d\sigma_1 \, \frac{1}{B_{11}}$$

$$- \frac{\xi'^2_L t^4 \sin^2 t}{8x^2} \cdot \int_0^1 d\tau_1 \int_0^1 d\sigma_1 \int_0^1 d\tau_2 \int_0^1 d\sigma_2$$

$$\vdots$$

$$\cdot \left(\frac{1}{B_{11}B_{22} - |B_{12}|^2} - \frac{1}{B_{11}B_{22}}\right)\right].$$

We cannot observe the behaviour of the electron because of the difficulties of an exponential part which cannot be performed analytically. It can be calculated numerically by the Monte Carlo method (appendix). However, the result obtained by this numerical integration is not sufficient to obtain a refined result of the density of states, because the values of the second cumulant can be very large and unphysical.

Conclusion

We did not meet with success while obtaining the result of the numerical calculation when we went to the second-order cumulant approximation. The technique of Simpson's rule is one way that can be used for numerical calculation, but it is not optimal here since it takes much longer time than the Monte Carlo method. Nevertheless, we can use the result for the first-order approximation which compares very well with experiments [15]. Some numerical results from the second-order cumulant approximation give unphysically large values for the function $f(t)$. We can say that the second-order cumulant approximation is not useful which is unlike the first-order cumulant approximation. There is a difference between the cumulant theory and a perturbation theory which we would like to point out.

We consider an expression (4.14)

$$\overline{K}(0,0;t) = K_0(0,0;t)\left\langle \exp\left\{\frac{i}{\hbar}(S-S_0)\right\}\right\rangle_{S_0},$$

and define
$$A = \left\langle \exp\left\{\frac{i}{\hbar}(S-S_0)\right\}\right\rangle_{S_0}. \tag{6.1}$$

We expand the average on the right hand side of expression (6.1) in the cumulant series, then we get

$$A = \exp\left[\frac{i}{\hbar}\langle S-S_0\rangle_{S_0} + \left(\frac{i}{\hbar}\right)^2 \frac{1}{2}\{\langle (S-S_0)^2\rangle_{S_0}\right.$$
$$\left. -\langle S-S_0\rangle_{S_0}^2\} + \cdots\right]. \tag{6.2}$$

Approximating expression (6.2) by the first cumulant, we get

$$A \equiv \exp\left[\frac{i}{\hbar}\langle S - S_0 \rangle_{S_0}\right] . \tag{6.3}$$

Perturbation theory is applicable if the effect of the random action, $(S - S_0)$, is very small. Suppose the random action is very small compared to $\hbar$. Then the expression (6.1) can be expanded as

$$A = 1 + \frac{i}{\hbar}\langle S - S_0 \rangle_{S_0} + \left(\frac{i}{\hbar}\right)^2 \frac{1}{2}\langle (S - S_0)^2 \rangle_{S_0} + \cdots .$$

$$\tag{6.4}$$

Approximating expression (6.4) according to the first-order perturbation approximation, we get

$$A \approx \exp\left[\frac{i}{\hbar}\langle S - S_0 \rangle_{S_0}\right] . \tag{6.5}$$

Comparing between (6.3) and (6.5), then we can say that the first-order cumulant approximation is similar to the first-order perturbation approximation. We cannot say they are equivalent, because the random action, $(S - S_0)$ in the cumulant expansion depends on a parameter, ξ_L' and we do not know how large this is and cannot prospect anything from the result of a perturbation theory. Since the result from the first cumulant approximation can compare very well with experiments, we can say it is useful. The cumulant expansion for the first-order is accurate since we can get the observed physical density of states. The second-order cumulant calculation is not

accurate since we get an unphysical density of states. We need to get all cumulants, which is not possible. There is no reason why the second-order cumulant approximation should give an accurate density of states.