CHAPTER III

Adaptive Manipulator Control

3.1 A Certainty Equivalence Control Law of a Manipulator

From (6) the manipulator control problem is formulated in the framework of (5) that the desired trajectory is considered to have its nominal value which can be obtained from computation using the nominal model ,i.e., a fixed-parameter model exploited in (5) or an identified model in (6). The nominal point of generalized coordinates and torques address a instantaneous operating point for linearization to derived linearized perturbed dynamics of a manipulator that will be considered valid in suitably close region to the operating point. The problem now will turn out to be the problem of regulating motion error around an equilibrium point at the origin of the new state space which is corresponding to the nominal point of the original problem.

3.1.1. Derivation of a Linearized Perturbed Model of Manipulator Dynamics

The general dynamic equations for a manipulator of *n* degrees-of-freedom can be presented in the form as follows,

$$\tau = \mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}), \qquad (3.1)$$

where τ represents the n-dimension vector of the generalized joint torques, $\mathbf{D}(\mathbf{q})$ is a manipulator mass matrix of dimension $\mathrm{n} \times \mathrm{n}$, $\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})$ is the vector of centripetal and colioris torques or forces also of n dimension, $\mathbf{g}(\mathbf{q})$ is the vector of forces or torques due to gravity and $\mathbf{q}$ denotes the corresponding generalized coordinate vector of n dimension.

This matrix-vector equation represents simplified nonlinear model of a manipulator. Its derivation does not account for other complicated dynamics except for torque or force due to gravitation. we will define it to represent the nominal dynamics of a manipulator that it will also describe the nominal trajectory when the generalized coordinates in the equation are provided from the reference trajectory. Rewrite it in close form of the acceleration vector that

$$\ddot{\mathbf{q}} = \mathbf{D}(\mathbf{q})^{-1}(\tau - \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{g}(\mathbf{q}))$$

$$\ddot{\mathbf{q}} = \mathbf{F}(\mathbf{q},\dot{\mathbf{q}},\tau) \qquad (3.2)$$

$\mathbf{F}(\mathbf{q},\dot{\mathbf{q}},\tau)$ is a nonlinear vector-value function with $\mathbf{q},\dot{\mathbf{q}},\tau$ as independent vector variables. Use the nominal trajectory to address instantaneous operating point which denoted by * , perturbation of joint coordinates away from the operating point can be defined as

$$\mathbf{q} = \mathbf{q}^* + \mathrm{d}\mathbf{q}$$

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}^* + \mathrm{d}\dot{\mathbf{q}}$$

$$\ddot{\mathbf{q}} = \ddot{\mathbf{q}}^* + \mathrm{d}\ddot{\mathbf{q}} \qquad (3.3)$$

To linearize the perturbed dynamics, we use Taylor's expansion to the first order terms as follows

$$d\ddot{\mathbf{q}} = \frac{\partial \mathbf{F}}{\partial \mathbf{q}}\Big|_{*} d\mathbf{q} + \frac{\partial \mathbf{F}}{\partial \dot{\mathbf{q}}}\Big|_{*} d\dot{\mathbf{q}} + \frac{\partial \mathbf{F}}{\partial \tau}\Big|_{*} d\tau \qquad (3.4)$$

The partial differential terms of $\mathbf{F}$ with respect to $\mathbf{q},\dot{\mathbf{q}},\tau$ are constant coefficients by known values of its arguments at the nominal point. The latest equation may be worded that the variational acceleration at a nominal point can be described by a linear combination of the first order perturbation of joint coordinates, velocity and the generalized torques. This represents a linearized perturbed model of a manipulator. In general, we may rewrite it as

$$d\ddot{\mathbf{q}} = \overline{\mathbf{A}}d\mathbf{q} + \overline{\mathbf{B}}d\dot{\mathbf{q}} + \overline{\mathbf{C}}d\tau, \overline{\mathbf{A}}, \overline{\mathbf{B}}, \overline{\mathbf{C}} \in \mathbf{R}^{n\times n}, \tag{3.5}$$

where

$$\overline{\mathbf{A}} = \frac{\partial \mathbf{F}}{\partial \mathbf{q}}\Big|_{*}, \overline{\mathbf{B}} = \frac{\partial \mathbf{F}}{\partial \dot{\mathbf{q}}}\Big|_{*}, \overline{\mathbf{C}} = \frac{\partial \mathbf{F}}{\partial \tau}\Big|_{*}.$$

If we assign the state $\mathbf{x}_1 = d\mathbf{q}, \mathbf{x}_2 = d\dot{\mathbf{q}}$ and the variational torque $d\tau$ as the control input $\mathbf{u}$, we have the model in state-space formulation,

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ \overline{\mathbf{A}} & \overline{\mathbf{B}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{\mathbf{C}} \end{bmatrix} \mathbf{u}, \tag{3.6}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \tag{3.7}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & I \\ \overline{\mathbf{A}} & \overline{\mathbf{B}} \end{bmatrix}, \mathbf{A} \in \mathbf{R}^{2n\times 2n},$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \overline{\mathbf{C}} \end{bmatrix}, \mathbf{B} \in \mathbf{R}^{2n\times n},$$

and its equivalence in discrete-time domain

$$\mathbf{x}_{k+1} = \mathbf{A}_d\mathbf{x}_k + \mathbf{B}_d\mathbf{u}_k, \mathbf{x}_k \in \mathbf{R}^{2n}, \mathbf{u}_k \in \mathbf{R}^{n}, \tag{3.8}$$

where

$$\mathbf{A}_d = e^{\begin{bmatrix} 0 & I \\ \overline{\mathbf{A}} & \overline{\mathbf{B}} \end{bmatrix} T_s}, \mathbf{A}_d \in \mathbf{R}^{2n \times 2n},$$

$$\mathbf{B}_d = \int_0^{T_s} e^{\begin{bmatrix} 0 & I \\ \overline{\mathbf{A}} & \overline{\mathbf{B}} \end{bmatrix} t} \mathbf{B} dt, \mathbf{B}_d \in \mathbf{R}^{2n \times n},$$

3.1.2. Recursive Identification of A Linear Discrete Model

The further step is to linearly parameterize the model for identification process. Since the model is already linear in physical parameters, adopting the recursive least square identification is then straight forward. Just rewrite the equation (3.8) as follows

$$\mathbf{x}_{k+1} = (\mathbf{A}_d \mathbf{B}_d)_k \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix}, \tag{3.9}$$

and define the followings for the sake of formulating identification problem.

$$\mathbf{z}_k^T = (\mathbf{x}_k^T \mathbf{u}_k^T). \tag{3.10}$$

$$\Phi_k = (\mathbf{A}_d \mathbf{B}_d)_k. \tag{3.11}$$

$$\mathbf{w}_k = \mathbf{x}_{k+1}. \tag{3.12}$$

Then, from (3.9) we obtain

$$\mathbf{w}_k = \Phi_k \mathbf{z}_k, \tag{3.13}$$

where the matrix Φ_k, of dimension $2n \times n$, represents $2n$ parameter vectors to be simultaneously identified at time k. If we rewrite (3.13) as

$$\mathbf{w}_k^T = \mathbf{z}_k^T \Phi_k^T, \tag{3.14}$$

and define a scalar value equation,

$$w^i = \mathbf{z}^T \phi^i, \tag{3.15}$$

where w^i is the ith element of the vector $\mathbf{w}$ and ϕ^i is the ith-row vector extracted from matrix Φ. At this point we shall drop the discrete time subscript k for a while as we now concern with the problem of recursive identification of a point-wise stationary system, by definition claimed from discrete-time approach, at each frozen time interval denoted by k. The subscript t used throughout identification formulation will stand for a number of measurement times applied within that frozen time interval. In the method, the prediction error function as follows

$$S[\phi^i] = \sum_{t=1}^{M} \left(w_t^i - \mathbf{z}_t^T \hat{\phi}^i \right)^2, \tag{3.16}$$

need to be minimized at a unique value of its argument $\hat{\phi}^i$, which is indeed the optimal estimate we seek. The number M referred to the number of input data measurements is dependent upon the choice of the designer related to the rate of change of physical parameters of the system being considered. The recursive solution to the problem for simultaneous findings of $2n$ parameter vectors is obtained by iterating the following algorithm.

$$\hat{\phi}_{t+1}^i = \hat{\phi}_t^i + \mathbf{k}_{t+1}^i s_{t+1}^i, \tag{3.17}$$

where

$$s_{t+1}^{i} = w_{t+1}^{i} - \mathbf{z}_t^T \hat{\phi}_t^i, \tag{3.18}$$

$$\mathbf{k}_{t+1}^{i} = \frac{\mathbf{P}_t \mathbf{z}_{t+1}}{1 + \mathbf{z}_{t+1}^T \mathbf{P}_t \mathbf{z}_{t+1}}, \tag{3.19}$$

$$\mathbf{P}_{t+1} = \left(I - \mathbf{P}_t \frac{\mathbf{z}_{t+1} \mathbf{z}_{t+1}^T}{1 + \mathbf{z}_{t+1}^T \mathbf{P}_t \mathbf{z}_{t+1}} \right) \mathbf{P}_t, \tag{3.20}$$

with t denotes the number of iteration. The choice of P_0 should be from *priori* knowledge of system characteristics or from experimental trials.

3.1.3. Discrete Optimal Regulator for Linearized Perturbed Dynamics

At this point the problem left here is now to construct a controller to regulate the system state just derived by identification to zero. This corresponds to forcing the nominal trajectory to track the desired trajectory as can be suggested in (3.3). In (3), they utilize optimal tracking criterion to formulate control algorithm via one-step-ahead horizon. This type of control synthesis is well-known as the linear quadratic optimal control which is thoroughly-understood and well-applied to classes of linear systems.

For a linear discrete system that can be described by the following state-space formulation.

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{u}_k, \tag{3.21}$$

we can derive a linear feedback law by formulating the problem of minimizing the performance criterion

$$J_k = \mathbf{x}_{k+1}^T \mathbf{Q} \mathbf{x}_{k+1} + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k . \tag{3.22}$$

This is the so-called one-step-ahead horizon control criterion tending to find the control input as an argument minimizing the above. Its solution can be derived in form of a linear combination of state feedbacks as

$$\mathbf{u}_k = \mathbf{K} \mathbf{x}_k , \tag{3.23}$$

where $\mathbf{K}$ is the feedback gain matrix,

$$\mathbf{K} = \left(\mathbf{R} + \mathbf{G}^T \mathbf{Q} \mathbf{G} \right)^{-1} \mathbf{G}^T \mathbf{Q} \mathbf{F} \tag{3.24}$$

The state feedbacks (3.23) incorporates with the system description (3.21) form a closed loop control system having the objective of zero state regulation. The above case is commonly referred as the Linear Quadratic Regulator (LQR).

3.1.4. A Linear Parameterized Model of Manipulator Dynamics

Due to the fact that the fixed-parameter model does not compensate for loading effects and uncertainty in parameters. Hence, in pragmatic situation, when these effects significantly perturb the nominal system away from the predicted or designated point , the fundamental assumptions for linearization of the perturbed dynamic model would be no longer valid. As a result, the controller designed, based upon this model, would loss its potential, in part or whole, to deal with the actual perturbed dynamics as it would now stay outside a valid region of the controller designed as before. There is an alternate to deal with this aspect of the problem. The point is to readdress the operating point more properly. At this time, it is not carried out from the analytical model computation but current plant information is incorporated to recast the nominal plant model continuously with time change. This can be done by using on-line recursive identification technique the same as presented above but applied to the nominal model of a manipulator and working in parallel with the one of the linearized perturbed model. This is the main contribution of (6) which we will follow its methodology for experimental evaluation of adaptive manipulator control on the Chula2 manipulator.

Before we can go on to formulate the nominal identification, the linear parameterized model of the nominal model for a manipulator have to be defined. Many forms of this type of models can be obtained by various simplified assumptions and methodology. In (5) they define the augmented joint coordinate vector as follows

$$\mathbf{z}^T = \left(\ddot{\mathbf{q}}^T \dot{\mathbf{q}}^T \tilde{\mathbf{q}}^T 1\right), \qquad (3.25)$$

where $\ddot{\mathbf{q}}$ and $\dot{\mathbf{q}}$ are the vectors of joint acceleration and velocity respectively, 1 is a constant scalar, and $\tilde{\mathbf{q}}$ is the vector of extended joint coordinates whose value is defined as follows

$$\tilde{\mathbf{q}}^T = \left(\mathbf{q}_1^T \mathbf{q}_2^T \ldots \mathbf{q}_n^T\right), \qquad (3.26)$$

where $\mathbf{q}_i^T = [\sin\theta_i \quad \cos\theta_i]$,for a rotary joint and $\mathbf{q}_i^T = [d_i \quad 0]$,for a prismatic joint, $i = 1,2,\ldots,n\,\theta_i$ and d_i are joint variables. From the above definition, the vector of generalized forces can be determined by

$$\tau = \mathbf{A}(\mathbf{q},\dot{\mathbf{q}})\mathbf{z}, \qquad (3.27)$$

where $\mathbf{A}(\mathbf{q},\dot{\mathbf{q}})$, of corresponding dimension, i.e., for an n-degree-of-freedom manipulator we obtain $\mathbf{z}$ having dimension of $4n+1$, and then $\mathbf{A} \in \mathbf{R}^{n\times(4n+1)}$, is the new dynamic operator. The existence of the expression (3.27) is supported by the proposed theorem (6) which states that for n degree-of-freedom manipulator having n joints of any combination of rotary or prismatic joints, the linear operator which maps the augmented joint coordinate vector into the vector of generalized forces, i.e, at any instant, the generalized torques can always be expressed as a linear combination of the elements of the augmented joint coordinates, always exists. However, this linear operator does not offer one-to-one mapping. Yet this model formulation can still be utilized in our case. The operator $\mathbf{A}(\mathbf{q},\dot{\mathbf{q}})$ is a group of parameter vector to be recursively identified, given the information on the vector τ and $\mathbf{z}$. The resulting identified model will be used in the subsequence time to compute the nominal torques for a given augmented joint coordinates from the reference trajectory. The final control scheme is depicted in Figure 3.1.

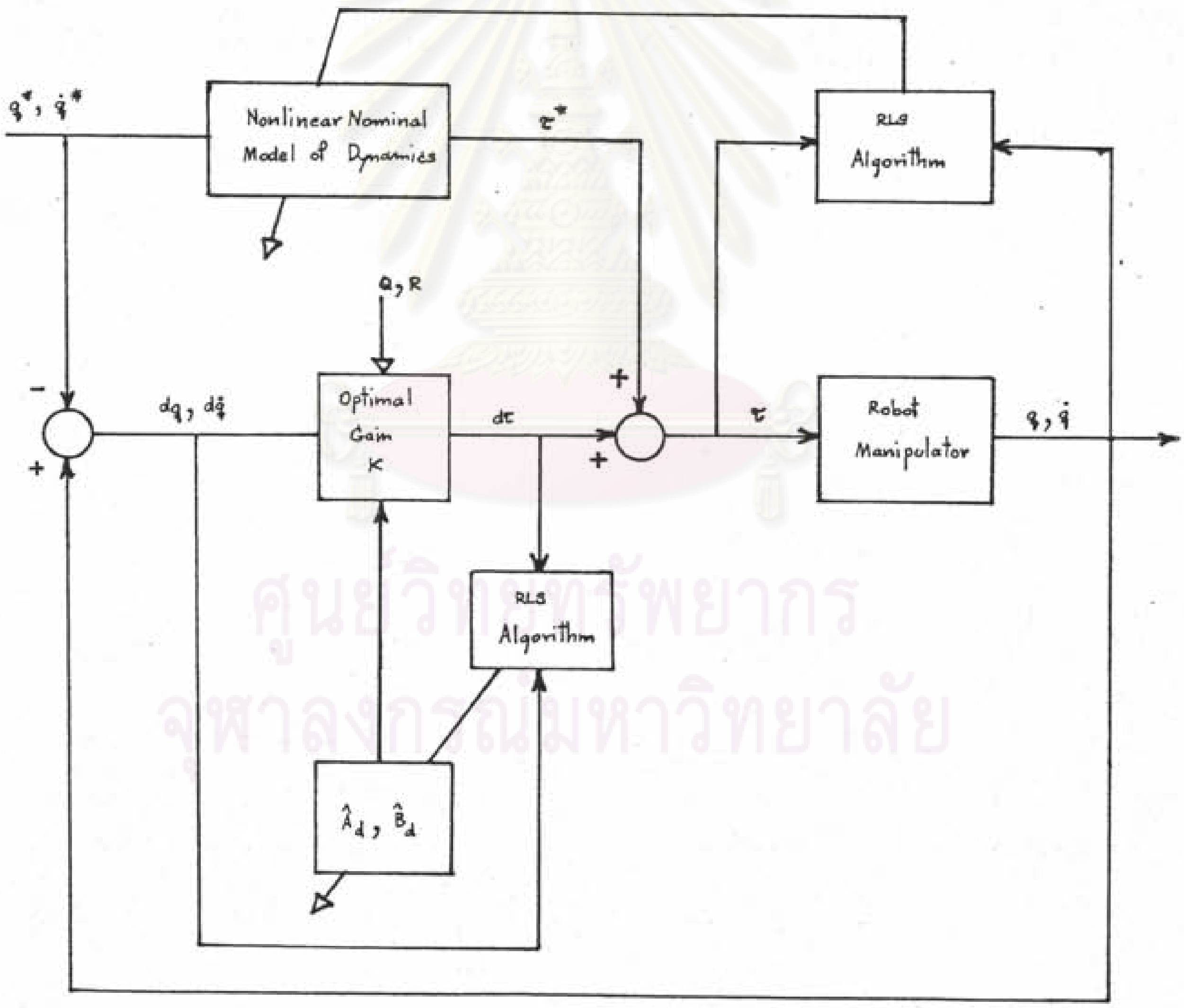

Figure 3.1: The certainty equivalence control law of a manipulator.