INTRODUCTION AND SCOPE OF THESIS

The path-integral formulation of quantum statistical mechanics has been accomplished several year ago by Feynman who extended his treatment of the quantum mechanical propagator to imaginary times, in order to get the analogous expression for the matrix elements of the unnormalized density operator in the coordinate representation. Theoretical methods trying to reduce quantum statistical mechanics calculation to classical one received large interest still at the beginning of quantum mechanics. The fundamental idea was to define suitable representations of the quantum operators.

In our opinion the path-integral formulation of statistical mechanics represents an ideal tool because it should permit expansion starting from the classical regime which is given by minimum of the euclidean action. The quantum character of the statistics is given by the relevance of other trajectories in addition to the classical ones. In particular, one peculiar method of the path-integral scheme, namely the variational approach [Feynman and Hibbs 1965, Feynman 1972], appears to be very appealing for constructing an *effective classical potential* to be inserted again in a configuration integral. In this way, at least in the low-coupling approximation, the calculation of the quantum partition function can be reduced to the easier classical one with a modified potential taking into account the quantum effect.

Feynman and Kleinert [Feynman and Kleinert 1986] proposed a new variational method for calculating statistical properties of quantum mechanical system. It is a generalization of a well-known variational given a long time ago by Feynman to which it reduces for a special, non-optimal choice of the variational parameters. It has been used to study free energies and particle densities for the one-dimensional anharmonic oscillator and double-well potential [Kleinert 1986, Janke and Kleinert 1987] and to investigate the radial distribution function of the three-dimensional Coulomb problem [Janke and Kleinert 1986]. In all case, the new approximation gives reasonably good results even at relatively low temperatures. Indeed, at that time, a similar line of improvement was given by Giachetti and Tognetti [Giachetti and Tognetti 1986] and that method has been successfully applied to one-dimensional models, like sine-Gordon, ϕ^4 and double sine-Gordon chain [Giachetti, et. al. 1988].

This thesis presents some detailed evaluations of the approximation treatment in the spirit of Feynman path integral for quantum statistical systems for which the path integral can not be done exactly. In Chapter I we review some useful concepts in classical mechanics and classical statistical mechanics bringing us to understanding the formulation of path integral in this thesis. In Chapter II we introduce the path integral formulation and show clearly about the exactly technical calculation for the free particle, harmonic oscillator and forced harmonic oscillator. The path integral approach in quantum statistical system is presented in Chapter III. Some results obtained in this chapter and previous one are always useful in the next chapters. In Chapter IV we summarize the original approximation developed by Feynman for long time ago. The importance of this part is the variational path integrals we, for main purpose of this thesis, point out the improvement. For Chapter V we explain some improved versions in detail of variational treatment developed by Feynman and Kleinert and also present the interesting application to anharmonic system. Moreover, we present the greatly superior improvement for increasing the power of the variational approach to path integrals. The extremely good approximation is recently given by Kleinert. And for the Chapter VI we give the detailed discussion about these methods and results of their applications.