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ภาคผนวก ก

Properties of Pure Sodium Chloride Brine

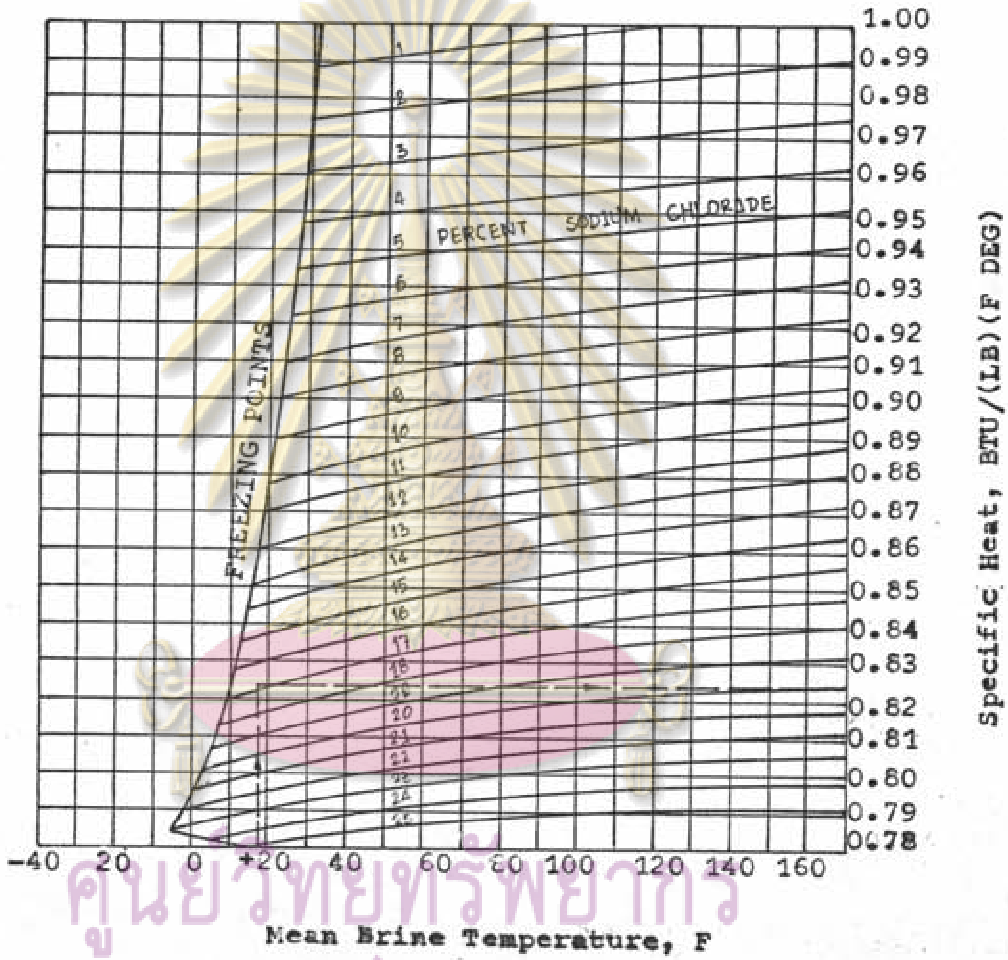


Fig. 1 Specific Heat of Sodium Chloride Brine

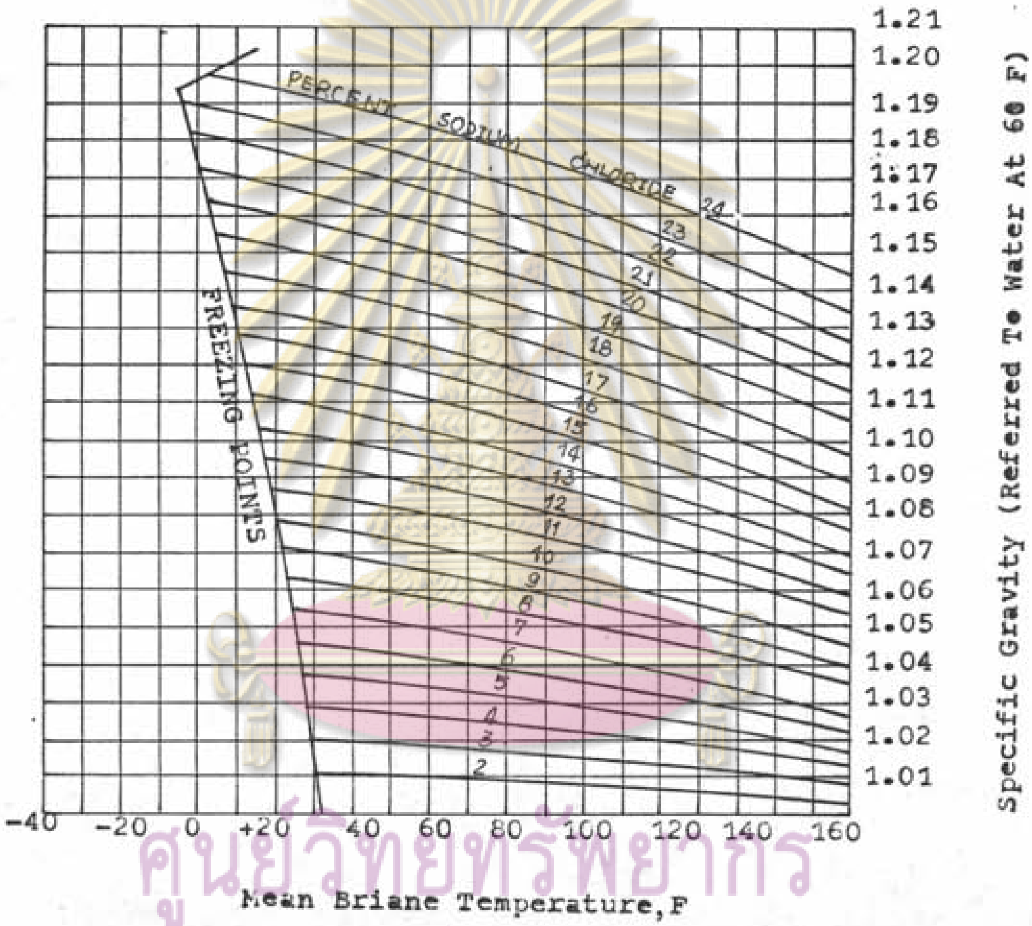


Fig. 2 Specific Gravity of Sodium Chloride Brine

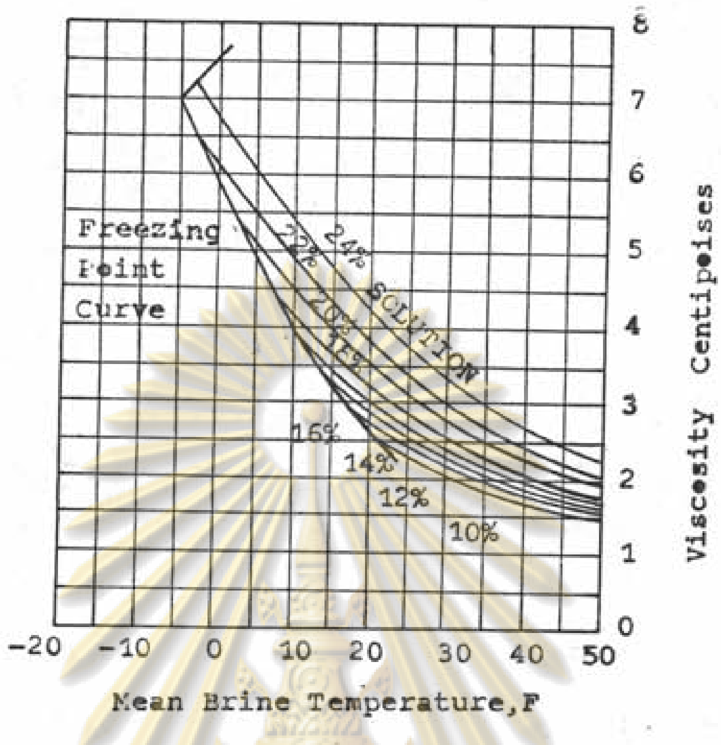


Fig. 3 Viscosity of Sodium Chloride Brine

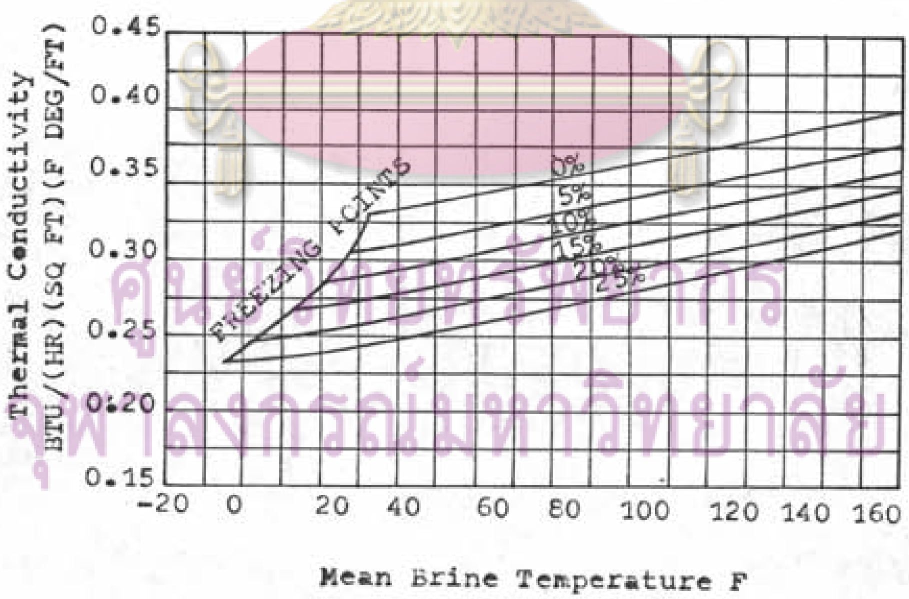


Fig. 4 Thermal Conductivity of Sodium Chloride Brine

Table 1. Properties of Pure Sodium Chlorid Brine

Pure NaCl % by wt	Specific gravity 59 F / 39 F	Baume density 60 F	Specific heat 59 F Btu/lb deg F	Crystallization starts F	Weight per galeen* NaCl lb/gal	Water lb/gal	Brine lb/gal	Weight per cubit feet* NaCl lb/cu ft	Water lb/cu ft	Brine lb/cu ft	Sp.Gravity of Other Temperature (Refers to water at 60 F) 14 F -10 C	32 F 0 C	50 F 10 C	68 F 20 C
0	1.000	0.0	1.000	32.0	0.000	8.34	8.34	0.000	62.40	62.4				
5	1.035	5.1	0.938	26.7	0.432	8.22	8.65	3.230	61.37	64.6		1.0382	1.0366	1.0311
6	1.043	6.1	0.927	25.5	0.523	8.19	8.71	3.906	61.19	65.1		1.0459	1.0440	1.0413
7	1.050	7.0	0.917	24.3	0.613	8.15	8.76	4.585	60.91	65.5		1.0536	1.0515	1.0460
8	1.057	8.0	0.907	23.0	0.706	8.11	8.82	5.280	60.72	66.0		1.0613	1.0590	1.0559
9	1.065	9.0	0.897	21.6	0.800	8.09	8.89	5.985	60.51	66.5		1.0601	1.0665	1.0633
10	1.072	10.1	0.888	20.2	0.895	8.05	8.95	6.690	60.21	66.9		1.0769	1.0741	1.0707
11	1.080	10.8	0.879	18.8	0.992	8.02	9.02	7.414	59.99	67.4		1.0849	1.0817	1.0782
12	1.087	11.7	0.870	17.3	1.000	7.99	9.08	8.136	59.66	67.8		1.0925	1.6897	1.0857
13	1.095	12.7	0.862	15.7	1.188	7.95	9.14	8,879	59.42	68.3		1.1004	1.0971	1.0933
14	1.103	13.6	0.854	14.0	1.291	7.93	9.22	9.632	59.17	68.8		1.1083	1.1048	1.1009
15	1.111	14.5	0.847	12.3	1.392	7.89	9.28	10.395	58.90	69.3	1.1195	1.1163	1.1126	1.1086
16	1.118	15.4	0.840	10.5	1.493	7.84	9.33	11.168	58.63	69.8	1.1277	1.1243	1.1205	1.1163
17	1.126	16.3	0.833	8.6	1.598	7.80	9.40	11.951	58.36	70.3	1.1359	1.1323	1.1284	1.1241
18	1.134	17.2	0.826	6.6	1.705	7.76	9.47	12.744	58.06	70.8	1.1442	1.1404	1.1363	1.1319
19	1.142	18.1	0.819	4.5	1.813	7.73	9.54	13.547	57.75	71.3	1.1535	1.1486	1.1444	1.1398
20	1.150	19.0	0.813	+ 2.3	1.920	7.68	9.60	14.360	57.44	71.8	1.1608	1.1562	1.1542	1.1478
21	1.158	19.9	0.807	- 0.0	2.031	7.64	9.67	15.183	57.12	72.3	1.1692	1.1651	1.1606	1.1559
22	1.166	20.8	0.802	- 2.3	2.143	7.60	9.74	16.016	56.78	72.8	1.1777	1.1734	1.1688	1.1640
23	1.175	21.7	0.796	- 5.1	2.256	7.55	9.81	16.854	56.45	73.3	1.1862	1.1818	1.1771	1.1721
24	1.183	22.5	0.791	+ 3.8	2.371	7.51	9.83	17.712	56.09	73.8	1.1948	1.1902	1.1854	1.1804
25	1.191	23.4	0.786	+16.1	2.488	7.46	9.95	18.575	55.72	74.3				
25.2	1.200			+32.0										

*Wt of commercial NaCl required = $\frac{\text{wt of pure NaCl required}}{\text{percent purity}}$

ภาคผนวก ข.

Table 1. - Albedos of Various Substances (6,pp.18)

Fresh snow cover	75-95	percent
Dense cloud cover	60-90	"
Old snow cover	40-70	"
Clear firm snow	50-60	"
Light sand dunes, surf	30-60	"
Clear glacier ice	30-46	"
Dirty firm snow	20-50	"
Dirty glacier ice	20-30	"
Sandy soil	15-40	"
Meadows and fields	12-30	"
Densely built up areas	15-25	"
Woods	5-20	"
Darks cultivated soil	7-10	"
Water surfaces, sea	3-10	"

Table 1. shows that water is one of the least reflecting substances covering the surface of the earth. This makes it about the best solar energy storing medium except for evaporation losses.

ภาคผนวก ค

The Normal Total Emissivity of Various Surfaces

Surfaces	°F	Emissivity
Brick:		
Red, rough, but no gross irregularities	70	0.93
Silica, unglazed, rough	1832	0.80
Silica, glazed, rough	2012	0.85
Grog, brick, glazed	2012	0.75
Refractory materials,		
poor radiators		0.65-0.70
Good radiators		0.80-0.90
Plaster, rough, lime	50,190	0.91
Rubber:		
Hard, glossy plate	74	0.945
Soft, gray, rough	76	0.859
Water	32,212	0.95-0.963
Carbon:		
T-carbon, 0.9 % ash	260,1160	0.81,0.79
carbon filament	1900,2560	0.526
candle soot	206, 520	0.952
Roofing Paper	69	0.91
Black shiny lacquer, sprayed on iron	76	0.875
Black lacquer	100, 200	0.80, 0.95

Adopted from: AUBREY, I.B. and SALVATORE, M.M., Introduction to heat transfer. Third Edition, McGRAW-HILL BOOK COMPANY, INC. 1958, pp. 57,58.

ภาคผนวก ง

Table 1 - Thermal Properties for Determining Bottom Losses from Solar Collectors

Material	$\sqrt{4 K_s \rho_s C_{ps}/\pi}$	Ks	ρ_s	C_{ps}
Soil at Puerto penasco	1.4	0.094	100	0.17
Sawdust (Typical)	0.54	0.034	12	0.55
Styrofoam	0.12	0.018	2	0.30
Soils				
Gravel, 0% Moisture	2.0	0.15	112	0.19
2.9 %	3.3	0.36	112	0.21
5.3 %	7.9	2.0	119	0.21
Coarse Sand, 0%	1.8	0.12	106	0.19
5.1 %	3.9	0.51	112	0.21
9.5 %	6.8	1.4	112	0.23
Medium Sand, 0%	1.7	0.12	100	0.19
7.4 %	3;8	0.48	106	0.26
13.7 %	6.3	1.1	106	0,26
Fine Sand, 0%	1.8	0.12	100	0.20
9.7 %	4.0	0.48	106	0.25
18.0 %	6.4	1.0	112	0.23
Sandy Loam, 0%	1.5	0.10	94	0.19
14.5 %	4.0	0.46	100	0.27
27.0 %	4.0	0.46	100	0.27
Silt Loam, 0%	1.2	0.073	87	0.18
17.8 %	4.3	0.53	100	0.27
33.0 %	6.5	0.92	106	0.34
Silty Clay Loam, 0%	1.2	0.073	81	0.20
21.1 %	4.8	0.60	94	0.31
39.0 %	6.7	0.87	106	0.38
Clay, 0%	1.3	0.073	81	0.22
28.1 %	1.7	0.089	100	0.36
52.0 %	6.9	0.75	112	0.44
Heavy Clay, 0%	1.3	0.073	87	0.21
35.9 %	4.9	0.46	112	0.37
66.5 %	6.8	0.68	94	0.57
Peat, 0%	0.5	0.029	12	0.45
22.4 %	1.5	0.090	25	0.78
56.8 %	4.1	0.24	50	1.10

where Q_s is the heat loss from the solar heater into the fround, Btu(ft)(day)

Δt is the increase in temperature at the soil surface, °F

K_s is the thermal conductivity of the soil, Btu $ft^{-1} hr^{-1} (°F)^{-1}$

ρ_s is the density of the soil , lb/(cu ft)

C_{ps} is the specific heat of the soil, Btu/ $lb(°F)^{-1}$

Adopted from : Solar Distillation Utilizing Mutiple - Effect Humidification, United States Department of the Interior, Research and development progress report No. 194. Office of Saline Water, Washington, D.C. pp. 43

ภาคผนวก จ.

ตารางแสดงค่าอุณหภูมิที่เกิดขึ้นในสระแสงอาทิตย์ได้จากการทดลอง

วันที่ 12 กันยายน พ.ศ. 2528

ความลึกของสระวัดจากก้นสระถึงผิวน้ำตอนบน	เวลา 09.00 น. T_a = 29.2 °ซ	เวลา 10.00 น. T_a = 29.6 °ซ	เวลา 11.00 น. T_a = 29.8 °ซ	เวลา 12.00 น. T_a = 31.3 °ซ	เวลา 13.00 น. T_a = 30.3 °ซ	เวลา 14.00 น. T_a = 31.2 °ซ	เวลา 15.00 น. T_a = 31.6 °ซ	เวลา 16.00 น. T_a = 30.9 °ซ	อุณหภูมิที่ผิวใยแก้วด้านนอก	ค่าความถ่วงจำเพาะของน้ำเกลือ
H (ซม.)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_S (°ซ)	Sp.Gr
10										1.110
20										1.110
30										1.110
40	57.4	57.5	57.6	57.8	58.3	58.5	58.7	59.00	33.2	1.110
50										1.105
60										1.102
70	55.7	55.5	55.6	56.7	56.4	56.8	57.1	57.6	32.6	1.099
80	53.2	52.7	52.8	52.8	52.9	53.0	53.4	52.8	32.3	1.066
90	42.6	42.6	43.0	43.3	43.2	43.5	43.7	43.1	32.0	1.052
100	34.3	34.7	35.0	35.4	35.6	35.8	36.6	36.1	30.9	1.034
110	29.4	30.2	31.0	31.8	32.3	32.6	33.7	33.4	29.8	1.000
120	29.2	30.3	31.6	33.2	34.1	34.8	36.2	33.5	29.3	1.000

วันที่ 21 กันยายน พ.ศ. 2528

ความลึกของสระ วัดจากก้นสระถึงผิวน้ำตอนบน	เวลา 09.00 น. T_a = 27.9 °ซ.	เวลา 10.00 น. T_a = 28.1 °ซ.	เวลา 11.00 น. T_a = 28.6 °ซ.	เวลา 12.00 น. T_a = 29.9 °ซ.	เวลา 13.00 น. T_a = 30.2 °ซ.	เวลา 14.00 น. T_a = 30.4 °ซ.	เวลา 15.00 น. T_a = 30.1 °ซ.	เวลา 16.00 น. T_a = 29.6 °ซ.	อุณหภูมิที่ผิวใยแก้วด้านนอก	ค่าความถ่วงจำเพาะของน้ำเกลือ
H (ซม.)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_{SG} (°ซ)	Sp.Gr
10										
20										
30										
40	60.00	60.20	60.30	60.45	60.55	60.60	60.70	60.85	32.6	1.110
50										
60										
70	56.80	56.80	56.90	57.00	57.20	57.25	57.30	57.40	32.00	1.098
80	48.20	48.20	48.30	38.50	48.60	48.90	49.10	49.40	31.20	1.085
90	39.20	39.20	39.40	39.90	40.40	39.60	40.90	41.60	30.40	1.060
100	32.60	32.60	32.95	33.20	33.60	33.90	34.40	34.60	30.10	1.032
110	29.20	30.20	30.60	31.30	31.50	31.90	32.40	32.80	29.60	1.002
120	28.90	30.40	31.60	32.50	33.20	34.70	35.10	35.90	29.80	1.000

วันที่ 30 กันยายน พ.ศ. 2528

ความลึกของสระ วัดจากก้นสระถึงผิวน้ำตอนบน	เวลา 09.00 น. T_a = 27.0 °ซ	เวลา 10.00 น. T_a = 27.3 °ซ	เวลา 11.00 น. T_a = 27.4 °ซ	เวลา 12.00 น. T_a = 27.6 °ซ	เวลา 13.00 น. T_a = 28.2 °ซ	เวลา 14.00 น. T_a = 28.5 °ซ	เวลา 15.00 น. T_a = 28.5 °ซ	เวลา 16.00 น. T_a = 28.4 °ซ	อุณหภูมิที่ผิวใยแก้วด้านนอก	ค่าความถ่วงจำเพาะของน้ำเกลือ
H (ซม.)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_R (°ซ)	T_{SG} (°ซ)	Sp.Gr
10										
20										
30										
40	59.80	59.92	60.50	60.18	60.33	60.48	6064	60.80	33.70	1.110
50										1.108
60										1.106
70	57.80	57.80	57.90	58.00	58.35	58.50	58.65	58.80	33.60	1.101
80	52.90	53.00	53.20	53.40	53.50	53.60	53.60	53.50	32.40	1.098
90	48.80	48.90	50.00	50.20	50.40	50.50	50.60	50.60	32.20	1.060
100	36.20	36.40	36.50	36.55	36.70	36.60	36.70	36.70	30.60	1.032
110	31.20	31.40	31.90	32.30	32.60	32.90	33.60	34.00	29.60	1.002
120	30.90	31.60	32.60	33.20	33.80	34.60	35.00	35.90	29.50	1.002

The instantaneous point source

The differential equation of conduction of heat,

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = \frac{1}{K}\frac{\partial v}{\partial t} \qquad (1)$$

is staisfied by $$v = \frac{Q}{8(\pi kt)^{\frac{3}{2}}} \cdot e^{-\{(x-x')^2+(y-y')^2+(z-z')^2\}/4kt} \qquad (2)$$

As $t \to 0$ this expression tends to zero at all points except (x',y',z'), where it becomes infinite.

Also the total quantity of heat in the infinite region is

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho c v \, dxdydz$$

$$= \frac{Q\rho c}{8(\pi kt)^{\frac{3}{2}}} \int_{-\infty}^{\infty} e^{-(x-x')^2/4kt}dx \int_{-\infty}^{\infty} e^{-(y-y')^2/4kt}dy$$

$$\int_{-\infty}^{\infty} e^{-(z-z')^2/4kt}dz$$

$$= Q\rho c \qquad (3)$$

Thus the solution (2) may be interpreted as the temperature in an infinite solid due to a quantity of heat $Q\rho c$ instantaneously generated at $t = 0$ at a point (x',y',z'). The solution (2) is called the temperature due to an instantaneous point source of strength Q at (x',y',z') at $t = 0$.

Instantaneous line source of strength Q at $t = 0$, parallel to the z-axis and passing through the point (x',y')

Here we consider a distribution of instantaneous point sources of strength $Q\,dz'$ at z' along the line. The temperature, obtained by integrating 10.2 (2), is

$$v = \frac{Q}{8(\pi kt)^{\frac{3}{2}}} \int_{-\infty}^{\infty} dz' \, e^{-\{(x-x')^2+(y-y')^2+(z-z')^2\}/4kt}$$

$$= \frac{Q}{4\pi kt} \cdot e^{-\{(x-x')^2+(y-y')^2\}/4kt} \qquad (4)$$

In this case the quantity of heat liberated per unit length of the line is $Q\rho c$.

Instantaneous plane source of strength Q at t = 0, parallel to the plane x = 0, and passing through the point x'

Here we distribute line sources of strength Q dy' along the line x = x'. Intergrating (1) we have

$$v = \frac{Q}{4\pi kt} \int_{-\infty}^{\infty} dy' \, e^{-\{(x-x')^2+(y-y')^2\}/4kt}$$

$$= \frac{Q}{2\sqrt{(\pi kt)}} \cdot e^{-(x-x')^2/4kt} \qquad (5)$$

The quantity of heat liberated is $Q_\rho c$ per unit area of the plane.

The continuous plane source

Suppose heat is liberated at the rate $\rho c\phi(t)$ per unit area per unit time in the plane x' starting at time t = 0. Then, from (5), the temperature at time t is

$$v = \frac{1}{2(\pi k)^{\frac{1}{2}}} \int_0^t e^{-(x-x')^2/4k(t-t')} \frac{\phi(t')dt'}{(t-t')^{\frac{1}{2}}} \qquad (6)$$

Application to surface heating of a semi-infinite region

Heat supply at the rate q per unit time per unit area for $t > 0$ over the half - plane

$$x < 0, \; -\infty < y < \infty, \; z = 0$$

The temperature v at the point x of the surface at time t is

$$v = \frac{q}{2\cdot\pi K}\int_{0}^{t}\frac{d\tau}{t-\tau}\int_{-\infty}^{0} e^{-(x-x')^{2}/4k(t-\tau)}\,dx' \qquad (7)$$

$$v = \frac{qk^{1/2}}{2K\,\pi^{1/2}}\int_{0}^{t}\frac{d\tau}{(t-\tau)^{1/2}}\cdot \operatorname{erfc}\frac{x}{2k^{1/2}(t-\tau)^{1/2}} \qquad (8)$$

$$v = \frac{\sqrt{k/\pi}}{2K}\int_{0}^{t} q\left[1-\exp\{x^{-2}/k(t-\tau)\}\right]\frac{d\tau}{(t-\tau)^{1/2}} \qquad (9)$$

when $k = \alpha$ $\quad q = \phi(h,\tau) = \text{Constant}$

$K = K$ $\quad x = h$ $\quad v = T_a(t)$

$$\therefore \quad T_a(t) = \frac{\sqrt{\alpha/\pi}}{2K}\int_{0}^{t}\frac{\phi(h,\tau)}{\sqrt{t-\tau}}\left[1-\exp\{-h^{2}/\alpha(t-\tau)\}\right]d\tau \qquad (10)$$

$$\therefore \quad T_c(t) = \frac{\sqrt{\alpha/\pi}}{2K}\int_{0}^{t}\frac{-U\tau}{\sqrt{t-\tau}}\left[1-\exp\{-h^{2}/\alpha(t-\tau)\}\right]d\tau \qquad (11)$$

when

T_c is the temperature fall at the bottom of the pond due to heat removal from the bottom of the pond. The express for T_c is identical to equation T_a except that the term $\phi(h,\tau)$ must be replaced by the rate of energy removal $-U\tau$

Semi - infinite solid. Radiation at the surface into a medium at zero temperature. Initial temperature constant

When the initial temperature is constant and equal to V, the equations for v are as follows:

$$\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2},$$

$$v = V \quad \text{when } t = 0,$$

$$-\frac{\partial v}{\partial x} + hv = 0, \text{ when } x = 0.$$

Let

$$\phi = v - \frac{1}{h}\frac{\partial v}{\partial x}.$$

Then we have

$$\frac{\partial \phi}{\partial t} = k \frac{\partial \phi}{\partial x^2}$$

$$\phi = V \quad \text{when } t = 0,$$

$$\phi = 0 \quad \text{when } x = 0.$$

Semi - infinite solid. Radiation at the surface into a medium at temperature f(t). Initial temperature zero

In this problem the temperature v has to satisfy

$$\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2}$$

$$-\frac{\partial v}{\partial x} + hv = hf(t) \text{ at } x = 0,$$

$$v = 0 \text{ when } t = 0.$$

Proceeding as in the last article, put

$$\phi = v - \frac{1}{h}\frac{\partial v}{\partial x}.$$

Then we have the following equations to determine :

$$\frac{\partial \phi}{\partial t} = k \frac{\partial^2 \phi}{\partial x^2},$$

$$\phi = f(t) \quad \text{at } x = 0,$$

$$\phi = 0 \text{ when } t = 0.$$

for to solving $T_b(t)$

Semi-infinite solid, Initial temperature zero. Surface at temperature $\phi(t)$

When the surface temperature varies with the time, the solution may be deduced, by Duhamel's theorem, from the case in which this temperature is constant.

Now, in the semi-infinite solid, where v has to satisfy

$$\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2},$$

$$v = 0 \text{ when } t = 0,$$

and

$$v = 1 \text{ at } x = 0,$$

the solution is given by

$$v = 1 - \frac{2}{\sqrt{\pi}}\int_0^{x/2\sqrt{(kt)}} e^{-\xi^2}\, d\xi = \frac{2}{\sqrt{\pi}}\int_{x/2\sqrt{(kt)}}^{\infty} e^{-\xi^2}\, d\xi$$

Therefore, if $\dfrac{\partial v}{\partial t} = k\dfrac{\partial^2 v}{\partial x^2}$,

$$v = 0 \text{ for } t = 0$$

and

$$v = \phi(t) \text{ at } x = 0,$$

the solution is given by

$$v = \int_0^t \phi(\lambda)\frac{\partial}{\partial t}F(x, t-\lambda)\,d\lambda,$$

where

$$F(x, t-\lambda) = \frac{2}{\sqrt{\pi}}\int_{x/2\sqrt{\{k(t-\lambda)\}}}^{\infty} e^{-\xi^2}\, d\xi$$

In this case

$$\frac{\partial}{\partial t}F(x, t-\lambda) = -\frac{2}{\sqrt{\pi}}e^{-x^2/4k(t-\lambda)}\frac{\partial}{\partial t}\left[\frac{x}{2\sqrt{\{k(t-\lambda)\}}}\right]$$

$$= \frac{x}{2\sqrt{\{\pi k(t-\lambda)^3\}}}e^{-x^2/4k(t-\lambda)}$$

Therefore the solution of our problem is

$$v = \frac{x}{2\sqrt{(\pi k)}} \int_0^t \phi(\lambda) \frac{e^{-x^2/4k(t-\lambda)}}{(t-\lambda)^{\frac{3}{2}}} d\lambda .$$

Putting

$$\frac{x}{2\sqrt{\{k(t-\lambda)\}}} = \mu ,$$

we have

$$t - \lambda = \frac{x^2}{4k\mu^2},$$

and

$$v_1 = \frac{2}{\sqrt{\pi}} \int_{x/2\sqrt{(kt)}}^{\infty} \phi\left(t - \frac{x^2}{4k\mu^2}\right) e^{-\mu^2} d\mu. \qquad (1)$$

When the boundary $x = 0$ is kept at constant temperature V and the initial temperature is zero,

$$v_2 = V\left\{1 - \operatorname{erf}\frac{x}{2\sqrt{(kt)}}\right\} \qquad (2)$$

Solution $\quad T_d(t) = v_1 + v_2$

when $\quad v = \tilde{T}_{2bar} \qquad k = \alpha$

$\qquad \phi = \tilde{T}_{bar} \qquad \mu = \nu$

$$\therefore T_d(t) = \tilde{T}_{2bar}\left(1 - \operatorname{erf}\frac{x}{2\sqrt{\alpha t}}\right) + \frac{2}{\sqrt{\pi}} \int_{x/2\sqrt{\alpha t}}^{\infty} \tilde{T}_{bar}\left(t - \frac{x^2}{4\alpha\nu^2}\right) e^{-\nu^2} d\nu$$

<u>Find. $\quad T_e(t)$</u>

$$\frac{\partial^2 v}{\partial x^2} \quad \frac{1}{k}\frac{\partial v}{\partial t} = 0 \qquad (1)$$

It is required to find the solution of the equation of linear flow of heat (1) in the infinite region $-\infty < x < \infty$ with the initial condition

$$v = f(x), \text{ when } t = 0.$$

The usual formal discussion of this problem is as follows: by consider the express

$$\frac{1}{2(\pi k t)^{1/2}} e^{-(x-x')^2/4kt} \qquad (2)$$

is a particular integral of (1).

The solution (2) has the properties.

$$u \to 0, \text{ as } t \to 0, \text{ for fixed } x \neq 0,$$

$$u \to \infty, \text{ as } t \to 0, \text{ if } x = 0,$$

$$\int_{-\infty}^{\infty} u\, dx = 2(\pi k)^{1/2} \text{ for all } t > 0,$$

Further, the equation being linear, the sum of any number of particular integrals is also an integral, and thus

$$v = \frac{1}{2\sqrt{(\pi k t)}} \int_{-\infty}^{\infty} f(x') e^{-(x-x')^2/4kt} dx',$$

satisfies the equation, assuming that this integral is convergent.

Putting $x' = x + 2\sqrt{(kt)}\xi$,

we find that $v = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f\{x + 2\sqrt{(kt)}\xi\} e^{-\xi^2} d\xi.$

In the limit when $t \to 0, f\{x+2\sqrt{(kt)}\xi\} = f(x)$, if this function is continuous; and it is assumed that the limiting value of this integral is given by

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x) e^{-\xi^2} d\xi,$$

which is equal to $f(x)$.

Therefore the temperature in the infinite solid at time t, due to the initial temperature $v = f(x)$, is given by

$$v = \frac{1}{2\sqrt{(\pi k t)}} \int_{-\infty}^{\infty} f(x') e^{-(x-x')^2/4kt} dx' \qquad (3)$$

Since $$\int_0^\infty e^{-a^2x^2}\cos 2bx\,dx = \frac{\sqrt{\pi}}{2a}e^{-b^2/a^2},$$

and therefore

$$\int_{-\infty}^{\infty} e^{-k\alpha^2 t}\cos\alpha(x'-x)\,d\alpha = \frac{\sqrt{\pi}}{2\sqrt{(kt)}}e^{-(x'-x)^2/4kt},$$

We may transform the expression for v into

$$\frac{1}{\pi}\int_{-\infty}^{\infty}dx'\int_0^\infty f(x')\cos\alpha(x'-x)e^{-k\alpha^2 t}d\alpha, \qquad (4)$$

a from which would be suggested by Fourier's integral for f(x).

Finally, some results of practical importance. which follow from, or extend (3) may be noted

If the plane x = 0 is impervious to heat, the solution takes the form

$$v = \frac{1}{2\sqrt{(\pi kt)}}\int_0^\infty f(x')\left\{e^{-(x-x')^2/4kt} + e^{-(x+x')^2/4kt}\right\}dx' \qquad (5)$$

when $v = T_e(t)$, $f(x') = T_i(x,0)$, $dx' = dx$

$k = \alpha$ $x = h$ $x' = x$

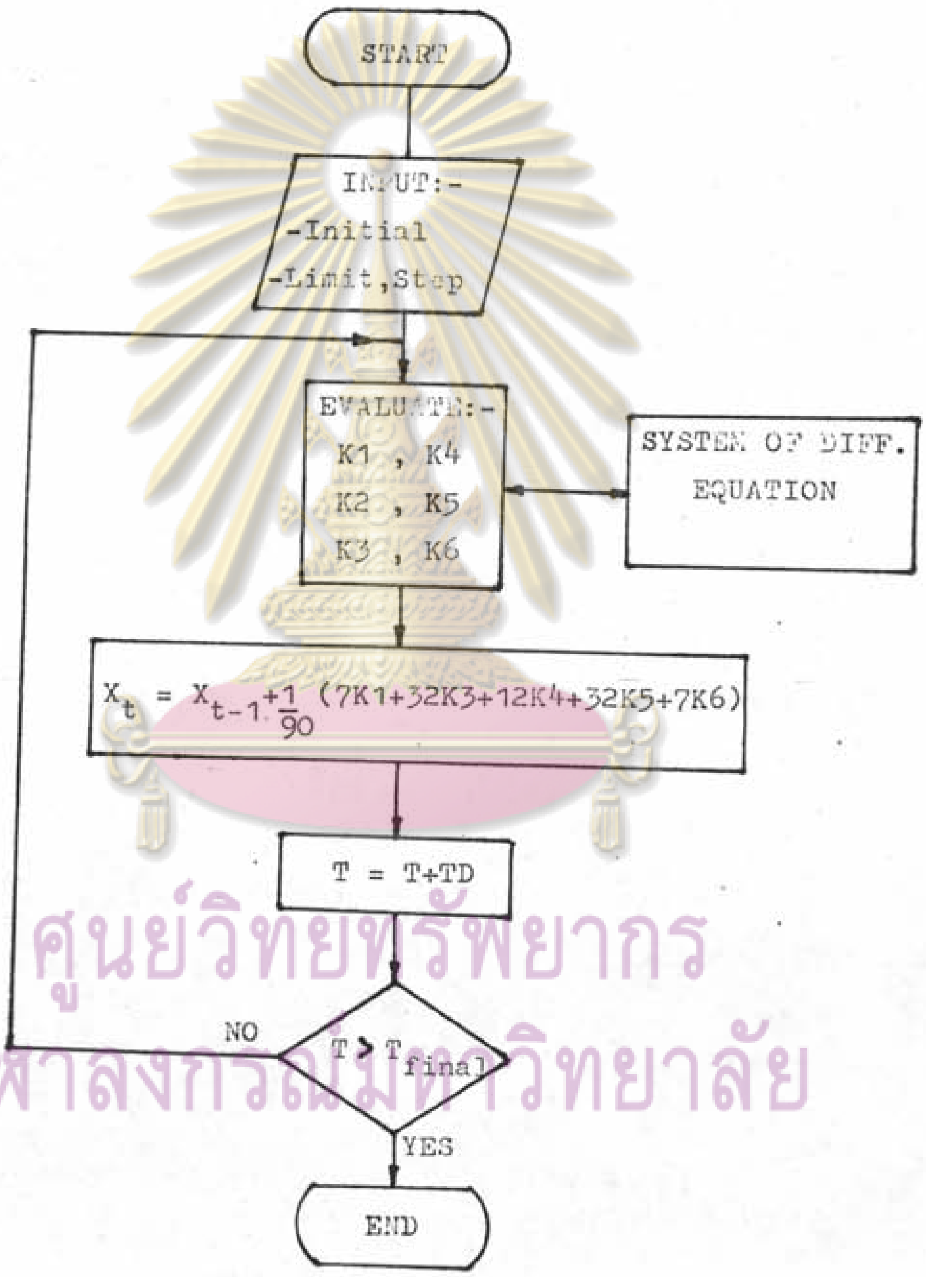
$$\therefore T_e(t) = \frac{1}{2\sqrt{\pi\alpha t}}\int_0^\infty T_i(x,0)\left\{e^{-(h-x)^2/4\alpha t} - e^{-(h+x)^2/4kt}\right\}dx$$

Reference : H.S. Carslaw and J.C. Jaeger. Conduction of heat in solids. 2nd

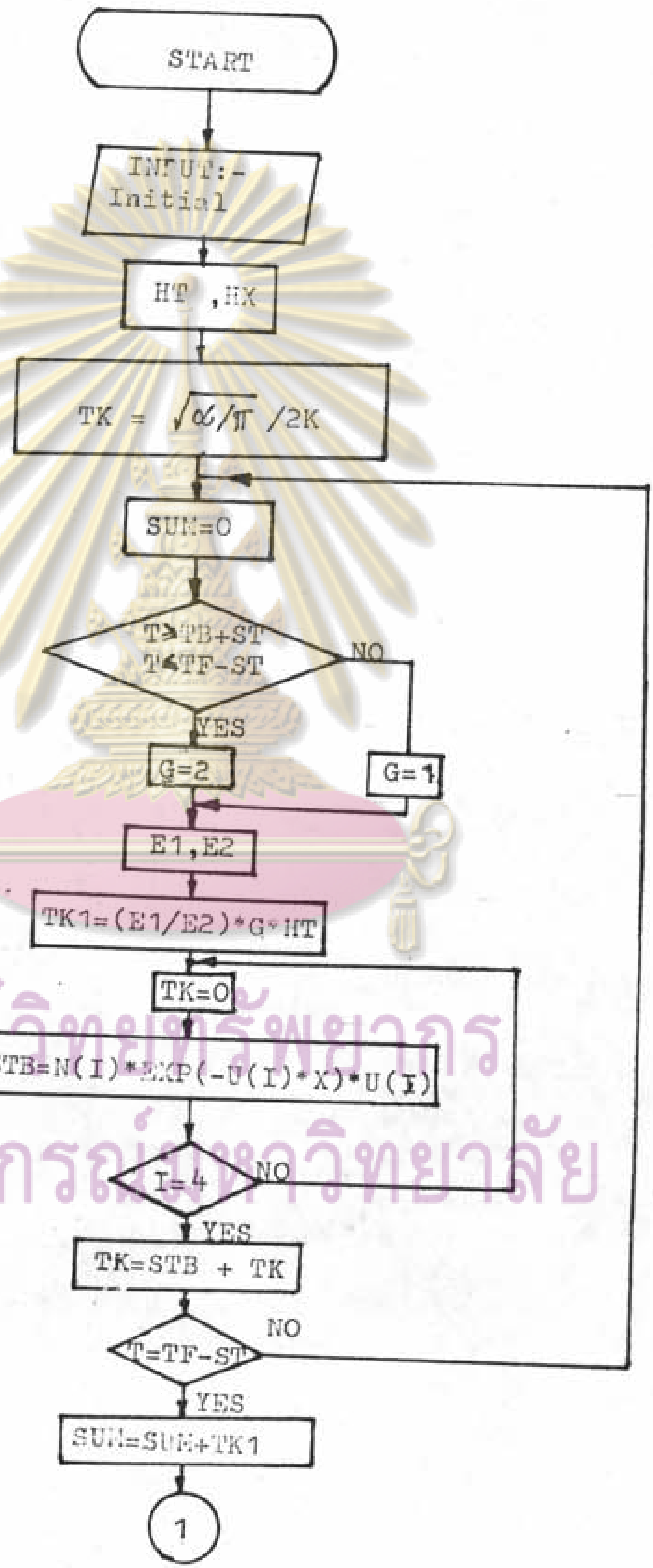
ภาคผนวก ฉ.

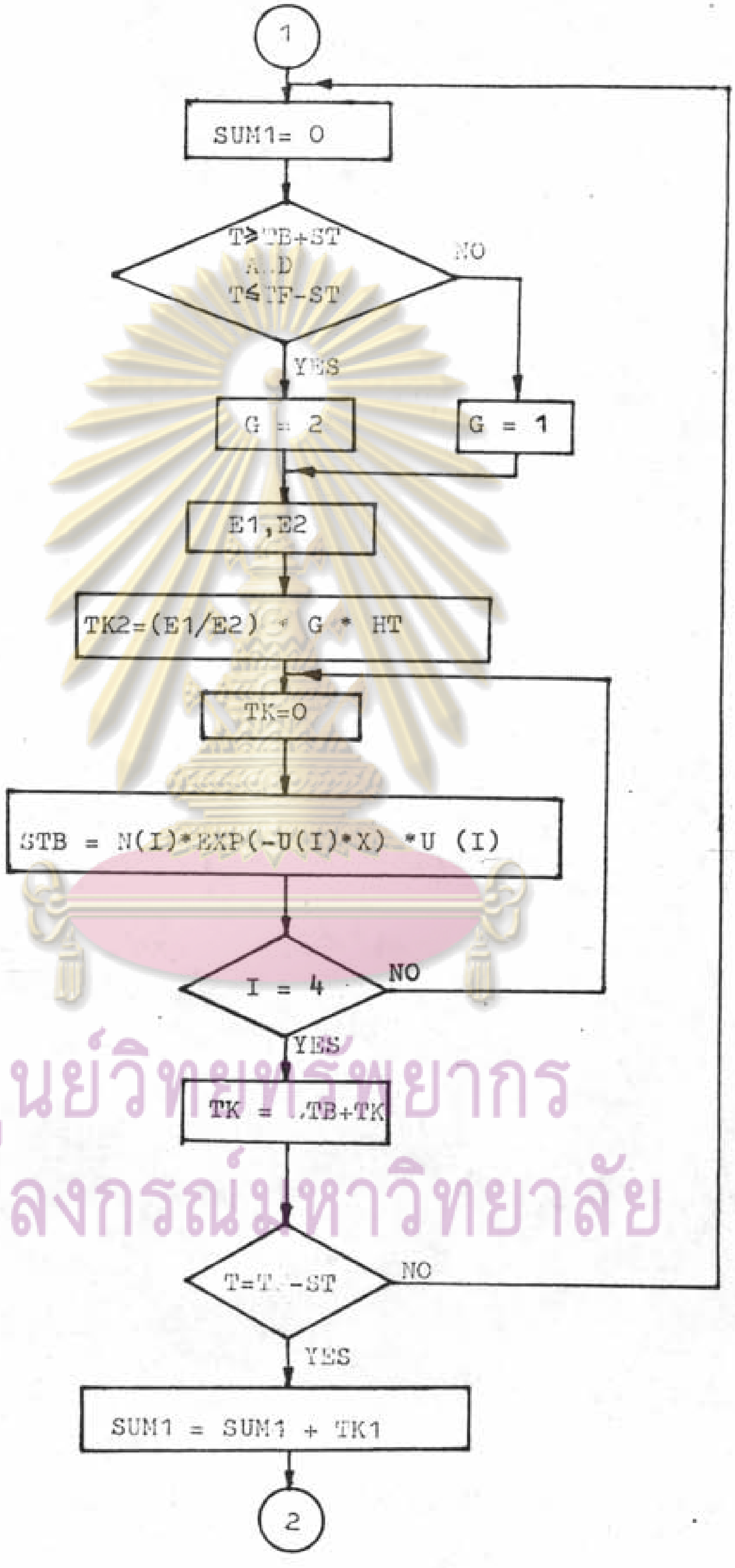
1. แสดงผังงานของโปรแกรมทั้ง 2 โปรแกรม

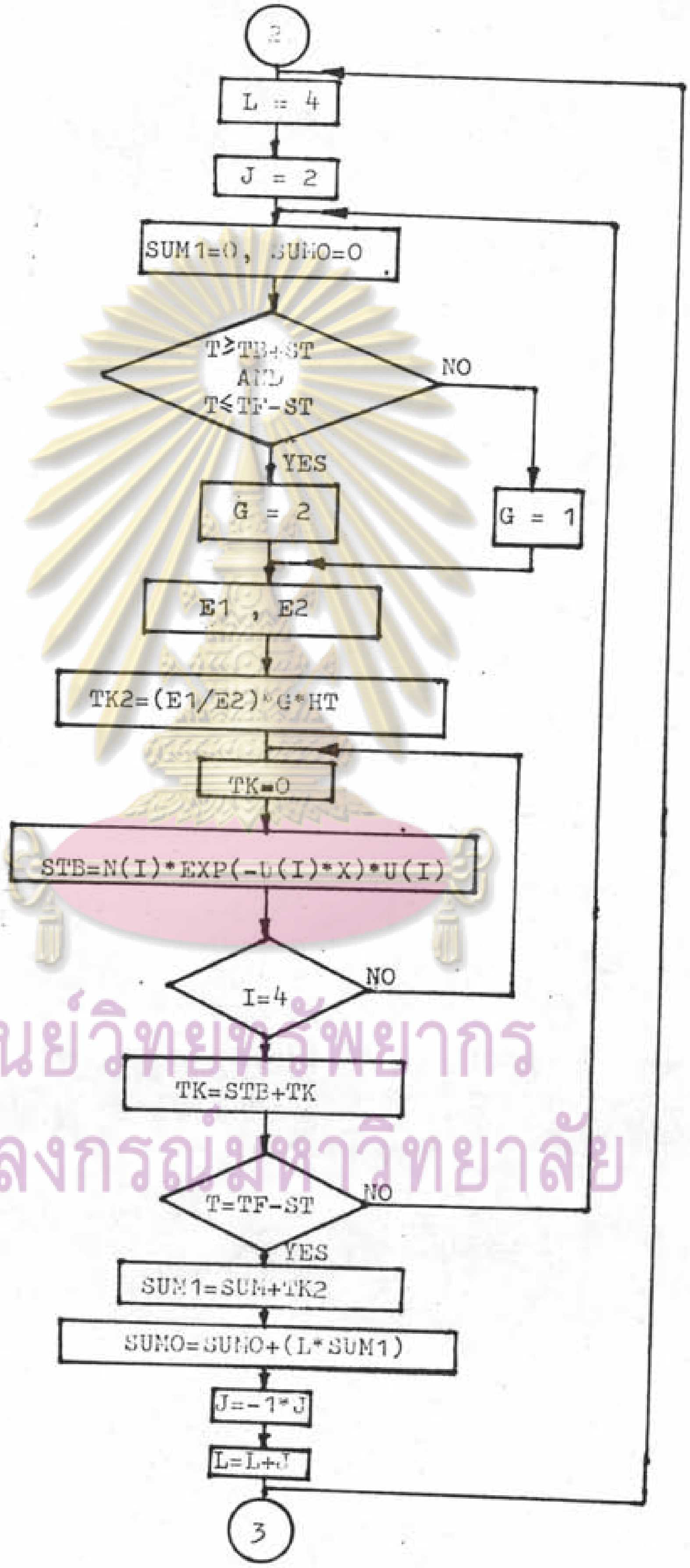
1. แสดงผังงานของโปรแกรมที่ 1

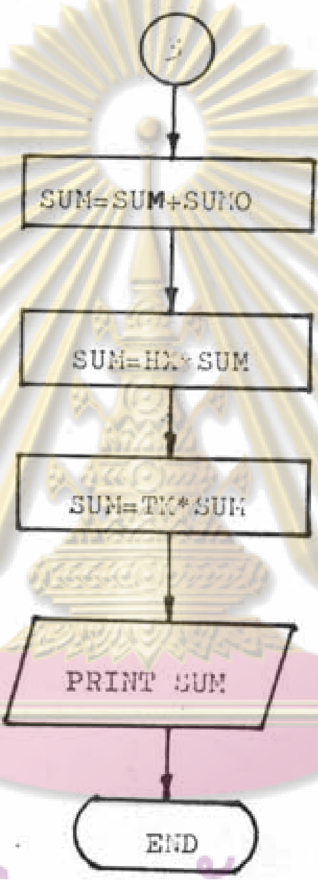


2. แสดงผังงานของโปรแกรมที่ 2




1
SUM1= 0
T≥TB+ST
T≤TF-ST
NO
YES
G = 2
G = 1
E1,E2
TK2=(E1/E2)
G
HT
TK=0
STB = N(I)*EXP(-U(I)*X)
U (I)
I = 4
NO
YES
TK =
TB+TK
T=T
-ST
NO
YES
SUM1 = SUM1 + TK1
2


2
L = 4
J = 2
SUM1=0, SUMO=0
T≥TB+ST
AND
T≤TF-ST
NO
YES
G = 2
G = 1
E1 , E2
TK2=(E1/E2)*G*HT
TK=0
STB=N(I)*EXP(-U(I)*X)*U(I)
I=4
NO
TK=STB+TK
T=TF-ST
NO
YES
SUM1=SUM+TK2
SUMO=SUMO+(L*SUM1)
J=-1*J
L=L+J
3


SUM=SUM+SUMO
SUM=HX*SUM
SUM=TK*SUM
PRINT SUM
END

ภาคผนวก ช.

2. แสดงโปรแกรมที่ใช้ในการคำนวณหาอุณหภูมิ 2 โปรแกรม

```
10 REM ********** SIXTH ORDER RUNGE-KUTTA METHOD ***********
20 REM                          FOR
30 REM       SIMULTANEOUS ORDINARY DIFFERENTIAL EQUATIONS
40 REM                  SEPTEMBER 30TH,1985 VERSION
50 REM ********* BY THANAKORN SOONTORNRATHANAKUL M.ENG (ME) CHULA ***
60 REM
70 DIM X(12),XX(12),DX(12),F(6,12)
80 HOME
90 VTAB(10)
100 PRINT TAB(22) "SIXTH ORDER RUNGE-KUTTA METHOD"
110 VTAB(20)
120 PRINT "You can input your system of ordinary differentials at LINE 1820"
130 PRINT "Type CTRL-C to exist from this program"
140 PRINT
150 PRINT "HAVE YOU INPUT YOUR SYSTEM OF EQUATIONS ALREADY ? [Y/N] ";
160 IP$=INKEY$ :IF LEN(IP$)= 0 THEN 160
170 IF IP$= "n" THEN IP$= "N"
180 IF IP$<> "N" THEN IP$= "Y"
190 PRINT IP$
200 IF IP$= "N" THEN END
210 HOME
220 VTAB(22)
230 INPUT "INPUT NUMBER OF ORDINARY DIFFERENTIAL EQUATIONS ";NUM
240 PRINT
250 PRINT "Do you want line printer output ? [Y/N] ";
260 PR$=INKEY$ :IF LEN(PR$)= 0 THEN 260
270 IF PR$= "y" THEN PR$= "Y"
280 IF PR$<> "Y" THEN PR$= "N"
290 HOME
300 PRINT "< CHOOSE YOUR DESIRED DATA OUTPUT >"
310 PRINT "Not more than 5 variables for the terminal and the FX-80 printer"
320 PRINT
330 PRINT "TYPE '1' FOR YOUR DESIRED VARIABLES OTHERWISE '0'"
340 FOR I= 1 TO NUM
350 PRINT
360       PRINT "DX(" I ") ";
370       INPUT MDX(I)
380       PRINT "X(" I ") ";
390       INPUT MX(I)
400 NEXT I
410 VTAB(22)
420 PRINT "Do you want to change any data ? [Y/N] ";
430 CH$=INKEY$ :IF LEN(CH$)= 0 THEN 430
440 IF CH$= "y" THEN CH$= "Y"
450 IF CH$<> "Y" THEN CH$= "N"
460 PRINT CH$
470 IF CH$= "Y" THEN 290
480 HOME
490 VTAB(22)
500 PRINT "Do you want to save OUTPUT DATA in a file ? [Y/N] ";
510 SA$= INKEY$ :IF LEN(SA$)= 0 THEN 510
520 IF SA$= "y" THEN SA$= "Y"
530 IF SA$<> "Y" THEN SA$= "N"
540 PRINT SA$
550 IF SA$<> "Y" THEN 570
560 INPUT "Input FILENAME (Not exceed 8 characters) ";FI$
570 HOME
580 PRINT "< INPUT INITIAL CONDITIONS >"
590 INPUT "T initial";T
```

```
600 FOR I= 1 TO NUM
610        PRINT "X(" I ")= ";
620        INPUT X(I)
630 NEXT I
631 INPUT "TO ";Z1
632 INPUT "H " ;Z2
633 INPUT " L "; Z3
634 INPUT " ALPHA ";ALPHA
640 TI=1
650 PRINT
660 PRINT "Do you want to change any data ? (Y/N) ";
670 CHG$= INKEY$ :IF LEN(CHG$) = 0 THEN 670
680 IF CHG$= "y" THEN CHG$= "Y"
690 IF CHG$<> "Y" THEN CHG$= "N"
700 PRINT CHG$
710 IF CHG$= "Y" THEN 570
720 INPUT "del T";TD
730 INPUT "T final";TMAX
740 HOME
750 PRINT "T",;
760 IF PR$= "N" THEN 780
770 LPRINT "T",;
780 FOR I= 1 TO NUM
790        IF MDX(I)<> 1 THEN 830
800        PRINT "      DX(" I ")",;
810        IF PR$= "N" THEN 830
820        LPRINT "      DX(" I ")",;
830        IF MX(I)<> 1 THEN 870
840        PRINT "      X(" I ")",;
850        IF PR$= "N" THEN 870
860        LPRINT "      X(" I ")",;
870 NEXT I
880 PRINT
890 PRINT "------------------------------------------------------------------------------------------
---------"
900 IF PR$= "N" THEN 930
910 LPRINT
920 LPRINT "------------------------------------------------------------------------------------------
---------"
930 IF SA$= "N" THEN 950
940 OPEN "O",#1,F1$
950 FOR I= 1 TO NUM
960        XX(I)= X(I)
970 NEXT I
980 PRINT T,;
990 IF PR$= "N" THEN 1010
1000 LPRINT T,;
1010 IF SA$= "N" THEN 1030
1020 PRINT#1,T,;
1030 FOR I= 1 TO NUM
1040        IF MDX(I)<> 1 THEN 1100
1050        PRINT USING "##.####^^^^";DX(I);
1060        IF PR$= "N" THEN 1080
1070        LPRINT USING "##.####^^^^";DX(I);
1080        IF SA$= "N" THEN 1100
1090        PRINT#1,USING "##.####^^^^";DX(I);
1100        IF MX(I)<> 1 THEN 1160
1110        PRINT USING "##.####^^^^    ";X(I);
1120        IF PR$= "N" THEN 1140
```

```
1130      LPRINT USING "##.####      ";X(I);
1140      IF SA$= "N" THEN 1160
1150      PRINT#1,USING "##.####      ";X(I);
1160 NEXT I
1170 PRINT
1180 IF PR$= "N" THEN 1200
1190 LPRINT
1200 IF SA$= "N" THEN 1220
1210 PRINT#1,
1220 REM    *** EVALUATE K1 ***
1230 T= TI
1240 GOSUB 1870
1250 FOR I= 1 TO NUM
1260      K(1,I)= TD*DX(I)
1270 NEXT I
1280 REM    *** EVALUATE K2 ***
1290 T= TI+TD/4
1300 FOR I= 1 TO NUM
1310      X(I)= XX(I)+K(1,I)/4
1320 NEXT I
1330 GOSUB 1870
1340 FOR I= 1 TO NUM
1350      K(2,I)= TD*DX(I)
1360 NEXT I
1370 REM    *** EVALUATE K3 ***
1380 T= TI+TD/4
1390 FOR I= 1 TO NUM
1400      X(I)= XX(I)+K(1,I)/8+K(2,I)/8
1410 NEXT I
1420 GOSUB 1870
1430 FOR I= 1 TO NUM
1440      K(3,I)= TD*DX(I)
1450 NEXT I
1460 REM    *** EVALUATE K4 ***
1470 T= TI+TD/2
1480 FOR I= 1 TO NUM
1490      X(I)= XX(I)-K(2,I)/2+K(3,I)
1500 NEXT I
1510 GOSUB 1870
1520 FOR I= 1 TO NUM
1530      K(4,I)= TD*DX(I)
1540 NEXT I
1550 REM    *** EVALUATE K5 ***
1560 T= TI+TD*3/4
1570 FOR I= 1 TO NUM
1580      X(I)= XX(I)+3*K(1,I)/16+9*K(4,I)/16
1590 NEXT I
1600 GOSUB 1870
1610 FOR I= 1 TO NUM
1620      K(5,I)= TD*DX(I)
1630 NEXT I
```

```
1640 REM    *** EVALUATE K6 ***
1650 T= TI+TD
1660 FOR I= 1 TO NUM
1670        X(I)= XX(I)-3/7*K(1,I)+2/7*K(2,I)+12/7*K(3,I)-12/7*K(4,I)+8/7*K(5,I)
1680 NEXT I
1690 GOSUB 1870
1700 FOR I= 1 TO NUM
1710   .    K(6,I)= TD*DX(I)
1720 NEXT I
1730 REM    *** EVALUATE X(I) ***
1740 FOR I= 1 TO NUM
1750        X(I)= XX(I)+1/90*(7*K(1,I)+32*K(3,I)+12*K(4,I)+32*K(5,I)+7*K(6,I))
1760 NEXT I
1770 TI= TI+TD
1780 T= TI
1790 IF T< TMAX+TD THEN 950
1800 CLOSE#1
1810 PRINT
1820 PRINT "Continue (Y/N) ? ";
1830 CON$=INKEY$:IF LEN(CON$)=0 THEN 1830
1840 IF CON$= "y" THEN CON$= "Y"
1850 IF CON$= "Y" THEN 480
1860 END
1870 REM    *** SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS ***
1880 DX(1)=Z1*(EXP(-(Z2-T)^2/4/ALPHA/Z3)-EXP(-(Z2+T)^2/4/ALPHA/Z3))/2/SQR(22/7*
LPHA)
1920 RETURN
1930 END
```

```
10 'INTEGRATE  2  DIMENSION
20 'BY SIMPSON  RULE
25 'BY THANAKOM  SOONTORNCHAINACKSAENG  M.ENG  CHULA
30 INPUT "LOWER LIMIT TB =",TB
40 INPUT "UPPER LIMIT TF =",TF
50 INPUT "LOWER LIMIT HB =",HB
60 INPUT "UPPER LIMIT HF =",HF
70 INPUT "NUMBER OF INTERVAL N IN X DIRECTION ",N
80 INPUT "NUMBER OF INTERVAL M IN T DIRECTION ",M
100 HT=(TF-TB)/N : HX=(HF-HB)/N : PRINT "HT=";HT,"HX=";HX
110 INPUT "ZETA =",ZETA
120 INPUT "PHI= ",PHI
140 INPUT "K =",K
145 INPUT "ALPHA =",ALPHA
147 INPUT "ST=",ST
148 INPUT "SH =",SH
150 N(1) = .237
160 N(2) = .193
170 N(3) = .163
180 N(4) = .179
190 U(1) = .32*10^(-3)
200 U(2) = 4.5*10^(-3)
210 U(3) = .03
220 U(4) = .35
290          TK=SQR(((ALPHA*7/22)/(2*K)))*ZETA*PHI
295 PRINT TK
310        SUM =0
330 FOR T=TB  TO  TF-ST  STEP  ST
332 IF T>=TB+ST  AND T<= TF-ST  THEN  GOTO  370
335 ON ERROR GOTO  410
350        G=1
360 GOTO  371
370        G=2
371 E1=(EXP(-(HF-HB)^2/(4*ALPHA*(TF-T))))
372 E2=(EXP(-(HF+HB)^2/(4*ALPHA*(TF-T))))
373 TK1=(E1-E2)/SQR(TF-T)*G*HT
374 LET TK=0:STB=0
375 FOR I=1  TO 4
376          STB=N(I)*EXP(-U(I)*HB)*U(I)
377          TK=STB+TK
378 NEXT I
379          TK1=TK1*TK
390        SUM = SUM+TK1
395 PRINT "T=";T," G=";G,"  TK1=";TK1," E1=";E1,"  E2=";E2," SUM=";SUM
410 NEXT T
420        SUM1 = 0
430 FOR T=TB  TO  TF-ST  STEP  ST
440 IF  T>=TB+ST AND T<=TF-ST  THEN GOTO 470
445 ON ERROR GOTO  510
450        G=1
460 GOTO 471
470        G=2
471 E1=(EXP(-(HF-HF)^2/(4*ALPHA*(TF-T))))
472 E2=(EXP(-(HF+HB)^2/(4*ALPHA*(TF-T))))
473        TK1=(E1-E2)/SQR(TF-T)*G*HT
474 LET TK=0:STB=0
475 FOR I=1  TO  4
476          STB=N(I)*EXP(-U(I)*HF*U(I)
477          TK=STB+TK
478 NEXT I
479          TK1=TK1*TK
```

```
490          SUM1=SUM1+TK1
510 NEXT T
520           SUM=SUM+SUM1
530        L=4
540        J=2  :SUM1=0  :SUMO=0
550 FOR X=(HB+SH) TO (HF-SH)   STEP SH
560 FOR T=TB   TO   TF-ST   STEP   ST
565 ON ERROR GOTO 640
570 IF T>=TB+ST   AND T<=TF-ST   THEN GOTO 600
580 .       G=1
590 GOTO 601
600         G=2
601 E1=(EXP(-(HF-X)^2/(4*ALPHA*(TF-T))))
602 E2=(EXP(-(HF+X)^2/(4*ALPHA*(TF-T))))
605         TK2 =  (E1-E2)/SQR(TF-T)*G*HT
606 LET TK=0:STB=0
607 FOR I=1   TO 4
608             STB=N(I)*EXP(-U(I)*X*U(I))
609             TK=STB+TK
610 NEXT I
611             TK2=TK2*TK
620         SUM1=SUM1+TK2
625 PRINT "T=";T," G=";G," TK2=";TK2," E1=";E1," E2=";E2," SUM1=";SUM1
640 NEXT T
650         SUMO=SUMO+(L*SUM1)
660         J=-1*J
670         L=L+J
675 PRINT "J=";J," L=";L
680 NEXT X
690         SUM=SUM+SUMO
700     .   SUM=SUM*HK
710         SUM=SUM*TK
720 PRINT SUM
730 END
```

ประวัติ

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