CHAPTER 2

BRIEF SUMMARY ON SHIP STABILITY THEORY

A ship will experience many forces (eg. wind and waves) trying to turn it over in the service. It must be capable of resisting these by what is termed its stability. It is also well known that too much stability is undesirable because it can cause unpleasant motions and be costly. Thus, stability is a factor to be compromised. Actually no ship can be guaranteed under all conditions. Because a ship will meet varied conditions during her life.

2.1 INITIAL STABILITY [14; 18]

A ship is a complex structure and is not in the mathematical sense a rigid body. However, for the purpose of studying stability it is permissible so to regard it in calm water and not underway [4]. When the ship is in waves or is underway, there are hydrodynamic forces acting on the ship which may affect the buoyancy forces. For fast motor boats the hydrodynamics forces predominate the assessing stability.

Any small disturbance can be resolved into three components of translation and three of rotations with reference to the ship's body axes. Fig. (2.1) shows the positive directions prescribed.

- Translation along the x-axis (SURGE) leads to no resultant force. The ship is in neutral equilibrium for this type of disturbance.

- Translation along the y-axis (SWAY) leads to no resultant force. The ship is also in neutral equilibrium for this type of disturbance.
- Translation along the z-axis (HEAVE) results in an augmented buoyancy force which will tend to move the ship in the opposite direction. The ship is thus stable for this type of disturbance. (Special case of totally sumbmerged body is not considered here)
- Rotation about the x-axis (ROLL), results in a moment acting on the ship about which no generalisation is possible. The ship may display stable, neutral or unstable equilibrium.
- Rotation about the y-axis (PITCH) leads to the condition similar of rolling
- Rotation about the z-axis (YAW), results in no resultant force or moment. The ship is in neutral equilibrium for this type of disturbance.

The only disturbances which herein demand study are the rotations about the x-and y-axes. Due to the waterplane characteristics are such that it is convenient to study the two separately :-

a) The stability exhibited by the ship for rotations about the x-axis referred to as its transverse stability.

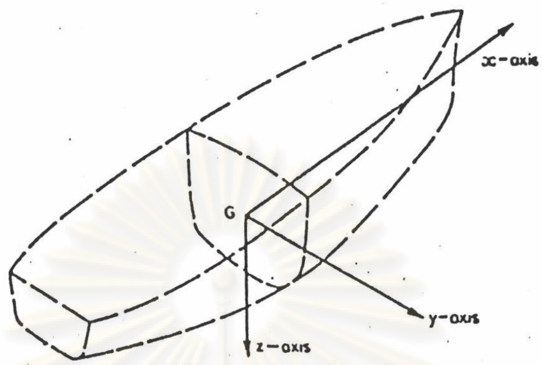

Fig. (2.1) Define ship's reference axes

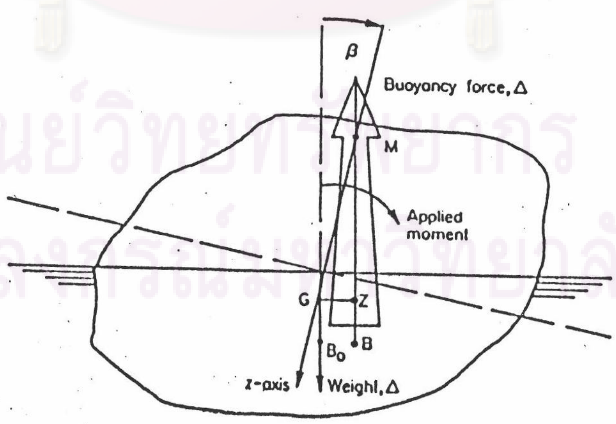

Fig. (2.2) Action of buoyancy and weight for small rotational disturbance

b) The stability exhibited by the ship for rotations about the y-axis referred to as its longitudinal stability.

Consider the irregular shaped body shown in Fig. (2.2) floating in a state of equilibrium. The centre of buoyancy, B_o, and the centre of gravity, G, must lie on the same vertical line, in the state of equilibrium. If the body is now subjected to a rotational disturbance, by turning it through a small angle at constant displacement, the centre of buoyancy will move to some new position, B. In the figure, angular disturbance is shown as being caused by a pure moment about G, but it will be realised that the condition of constant displacement will, in general, require translational movements of G by forces as well.

The weight and buoyancy forces continue to act vertically after rotation but, in general, are separated so that the body is subject to a moment ΔGZ where Z is the foot of the normal from G on to the line of action of the buoyancy force. As drawn, this moment tends to restore the body to the original position. The couple is termed the righting moment and GZ is termed the righting lever.

Another way of defining the line of action of the buoyancy force is to use its point of intersection, M, with the z-axis. As the angle is indefinitely diminished

M tends to a limiting position termed the metacentre. For small values of Beta, it follows that

$$GZ = GM \sin\beta$$
$$\cong GM\beta$$

The distance GM is termed the metacentric height and is said to be positive when M lies above G. This is the condition of stable equilibrium, for should M lie below G, the moment acting on the body tends to increase β, the body is unstable. If M and G coincide the equilibrium is neutral. See Fig. (2.3)

<u>The method of form changing [50]</u>

Alteration of the ship's dimensions (ie.in x, y or z directions) can lead to the adjustment of transverse metacentric height. The method of form changing has the advantage of maintaining the coefficients of fineness unaltered and is often used in the early stages of ship design.

$$\Delta = (\text{const.}) \times L \times B \times T$$

Hence

$$\log \Delta = \log (\text{const.}) + \log L + \text{Log } B + \text{Log } T$$

Differentiating

$$\frac{d\Delta}{\Delta} = \frac{dL}{L} + \frac{dB}{B} + \frac{dT}{T}$$

Thus, if the percentage changes in the main dimensions are small, their sum will provide the percentage change in the displacement. It can also be shown that

$$BM = \frac{I_T}{\nabla} = (\text{const.}) \times \frac{B^2}{T}$$

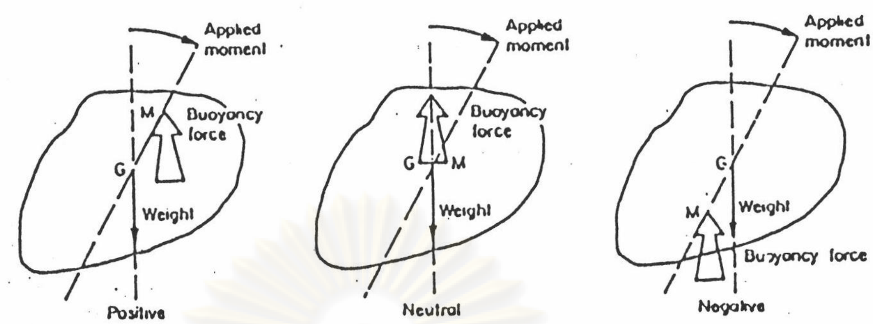

Fig. (2.3) Stability conditions

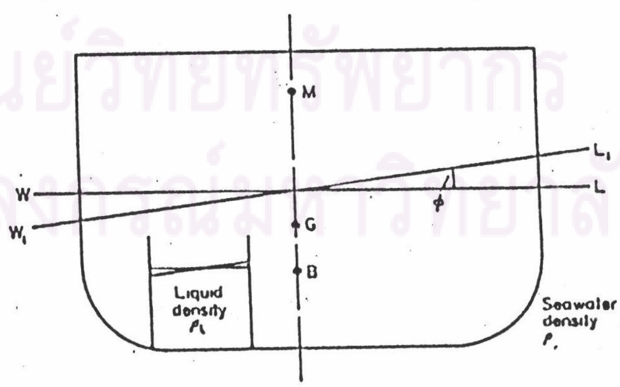

Fig. (2.4) Liquid free surface

By the same process

$$\frac{dBM}{BM} = \frac{2dB}{B} - \frac{dT}{T}$$

Also (Fig. 2.2)

$$\frac{dBM}{BM} = \frac{dBG + dGM}{BM}$$

$$= \frac{dBG}{BG} + \frac{dGM}{GM}$$

Some special cases are worth considering :

a) To change the vessel's beam only, the increase in displacement is given by

$$d\Delta = \Delta\frac{dB}{B}$$

GM is deduced by

$$\frac{dBG + dGM}{BM} = 2\,\frac{dB}{B}$$

If KG is const., then dBG = 0

Hence $$dGM = 2BM\,\frac{dB}{B}$$

Normally BM is greater than GM, the metacentric height is normally increase more than twice that in beam.

b) To keep the displacement and draught constant.

$$-\frac{dL}{L} = \frac{dB}{B}$$

Thus, the length must be decreased in the same ratio as the beam is increased. Provided, KG = const. The change in metacentric height is according to case (a).

c) To change the vessel's draught while keeping the length and displacement constant

$$\frac{dT}{T} = \frac{-dB}{B}$$

Normally KG varies as the draught, hence two possibilities exist :-

(i) $KG = \text{const.}$

Then $BG = KG - KB$

$$dBG = -\text{ (Change in KB)}$$

$$= -KB \cdot \frac{dT}{T}$$

$$= KB \cdot \frac{dB}{B}$$

Hence, from the previous expression

$$\frac{dBG + dGM}{BG + GM} = \frac{2dB}{B} - \frac{dT}{T} = -\frac{3dB}{B}$$

$$dBG + dGM = 3\ (BG + GM)\ \frac{dB}{B}$$

$$dGM = (4BG + 3GM - KG)\ \frac{dB}{B}$$

(ii) $KG = (\text{const.}) \times T$

But $KB = (\text{const.}) \times T$

Also $BG = KG - KB = (\text{const.}) \times T$

The above constants of proportionality are not necessarily the same.

Hence $$\frac{dBG}{BG} = \frac{dT}{T} = \frac{-dB}{B}$$

It has been shown that

$$\frac{dBG + dGM}{BG + GM} = \frac{2dB}{B} - \frac{dT}{T} = \frac{3dB}{B}$$

Hence

$$-BG\frac{dB}{B} + dGM = 3(BG + GM)\frac{dB}{B}$$

and

$$dGM = (4BG + 3GM)\frac{dB}{B}$$

Effect of free surfaces of liquids

Let the ship be floating initially at a waterline WL and let it be heeled through a small angle ϕ to a new waterline W_1L_1. Since, the surface of the liquid in the tank is free, it will also change its surface inclination relative to the tank, by the same angle ϕ Fig. (2.4).

For small angles of inclination, it can be shown that the transfer of buoyancy is given approximately by the expression.

$$\frac{\Delta_s}{\nabla s} Is \cdot \phi = \rho_s I_s \phi \cdot g$$

Similarly, the transfer of weight due to the movement of the liquid in the tank is approximately

$$\frac{\Delta_1}{\nabla 1} I_1 \cdot \phi = \rho_1 I_1 \phi \cdot g$$

Transferring of weight opposes the righting moment due to the transfer of buoyancy and results in the effective righting moment acting on the ship being reduced to

$$\Delta_s GMo - \rho_1 I_1 \cdot \phi \cdot g$$

Let GM_F be the effective metacentric height allowing for the action of the liquid free surface

$$\Delta_s GM_F \phi = \Delta_s GM_s \phi - \rho_1 I_1 \cdot \phi g$$

Therefore

$$GM_F = GM_s - \frac{\rho_1 I_1}{\nabla_s} \cdot g \quad s$$

$$= GM_s - \left(\frac{\rho_1}{\rho_s}\right) \frac{I1}{\nabla_s}$$

Effect of the free surface is independent of the tank's position. The effect is also independent of the amount of liquid in the tank provided the second moment of area of the free surface is substantially unchanged when inclined.

Normally, the free surface effect is regarded as being a virtual rise of centre of gravity of the ship.

<u>The wall-sided formula</u>

A ship may be regarded as wall-sided if those portions of the outer bottom covered or uncovered by the moving waterplane are vertical with the ship upright.

No practical ships are truly wall-sided but many may be regarded as such for small angles of inclination .

Let the ship be inclined from its initial water WL to a new waterline W_1L_1 by being heeled through a small angle ϕ. Since the vessel is wall-sided, WL and W_1L_1 must intersect on the centre line. See Fig. (2.5)

The volume transferred in an elemental wedge of length δL, where the beam is b, is

$$\delta L \left\{\frac{1}{2} \times \frac{b}{2} \times \frac{b}{2} \tan \phi\right\} = \left\{\frac{b^2}{8} \tan \phi\right\} \delta L$$

Moment of transfer of volume for this wedge in a direction parallel to WL

$$= \frac{b^2}{8} \tan \phi \, \frac{2}{3} b . \delta L$$

Hence, for the whole ship, moment of transfer of volume is

$$\frac{b^3}{12} \tan \phi \, . \, dL$$

Horizontal component of shifting of B, BB'is given by

$$BB' = \frac{1}{\nabla} \int_0^L \frac{b^3}{12} \tan \phi \, . \, dL$$

$$= \frac{I}{\nabla} . \tan \phi = BM . \tan \phi$$

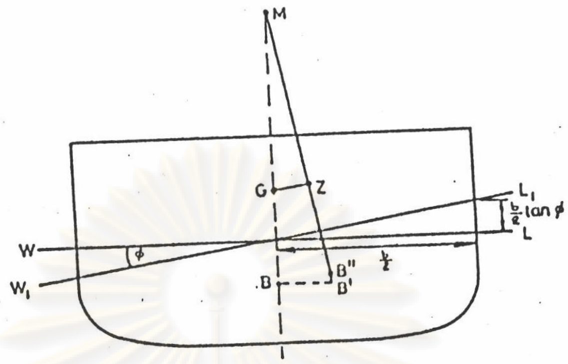

Fig. (2.5) Wall-sided formula

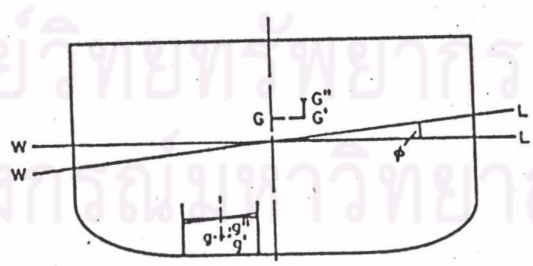

Fig. (2.6) Wall-sided ship with liquid contained in a wall-sided tank

Similarity, vertical shift B'B" is given by

$$B'B'' = \frac{1}{\nabla}\int_0^L \frac{b^2}{8} \cdot \tan\phi \, \frac{1}{3} b \cdot \tan\phi \cdot dL$$

$$= \frac{I}{2\nabla} \cdot \tan^2\phi$$

$$= \frac{BM}{2} \cdot \tan^2\phi$$

By projection on to a plane parallel to W_1L_1

$$GZ = BB' \cos\phi + B'B'' \sin\phi - BG \sin\phi$$

$$= BM [\sin\phi + \frac{1}{2} \tan^2\phi \cdot \sin\phi\} - BG \sin\phi$$

$$= \sin\phi \{BM - BG + \frac{1}{2} BM \cdot \tan^2\phi]$$

$$GZ = \sin\phi [GM + \frac{1}{2} BM \cdot \tan^2\phi]$$

<u>Wall-sided vessel containing a tank with vertical sides containing liquid</u>

Let the density of the liquid in tank be ρ times that the water in which the ship is floating

Fig.(2.6), horizontal and vertical shifts of B will be as calculated for the ship without a tank. The centroid of mass of the liquid in the tank will suffer movements

$$gg' = \frac{I1}{\nabla 1} \cdot \tan \phi$$

and

$$g'g'' = \frac{I1}{2\nabla 1} \cdot \tan^2 \phi$$

The centre of gravity of the ship, G, will suffer movements of

$$GG' = \frac{I1}{\nabla_1} \cdot \tan \phi \times \rho \frac{\nabla_1}{\nabla_s} = \rho \frac{I1}{\nabla_s} \cdot \tan \phi$$

and

$$G'G'' = \rho I1 \cdot \tan^2 \phi$$

The presence of the free surface will effectively reduce GZ to GZ_F

where

$$GZ_F = \sin \phi \left[\left(GM - \rho \frac{I1}{\nabla_s} \right) + \left(BM - \rho \frac{I1}{\nabla_s} \right) \frac{1}{2} \tan^2 \phi \right]$$

2.2 LARGE ANGLE STABILITY [65]

Cross Curves of Stability

In principle, the concepts of large angle transverse stability are equally applicable to longitudinal stability but, in practice, they are not normally required because of the relatively small angles of trim a ship can accept for reasons other than stability.

Let the ship be floating initially at a waterline WL. Fig. (2.7). Now Let let it be heeled through some angle 0 to a new waterline W_1L_1 such that the displacement remains constant. The buoyancy force will act through B_1 the new position of the centre of buoyancy, its line of action being perpendicular to W_1L_1. If Z_1 is the foot of the perpendicular dropped from G on to the line of action of the buoyancy force then the righting moment acting on the ship is given by ΔGZ_1.

For large angles of stability, the concept of a metacentre can no longer be used as the buoyancy force does not intersect the ship's middle line plane in a fixed point.

In general, a ship when heeled will also trim to maintain its longitudinal equilibrium. This can usually be ignored but where it is significant. The points shown in Fig. (2.7) must be regarded as projections of the true points on to a transverse plane of the ship.

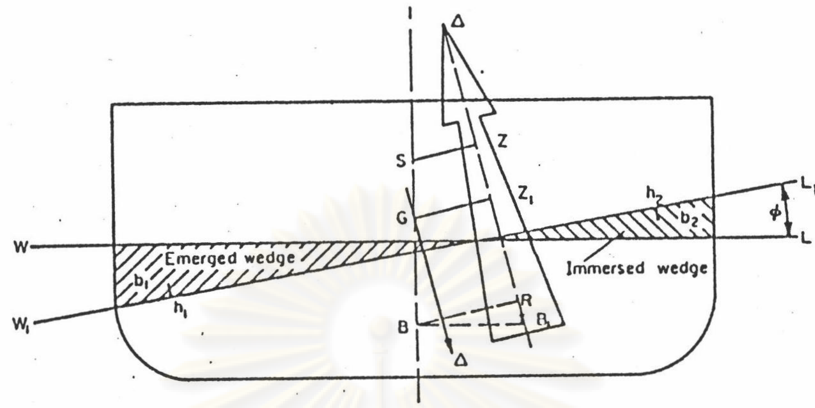

Fig. (2.7) Stability at large angles

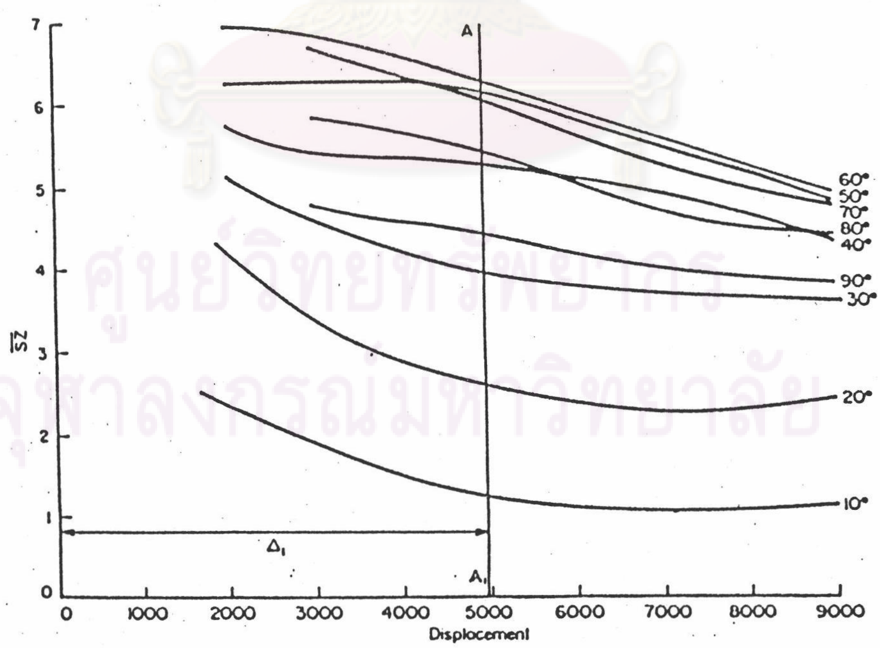

Fig. (2.8) Cross curves of stability

Since the displacement is the same, it follows that the volumes of the immersed and emerged wedges are equal

Let δ = buoyancy force associated with immersed wedge

b_1; b_2 = centroids of volume of the emerged and immersed wdeges respectively

h_1; h_2 = feet of perpendiculars from b_1; b_2 onto W_1L_1

R = Foot of perpendicular from B on to the line of action of the buoyancy force through B_1

Then $\Delta_{BR} = \delta_{h_1 h_2}$

The righting moment acting is given by

$$\Delta_{GZ} = \Delta_{BR} - \Delta_{BG} \text{ Sin } \phi$$

$$= \Delta \left[\frac{\delta}{\Delta} h_1 h_2 - \text{BG Sin } \phi \right]$$

The above formula is known as Atwood's Formula. The point G depends upon the loading of the ship and is not a fixed point. It is more convenient to think in terms of an arbitrary, but fixed, pole S and its perpendicular distance SZ from the line of action of the buoyancy force SZ is actually a function of the ship's geometry and can be calculated for various angles of heel and displacements without reference to a particular loading condition.

Then

$$GZ_1 = SZ + SG \sin \phi$$

The position of G can be calculated for a particular condition of loading. The values of SZ can be plotted against displacement for each of a number of angles of

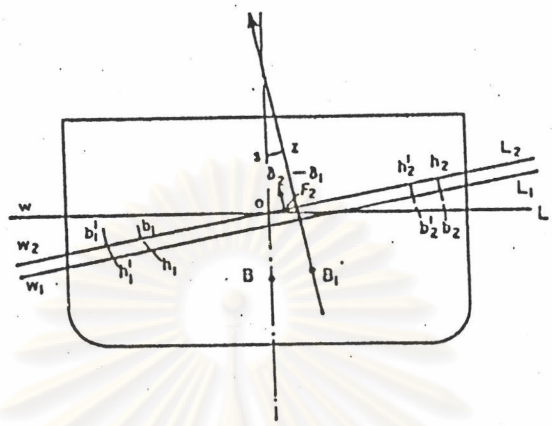

Fig.(2.9) Derivation of SZ values for constant displacement

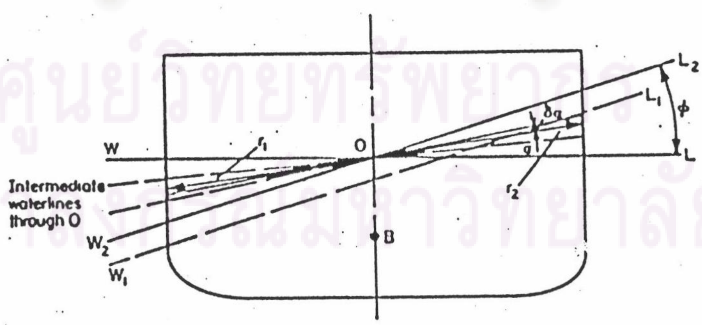

Fig. (2.10) Radial integration for Barnes's method of deriving statistical stability data inclination as shown in Fig. (2.8). The curves are, commonly, called "Cross Curves of Stability"

For constant displacement, the SZ curve may be determined as follows. Fig. (2.9) let WL be the upright waterline, W_1L_1 the inclined water line at the same displacement and W_2L_2 the waterline at the inclination of W_1L_1 passing through the intersection of WL with the centerline of the ship.

Let

δ = buoyancy of immersed or emerged wedge between WL and W_1L_1

δ_1 = emerged buoyancy between WL and W_2L_2

δ_2 = immersed buoyancy between WL and W_2L_2

Layer of buoyancy between W_1L_1 and W_2L_2 is $\delta_2 - \delta_1$ and let this force act through point F_2 in W_2L_2. If the layer is thin, then F_2 may be regarded as the centroid of the waterplane W_2L_2.

Let

b_1 ; b_2 be the centroids of the wedges between WL and W_1L_1

b_1' ; b_2' be the centroids of the wedges δ_1 and δ_2

h_1 ; h_2 projections of b_1 ; b_2 on to W_2L_2

h_1' ; h_2' projections of b_1' ; b_2'; on to W_2L_2

Then

$$\delta h_1h_2 = \delta_1 Oh_1' + \delta_2 Oh_2' - (\delta_2 - \delta_1) OF_2$$

There area and first moment of area of waterplane W_2L_2 are given by the expressions.

$$\text{Area} \quad = \quad \int_0^L r_\phi \, dx$$

First moment of area about fore and off axis through O

$$= \quad \int_0^L \frac{1}{2} \; r_\phi^2 \cdot dx$$

By dividing the moment by the area the value of OF_2 is found. Substituting in Atwood's formula

$$SZ \quad = \quad \frac{\delta_1 Oh_1' + \delta_2 Oh_2' \; - (\delta_2 - \delta_1) \, OF_2}{\Delta} - BS \, Sin\phi$$

Evaluating this expression for various values of ϕ gives the values of SZ at constant displacement.

An adaptation of Barne's method, if a whole range of ship displacements are to be covered, would be to find the displacement Δ_ϕ corresponding to waterline W_2L_2 through O and the corresponding SZ

$$\Delta_\phi \quad = \Delta + \delta_2 \quad - \delta_1$$

$$SZ_\phi \quad = \quad \frac{\delta_1 Oh_1' + \delta_2 Oh_2'}{\Delta_\phi} - BS \, Sin\phi$$

Other methods to derive cross curves of stability

It is quite common to derive data for the plotting of cross curves from a digital computer. Some methods will be described here.

a) Integrator methods

An integrator is essentially a machine which measures the area lying within a closed curve and the first and second moments of that are about a datum line - the axis of the integrator. Two methods to be used- the all-round and the figure of eight method.

b) Barnes's method

From the given expression :-

$$\delta h_1h_2 = \delta_1 Oh_1' + \delta_2 Oh_2' - (\delta_2 - \delta_1) \cdot OF_2$$

and the various terms are evaluated using radial integration.

By considering the element of angle in Fig.(2.10) and applying the principles of radial integration.

$$\delta_1 Oh_1' + \delta_2 Oh_2' = \int_0^L \int_0^{\phi} \left[\frac{1}{2} r_1^2 \cdot d\alpha \frac{2}{3} r_1 + \frac{1}{2} r_2^2 \, d\alpha \frac{2}{3} r_2\right] \cos(\phi - \alpha) dx$$

$$= \int_0^L \int_0^{\phi} \left[\frac{r_1^3 + r_2^3}{3}\right] \cos(\phi - \alpha) \, dx \, d\alpha$$

This double integral can be evaluated by drawing intermediate radial waterlines at equal increments of heel and measuring the offsets "r" at the appropriate stations. The same process can be followed to evaluate.

$$\delta_2 = \int_0^L \int_0^{\phi} \frac{1}{2} r_2^2 \, d\alpha \cdot dx$$

$$\delta_1 = \int_0^L \int_0^{\phi} \frac{1}{2} r_1^2 \, d\alpha \, dx$$

c) Reech's Method

Let B_α and M_α be the positions of the centre of buoyancy and metacentre corresponding to inclined waterplane W_α L_α. From the definition of pro-metacentre, if B_α moves to $B_{\alpha+\delta\alpha}$ for a very small additional inclination

$$B_\alpha B_{\alpha+\delta\alpha} = B_\alpha M_\alpha \delta\alpha \quad \text{, and } B_\alpha B_{\alpha+\delta\alpha} \text{ is parallel to } W_\alpha L_\alpha$$

$$\delta y = B_\alpha M_\alpha \cos\alpha\,\delta\alpha$$

$$\delta z = - B_\alpha M_\alpha \sin\alpha\,\delta\alpha$$

It follows by radial integration that from waterplane W_1L_1 the position of B_1 relative to B can be expressed as (y,z) where

$$y = \int_0^\phi B_\alpha M_\alpha \cos\alpha\, d\alpha$$

$$z = -\int_0^\phi B_\alpha M_\alpha \sin\alpha\, d\alpha$$

The negative sign for z signifies only that B_1 is "higher" than B. Hence

$$SZ = y \cos\phi - z \sin\phi - BS \sin\phi$$

$$= \cos\phi \int_0^\phi B_\alpha M_\alpha \cos\alpha\, d\alpha + \sin\phi \int_0^\phi B_\alpha M_\alpha \sin\alpha\, d\alpha - BS \sin\phi$$

To evaluate this expression, it is necessary to determine a series of inclined waterlines at constant displacement and for each waterplane obtain the value of BM by finding the values of the second moment of area and dividing by the volume of displacement.

d) Prohaska's method (53)

Prohaska considers the stability lever GZ as composed of two parts :-

$$GZ = GM \sin\phi + MS$$

MS is the distance from the upright metacentre to the line of action of the buoyancy force when view from projection on a transverse plane.

MS is normally called the residuary stability lever. Fig. (2.12)

Define : $C_{RS} = \frac{MS}{BM_{upright}}$

Coordinates of the centre of buoyancy for 90 and 180 degrees inclination can be determined in relative to the upright ship. Thus the metacentric radii for 0, 90 and 180 degrees can be known.

Using this data and other geometric form data, Prohaska reduces the derivation of the residual stability curve to a calculation in tabular form. The tables used are based on calculations performed on forty-two forms having form characteristics covering the usual merchant ship field, backed up by mathematical treatments assuming that the stability curve can be approximated by a trigonometric series. By the nature, this method is approximate, but for general ship forms lying within the range of parameters considered, accuracy comparable with that achieved by the so-called exact methods is obtained.

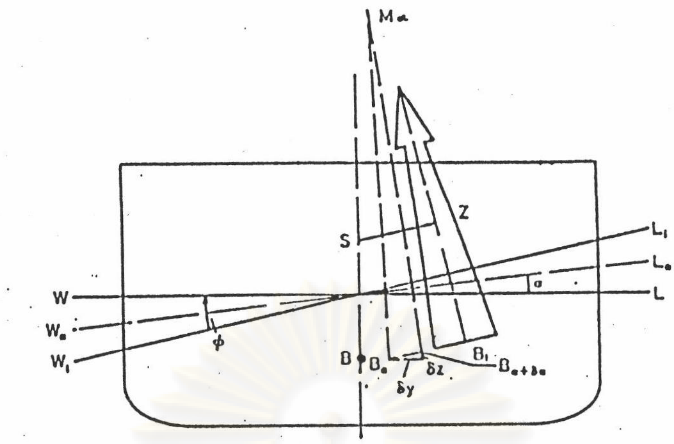

Fig. (2.11) Reech's method

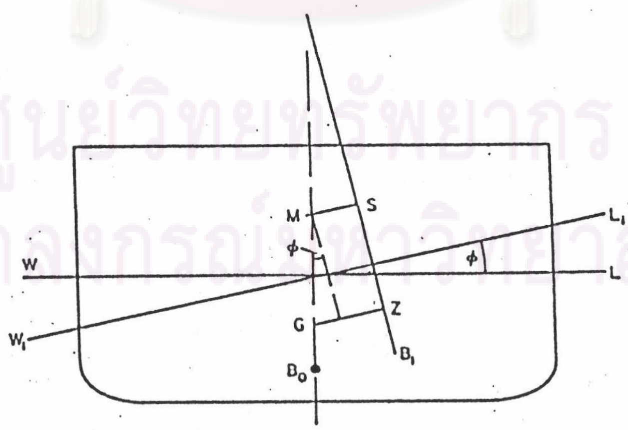

Fig. (2.12) Prohaska's method

Physical Modelling Techniques

An assumption that has been used extensively in the above described method is the neglect trim induced by the heeling of the ship. This can be overcome by finding the true equilibrium waterplane at each inclined waterplane but this leads to considerable complication. One method of automatically allowing for the effects of trim is to use a scale model of the ship, applying to it known heeling moments and noting the resulting angles of heel.

Normally, the method is used where a model already exists for other purpose such as the conduct of ship tank tests. When there are large changes in waterplane shape for small changes in heel, the method shows its advantage.

Curves of Statical Stability

For practical applications, it is necessary to present stability in the form of righting moments or levers about the centre of gravity, as the ship is heeled at constant displacement. Fig. (2.13) shows the plotting, and is known as statical stability curve. It is a curve derived from cross curves of stability for a particular displacement, Δ_1

$$GZ = SZ - SG \sin\phi$$

For a particular angle of inclinations, SG is known for the particular loading of the ship.

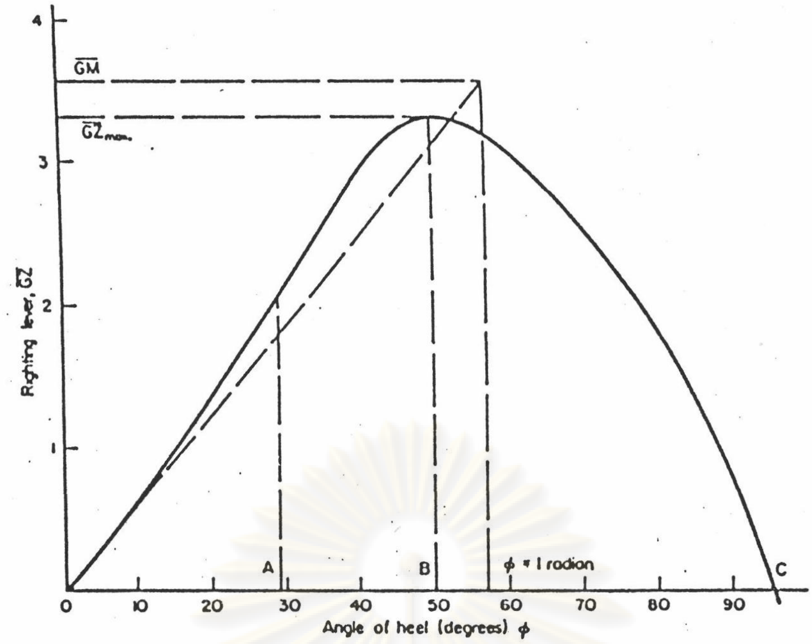

Fig. (2.13) Curve of statical stability

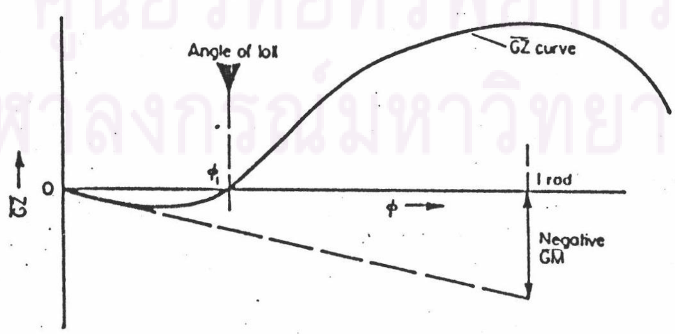

Fig. (2.14) Angle of loll

Main features of the GZ curve are as follows :-

a) Slope at the origin.

For small angles of heels, the righting lever is proportional to the angle of inclination, the metacentre being effectively a fixed point. The tangent to the GZ curve at the origin represents the metacentric height.

b) Maximum GZ

GZ_{max} is proportional to maximum steady heeling moment that a ship can sustrain without capsizing.

c) Range of Stability

The GZ reduces to be zero at some angle, normally greater than 90 degrees, and becomes negative for larger inclinations. The angle, which GZ is zero, is termed angle of vanishing stability and the range of angle (OC in the figure) for which GZ maintains positive value is called the range of stability. A ship will return to upright position with this range of stability if heeling moment is removed.

d) Angle of deck edge immersion

At point A in the Fig. (2.13) corresponds to the angle at which the deck edge becomes immersed. It is normally corresponded to the lowest point on deck edge. It provides guidance to the designer upon the possible effect on stability.

e) Area under the curve

This area represents the ability of the ship to absorb energy imparted to it by external loads ie. winds, waves etc.

Angle of Loll

A special case arises when GM is negative but GZ become positive at some reasonable angle of heel. This is illustrated in Fig.(2. 14)as ϕ_1. If the ship is momentarily at some angle of heel less than ϕ_1, the moment acting due to GZ tends to increase the heel. If the angle is greater than ϕ_1, the moment tends to reduce the heel. Thus the angle ϕ_1 is a position of stable equilibrium. Unfortunately, since the GZ curve is symmetrical about the origin, as ϕ_1 is decreased, the ship eventually passes through the upright condition and will then suddenly lurch over towards the angle ϕ_1 on the opposite side and overshoot this motion which is often the only direct indication that the heel to one side is due to a negative GM rather than to a positive heeling moment acting on the ship.

As a special case, consider a wall-sideds vessel with negative GM. In this case,

$$GZ = \sin\phi \left(GM + \frac{1}{2} BM \tan^2 \phi\right)$$

GZ is zero when $\sin\phi = 0$. This merely demonstrates that the upright condition is one of equilibrium. GZ is also zero when $GM + \frac{1}{2} BM \tan^2\phi = 0$. i.e. when

$$\tan\phi = \pm\sqrt{\frac{-2GM}{BM}}$$

Also, in this case, the slope of the GZ curve at ϕ_1 will be given by

$$\frac{dGZ}{d\phi} = \cos\phi\,(GM + \frac{1}{2}\,BM\,\tan^2\phi) + \sin\phi\,BM\,\tan\phi\,\sec^2\phi$$

$$= 0 + BM\,\tan^2\phi_1/\cos\phi_1 \text{ (putting } \phi = \phi_1$$

$$= -2\,GM/\cos\phi_1$$

2.3 DYNAMICAL STABILITY

The dynamical stability of a ship, at a given angle of heel, is defined as the work done in heeling the ship to that angle very slowly and at constant displacement.

Let the righting moment at an angle of heel ϕ be M_o. Fig.(2.15). Then, the work done in healing the ship through an additional small angle $\delta\phi$ is given approximately by

$$M_\phi \, \delta_\phi$$

total work done, up to an angle ϕ, is

$$= \int_0^\phi M_\phi \, d\phi \quad = \quad GZ_o . d\phi$$

Hence, the dynamical stability at an angle, is proportional to the area under the statical stability curve up to that angle.

An alternative way of determining the work done in heeling a ship is to investigate the potential energy of the ship. Consider Fig. (2.7), weight and displacement forces remain constant during the process but they are separated vertically, thereby raising the potential energy of the ship.

When the ship is heeled from waterline WL to waterline W_1L_1, Let B move to B_1

Increased vertical separation $= B_1Z_1 - BG$

Dynamical stability (at angle ϕ) $= \Delta(B_1Z_1 - BG)$

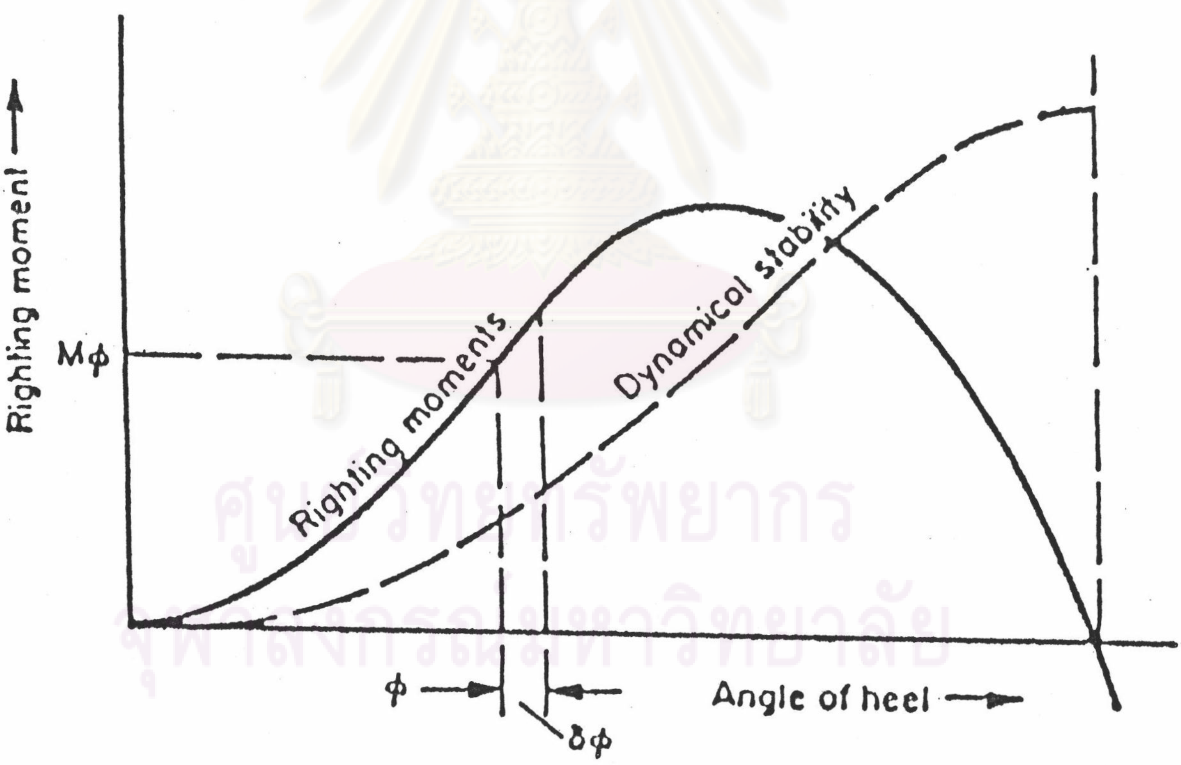

Fig. (2.15) dynamical stability

Let v be the volume of the immersed or emerged wedge, let their centroids of volume be at b_1 and b_2 and let h_1 and h_2 be the feet of perpendiculars dropped from b_1 and b_2 onto W_1L_1.

$$B_1R = \frac{v(b_1h_1 + b_2h_2)}{\nabla}$$

and $B_1Z_1 = B_1R + BG \cos \phi$

Therefore

$$B_1Z_1 = \frac{v(b_1h_1 + b_2h_2)}{\nabla} + BG \cos \phi$$

$$\text{Dynamical stability} = \Delta\left[\frac{(b_1h_1 + b_2h_2)}{\nabla} - BG(1-\cos \phi)\right]$$

The above formula is known as Moseley's Formula. Curves of dynamical stability can be drawn by using this expression but it is time-consuming and laborious. It is not often used in practice.

Readers, who want to learn more in ship's stability, are requested to consult [14].