บรรณานุกรม

ภาษาไทย

หนังสือ

ธีระพร วีระถาวร. ความน่าจะเป็นเบื้องต้น. กรุงเทพมหานคร : ภาควิชาสถิติ คณะพาณิชยศาสตร์และการบัญชี จุฬาลงกรณ์มหาวิทยาลัย, 2529.

มนตรี นิริยะกุล. ทฤษฎีสถิติ 2. กรุงเทพมหานคร : ภาควิชาสถิติ คณะวิทยาศาสตร์ มหาวิทยาลัยรามคำแหง, 2524.

วิทยานิพนธ์

สอาด นิวิศพงศ์. "การเปรียบเทียบอำนาจการทดสอบของตัวสถิติบางตัวที่ใช้ทดสอบการแจกแจงแบบเอกซ์โปเนนเชียล." วิทยานิพนธ์ ปริญญามหาบัณฑิต ภาควิชาสถิติ บัณฑิตวิทยาลัย จุฬาลงกรณ์มหาวิทยาลัย, 2532.

สุพรรณี อร่ามวัฒนกุล. "สถิติทดสอบที่มีความแกร่งสำหรับทดสอบความเท่ากันของความแปรปรวนระหว่างประชากรสองชุด." วิทยานิพนธ์ ปริญญามหาบัณฑิต ภาควิชาสถิติ บัณฑิตวิทยาลัย จุฬาลงกรณ์มหาวิทยาลัย, 2531.

สมชัย ยืนนาน. "การศึกษาโดยวิธีมอนติคาร์โลเปรียบเทียบอำนาจของการทดสอบการเท่ากัน ของความแปรปรวนระหว่างประชากรสองกลุ่ม." วิทยานิพนธ์ ปริญญามหาบัณฑิต ภาควิชาสถิติ บัณฑิตวิทยาลัย จุฬาลงกรณ์มหาวิทยาลัย, 2528.

ภาษาต่างประเทศ

หนังสือ

Hajek,J., and Sidak, Z. Theory of Rank Tests. New York : Academic Press, 1967.

Lawless, J.F. Statistacal Models & Methods for Lifetime Data. New York : John Wiley, 1982.

Mann, N.R., Schafer, R.E., and Singpurwalla, N.D. Methods for Statistical Analysis of Relability and Life Data. New York : John Wiley, 1974.

Vic Barnett and Toby Lewis. Outliers in statistical data. 2nd edition, University of Sheffield and the open university, 1987.

บรรณานุกรม (ต่อ)

บทความภาษาอังกฤษ

Birnbaum, A., and Laska, E. "Efficiency robust two-sample rank tests." Journal of the American Statistical Association 62 (1967) 1241-1251.

Chernoff, H., and Savage, I.R. "Asymptotic normality and efficiency of certain nonparametric test statistics." Annals of Mathematical Statistical Statistics 29 (1958) 972-994.

Duran, B.S., and Mielke, P.M. "Robustness of sum of squared ranks test." Journal of the American Statistical Association 63 (1968) 338-334.

Gastwirth, J.L. "On robust procedures." Journal of the American Statistical Association 61 (1966) 929-943.

Joseph L.Gastwirth and Hosam Mahmond "An efficiency robust nonparametric test or scale change for data from a gamma distribution." Technometrics 128 (1986) 81-84.

Ram C. Dahiya and John Gurland. "Goodness of fit tests for the gamma and exponential distributions." Technometrics (1972) 791-801.

Savage, I.R. "Contributions to the theory rank order statistics the two samples case." Annals of Mathematical Statistics 27 (1956) 590-615.

Shiue, W.K., and Bain, L.J. "A two sample test of equal gamma distribution scale parameters with unknown common shape parameter." Technometrics 25 (1983) 377-381.

Woinsky, M.N. "A composite non-parametric test for a scale slippage alternative." Annals of Mathematical Statistics 43 (1972b) 65-73.

ภาคผนวก

ภาคผนวก ก

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C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$       A  COMPARISON  ON  THE  POWER  OF  TEST  STATISTICS            $$
C$$                      FOR  GAMMA  DISTRIBUTION                        $$
C$$            BY   MS.  DAORADA  TORNTHAM    ID: C022888                $$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C##########################################################################$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$  MAIN  PROGRAM FOR COMPUTING TYPE I ERROR AND POWER OF TEST          $$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      DOUBLE PRECISION QH(3,3),QHTRAN(3,3),QW(3,3),QWTRAN(3,3),QI(3,3),
     *QSIG(3,3),QWTSIG(3,3),QWTSW(3,3),QWWSW(3,3),QR(3,3),QIR(3,3),
     *QA(3,3),QHA(3,3),QHAH(3,3)
      DIMENSION X(200),W(200),Y(200),SA(200),U(200),Z(200),ZJ(200),
     *U1(200), U2(200)
      COMMON /SEED /IX /SELECT /KK
      REAL NORMAL
      G05    =  0.0
      G10    =  0.0
     TN05    =  0.0
     TN10    =  0.0
      Q05    =  0.0
      Q10    =  0.0
      SFT    =  0.0
    ROUND    =  0.0
        N    =  100
        G    =  0.0
       G1    =  0.0
       G2    =  0.0
      IJK    =  2
      TOT    =  0.0
```

```
      ALPHA   =  2
       BETA   =  2
         IA   =  65539
         IX   =  973253
         KK   =  0
C ----------------------------------------------------
      DO 500 L = 1,500
      GOTO (20,40,60),IJK
C ####################################################
C ##         SELECT   SAMPLE   POPULATION          ##
C ####################################################
C       IJK  = 1  IS  GAMMA  DISTRIBUTION
C       IJK  = 2  IS  WEIBULL  DISTRIBUTION
C       IJK  = 3  IS  LOGNORMAL  DISTRIBUTION
   20 WRITE(6,222)
  222 FORMAT(10X, '*------GAMMA DIST------*')
      DO 30 J1 = 1,N
      X(J1) = GAMMA1 (ALPHA, BETA)
      Y(J1) = GAMMA1 (ALPHA, BETA)
   30 CONTINUE
      GO TO 100
   40 WRITE (6,444)
  444 FORMAT (10X, '*-----WEIBULL DIST-----*')
      DO 50 J2 = 1,N
         X(J2) = WEIBULL (ALPHA, BETA)
         Y(J2) = WEIBULL (ALPHA, BETA)
   50 CONTINUE
      GO TO 100
   60 WRITE (6,666)
  666 FORMAT (10X, '*------LOGNORMAL DIST-----*')
      DO 80 J3 = 1,N
         DMEAN = 0.0
```

```
      SIGMA = 0.3162
         A1 = NORMAL (DMEAN,SIGMA,AX,)
      X(J3) = EXP(AX)
         A2 = NORMAL (DMEAN,SIGMA,AX)
      Y(J3) = EXP(AX)
   80 CONTINUE
  100 DO 90 I3 = 1,N
   90 SA(I3) = X(I3)
        N2 = N/2
      DO 1 I4 = 1, N2
    1   Y(I4) = X(I4)
      CALL RANK(N,X)
      CALL RANK(N,SA)
C#######################################################
C##              GINI  STATISTICS  TEST              ##
C#######################################################
      SLT  =  0.0
      SFT  =  0.0
      WRITE(6,2013)
 2013 FORMAT (15X, '###### GINI TEST #####')
      NA = N-1
      DO 240 I5 = 1,N
            FT = N-I5+1
             C = I5-1
      IF (C) 220,220,230
  220 W(1) = FT*X(1)
      GO TO 240
  230 W(I5) = FT*(X(I5)-X(C))
  240 CONTINUE
      DO 250 MN = 1,NA
             KJ = MN+1
            SFT = SFT + MN*W(KJ)
```

```
  250 CONTINUE
      DO 260 MN1 = 1,N
              SLT = SLT + W(MN1)
  260 CONTINUE
              SLT = SLT*NA
               G1 = SFT/SLT
      IF (N.LE.20) GOTO 1099
         G2 = (12.0*(N-1))**0.5
         G1 = G2*(G1 - 0.5)
 1099 G = ABS(G1)
C     IF (G.GT. 0.62952) G05 = G05 + 1
C     IF (G.GT. 0.60902) G10 = G10 + 1
      IF (G.GT. 1.96) G05 + 1
      IF (G.GT. 1.645) G10 = G10 + 1
C ####################################################
C ###              The   SAVAGE   Test             ##
C ####################################################
      NN = N
      DO 320 I6 = 1,NN
             AI = I6
             BI = NN + 1
          U(I6) = AI/BI
C ***********  ZJ(I6) = (-ALOG (1-U(I6)))-1*******
         U1(I6) = 1-U(I6)
         U2(I6) = -ALOG(U1(I6))
         ZJ(I6) = U2(I6)-1
      DO 340 KI = 1,N2
      IF (SA(I6) .EQ. Y(KI)) GOTO 310
  340 CONTINUE
      GO TO 320
  310 Z (I6) = 1
  320 CONTINUE
```

```
C ##################################################
C ##                Q  STATISTICS  TEST           ##
C ##################################################
      WRITE (6.2023)
 2023 FORMAT (10X, '#### Q TEST ####')
      DO 1000 IVAR1 = 1,3
        DO 1000 IVAR2 = 1,3
          QW(IVAR1,IVAR2)          =      0.0
          QI(IVAR1,IVAR2)          =      0.0
          QWTRAN(IVAR1,IVAR2)      =      0.0
          QHTRAN(IVAR1,IVAR2)     .=      0.0
          QH(IVAR1,IVAR2)          =      0.0
          QSIG(IVAR1,IVAR2)        =      0.0
          QWTSIG(IVAR1,IVAR2)      =      0.0
          QWTSW(IVAR1,IVAR2)       =      0.0
          QWWSW(IVAR1,IVAR2)       =      0.0
          QR(IVAR1,IVAR2)          =      0.0
          QIR(IVAR1,IVAR2)         =      0.0
          QA(IVAR1,IVAR2)          =      0.0
          QHA(IVAR1,IVAR2)         =      0.0
          QHAH(IVAR1,IVAR2)        =      0.0
 1000 CONTINUE
C  ######## W #######
      QW(1,1) = 1.0
      QW(2,2) = 1.0
      QW(3,2) = 2.0
C  #### IDENTITY ####
      QI(1,1) = 1.0
      QI(2,2) = 1.0
      QI(3,3) = 1.0
```

```
C     ####### H #######
         SUMXI1  = 0.0
         SUMXI2  = 0.0
         SUMXI3  = 0.0
         DO 2010 M01 = 1,N
            SUMXI1   =  SUMXI1 + X(M01)
            SUMXI2   =  SUMXI2 + X(M01)**2
            SUMXI3   =  SUMXI3 + X(M01)**3
   2010 CONTINUE
            QH (1,1) =  SUMXI1/N
            QH (2,1) =  SUMXI2/SUMXI1
            QH (3,1) =  SUMXI3/SUMXI2
C     ########  TRANSPOST ########
C     --------   QW Σ QH   --------
         DO 2020  M02  =  1,3
            QHTRAN (1,M02)  =  QH(M02,1)
            DO  2020  M03   =  1,2
            QWTRAN (M03,M02)  =  QW (M02,M03)
   2020 CONTINUE
C    ######## CALL R #######
C    -------  SIGMA   ------
         GO TO (2210,2220,2230),IJK
C    ##### IJK --- 1.GAMMA, 2.WEIBULL, 3.LOGNORMALL #####
C    **** GAMMA **** (ค่าตัวเลขข้างล่างจะเปลี่ยนตามค่า α และ β) ****
   2210 QSIG (1,1)  =   6.0
        QSIG (1,2)  =  10.0
        QSIG (1,3)  =  14.0
        QSIG (2,1)  =  10.0
        QSIG (2,2)  =  30.0
        QSIG (2,3)  =  60.0
        QSIG (3,1)  =  14.0
        QSIG (3,2)  =  60.7
```

```
      QSIG (3,3)  = 134.4
      GO  TO 2300
C  **** WEIBULL **** (ค่าตัวเลขข้างล่างจะเปลี่ยนตามค่า α และ β )****
 2220 QSIG (1,1)  =    0.86
      QSIG (1,2)  =    0.90
      QSIG (1,3)  =    0.94
      QSIG (2,1)  =    0.90
      QSIG (2,2)  =   -4.03
      QSIG (2,3)  =   16.07
      QSIG (3,1)  =    0.94
      QSIG (3,2)  =   16.07
      QSIG (3,3)  =  -16.03
      GO TO 2300
C  ****  LOGNORMAL **** (ค่าตัวเลขข้างล่างจะเปลี่ยนตามค่า σ²)****
 2230 QSIG (1,1)  =  0.12
      QSIG (1,2)  =  0.14
      QSIG (1,3)  =  0.17
      QSIG (2,1)  =  0.14
      QSIG (2,2)  =  0.21
      QSIG (2,3)  =  0.30
      QSIG (3,1)  =  0.17
      QSIG (3,2)  =  0.30
      QSIG (3,3)  =  0.51
C  ######  INV OF QSIG ----- QSIG ######
 2300 CALL INVS(3,QSIG)
C  ######  CAL R -- QR  #####
      CALL MULT(2,3,3,QWTRAN,QSIG,QWTSIG)
      CALL MULT(2,2,3,QWTSIG,QW,QWTSW)
      CALL INVS(2,QWTSW)
      CALL MULT(3,2,2,QW,QWTSW,QWWSW)
      CALL MULT(3,3,2,QWWSW,QWTSW,QR)
```

```
C  ###### CAL A --- QA ######
       DO 2310 M05 = 1,3
        DO 2310 M06 = 1,3
          QIR(M05,M06)  =  QI(M05,M06) - QR(M05,M06)
  2310 CONTINUE
       CALL MULT(3,3,3,QSIG,QIR,QA)
C  ##### CAL QHAB --- QH #####
       CALL MULT(1,3,3,QHTRAN,QA,QHA)
       CALL MULT(1,1,3,QHA,QH,QHAH)
       QHAB  =  N*QHAH(1,1)
       IF (QHAB.GT.3.84) Q05 = Q05+1
       IF (QHAB).GT.2.71) Q10 = Q10 + 1
       LR = L
   500 CONTINUE
       WRITE (6,58) IJK
    58 FORMAT (//20X,'DISTRIBUTION = ',I2/)
       WRITE (6,61) N,ALPHA,BETA
    61 FORMAT (5X,'##### TEST POWER ','N = I4,
C   ######################################################
C   ##     COMPUTER TYPE I ERROR AND POWER OF TEST      ##
C   ######################################################
       ROUND = LR
       PG05  =  G05/ROUND
       PG10  =  G10/ROUND
       PTN05 =  TN05/ROUND
       PTN10 =  TN10/ROUND
       PQ05  =  Q05/ROUND
       PQ10  =  Q10/ROUND
       WRITE (6,1500) PG05,PG10
  1500 FORMAT (/10X,'PG05  = ',F10.2,10X,'PG10 = ',F10.2)
       WRITE (6,1501) PTN05,PTN10
  1501 FORMAT (/10X,'PTN05 = 'F10.2,10X,'PTN10 = ',F10.2)
```

```
      WRITE (6,1502) PQ05,PQ10
 1502 FORMAT (/10X,'PQ05 = ',F10.2,10X,'PQ10  = ',F10.2//)
      STOP
      END
C   ##################################################
C   ##         SUBROUTINE  RANDOM  VARIABLE         ##
C   ##################################################
      SUBROUTINE RAND(IX,IY,YFL)
      IY   =   IX * 65539
      IF (IY) 5,6,6
    5 IY   =  IY + 2147483647 + 1
    6 YFL  =  IY
      YFL  =  YFL / 2147483647
      IX   =  IY
      RETURN
      END
C   ################################################
C   ##       GAMMA  DISTRIBUTION  FUNCTION        ##
C   ################################################
      FUNCTION  GAMMA1(ALPHA1,BETA1)
      COMMON / SEED / IX
      ALPHA  =  ALPHA1
         U   =  0.0
    5 CALL RAND(IX,IY,YFL)
         V   =  -ALOG(YFL)
         U   =   U + V
      IF (ALPHA.EQ.1.0) GOTO 10
      ALPHA  =  ALPHA - 1:0
         GOTO  5
   10 GAMMA1 =  BETA1*U
      RETURN
      END
```

```
C    ################################################
C    ##        WEIBULL  DISTRIBUTION  FUNCTION     ##
C    ################################################
        FUNCTION  WEIBUL (ALPHA1,BETA1)
        COMMON / SEED / IX
        CALL RAND(IX,IY,YFL)
        WEIBUL  =  BETA1*(-ALOG(1.0-YFL))**(1.0/ALPHA 1)
        RETURN
        END
C    ############################################################
C    ##       NORMAL (DMEAN,SIGMA)  DISTRIBUTION   FUNCTION    ##
C    ############################################################
        FUNCTION NORMAL(DMEAN,SIGMA,AX)
        REAL NORMAL
        COMMON /SEED/IX /SELECT/KK
        PI  =  3.1415926
        IF (KK.EQ.1) GOTO 10
           CALL  RAND(IX,IY,YFL)
           RONE   =  YFL
           CALL RAND (IX,IY,YFL)
           RTWO  =  YFL
           ZONE  =  SQRT(-2*ALOG(RONE))*COS(2*PI*RTWO)
           ZTWO  =  SQRT(-2*ALOG(RONE))*SIN(2*PI*RTWO)
         NORMAL  =  ZONE*SIGMA + DMEAN
             KK  =  1
             AX  =  NORMAL
         RETURN
      10 NORMAL  =  ZTWO*SIGMA+DMEAN
             KK  =  0
             AX  =  NORMAL
         RETURN
         END
```

```
C     ##################################################
C     ##          SUBROUTINE FOR SORTING DATA          ##
C     ##################################################
      SUBROUTINE  RANK(N,X)
      DIMENSION  X(200)
      N1 = N-1
      DO 10 I = 1,N1
        II = I+1
      DO 10 K = II,N
      IF (X(I) .LE. X(K)) GOTO 10
            T = X(I)
        X(I) = X(K)
        X(K) = T
   10 CONTINUE
      RETURN
      END
C     ##################################################
C     ##         SUBROUTINE  A(I)*B(I) = C(I)          ##
C     ##################################################
      SUBROUTINE  MULT(III,KKK,LLL,AA1,BB1,CC1)
      DOUBLE PRECISION AA1(3,3),BB1(3,3),CC1(3,3)
      DO 11 II1 = 1,III
        DO II KK1 =  1,KKK
          DO 11 LL1 = 1,LLL
   11         CC1(II1,KK1)=CC1(II1,KK1)+AA1(II1,LL1)*BB1(LL1,KK1)
      RETURN
      END
```

```
C     ##################################################
C     ##        SUBROUTINE   INVERSE   MATRIX         ##
C     ##################################################
          SUBROUTINE INVS(M,A)
          DOUBLE PRECISION A(3,3)
          DO 20 K   =  1,M
            A(K,K) =  -1.0/A(K,K)
          DO  5 I   =  1,M
            IF(I-K) 3,5,3
        3 A(I,K)     =  -A(I,K)*A(K,K)
        5 CONTINUE
          DO 10 I = 1,M
          DO 10 J = 1,M
          IF((I-K)*(J-K)) 9,10,9
        9 A(I,J)  = A(I,J)-A(I,K)*A(K,J)
       10 CONTINUE
          DO 20 J = 1,M
          IF (J-K) 18,20,18
       18 A(K,J)  = -A(K,J)*A(K,K)
       20 CONTINUE
          DO 25 I = 1,M
            DO 25 J = 1,M
       25 A(I,J)  = -A(I,J)
          DO 27 I = 1,M
            DO 27 J = 1,M
       27 CONTINUE
          RETURN
          END
          /*
         //
```

ภาคผนวก ข

<u>วิธีการคำนวณค่า Σ ของตัวสถิติ Q</u>

1. <u>กรณีตัวแปรสุ่มมีการแจกแจงแบบแกมม่า</u>

ใช้สูตร $\hat{\Sigma} = JGJ'$

$$J = \begin{bmatrix} \frac{\partial \mu_1'}{\partial \mu_1'} & \frac{\partial \mu_1'}{\partial \mu_2'} & \frac{\partial \mu_1'}{\partial \mu_3'} \\ \frac{\partial(\mu_2'/\mu_1')}{\partial \mu_1'} & \frac{\partial(\mu_2'/\mu_1')}{\partial \mu_2'} & \frac{\partial(\mu_2'/\mu_1')}{\partial \mu_3'} \\ \frac{\partial(\mu_3'/\mu_2')}{\partial \mu_1'} & \frac{\partial(\mu_3'/\mu_2')}{\partial \mu_2'} & \frac{\partial(\mu_3'/\mu_2')}{\partial \mu_3'} \end{bmatrix}_{3\times 3}$$

$$J = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-\mu_2'}{\mu_1'} & \frac{1}{\mu_2'} & 0 \\ 0 & \frac{-\mu_3'}{\mu_2'} & \frac{1}{\mu_2'} \end{bmatrix}_{3\times 3}$$

ค่า μ' ต่างๆ สามารถหาได้จากฟังก์ชั่น ในรูปของ "โมเมนต์" (Moment) เช่นถ้า $f(x) = x^{\nu}$ จะเรียก $E(x^{\nu})$ ว่า "โมเมนต์ที่ ν ของ x รอบจุดกำเนิด " นิยมเขียนโดยใช้สัญญลักษณ์ μ_ν' ซึ่ง $\mu_\nu' = E(x^{\nu})$

กรณีของการแจกแจงแบบแกมม่า

$$\mu_{\nu}' = \beta\ \Gamma(\alpha+\nu)$$

$$\mu_1' = \beta\Gamma(\alpha+1) = \alpha\beta$$

$$\mu_2' = \frac{\beta^2\Gamma(\alpha+2)}{\Gamma(\alpha)} = \frac{\beta^2(\alpha+1)!}{(\alpha-1)!} = \beta^2\alpha(\alpha+1)$$

$$\mu_3' = \frac{\beta^3\Gamma(\alpha+3)}{\Gamma(\alpha)} = \frac{\beta^3(\alpha+2)!}{(\alpha-1)!} = \beta^3(\alpha+2)(\alpha+1)\alpha$$

$$\mu_4' = \frac{\beta^4\Gamma(\alpha+4)}{\Gamma(\alpha)} = \frac{\beta^4(\alpha+3)!}{(\alpha-1)!} = \beta^4(\alpha+3)(\alpha+2)(\alpha+1)\alpha$$

$$\mu_5' = \frac{\beta^5\Gamma(\alpha+5)}{\Gamma(\alpha)} = \frac{\beta^5(\alpha+4)!}{(\alpha-1)!} = \beta^5(\alpha+4)(\alpha+3)(\alpha+2)(\alpha+1)\alpha$$

$$\mu_6' = \frac{\beta^6\Gamma(\alpha+6)}{\Gamma(\alpha)} = \frac{\beta^6(\alpha+5)!}{(\alpha-1)!} = \beta^6(\alpha+5)(\alpha+4)(\alpha+3)(\alpha+2)(\alpha+1)\alpha$$

คำนวณค่าตัวเลขในเมตริกซ์ J

$$\frac{\mu}{\mu^{2\prime}} = \frac{\alpha\beta^2(\alpha+1)}{\alpha^2\beta^2} = \frac{-(\alpha+1)}{\alpha}$$

$$\frac{1}{\mu_1'} = \frac{1}{\alpha\beta}$$

$$\frac{-\mu_3'}{\mu_2^{2\prime}} = -\left[\frac{\beta^3(\alpha+2)(\alpha+1)\alpha}{\{\alpha\beta^2(\alpha+1)\}^2}\right] = \frac{-(\alpha+2)}{\alpha\beta(\alpha+1)}$$

$$\frac{1}{\mu_2'} = \frac{1}{\alpha\beta^2(\alpha+1)}$$

$$J = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-(\alpha+1)}{\alpha} & \frac{1}{\alpha\beta} & 0 \\ 0 & \frac{-(\alpha+2)}{\alpha\beta(\alpha+1)} & \frac{1}{\alpha\beta^2(\alpha+1)} \end{bmatrix}_{3\times3}$$

$$J' = \begin{bmatrix} 1 & \frac{-(\alpha+1)}{\alpha} & 0 \\ 0 & \frac{1}{\alpha\beta} & \frac{-(\alpha+2)}{\alpha\beta(\alpha+1)} \\ 0 & 0 & \frac{1}{\alpha\beta^2(\alpha+1)} \end{bmatrix}_{3\times3}$$

ค่าของเมตริกซ์ G คำนวณจากค่า g_{ij}
โดยที่ $g_{ij} = \mu'_{i+j} - \mu_i'\mu_j'$

ดังนั้น $$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}_{3\times3}$$

$$
\begin{aligned}
g_{11} &= \mu_2' - \mu_1'\mu_1' \\
g_{12} &= \mu_3' - \mu_1'\mu_2' \\
g_{13} &= \mu_4' - \mu_1'\mu_3' \\
g_{21} &= \mu_3' - \mu_2'\mu_1' \\
g_{22} &= \mu_4' - \mu_2'\mu_2' \\
g_{23} &= \mu_5' - \mu_2'\mu_3' \\
g_{31} &= \mu_4' - \mu_3'\mu_1' \\
g_{32} &= \mu_5' - \mu_3'\mu_2' \\
g_{33} &= \mu_6' - \mu_3'\mu_3'
\end{aligned}
$$

$$
G = \begin{bmatrix}
\mu_2' - \mu_1'\mu_1' & \mu_3' - \mu_1'\mu_2' & \mu_4' - \mu_1'\mu_3' \\
\mu_3' - \mu_2'\mu_1' & \mu_4' - \mu_2'\mu_2' & \mu_5' - \mu_2'\mu_3' \\
\mu_4' - \mu_3'\mu_1' & \mu_5' - \mu_3'\mu_2' & \mu_6' - \mu_3'\mu_3'
\end{bmatrix}_{3\times 3}
$$

$$
\begin{aligned}
g_{11} = \mu_2' - \mu_1'\mu_1' &= \alpha\beta^2(\alpha+1) - (\alpha\beta)^2 \\
&= \alpha^2\beta^2 + \alpha\beta^2 - \alpha^2\beta^2 \\
&= \alpha\beta^2
\end{aligned}
$$

$$
\begin{aligned}
g_{12} = \mu_3' - \mu_1'\mu_2' &= \alpha\beta^3(\alpha+2)(\alpha+1) - \alpha\beta\{\alpha\beta^2(\alpha+1)\} \\
&= \alpha\beta^3(\alpha+2)(\alpha+1) - \alpha^2\beta^3(\alpha+1) \\
&= \alpha\beta^3(\alpha+1)\{(\alpha+2)-\alpha\} \\
&= 2\alpha\beta^3(\alpha+1)
\end{aligned}
$$

$$
\begin{aligned}
g_{13} = \mu_4' - \mu_1'\mu_3' &= \alpha\beta^4(\alpha+3)(\alpha+2)(\alpha+1) - \alpha\beta\{\alpha\beta^3(\alpha+2)(\alpha+1)\} \\
&= \alpha\beta^4(\alpha+3)(\alpha+2)(\alpha+1) - \alpha^2\beta^4(\alpha+2)(\alpha+1) \\
&= \alpha\beta^4(\alpha+2)(\alpha+1)\{(\alpha+3)-\alpha\} \\
&= 3\alpha\beta^4(\alpha+2)(\alpha+1)
\end{aligned}
$$

$$
g_{21} = \mu_3' - \mu_2'\mu_1' = g_{12}
$$

$$
\begin{aligned}
g_{22} = \mu_4' - \mu_2'\mu_2' &= \alpha\beta^4(\alpha+3)(\alpha+2)(\alpha+1) - \alpha\beta^2(\alpha+1)\alpha\beta^2(\alpha+1) \\
&= \alpha\beta^4(\alpha+3)(\alpha+2)(\alpha+1) - \alpha^2\beta^4(\alpha+1)^2 \\
&= \alpha\beta^4(\alpha+1)\{(\alpha+3)(\alpha+2)-\alpha(\alpha+1)\} \\
&= \alpha\beta^4(\alpha+1)(5\alpha+6-\alpha) \\
&= \alpha\beta^4(4\alpha+6)(\alpha+1) \\
&= 2\alpha\beta^4(2\alpha+3)(\alpha+1)
\end{aligned}
$$

$$
\begin{aligned}
g_{23} = \mu_5' - \mu_2'\mu_3' &= \alpha\beta^5(\alpha+4)(\alpha+3)(\alpha+2)(\alpha+1) - \alpha\beta^2(\alpha+1)\{\alpha\beta^3(\alpha+2)(\alpha+1) \\
&= \alpha\beta^5(\alpha+4)(\alpha+3)(\alpha+2)(\alpha+1) - \alpha^2\beta^5(\alpha+2)(\alpha+1)^2 \\
&= \alpha\beta^5(\alpha+2)(\alpha+1)\{(\alpha+4)(\alpha+3)-\alpha(\alpha+1)\} \\
&= \alpha\beta^5(\alpha+2)(\alpha+1)(7\alpha+12-\alpha) \\
&= \alpha\beta^5(\alpha+2)(\alpha+1)(6\alpha+12) \\
&= 6\alpha\beta^5(\alpha+2)^2(\alpha+1)
\end{aligned}
$$

$$g_{31} = \mu_4' - \mu_3'\mu_1' = g_{13}$$

$$g_{32} = \mu_5' - \mu_3'\mu_2' = g_{23}$$

$$
\begin{aligned}
g_{33} = \mu_6' - \mu_3'\mu_3' &= \alpha\beta^6(\alpha+5)(\alpha+4)(\alpha+3)(\alpha+2)(\alpha+1) - \{\alpha\beta^3(\alpha+2)(\alpha+1)\}^2 \\
&= \alpha\beta^6(\alpha+5)(\alpha+4)(\alpha+3)(\alpha+2)(\alpha+1) - \alpha^2\beta^6(\alpha+2)^2(\alpha+1)^2 \\
&= \alpha\beta^6(\alpha+2)(\alpha+1)\{(\alpha+5)(\alpha+4)(\alpha+3) - \alpha(\alpha+2)(\alpha+1)\} \\
&= \alpha\beta^6(\alpha+2)(\alpha+1)\{\alpha^2+9\alpha+20)(\alpha+3) - \alpha(\alpha^2+3\alpha+2) \\
&= \alpha\beta^6(\alpha+2)(\alpha+1)\{9\alpha^2+27\alpha+20\alpha+60-2\alpha\} \\
&= \alpha\beta^6(\alpha+2)(\alpha+1)(6\alpha^2+45\alpha+60) \\
&= 3\alpha\beta^6(2\alpha^2+15\alpha+20)(\alpha+2)(\alpha+1)
\end{aligned}
$$

$$
G = \begin{bmatrix}
\alpha\beta^2 & 2\alpha\beta^3(\alpha+1) & 3\alpha\beta^4(\alpha+2)(\alpha+1) \\
2\alpha\beta^3(\alpha+1) & 2\alpha\beta^4(2\alpha+3)(\alpha+1) & 6\alpha\beta^5(\alpha+2)^2(\alpha+1) \\
3\alpha\beta^4(\alpha+2)(\alpha+1) & 6\alpha\beta^5(\alpha+2)^2(\alpha+1) & 3\alpha\beta^6(2\alpha^2+15\alpha+20)(\alpha+2)(\alpha+1)
\end{bmatrix}_{3\times 3}
$$

$$JG = \begin{bmatrix} 1 & 0 & 0 \\ -(\alpha+1)/\alpha & 1/\alpha\beta & 0 \\ 0 & -(\alpha+2)/\alpha\beta(\alpha+1) & 1/\alpha\beta^2(\alpha+1) \end{bmatrix}_{3\times 3} \begin{bmatrix} \alpha\beta^2 & 2\alpha\beta^3(\alpha+1) & 3\alpha\beta^4(\alpha+2)(\alpha+1) \\ 2\alpha\beta^3(\alpha+1) & 2\alpha\beta^4(2\alpha+3)(\alpha+1) & 6\alpha\beta^5(\alpha+2)^2(\alpha+1) \\ 3\alpha\beta^4(\alpha+2)(\alpha+1) & 6\alpha\beta^5(\alpha+2)^2(\alpha+1) & 3\alpha\beta^6(2\alpha^2+15\alpha+20)(\alpha+2)(\alpha+1) \end{bmatrix}_{3\times 3}$$

$$JG = \begin{bmatrix} \alpha\beta^2 & 2\alpha\beta^3(\alpha+1) & 3\alpha\beta^4(\alpha+2)(\alpha+1) \\ \beta^2(\alpha+1) & 2\beta^3(\alpha+2)(\alpha+1) & 3\beta^4(\alpha+3)(\alpha+2)(\alpha+1) \\ \beta^2(\alpha+2) & 2\beta^3(\alpha+3)(\alpha+2) & 3\beta^4(\alpha+2)(7\alpha+12) \end{bmatrix}_{3\times 3}$$

$$JGJ' = \begin{bmatrix} \alpha\beta^2 & 2\alpha\beta^3(\alpha+1) & 3\alpha\beta^4(\alpha+2)(\alpha+1) \\ \beta^2(\alpha+1) & 2\beta^3(\alpha+2)(\alpha+1) & 3\beta^4(\alpha+3)(\alpha+2)(\alpha+1) \\ \beta^2(\alpha+2) & 2\beta^3(\alpha+3)(\alpha+2) & 3\beta^4(7\alpha+12)(\alpha+2) \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 & -(\alpha+1)/\alpha & 0 \\ 0 & 1/\alpha\beta & -(\alpha+2)/\alpha\beta(\alpha+1) \\ 0 & 0 & 1/\alpha\beta^2(\alpha+1) \end{bmatrix}_{3\times 3}$$

$$\hat{\Sigma} = JGJ' = \begin{bmatrix} \alpha\beta^2 & \beta^2(\alpha+1) & \beta^2(\alpha+2) \\ \beta^2(\alpha+1) & \frac{\beta^2}{\alpha}(\alpha+3)(\alpha+1) & \frac{\beta^2}{\alpha}(\alpha+5)(\alpha+2) \\ \beta^2(\alpha+2) & \frac{\beta^2}{\alpha}(\alpha+5)(\alpha+2) & \frac{\beta^2(\alpha+2)(-2\alpha^2+11\alpha+24)}{\alpha(\alpha+1)} \end{bmatrix}_{3\times 3}$$

ใช้ค่าพารามิเตอร์ (α,β) แทนลงใน $\hat{\Sigma}$ ก็จะสามารถทราบค่าของ $\hat{\Sigma}$ ในพารามิเตอร์นั้น ๆ

2. <u>กรณีตัวแปรสุ่มมีการแจกแจงแบบไวบูลล์</u>

ใช้สูตรการคำนวณค่า $\hat{\Sigma}$ เช่นเดียวกับที่ใช้ในการคำนวณค่า $\hat{\Sigma}$ ของการแจกแจงแบบแกมม่า แต่ค่าภายในเมตริกซ์ต่างๆ จะแตกต่างกันไปตามค่าของฟังก์ชั่นโมเมนต์ ที่ใช้ในแต่ละการแจกแจง

สูตร $\hat{\Sigma} = JGJ'$

คำนวณค่า J

$$J = \begin{bmatrix} 1 & 0 & 0 \\ -\mu_2'/\mu_1' & 1/\mu_1' & 0 \\ 0 & -\mu_3'/\mu_2'^2 & 1/\mu_2' \end{bmatrix}_{3\times 3}$$

$$\mu_\nu' = E(x^\nu)$$

กรณีของการแจกแจงแบบไวบูลล์

$$\mu_\nu' = \beta^\nu\Gamma(1+\nu/\alpha)$$

$$\mu_1' = \beta\Gamma(1+1/\alpha)$$

$$\mu_2' = \beta^2\Gamma(1+2/\alpha)$$

$$\mu_3' = \beta^3\Gamma(1+3/\alpha)$$

$$\mu_4' = \beta^4\Gamma(1+4/\alpha)$$

$$\mu_5' = \beta^5\Gamma(1+5/\alpha)$$

$$\mu_6' = \beta^6\Gamma(1+6/\alpha)$$

คำนวณค่าตัวเลขในเมตริก J

$$-\frac{\mu_2'}{\mu_1^{2\prime}} = \frac{-\beta^2\Gamma(1+2/\alpha)}{(\beta\Gamma(1+1/\alpha))^2}$$

$$= -\frac{\Gamma(1+2/\alpha)}{(\Gamma(1+1/\alpha))^2}$$

$$\frac{1}{\mu_1'} = \frac{1}{\beta\Gamma(1+1/\alpha)}$$

$$-\frac{\mu_3'}{\mu_2^{2\prime}} = \frac{-\beta^3\Gamma(1+3/\alpha)}{(\beta^2\Gamma(1+2/\alpha))^2}$$

$$= -\frac{\beta^3\Gamma(1+3/\alpha)}{\beta^4\Gamma^2(1+2/\alpha)}$$

$$= -\frac{1\Gamma(1+3/\alpha)}{\beta(\Gamma 1+2/\alpha)^2}$$

$$\frac{1}{\mu_2'} = \frac{1}{\beta^2\Gamma 1+2/\alpha}$$

$$J = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-\Gamma 1+2/\alpha}{(\Gamma 1+1/\alpha)^2} & \frac{1}{\beta(\Gamma 1+1/\alpha)} & 0 \\ 0 & \frac{-\Gamma 1+3/\alpha}{\beta(\Gamma 1+2/\alpha)^2} & \frac{1}{\beta^2(\Gamma 1+2/\alpha)} \end{bmatrix}_{3\times 3}$$

$$J' = \begin{bmatrix} 1 & \dfrac{-\Gamma(1+2/\alpha)}{(\Gamma 1+1/\alpha)^2} & 0 \\ 0 & \dfrac{1}{\beta\Gamma(1+1/\alpha)} & \dfrac{-\Gamma(1+3/\alpha)}{\beta\Gamma^2(1+2/\alpha)} \\ 0 & 0 & \dfrac{1}{\beta^2\Gamma(1+2/\alpha)} \end{bmatrix}_{3\times 3}$$

ค่าของเมตริกซ์ G คำนวณจากค่า g_{ij} เช่นเดียวกับการแจกแจงแบบแกมม่า ซึ่ง $g_{ij} = \mu'_{i+j} - \mu'_i\mu'_j$

$$\text{ดังนั้น } G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}_{3\times 3}$$

$$\text{หรือ } G = \begin{bmatrix} \mu_2'-\mu_1'\mu_1' & \mu_3'-\mu_1'\mu_2' & \mu_4'-\mu_1'\mu_3' \\ \mu_3'-\mu_2'\mu_1' & \mu_4'-\mu_2'\mu_2' & \mu_5'-\mu_2'\mu_3' \\ \mu_4'-\mu_3'\mu_1' & \mu_5'-\mu_3'\mu_2' & \mu_6'-\mu_3'\mu_3' \end{bmatrix}_{3\times 3}$$

$$\begin{aligned} g_{11} &= \mu_2'-\mu_1'\mu_1' = \beta^2\Gamma(1+2/\alpha) - \beta^2\Gamma^2(1+1/\alpha) \\ &= \beta^2\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha) \end{aligned}$$

$$\begin{aligned} g_{12} &= \mu_3'-\mu_1'\mu_2' = \beta^3\Gamma(1+3/\alpha) - \beta\Gamma(1+1/\alpha)\beta^2\Gamma(1+2/\alpha) \\ &= \beta^3\Gamma(1+3/\alpha) - \Gamma(1+1/\alpha)\Gamma(1+2/\alpha) \end{aligned}$$

$$\begin{aligned} g_{13} &= \mu_4'-\mu_1'\mu_3' = \beta^4\Gamma(1+4/\alpha) - \beta\Gamma(1+1/\alpha)\beta^3\Gamma(1+3/\alpha) \\ &= \beta^4\Gamma(1+4/\alpha) - \beta\Gamma(1+1/\alpha)\beta^3\Gamma(1+3/\alpha) \end{aligned}$$

$$g_{21} = \mu_3' - \mu_2'\mu_1' = g_{12}$$

$$g_{22} = \mu_4' - \mu_2'\mu_2' = \beta^4\Gamma(1+4/a) - (\beta^2\Gamma(1+2/a))^2$$
$$= \beta^4\{\Gamma(1+4/a) - \Gamma^2(1+2/a)\}$$

$$g_{23} = \mu_5' - \mu_2'\mu_3' = \beta^5\Gamma(1+5/a) - \beta^2\Gamma(1+2/a)\beta^3\Gamma(1+3/a)$$
$$= \beta^5\{\Gamma(1+5/a) - \Gamma(1+2/a)\Gamma(1+3/a)\}$$

$$g_{31} = \mu_4' - \mu_3'\mu_1' = g_{13}$$

$$g_{32} = \mu_5' - \mu_3'\mu_2' = g_{23}$$

$$g_{33} = \mu_6' - \mu_3'\mu_3' = \beta^6\Gamma(1+6/a) - (\beta^3\Gamma(1+3/a))^2$$
$$= \beta^6\{\Gamma(1+6/a) - \Gamma^2(1+3/a)\}$$

$$G = \begin{bmatrix} \beta^2\{\Gamma(1+2/a)-\Gamma^2(1+1/a)\} & \beta^3\Gamma(1+3/a)-\Gamma(1+1/a)\Gamma(1+2/a) & \beta^4\Gamma(1+4/a)-\Gamma(1+1/a)\Gamma(1+3/a) \\ \beta^3\Gamma(1+3/a)-\Gamma(1+1/a)\Gamma(1+2/a) & \beta^4\{\Gamma(1+4/a)-\Gamma^2(1+2/a)\} & \beta^5\Gamma(1+5/a)-\Gamma(1+2/a)\Gamma(1+3/a) \\ \beta^4\Gamma(1+4/a)-\Gamma(1+1/a)\Gamma(1+3/a) & \beta^5\Gamma(1+5/a)-\Gamma(1+2/a)\Gamma(1+3/a) & \beta^6\{\Gamma(1+6/a)-\Gamma^2(1+3/a)\} \end{bmatrix}_{3\times 3}$$

$$JG = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-\Gamma(1+2/a)}{\Gamma^2(1+1/a)} & \frac{1}{\beta\Gamma(1+1/a)} & 0 \\ 0 & \frac{-1}{\beta\{\Gamma(1+2/a)/\Gamma^2(1+2/a)\}} & \frac{1}{1/\beta^2\Gamma(1+2/a)} \end{bmatrix} \begin{bmatrix} \beta^2\{\Gamma(1+2/a)-\Gamma^2(1+1/a)\} & \beta^3\Gamma(1+3/a)-\Gamma(1+1/a)\Gamma(1+2/a) & \beta^4\Gamma(1+4/a)-\Gamma(1+1/a)\Gamma(1+3/a) \\ \beta^3\Gamma(1+3/a)-\Gamma(1+1/a\Gamma(1+2/a) & \beta^4\{(\Gamma(1+4/a-(\Gamma(1+2/a)^2\} & \beta^5(\Gamma(1+5/a)-\Gamma(1+2/a)\Gamma(1+3/a) \\ \beta^4\Gamma(1+4/a)-\Gamma(1+1/a)\Gamma(1+3/a) & \beta^5\Gamma(1+5/a)-\Gamma(1+2/a)\Gamma(1+3/a) & \beta^6\{\Gamma(1+6/a)-\Gamma^2(1+3/a)\} \end{bmatrix}_{3\times 3}$$

$$
JG=\begin{bmatrix}
\beta^2\{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)\} & \beta^3\{\Gamma(1+3/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+2/\alpha) & \beta^4\Gamma(1+4/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+3/\alpha) \\
\begin{array}{l}\frac{\beta^2}{\Gamma(1+1/\alpha)}[-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}\Gamma(1+2/\alpha)-\{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)\}+\\ \{\Gamma(1+3/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+2/\alpha)\}]\end{array} & \begin{array}{l}\frac{\beta^3}{\Gamma(1+1/\alpha)}[-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}\Gamma(1+3/\alpha)-\Gamma(1+1/\alpha)+\\ \Gamma(1+2/\alpha)+\{\Gamma(1+4/\alpha)-\Gamma^2(1+2/\alpha)\}]\end{array} & \begin{array}{l}\frac{\beta^4}{\Gamma(1+1/\alpha)}[-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}\Gamma(1+4/\alpha)-\Gamma(1+1/\alpha)+\\ \Gamma(1+3/\alpha)+\Gamma(1+5/\alpha)-\Gamma(1+2/\alpha)\Gamma(1+3/\alpha)\end{array} \\
\begin{array}{l}\frac{\beta^2}{\Gamma(1+2/\alpha)}[-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}\Gamma(1+2/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+2/\alpha)\\ \Gamma(1+4/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+3/\alpha)]\end{array} & \begin{array}{l}\frac{\beta^3}{\Gamma(1+2/\alpha)}[-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}\{\Gamma(1+4/\alpha)-\Gamma^2(1+2/\alpha)\}+\\ \Gamma(1+5/\alpha)-\Gamma(1+2/\alpha)\Gamma(1+3/\alpha)]\end{array} & \begin{array}{l}\frac{\beta^4}{\Gamma(1+2/\alpha)}(-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}\Gamma(1+5/\alpha)-\Gamma(1+2/\alpha)+\\ \Gamma(1+3/\alpha)+\Gamma(1+6/\alpha)-\Gamma^2(1+2/\alpha)]\end{array}
\end{bmatrix}_{3\times 3}
$$

$$
\hat{\Sigma}=\begin{bmatrix}
\beta^2\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha) & \begin{array}{l}\frac{\beta^3}{\Gamma(1+1/\alpha)}[-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}\Gamma(1+2/\alpha)-\\ \Gamma^2(1+1/\alpha)+\\ \Gamma(1+3/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+2/\alpha)]\end{array} & \begin{array}{l}\frac{\beta^3}{\Gamma(1+2/\alpha)}[-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}\Gamma(1+3/\alpha)-\\ \Gamma(1+1/\alpha)\Gamma(1+2/\alpha)+\\ \Gamma(1+4/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+3/\alpha)\end{array} \\
\begin{array}{l}\frac{\beta^2}{\Gamma(1+1/\alpha)}[-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}\Gamma(1+2/\alpha)-\\ \Gamma^2(1+1/\alpha)+\\ \Gamma(1+3/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+2/\alpha)]\end{array} & \begin{array}{l}\frac{\beta^4}{\Gamma^2(1+2/\alpha)}[-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}(-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}+\\ (\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)+\Gamma(1+3/\alpha)-\Gamma(1\\ +1/\alpha)\Gamma(1+2/\alpha)+\Gamma(1+4/\alpha)-\Gamma^2(1+2/\alpha))]\end{array} & \begin{array}{l}\frac{\beta^5}{\Gamma(1+2/\alpha)\Gamma(1+1/\alpha)}[-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}(-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}+\\ \Gamma(1+3/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+2/\alpha)+\\ \Gamma(1+4/\alpha)-\Gamma^2(1+2/\alpha)+(-\Gamma(1+2/\alpha)\Gamma(1+4/\alpha)-\\ \Gamma(1+1/\alpha)\Gamma(1+3/\alpha)+\Gamma(1+5/\alpha)-\Gamma(1+2/\alpha)\Gamma(1+3/\alpha))]\end{array} \\
\begin{array}{l}\frac{\beta^2}{\Gamma(1+2/\alpha)}[-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}\Gamma(1+3/\alpha)-\\ \Gamma(1+1/\alpha)\Gamma(1+2/\alpha)+\\ \Gamma(1+4/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+3/\alpha)\end{array} & \begin{array}{l}\frac{\beta^5}{\Gamma(1+2/\alpha)\Gamma(1+1/\alpha)}[-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}(-\frac{\Gamma(1+2/\alpha)}{\Gamma(1+1/\alpha)}+\\ \Gamma(1+3/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+2/\alpha)+\\ \Gamma(1+4/\alpha)-\Gamma^2(1+2/\alpha)+(-\Gamma(1+2/\alpha)\Gamma(1+\\ 4/\alpha)-\Gamma(1+1/\alpha)\Gamma(1+3/\alpha)+\Gamma(1+5/\alpha)-\\ \Gamma(1+2/\alpha)\Gamma(1+3/\alpha))]\end{array} & \begin{array}{l}\frac{\beta^6}{\Gamma^2(1+2/\alpha)}[-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}(-\frac{\Gamma(1+3/\alpha)}{\Gamma(1+2/\alpha)}+\\ \{\Gamma(1+4/\alpha)-\Gamma^2(1+2/\alpha)\}+\Gamma(1+4/\alpha)-\\ \Gamma^2(1+2/\alpha)+\Gamma(1+5/\alpha)-\Gamma(1+2/\alpha)\Gamma(1+3/\alpha)+(-\Gamma(1+\\ 3/\alpha)+\Gamma(1+5/\alpha)-\Gamma(1+2/\alpha)\Gamma(1+3/\alpha)+\Gamma(1+2/\alpha)\\ (\Gamma(1+6/\alpha)-\Gamma(1+3/\alpha))^2)]\end{array}
\end{bmatrix}_{3\times 3}
$$

$\hat{\Sigma} = JGJ'$

ใช้ค่าพารามิเตอร์ (α,β) แทนลงใน $\hat{\Sigma}$ ก็จะสามารถหาค่าของ $\hat{\Sigma}$ ในพารามิเตอร์นั้นๆ

3. <u>กรณีตัวแปรสุ่มมีการแจกแจงแบบลอกนอร์มอล</u>

ใช้สูตรการคำนวณค่า $\hat{\Sigma}$ เช่นเดียวกับที่ใช้ในการคำนวณ $\hat{\Sigma}$ ของการแจกแจงแบบแกมม่า และไวบูลล์ แต่ค่าภายในเมตริกซ์ จะแตกต่างกันไป ตามค่าของฟังก์ชั่นโมเมนต์ ที่ใช้ในแต่ละการแจกแจง

$$\text{สูตร} \quad \hat{\Sigma} = JGJ'$$

คำนวณค่า J

$$J = \begin{bmatrix} 1 & 0 & 0 \\ -\dfrac{\mu_2'}{\mu_1'^2} & \dfrac{1}{\mu_2'} & 0 \\ 0 & -\dfrac{\mu_3'}{\mu_2'^2} & \dfrac{1}{\mu_2'} \end{bmatrix}_{3\times 3}$$

$$\mu_\nu' = E(X^\nu)$$

กรณีของการแจกแจงแบบลอกนอร์มอล

$$\mu_\nu' = e^{\mu\nu + \frac{\sigma^2}{2}\nu^2}$$

$$\mu_1' = e^{\mu + \frac{\sigma^2}{2}}$$

$$\mu_2' = e^{2\mu + 2\sigma^2}$$

$$\mu_3' = e^{3\mu + \frac{9\sigma^2}{2}}$$

$$\mu_4' = e^{4\mu + 8\sigma^2}$$

$$\mu_5' = e^{5\mu + \frac{25\sigma^2}{2}}$$

$$\mu_6' = e^{6\mu + 18\sigma^2}$$

คำนวณค่าเมตริกซ์ J

$$\frac{-\mu_2'}{\mu_1'^{2}} = \frac{-e^{2\mu + 2\sigma^2}}{(e^{\mu + \frac{\sigma^2}{2}})^2}$$

$$= e^{2\mu + 2\sigma^2 - 2\mu - \sigma^2}$$

$$= -e^{\sigma^2}$$

$$\frac{1}{\mu_1'} = \frac{1}{e^{\mu + \frac{\sigma^2}{2}}}$$

$$= e^{-\mu - \frac{\sigma^2}{2}}$$

$$\frac{-\mu_3'}{\mu_2'^{2}} = \frac{-e^{3\mu + 9\frac{\sigma^2}{2}}}{(e^{2\mu+2\sigma^2})^2}$$

$$= -e^{3\mu + \frac{9\sigma^2}{2} - 4\mu - 4\sigma^2}$$

$$= -e^{-\mu + \frac{\sigma^2}{2}}$$

$$\frac{1}{\mu_2'} = \frac{1}{e^{2\mu + 2\sigma^2}}$$

$$= e^{-2\mu - 2\sigma^2}$$

$$J = \begin{bmatrix} 1 & 0 & 0 \\ -e^{\sigma^2} & e^{-\mu-\frac{\sigma^2}{2}} & 0 \\ 0 & -e^{-\mu+\frac{\sigma^2}{2}} & e^{-2\mu-2\sigma^2} \end{bmatrix}_{3\times 3}$$

ในที่นี้ศึกษาโดยกำหนดให้ $\mu = 0$ ส่วน σ^2 จะมีค่าต่างๆ ดังรายละเอียดที่กล่าวมาแล้วในบทที่ 1 กรณี $\mu = 0$

$$J = \begin{bmatrix} 1 & 0 & 0 \\ -e^{\sigma^2} & e^{-\frac{\sigma^2}{2}} & 0 \\ 0 & -e^{\frac{\sigma^2}{2}} & e^{-2\sigma^2} \end{bmatrix}_{3\times 3}$$

$$J' = \begin{bmatrix} 1 & -e^{\sigma^2} & 0 \\ 0 & e^{-\frac{\sigma^2}{2}} & -e^{\frac{\sigma^2}{2}} \\ 0 & 0 & e^{-2\sigma^2} \end{bmatrix}_{3\times 3}$$

ค่าของเมตริกซ์ G คำนวณจากค่า g_{ij} เช่นเดียวกับการแจกแจงแบบแกมม่าและไวบูลล์

ซึ่ง $g_{ij} = \mu_{i+j}' - \mu_i'\mu_j'$

ดังนั้น $$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}_{3\times 3}$$

$$G = \begin{bmatrix} \mu_2' - \mu_1'\mu_1' & \mu_3' - \mu_1'\mu_2' & \mu_4' - \mu_1'\mu_3' \\ \mu_3' - \mu_2'\mu_1' & \mu_4' - \mu_2'\mu_2' & \mu_5' - \mu_2'\mu_3' \\ \mu_4' - \mu_3'\mu_1' & \mu_5' - \mu_3'\mu_2' & \mu_6' - \mu_3'\mu_3' \end{bmatrix}_{3\times 3}$$

ในการคำนวณค่า g_{ij} ต่างๆ จะแทนค่าของค่าเฉลี่ย (μ) เป็น 0 ทุกตัว

$$g_{11} = \mu_2' - \mu_1'\mu_1' = e^{2\sigma^2} - (e^{\frac{\sigma^2}{2}})^2$$

$$= e^{2\sigma^2} - e^{\sigma^2}$$

$$= e^{\sigma^2}(e^{\sigma^2} - 1)$$

$$g_{12} = \mu_3' - \mu_1'\mu_2' = e^{\frac{9\sigma^2}{2}} - (e^{\frac{\sigma^2}{2}})(e^{2\sigma^2})$$

$$= e^{\frac{9\sigma^2}{2}} - e^{\frac{\sigma^2}{2} + 2\sigma^2}$$

$$= e^{\frac{9\sigma^2}{2}} - e^{\frac{5\sigma^2}{2}}$$

$$= e^{\frac{5\sigma^2}{2}}(e^{2\sigma^2} - 1)$$

$$g_{13} = \mu_4' - \mu_1'\mu_3' = e^{8\sigma^2} - (e^{\frac{\sigma^2}{2}})(e^{\frac{9\sigma^2}{2}})$$

$$= e^{8\sigma^2} - e^{5\sigma^2}$$

$$= e^{5\sigma^2}(e^{3\sigma^2} - 1)$$

$$g_{21} = \mu_3' - \mu_2'\mu_1' = g_{12}$$

$$g_{22} = \mu_4' - \mu_2'\mu_2' = e^{8\sigma^2} - (e^{2\sigma^2})^2$$

$$= e^{4\sigma^2}(e^{4\sigma^2} - 1)$$

$$g_{23} = \mu_5' - \mu_2'\mu_3' = e^{25\sigma^2} - (e^{2\sigma^2})(e^{\frac{9\sigma^2}{2}})$$

$$= e^{25\sigma^2} - e^{\frac{13\sigma^2}{2}}$$

$$= e^{\frac{13\sigma^2}{2}}(e^{6\sigma^2} - 1)$$

$$g_{31} = \mu_4' - \mu_3'\mu_1' = g_{13}$$

$$g_{32} = \mu_5' - \mu_3'\mu_2' = g_{23}$$

$$g_{33} = \mu_6' - \mu_3'\mu_3' = e^{18\sigma^2} - (e^{\frac{9\sigma^2}{2}})^2$$

$$= e^{18\sigma^2} - e^{9\sigma^2}$$

$$= e^{9\sigma^2}(e^{9\sigma^2} - 1)$$

$$G = \begin{bmatrix} e^{\sigma^2}(e^{\sigma^2}-1) & e^{\frac{5\sigma^2}{2}}(e^{2\sigma^2}-1) & e^{5\sigma^2}(e^{3\sigma^2}-1) \\ e^{\frac{5\sigma^2}{2}}(e^{2\sigma^2}-1) & e^{4\sigma^2}(e^{4\sigma^2}-1) & e^{\frac{13\sigma^2}{2}}(e^{6\sigma^2}-1) \\ e^{5\sigma^2}(e^{3\sigma^2}-1) & e^{\frac{13\sigma^2}{2}}(e^{6\sigma^2}-1) & e^{9\sigma^2}(e^{9\sigma^2}-1) \end{bmatrix}_{3\times3}$$

$$JG = \begin{bmatrix} 1 & 0 & 0 \\ -e^{\sigma^2} & e^{-\frac{\sigma^2}{2}} & 0 \\ 0 & -e^{\frac{\sigma^2}{2}} & e^{-2\sigma^2} \end{bmatrix}_{3\times3} \begin{bmatrix} e^{\sigma^2}(e^{\sigma^2}-1) & e^{\frac{5\sigma^2}{2}}(e^{2\sigma^2}-1) & e^{5\sigma^2}(e^{3\sigma^2}-1) \\ e^{\frac{5\sigma^2}{2}}(e^{2\sigma^2}-1) & e^{4\sigma^2}(e^{4\sigma^2}-1) & e^{\frac{13\sigma^2}{2}}(e^{6\sigma^2}-1) \\ e^{5\sigma^2}(e^{3\sigma^2}-1) & e^{\frac{13\sigma^2}{2}}(e^{6\sigma^2}-1) & e^{9\sigma^2}(e^{9\sigma^2}-1) \end{bmatrix}_{3\times3}$$

$$JG = \begin{bmatrix} e^{\sigma^2}(e^{\sigma^2}-1) & e^{\frac{5\sigma^2}{2}}(e^{2\sigma^2}-1) & e^{5\sigma^2}(e^{3\sigma^2}-1) \\ e^{3\sigma^2}(e^{\sigma^2}-1) & e^{\frac{11\sigma^2}{2}}(e^{2\sigma^2}-1) & e^{9\sigma^2}(e^{3\sigma^2}-1) \\ e^{5\sigma^2}(e^{\sigma^2}-1) & e^{\frac{17\sigma^2}{2}}(e^{2\sigma^2}-1) & e^{13\sigma^2}(e^{3\sigma^2}-1) \end{bmatrix}_{3\times3}$$

$$\hat{\Sigma} = JGJ' = \begin{bmatrix} e^{\sigma^2}(e^{\sigma^2} - 1 & e^{3\sigma^2}(e^{\sigma^2} - 1) & e^{5\sigma^2}(e^{\sigma^2} - 1) \\ e^{3\sigma^2}(e^{\sigma^2} - 1) & e^{4\sigma^2}(e^{3\sigma^2} - 2e^{\sigma^2} + 1) & e^{8\sigma^2}(e^{4\sigma^2} - e^{2\sigma^2} - e^{\sigma^2} + 1) \\ e^{5\sigma^2}(e^{\sigma^2} - 1) & e^{8\sigma^2}(e^{4\sigma^2} - e^{2\sigma^2} - e^{\sigma^2} + 1) & e^{9\sigma^2}(e^{5\sigma^2} - 2e^{2\sigma^2} + 1) \end{bmatrix}_{3x3}$$

ใช้ค่านารามิเตอร์ (σ^2) แทนลงใน $\hat{\Sigma}$ ก็จะสามารถทราบค่าของ $\hat{\Sigma}$ ในนารามิเตอร์นั้นๆ.

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