CHAPTER V

T_c EQUATIONS

In this chapter we will show how to obtain T_c formulas from Eq. (4.38) and discuss its limting cases. First, we will compare our limiting cases result with the other works in literature and then we will derive the new T_c formula of antiferromagnetic superconductor sandwiches.

Case I

In this case, to recover the T_c formula of the AFS which have been derived by many authors before.

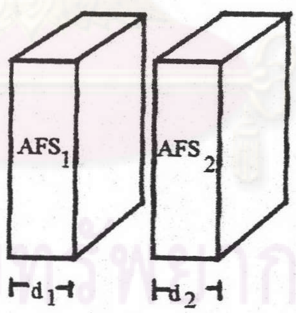

Fig. 5.1 The diagram of AFS's when not in proximity.

When the two films are not in proximity.Eq.(4.38) indicates $\Gamma_\alpha = R_\alpha = 0$, $K_\alpha = H_{Q\alpha}$

$$P_\alpha = 2\pi T \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T} \left[(\omega_n)^2/[(\omega_n)^2+(K_\alpha)^2]^{3/2} - 1/\omega_n\right] \qquad (5.1)$$

and

$$[\,(\lambda)^{-1} - f(T) - P_1]\,[\,(\lambda)^{-1} - f(T) - \ln(\omega_{D2}/\omega_{D1}) - P_2] \quad = \quad 0 \qquad (5.2)$$

From the self-consistency BCS order parameter equation.

$$(\lambda_\alpha)^{-1} \quad = \quad 2\,\pi\,T_c \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} 1/\,\omega_n \quad = \quad \ln(2\gamma\omega_{D\alpha}/\,\pi T_c) \qquad (5.3)$$

where $\gamma = 1.781$.

Solving Eq. (5.2) by using Eq. (5.1) and Eq. (5.3), we get

$$\ln(T_c/T_{c\alpha}) \quad = \quad 2\,\pi\,T_c \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} \Big[(\omega_n)^2/[(\omega_n)^2+(K_\alpha)^2]^{3/2} \; - \; 1/\omega_n\Big] \qquad (5.4)$$

Eq. (5.4) is the T_c formula of two different AFS layers in seperation which agrees exactly with the result of Suzumura and Nagi(Suzumura and Nagi,1981)

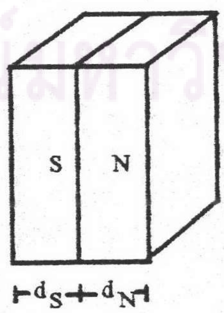

Fig. 5.2 The diagram of an S-N sandwich.

For an S-N sandwich. Here $\Gamma_\alpha \neq 0$, $\lambda_2 = H_{Q\alpha} = 0$. According to Eqs. (4.24) and (4.27), we have

$$K_\alpha = 0$$

$$P_\alpha = 2\pi T \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T} \left[1/[\Gamma_\alpha + \omega_n - \Gamma_\alpha \Gamma_\beta/(\Gamma_\beta + \omega_n)] - 1/\omega_n \right] \qquad (5.5.1)$$

$$R_\alpha = 2\pi T \Gamma_\alpha \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T} 1/[(\Gamma_\alpha + \omega_n)(\Gamma_\beta + \omega_n) - \Gamma_\alpha \Gamma_\beta] \qquad (5.5.2)$$

and

$$1 - \lambda_1 [f(T) + P_1] = 0 \qquad (5.5.3)$$

By inserting Eqs. (5.3), (5.5.1), and (5.5.2) into (5.5.3), we have

$$\ln(T_c/\mathring{T}_{c1}) = P_1$$

here

$$P_1 = 2\pi T_c \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} \left[[\Gamma_2 + \omega_n]/[\Gamma_1 + \omega_n - \Gamma_1\Gamma_2] - 1/\omega_n \right]$$

$$= 2\pi T_c \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} [-\Gamma_1/(\Gamma_1 + \Gamma_2)] [1/\omega_n - 1/(\omega_n + \Gamma_1 + \Gamma_2)]$$

$$= -\gamma_1 H_1(\rho_c)$$

$$\text{where } H_1(\rho_c) = 2\pi T_c \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} [\, 1/\omega_n - 1/(\omega_n + \Gamma_1 + \Gamma_2)\,]$$

$$= \Psi(-1/2 + \rho_c/2) - \Psi(-1/2) - \ln[\, 1 + (\Gamma_1 + \Gamma_2)/\omega_{D1}\,]$$

Here $\gamma_1 = [-\Gamma_1/(\Gamma_1 + \Gamma_2)]$, $\rho_c = [(\Gamma_1 + \Gamma_2)/\pi T_c]$, $\gamma = 1.781$, and $\Psi(Z)$ is the digamma function (Davis,1965).

Finally, we find

$$\ln [T_c / T^{\circ}_{c1}] = -\gamma_1 H_1(\rho_c) \tag{5.6}$$

Eq. (5.6) is the formula of McMillan ' s S-N bilayer (McMillan,1968).

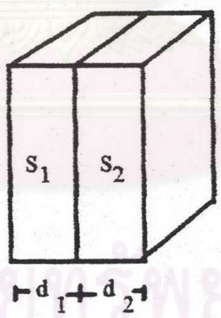

Fig. 5.3 The diagram of an S_1-S_2 sandwich.

For an S_1-S_2 sandwich, here $\Gamma_\alpha \neq 0$, $\lambda_\alpha \neq 0$, $H_{Q\alpha} = 0$.

Then

$$K_\alpha = 0 \tag{5.7.1}$$

$$P_\alpha = -\gamma_\alpha H_\alpha(\rho) \qquad (5.7.2)$$

$$R_\alpha = P_\alpha \qquad (5.7.3)$$

and

$$[(\lambda_1)^{-1} - f(T) - P_1][(\lambda_2)^{-1} - f(T) - \ln(\omega_{D2}/\omega_{D1}) - P_1] = P_1 P_2 \qquad (5.8)$$

Solving Eq. (5.8), we obtain

$$f(T) = (1/2)[(\lambda_1)^{-1} + (\lambda_2)^{-1} - \ln(\omega_{D2}/\omega_{D1}) + \gamma_1 H_1(\rho) + \gamma_2 H_2(\rho) - \{[(\lambda_1)^{-1} + (\lambda_2)^{-1} - \ln(\omega_{D2}/\omega_{D1}) + \gamma_1 H_1(\rho) + \gamma_2 H_2(\rho)]^2 - 4[(\lambda_1\lambda_2)^{-1} + \gamma_2 H_2(\rho)(\lambda_2)^{-1} - \ln(\omega_{D2}/\omega_{D1})((\lambda_1)^{-1} + \gamma_1 H_1(\rho))]\}^{1/2}] \qquad (5.9)$$

we define

$$\ln(T^{*}_{c1}/T_c) = f(T_c) - (\lambda_1)^{-1} \qquad (5.10.1)$$

$$E = (\lambda_1)^{-1} + (\lambda_2)^{-1} - \ln(\omega_{D2}/\omega_{D1}) \qquad (5.10.2)$$

$$F = \gamma_1 H_1(\rho_c) + \gamma_2 H_2(\rho_c) \qquad (5.10.3)$$

$$G = \gamma_2 H_2(\rho_c) - \gamma_1 H_1(\rho_c) \qquad (5.10.4)$$

we get

$$\ln(T^{\circ}_{c1}/T_c) \;=\; (1/2)\left[\; E + F - \left[E^2 + F^2 + 2EG \right]^{1/2} \;\right] \qquad (5.11)$$

Eq. (5.11) is the Mohabir and Nagi's result (Mohabir and Nagi,1979) for an S_1-S_2 bilayer that shown in Eq.(3.18).

Case II

In this section, we will derive a new T_c formula of an AFS-N sandwich by considering the limiting cases of Eq. (4.38).

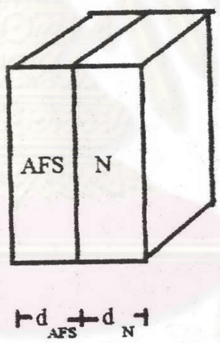

Fig. 5.4 The diagram of an AFS-N sandwich.

For an AFS-N sandwich. $\Gamma_\alpha \neq 0$, $\lambda_1 \neq 0$, $H_{Q1} \neq 0$, $H_{Q2} = \lambda_2 = 0$.

Then Eq.(4.24.2) gives

$$K_1 \;=\; [H_{Q1}\sqrt{(\omega_n)^2+(K_2)^2} + \Gamma_1 K_2] \,/\, [\,\Gamma_1 + \sqrt{(\omega_n)^2+(K_2)^2}\,] \qquad (5.12.1)$$

$$K_2 \;=\; [\,\Gamma_2 K_1] \,/\, [\,\Gamma_2 + \sqrt{(\omega_n)^2+(K_1)^2}\,] \qquad (5.12.2)$$

P_α and R_α are given by Eq. (4.37) .

and

$$1 - \lambda_1 \ (f(T) - P_1) = 0 \tag{5.13}$$

Because Eq. (5.13) is the same as Eq. (5.5.3), then

$$\ln(T_c/T^{\circ}_{c1}) = P_1$$

Inserting Eq. (5.12) into Eq. (4.37.1) and using ω_n=2πT(n+1/2), where n is an integer

$$P_\alpha = \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} \Big[[(n+1/2)^2 A_\beta/(A_\alpha)^3] \,/\, [A_\beta + B_\alpha - (n+1/2)^4 B_\alpha B_\beta \,/[(A_\beta A_\alpha)^2 (A_\alpha + B_\beta)]] - 1/(n+1/2) \Big] \tag{5.14}$$

$$A_\alpha = \sqrt{(n+1/2)^2 + (K_\alpha /2\pi T_c)^2} \tag{5.15.1}$$

$$B_\alpha = \Gamma_\alpha/2\pi T_c \tag{5.15.2}$$

$$K_1 = [H_{Q1} A_2 + B_1 K_2] / [B_1 + A_2] \tag{5.15.3}$$

$$K_2 = [B_2 K_1] / [B_2 + A_1] \tag{5.15.4}$$

we find the T_c formula of an AFS-N sandwich as

$$\ln(T_c/T^{\circ}_{c1}) = \sum_{n\geq 0}^{\omega_{D1}/2\pi T_c} \left[[(n+1/2)^2 A_2/(A_1)^3] \,/\, [A_2+B_1-(n+1/2)^4 B_1 B_2 \,/\, [(A_1A_2)^2(A_1+B_2)]] - 1/(n+1/2) \right] \qquad (5.16)$$

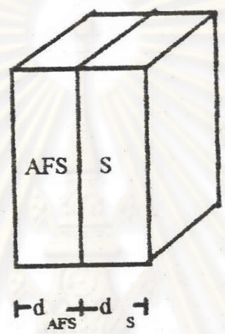

Fig. 5.7 The diagram of an AFS-S sandwich.

For an AFS-S sandwich. $\Gamma_\alpha \neq 0$, $\lambda_\alpha \neq 0$, $H_{Q1} \neq 0$, $H_{Q2}=0$.

From Eq. (3.24.2)

$$K_1 = [H_{Q1}\sqrt{(\omega_n)^2+(K_2)^2} + \Gamma_1 K_2] \,/\, [\Gamma_1+\sqrt{(\omega_n)^2+(K_2)^2}] \qquad (5.17.1)$$

$$K_2 = [\Gamma_2 K_1] \,/\, [\Gamma_2 + \sqrt{(\omega_n)^2+(K_1)^2}] \qquad (5.17.2)$$

In these equations

$$A_\alpha = \sqrt{(n+1/2)^2 + (K_\alpha/2\pi T_c)^2} \qquad (5.17.3)$$

$$B_\alpha = \Gamma_\alpha/2\pi T_c \qquad (5.17.4)$$

$$K_1 = [\, H_{Q1}A_2+B_1K_2 \,] / [\, B_1+A_2 \,] \tag{5.17.5}$$

$$K_2 = [\, B_2K_1 \,] / [\, B_2+A_1 \,] \tag{5.17.6}$$

The functions P_α and R_α are given by

$$P_\alpha = \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} [\ \{(n+1/2)^2 A_\beta/(A_\alpha)^3] / [A_\beta+B_\alpha-(n+1/2)^4 B_\alpha B_\beta / [(A_\alpha A_\beta)^2(A_\alpha+B_\beta)]\} - 1/(n+1/2)\ ] \tag{5.18.1}$$

$$R_\alpha = \sum_{n\geq 0}^{\omega_{D\alpha}/2\pi T_c} [\ [(n+1/2)^4/[\ (A_\alpha A_\beta)^2(A_\alpha+B_\beta)(A_\beta+B_\alpha)-(n+1/2)^4 B_\alpha B_\beta\ ]\ ] \tag{5.18.2}$$

Solving Eq. (4.38), we obtain

$$f(T) = (1/2)\ [(\lambda_1)^{-1}+(\lambda_2)^{-1}-P_1-P_2-\ln(\omega_{D2}/\omega_{D1})-\{[(\lambda_1)^{-1}+(\lambda_2)^{-1}-P_1-P_2 -\ln(\omega_{D2}/\omega_{D1})]^2-4\ [(\ (\lambda_1)^{-1}\ -P_1)(\ (\lambda_2)^{-1}-\ln(\omega_{D2}/\omega_{D1})-P_2)-R_1R_2]\}^{1/2}] \tag{5.19}$$

we define

$$\ln(T_c / T^{\circ}_{c1}) = (\lambda_1)^{-1} - f(T_c)$$

where $\alpha=1,2 : \beta=1,2$ and $\alpha \neq \beta$.

The discussion of the T_c result of AFS sandwiches will be done in the next chapter.