CHAPTER I

SUPERCONDUCTIVITY

The subject of superconductivity was discovered over half a century ago by Kamerlingh Onnes (Kamerlingh Onnes,1911). Superconductivity is now very well known. It is a field with many practical and theoretical aspects. Thus it will be quite valuable to review the basis phenomenology of superconductivity.

DC Electrical Resistance

The general behavior of a normal conductor and a superconductor is shown in Fig. 1.1

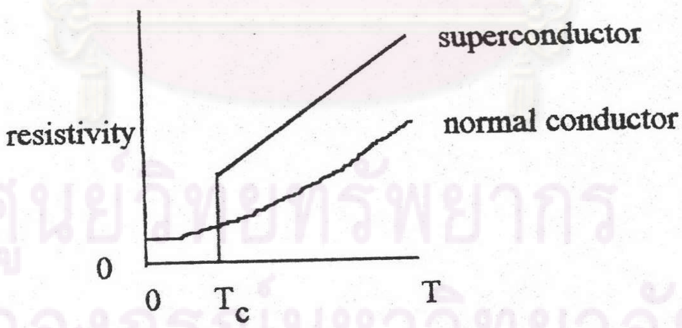

Fig 1.1 The general behavior of a normal conductor and a superconductor (Lynn et al., 1988)

The superconducting state is associated with the precipitous drop of the resistance to an immeasurably small value at a specific critical temperature T_c. The most familiar property of superconductor is the lack of any resistance to the flow of electrical current. Classically we call this perfect conductivity. However, the resistanceless state is much more than just perfect conductivity, and in fact cannot be understood at all on the basis of classical physics , hence the name superconductor.

Now consider initiating a current in a closed loop of wire. For a perfect conductor we might at first expect the current to continue forever. However, the electrons circulating in the loop of wire are in an accelerating reference frame, and an accelerating charge radiates energy. Including these classical radiation effects yields a current which decays with time, and hence there is resistance. In a superconductor, there is no observable decay of the supercurrent (with an experimental half-life exceeding 10^5 years (File and Mills,1963)).

London Theory

Soon after the discovery of superconductivity, Onnes observed that the electrical resistance reappears if a sufficiently large current is passed through the conductor, or if a magnetic field above a critical value is applied to the sample. The phase diagram for a typical elemental superconductor is shown in Fig. 1.2

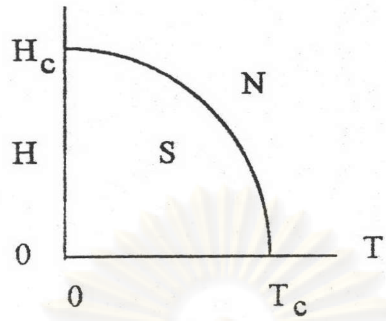

Fig. 1.2 Effect of magnetic field on typical elemental superconductor (Lynn et al., 1988) N = normal state, S = superconducting state .

Note that the value of the critical field H_c approaches zero in continuous fashion as a function of temperature, which contrasts with the apparent sudden drop of the electrical resistivity at T_c shown in Fig. 1.1 . For a simple superconductor there is a direct relation between the critical current and the critical field. Consider a superconductor which is initially in a field-free region, and then apply a magnetic field to system. Persistent current will set up. The currents in turn generate a magnetic field of their own, which opposes the applied field. A simple analysis of this problem starts with Newton's law ;

$$m \frac{d\vec{v}}{dt} = -e\vec{E} \qquad (1.1)$$

where m is the effective mass of the electron, and -e is the elementary charge. If we multiply both sides by -e n_s, where n_s is the density of electrons, then we have

$$\frac{d\vec{j}}{dt} = n_s e^2 \vec{E} / m \qquad (1.2)$$

where $\vec{j} = -en_s\vec{v}$ is the induced current. Taking the curl of both sides gives

$$\frac{d}{dt}\left[\nabla \times \vec{j}\,\right] - (n_s e^2/m)\,\nabla \times \vec{E} \;=\; \frac{d}{dt}\left[\nabla \times \vec{j} + (n_s e^2/mc)\,\vec{B}\,\right] \;=\; 0 \qquad (1.3)$$

where we have used Faraday's law

$$\nabla \times \vec{E} \;=\; \frac{-1}{c}\frac{\partial \vec{B}}{\partial t} \qquad (1.4)$$

we can relate B and j by another of Maxwell's equations

$$\nabla \times \vec{B} \;=\; \frac{4\pi}{c}\,\vec{j} + \frac{\partial \vec{D}}{\partial t} \qquad (1.5)$$

we obtain finally

$$\frac{d}{dt}\left\{\nabla^2\vec{B} - (4\pi n_s e^2/mc^2)\,\vec{B}\right\} \;=\; 0 \qquad (1.6)$$

Clearly any $\vec{B}$ field which is time independent will satisfy the above equation. Here if $\vec{B}=0$, the field inside the material will remain zero when a field is applied.

Consider an experiment where we first apply a field to the sample in the normal state, and then cool it below T_c . The classical result dictates that the field will still be time-independent, so that there would be no change in $\vec{B}$, and the flux will be trapped in the conductor. However, Meissner and Oschenfeld (Meissner and Oschenfeld,1933) discovered in 1933 the magnetic field is completely expelled from the interior of the superconductor, in contradiction with the classical expectation. The sample in the superconducting state exhibits perfect diamagnetism, with the magnetic induction $\vec{B}=0$.

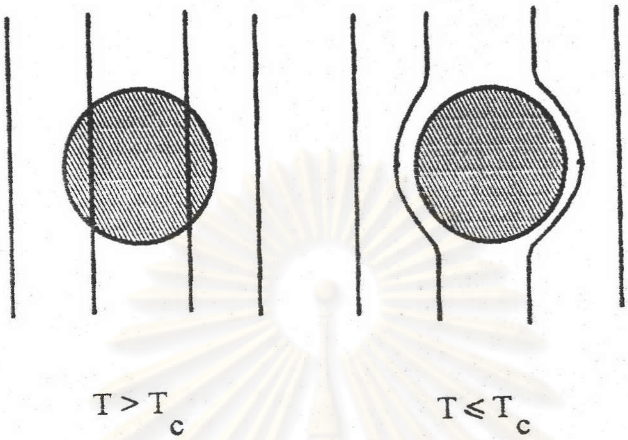

Fig. 1.3 Meissner effect (Kittel, 1991)

The London brothers (London and London,1935) provided an explanation for the Meissner effect by proposing that not only must the time derivative of the expression in Eq. (1.6) be zero, but the bracket itself must vanish. Hence we have

$$\nabla^2\vec{B} - (4\pi n_s e^2/mc^2)\vec{B} = \nabla^2\vec{B} - \vec{B}/\lambda^2 = 0 \qquad (1.7)$$

The specific form of the solution to Eq. (1.7) depends on the particular geometry and boundary condition, but is typically of form $B_0 \exp(-x/\lambda_L)$,where $\lambda_L = \{ mc^2/4\pi n_s e^2 \}^{1/2}$, m is electron effective mass, and n_s is the density of electrons. Hence the field inside the superconductor decays exponentially with distance, with a characteristic length scale given by the London penetration depth λ_L .

Magnetic Levitation

One of the most fascinating demonstrations of superconductivity is the levitation of a superconducting particle over a magnet (or vice versa). Typically this is done by dropping a particle of superconducting materials in a dish of liquid nitrogen, with a magnet underneath, and watching the particle jump and hover above the magnet when the temperature drops below T_c .

The repulsion of the particle from the magnet is caused by the flux exclusion from the interior of the material. We assume for simplicity that the particle is spherical with radius R and that R >> λ_L so that we may neglect surface effects. We have (Lynn et al.,1988)

$$h = [\ B^2(a)\, a^2/4\pi\rho g\ ]^{1/3} \tag{1.8}$$

where B(a) is the value of the field at the surface of the magnet, a locates the surface of the magnet, and h is the height of the sphere above the magnet. We also assume that the average value of B may be taken at center of the sphere (h>>R).

Note that this result does not depend on the size of the particle and in fact the only material-dependent parameter is the density ρ.This equation is valid only in the regime that $h >> R >> \lambda_L$.

Flux Quantization

The current density in any conductor is defined by $\vec{j} = nq\vec{v}$, where n is the density of carriers, q is the charge, $\vec{v}$ is their average velocity. In the presence of a magnetic field we can write this in term of the vector potential $\vec{A}$ as

$$\vec{j} \quad = \quad n q [\vec{P}/m \; - q \vec{A}/mc] \tag{1.9}$$

where $\vec{P}$ is the momentum of a carrier.

If we integrate around a closed path deep inside the superconductor where $\vec{j} = 0$, use the relation $\int \vec{A} \cdot d\vec{l} = \phi$, ϕ is the magnetic flux, and apply the Bohr-Sommerfeld quantization condition as London (London,1950) did, then we find that magnetic flux is quantized

$$\phi \quad = \quad hc / 2e \quad = \quad 2.07 \times 10^{-7} \quad \text{gauss-cm}^2 \tag{1.10}$$

where the charge q for a Cooper pair is $2e$.

Another way to obtain the quantization condition is to employ the theory of Ginzburg and Laudau (Ginzburg and Landau,1950) , which is a general thermodynamical approach to the theory of phase transition.

Energy Gap

One of the central features of a superconductor is that there exists an energy gap in the excitation spectrum for electron, which was first discovered in specific heat measurements. In a normal metal the specific heat at low temperatures is given by (Lynn et al.,1988)

$$C \quad = \quad \gamma T \quad + \quad \beta T^3 \tag{1.11}$$

where the linear term is due to electron excitations, and the cubic term originates from phonon excitation. Below the superconducting transition, the electronic term was found to be of the form $\exp(-\Delta/k_B T_c)$ which is characteristic of a system with a gap in the excitation spectrum of energy $2\,\Delta$. The gap is directly related to the superconducting order parameter, and hence we might expect that $\Delta \rightarrow 0$ as $T \rightarrow T_c$.

Electron-Phonon Interaction and Cooper Pairing

Another very important discovery (Maxwell,1950 ; Reynolds et al. ,1950) was that the superconducting transition temperature depends on the isotopic mass of the material. The simplest form of the isotope effect given

$$T_c \quad \alpha \quad (M)^{-1/2} \tag{1.12}$$

where M is the ionic mass. The effect demonstrates that lattice vibration plays an essential role in the formation of the superconducting state.

In order to obtain an interaction which has an attractive part, and at least a chance of producing a bound state, the dynamics of the system must be taken into account.Considering that the lattice deformation caused by the presence of an electron takes a finite time to relax, and can therefore influence a second electron passing by at a later time.

The crucial point Cooper (Cooper,1956) emphasized is that we are not dealing with a two-electron system, but with an N-electron system. The central result he obtained is that the ground state of an electron gas is unstable to the formation of bound pairs if an attractive electron-phonon interaction exists, regardless of the strength of the potential. Hence he showed that the weak electron-phonon interaction could in fact produce bound electron pairs.

BCS Theory

We consider the theory introduced by Bardeen, Cooper, and Schrieffer (BCS) in 1957 (Bardeen, Cooper, and Schrieffer,1957). The BCS theory is a model Hamiltonian set up to explain characteristics of the superconducting state.This theory incorporates the assumption of a weak net attractive interaction force. An attractive force among conduction electrons combines two electrons with momenta $\vec{p}$ and $-\vec{p}$ into a Cooper pair. The electron-lattice interaction is the orgin of this attractive force. The simple model (Golovashkin et al.,1981) which permits such behavior is given by the BCS "reduced" Hamiltonian , $H_0 + H_{red}$,where

$$H_0 = \sum_{k\sigma} \varepsilon_k C^{+}_{k\sigma} C_{k\sigma} \qquad (1.13.1)$$

$$H_{red} = \sum_{kk'} V_{kk'} C^{+}_{k\uparrow} C^{+}_{-k\downarrow} C_{-k'\downarrow} C_{k'\uparrow} \qquad (1.13.2)$$

where ε_k is the energy of the conduction electron. $C^{+}_{k\sigma}$ ($C_{k\sigma}$) is the creation (annihilation) operator for electron. $V_{kk'}$ is interaction matrix elements. σ is spin index. Interaction (1.13.2) contain terms that scatter pairs of electron from one pair state ($k\uparrow,-k\downarrow$) to a different one ($k'\uparrow,-k'\downarrow$). The interaction matrix elements $V_{kk'}$ are at this state unspecified. Bardeen, Cooper and Schrieffer had in mind the Frohlich or Bardeen-Pines (Bardeen and Pines,1955) effective phonon-induced interaction which $V_{kk'}$ is negative.

The characteristic BCS pair-interaction Hamiltonian will lead to a ground state which is some phase-coherent superposition of many-body states with pairs of state ($k\uparrow,-k\downarrow$) occupied or unoccupied as units. Because of the coherence, operators such as $C_{-k\downarrow} C_{k\uparrow}$ can have nonzero expectation values $< C_{-k\downarrow} C_{k\uparrow} >$ .The bracket $< >$ denotes the thermal average. We define

$$\Delta = \sum_{k} V_k < C_{-k\downarrow} C_{k\uparrow} > \qquad (1.14)$$

In terms of Δ , the model Hamiltonian becomes

$$H = \sum_{k\sigma} \varepsilon_k C^{+}_{k\sigma} C_{k\sigma} + \sum_{k} \Delta \left(C^{+}_{k\uparrow} C^{+}_{-k\downarrow} + \text{h.c.} \right) \qquad (1.15)$$

We define the Green's function as

$$G(k,\omega_n) = \langle -T_\tau \, \psi_k(\tau) \, \psi_k^{+}(0) \rangle \tag{1.16}$$

where $\psi_k^{+} = (\; C_{k\uparrow}^{+} \quad C_{-k\downarrow} \;)$ and T_τ is the time ordering operator for the imaginary time $\tau = it$, we find the single particle Green's function of a superconductor as

$$G(k,\omega_n) = (\; i\omega_n - \varepsilon_k \tau_3 + \Delta \, \tau_1 \;)^{-1} \tag{1.17}$$

where τ_1 and τ_3 are well known Pauli matrices.

Although, the original theory contains a number of approximations such as the electron-phonon interaction is constant, yet it works very well for simple metals and contains all the essential elements of the superconducting state. Here we will simply summarize some of the central results of the theory (Lynn et al., 1988).

In a BCS superconductor, there is an energy gap in the one-electron spectrum at zero temperature. With increasing temperature the energy gap monotonically decreases, and approaches zero as $T \rightarrow T_c$. In the vicinity of the transition temperature the theory yields

$$\Delta(T) = 1.74 \;\; \Delta(0) \;\; [\; 1 - T/T_c \;]^{1/2} \tag{1.18}$$

Here $\Delta(T)$ and $\Delta(0)$ are the energy gaps at temperature T and zero, respectively. Along with the gap collapsing to zero, the critical magnetic field above which the superconductivity is quenched follows

$$H_c(T) \quad = \quad H_c(0)\ [\ 1\ -\ a\,(\ T/T_c)^2\ ] \qquad (1.19)$$

where a is a constant which is weakly dependent on temperature. Like the energy gap, the critical field approaches zero continuously.

The electronic term in the heat capacity at low temperature is

$$C_s \quad = \quad 1.34\,\gamma\, T_c\ (\ \Delta(0)\,/T\ )^{3/2}\ \exp(-\ \Delta(0)\,/T) \qquad (1.20)$$

where γ is the electronic specific heat coefficient in normal state. Eq.(1.20) is characteristic of a system which possesses an energy gap in the excitation spectrum. On the other hand at the onset of superconductivity there is a jump in the value of the specific heat which is given by

$$(\ C_s\ -\ C_n\ )\,/\,C_n \quad = \quad 1.43 \qquad (1.21)$$

where C_n is the specific heat in normal state, just above T_c , and C_s is the specific heat just below T_c .

The BCS theory also makes a prediction for the ordering temperature in terms of the parameters of the theory. The transition temperature T_c is given by

$$k_B T_c = 1.13 \hbar \omega_D \exp(-1/N(0)V) \tag{1.22}$$

where $\hbar \omega_D$ is the Debye energy for phonon. Since the phonon frequency is inversely proportional to the square root of the mass for a simple metal, this prediction is in agreement with the isotope effect.

Finally, the theory gives a relationship between the zero temperature energy gap and the ordering temperature

$$\Delta(0) / k_B T_c = 1.764 \tag{1.23}$$

This ratio is independent of the parameters of the theory, and agrees quite well with many of the simple elemental superconductors.