CHAPTER VI

CONCLUSION AND DISCUSSION

From the starting to the end of this work we deal with the concept of reduced density matrices and its applications. In a many-body system of either bosons or fermions, it is possible to have an off-diagonal long-range order in the coordinate representation. Because of this order it leads to a new thermodynamic phase of the system. Therefore it is reasonable to assume that the properties of superfluid He II and superconductors are characterized by such an order.

In order to study the properties of reduced density matrices and using these properties to a system of identical particles, here our purpose is restricted to a system of fermions. We have thus summarize the brief history of the theory of superconductivity. We then study the properties of reduced density matrices and off-diagonal long-range order, especially for the two-body reduced density matrix which is responsible for the characterization of fermions system. The theory of two-fluid model has been reviewedin order to use some definitions that have been defined earlier. Attention is fully focused in the applications of reduced density matrices in describing the thermo-hydrodynamic equations for superconductivity, as shown in Chapter V.

In defining some terms we define them in analogy to the theory of superfluid such as the condensate macroscopic wave function. But in order to define the condensate current density we have two choices. The first is that of Eq. (4.6) where we define the condensate current density by considering the superfluid at rest and treat the whole density as condensate density with velocity $\vec{v}_s$ and define $\vec{v}_n$ via the velocity of the normal fluid. The second is that of Eq. (4.13) where we use the relation in thermodynamics and the Galilean transformation of the wave function to define the condensate and normal current density. The difference between these two methods lies in the definition of the condensate term. However we incline to use the notion of coupled pair in the center of mass coordinate. Then we can see easily the meaning of the current density as consists of the condensate coupled pair and the normal component that is not bounded into pair. Thus we use Eq.(4.13) instead of Eq.(4.6)

When we derive the wave equation of macroscopic wave function by using Heisenberg equation of motion, we show that this wave equation remains unchanged under the gauge transformations provided that the gauge choice $\chi = \frac{2\pi\hbar c}{e} n$ where n = 0, $\pm$ 1, $\pm$ 2. The gauge invariance is one of the properties of superconductivity which yields the quantization condition $\chi = \frac{hc}{e} n$ where $n = 0, \pm 1, \pm 2, ...$ This means that so far our wave equation is all right to this point.

For another confirmation we consider the Fourier transform of the wave equation. From this transformation we can show that our wave equation is satisfied in equilibrium by the conventional microscopic theory, that is the BCS theory.

Finally from the wave equation we can find the two thermo-hydrodynamic equations, namely the equation of motion for the condensate velocity and the condensate density . From now on we can define

$$\rho_n v_n = \frac{\hbar}{4mi} \lim_{\substack{x_1 \to x_1'' \\ x_2 \to x_2''}} \Big[\nabla_{x_1} + \nabla_{x_2} - \nabla_{x_1''} - \nabla_{x_2''} - \frac{ie}{\hbar c} \big(A(x_1) + A(x_2) + A(x_1'') + A(x_2'') \big) \Big] \tilde{\Omega}_2(x_1, x_2, x_1'', x_2'') \qquad (6.1)$$

Where

$$\tilde{\Omega}_2(x_1, x_2, x_1'', x_2'') = \rho_n(x_1, x_2, x_1'', x_2'') \exp i\chi(x_1, x_2, x_1'', x_2'') \qquad (6.2)$$

From Eqs. (6.1) and (6.2) we find that

$$\rho_n v_n = \Big\{ \frac{\hbar}{4m} \lim_{R \to R''} (\nabla_R - \nabla_{R''}) \chi(R, r, R'', r'') - \frac{e}{mc} A(R, r) \Big\} \rho_n$$

Where $\vec{R} = \frac{1}{2}(\vec{x}_1 + \vec{x}_2)$, $\vec{r} = \vec{x}_1 - \vec{x}_2$ and $\vec{R}'' = \frac{1}{2}(\vec{x}_1'' + \vec{x}_2'')$, $\vec{r}'' = \vec{x}_1'' - \vec{x}_2''$.

Also we use the fact that $\chi(\vec{x}_1, \vec{x}_2, \vec{x}_1'', \vec{x}_2'')$ is an odd function. Then the normal velocity is defined by

$$\vec{v}_n = \frac{\hbar}{4m} \lim_{\vec{R} \to \vec{R}''} (\nabla_{\vec{R}} - \nabla_{\vec{R}''}) \chi(\vec{R}, \vec{r}, \vec{R}'', \vec{r}'') - \frac{e}{mc} \vec{A}(\vec{R}, \vec{r}) \qquad (6.3)$$

Then from the continuity equation, with Eq. (5.31) we can find the equation of motion for the normal density. But we must find a suitable equation in statistical mechanics in finding the equation of motion for the normal velocity. We thus leave this problem for the next work.