CHAPTER I

INTRODUCTION

The hydrogen atom problem has always excited interest because of its relevance to understanding the behavior of hydrogenic atoms (one electron atoms), hydrogenic impurities in semiconductors, positronium (an unstable union of a positron and electron which gives a hydrogenic atom whose proton has been replaced by positron), and Wannier excitons in solids (1). The interest in the three dimensional solution is obvious because of its relevance to the problem of the real hydrogen atom. In this case, the interaction between the negative and positive charges in the atom are via an attractive Coulomb potential

$$V(r) = -\frac{Ze^2}{r} = -eA_o \qquad (1.1)$$

The solution of the nonrelativistic three dimensional hydrogen atom problem has already been well-known. The energy spectrum and the wave functions can be obtained either by means of the Schroedinger wave mechanics (2) or by Feynman's path integration (3). Moreover, the energy spectrum can also be obtained with in the framework of the Heisenberg matrix mechanics (4)

The solution of the hydrogen atom problem in one and two dimensions have become of interest because of the problem of electrons which are spatially confined to move in one or two dimensions (5-7). With the development of molecular beam epitaxy techniques (MBE - A technique of growing single crystals in which beams of atoms or molecules are made to strike a single crystalline substrate in a vacuum, giving rise to crystals whose crystallographic orientation is related to that of substrates), there has been a growing interest in quantum-well structures in which the electrons are confined to move in two dimensions. This confinement of the electrons implied that if one is interested in the binding energies of hydrogenic impurities or excitons in these structures, one must solve the hydrogen atom problem for a quasi- two-dimensional system.

The Schroedinger equation for the two dimensional hydrogen atom, which is usually defined to be a system in which the motion of the electron around the nucleus under the influence of an attractive Coulomb potential is constrainted to be planar, has already been solved both in momentum space (8) and in position space (9-11). This problem has also been solved with in the framework of Feynman path integral (3). However, these treatments are rather incomplete since the spin-orbit interaction and the relativistic effects have not been included in the consideration .

Recently Asturias and Aragon (12) have presented discussion of the two dimensional hydrogen atom with the logarithmic potential

$$V(\rho) = Ze^2 \ln \left(\frac{\rho}{\rho_o}\right) , \qquad (1.2)$$

where ρ_o is a suitably defined scale constant. This potential is predicted by electrodynamics extended in a covariant fashion to two dimensions, i.e., it satisfies Gauss' theorem in the two dimensional space ;

$$\nabla^2 V(\vec{\rho}) = - 4\pi D(\vec{\rho}) \ , \ D(\vec{\rho}) = \text{charge density.}$$

But, to date, there is no known exact solution to the Schroedinger equation of the two dimensional hydrogen atom with the logarithmic potential.

The interest in the solution of the hydrogen atom problem in one dimension arises because of the problem of excitons in semiconductors in the presence of strong magnetic fields (5). More recently there has been a growing interest in the problem of the hydrogen atom in the presence of the intense magnetic fields which can occur under astrophysical conditions, such as, for example, in the vacinity of a pulsar (13). Under these conditions the motion of the electron-hole pair in the exciton or the electron in the hydrogen atom is spatially confined by the strong magnetic fields.

Loudon (5) solved the Schroedinger equation for the one dimensional hydrogen atom because of his interest in the behavior of excitons in semiconductors in the presence of strong magnetic fields. He assumed that the electron-hole pair in the exciton interacted via an attractive Coulomb potential which fell off inversely as the distance between the electron-hole pair ;

$$V(x) = - \frac{Ze^2}{|x|} \quad . \qquad (1.3)$$

This problem has also been studies by Andrews (14), Haines and Roberts (15), and by Gomes and Zimmerman (16). Apart from an attractive Coulomb potential, an attractive delta function potential has also been used to model the one dimensional hydrogen atom (17);

$$V(x) = - Ze^2 \, \delta(x) \ . \qquad (1.4)$$

The solution of the Schroedinger equation for the general N dimensional hydrogen atom with an attractive Coulomb potential in N-space dimensions,

$$V(r) = - eA_o = - \frac{Ze^2}{r}, \qquad (1.5)$$

where

$$r^2 = \sum_{i=0}^{N} x_i^2 , \qquad (1.6)$$

was given some years ago by Alliluev and more recently by Nieto (18).

For systems in which the motion of the particles have the speed approaching that of light, relativistic effects should be important. Thus, in studying such systems, it is necessary to use relativistic quantum mechanics. The solutions of the Dirac equation for the relativistic three dimensional hydrogen atom are well-known (19-22). They was first given by Darwin and by Gordon in 1928. The problem of the relativistic one dimensional hydrogen atom has also been studied by Lapidus (23), Capri and Ferrari (24), and by Spector and Lee (25). Lapidus solved the Dirac equation for the relativistic one dimensional hydrogen atom but in his treatment he took the interaction potential to be a delta function (1.4) rather than an attractive Coulomb potential(1.3). Capri and Ferrari also solved the Dirac equation but with the potential proportional to the distance between the electron and the nucleus ;

$$V(x) = g\,|x| . \qquad (1.7)$$

These treatments are thus not consistent with the case in which the Coulomb potential is three dimensional but the electron is confined to move in only one dimension. Spector and Lee considered the problem of the relativistic one dimensional hydrogen atom with an attractive Coulomb potential (1.3). But, since their main interest was about the change in the ground state binding energy due to the relativistic effects, they solved the Klein-Gordon equation rather that the Dirac equation.

In this thesis we will first give, in the next chapter, a brief review about the theories of the three dimensional hydrogen atom. Next, in Chapter III, the Schroedinger nonrelativistic wave mechanical treatment of the two dimensional hydrogen atom will be discussed with the extension to include the spin-orbit interaction and also the effect of the relativistic variation of mass with velocity (Sections 3.2 and 3.3, respectively). As the purpose of this thesis is to solve the Dirac equation for the two dimensional hydrogen atom with an attractive Coulomb potential to see whether the energy eigenvalues and the eigenfunctions are effected by taking relativistic effects into account, the Dirac relativistic wave mechanical treatment of the two dimensional hydrogen atom will be worked out in Chapter IV. Finally, the conclusion and discussion will be given in Chapter V.