CHAPTER V

DISCUSSIONS AND CONCLUSIONS

The most outstanding success of quantum mechanics has been in the understanding of the details of some simple atoms, especially, the hydrogen atom which is the simplest. In this thesis we have taken our quantum mechanics to the point of this important achievement, i.e., to an understanding of the spectrum of the hydrogen atom. The solution of the hydrogen atom problem is well-known. The energy spectrum and the wave functions can be obtained easily from Schroedinger's wave mechanics. The hydrogen atom problem has also been discussed in terms of Feynman's path integral. The main idea of Feynman approach is to convert the path integral for the Green's function of the hydrogen atom into that of a harmonic oscillator by transforming the time parameter as well as coordinate variable (10). Such a conversion can be achieved by using the mapping called the Kustaanheimo-Stiefel transformation which converts the hydrogen atom problem into an istropic oscillator problem in R^4 subjected to a constraint for the dimensional reduction. The discrete energy spectrum is deducible from the poles of the gamma function in the exactly transformed Green's function.

Let us now recall the treatment given in this thesis. We have discussed the method used by Pauli to obtain the hydrogen spectrum within the frameworks of matrix mechanics. Though, the matrix method is theoretically simple, but, in the pratice, it involves very complicated mathematics. In the matrix mechanics, Hamiltonian of classical mechanics reduce to relations between infinite matrices and are extremely cumbersome to solve. Even the problem of harmonic oscillator, which in classical mechanics is one of the simplest, becomes complicated when treated by the matrix method. The difficulties are further aggravated when we deal with systems having several degrees of freedom. The success of the Pauli's treatment, which gives the hydrogen spectrum, is an elegant illustration of how matrix mechanics can be used. The solution of the hydrogen atom problem itself is not particularly important since it can be obtained easily from Schroedinger's equation or from Feynman's path integration. The main point is to see how matrix mechanics can be applied to solve the hydrogen atom problem.

In his treatment, Pauli had used the conserved vector in the Kepler problem, i.e., Lenz vector and constructed matrix equations which comprise the constant quantities in the problem. Nevertheless, this treatment is rather incomplete since, in the "real" hydrogen atom problem, a spin-orbit interaction has to be included in the consideration. In the presence of the spin-orbit interaction, $\vec{L}$ is no longer a constant of the motion. When an spin angular momentum $\vec{S}$ of the electron is added to the orbital angular momentum $\vec{L}$, the total angular momentum

$$\vec{J} = \vec{L} + \vec{S}$$

will be a constant of the motion. One should extend the present treatment to the real hydrogen atom problem by adding up the spin-orbit effect in order to obtain the realistic solutions of the hydrogen atom.