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CHAPTER II
THEORY

II.1 Radio wave refractivity of the atmosphere

The troposphere[6] is the region of the atmosphere extending from the surface of the earth up to the height of about 8 to 10 kilometers at the polar latitudes, 10 to 12 kilometers at moderate latitudes, and up to 16 to 18 kilometers at the equator. The most important part of the troposphere is called the tropopause[6] which is the narrow region of constant temperature. The average pressure measured at the earth surface is 1014 millibars. At an altitude of 5 kilometers, it is decreased to nearly 507 millibars, and at 11 kilometers above the earth's surface, it is 225 millibars. The water vapour contained in the troposphere comes from the evaporation of water from the surface of the oceans, seas, and other water reservoirs. The water vapour content also rapidly decreases with height. The important factors to be considered in this thesis are pressure, temperature, relative humidity expressed in percent, and the saturated water vapour pressure (e_s)

From the refractive index point of view, the troposphere may be treated as a mixture of dry air and water vapour. M.Dolukhanov[6] stated in his book that the refractive index of gas, is

$$n = 1 + \rho\left(A + \frac{B}{T}\right) \qquad \text{------------- (2-1)}$$

Where

n = the refractive index

ρ = the gas density in kilogram per cubic metre.

A = the constant (expressed in cubic meters per kilogram) depending on the polarization of the gas molecules in an external electric field.

B = the constant (expressed in cubic meters x degree per kilopgram) decided by the content of permanent molecular dipole moments in the gas.

T = the absolute temperature in degree Kelvin

Since, by Clapegron's law, the gas density of a gas is directly proportional to its partial pressure and inversely proportional to its temperature in absolute degree Kenvin, Eq. (2-1) becomes

$$n = CP_p \left(A + \frac{B}{T}\right) \qquad \text{------------------} \quad (2\text{-}2)$$

C = the proportionally factor, expressed in kilograms degrees per cubic meters-millibars.

P_p = partial pressure (mb)

As the value of refractive index, n, is very nearly to unity the difference amoung various values are difficult to observed. Internationally, the value of refractive index is transfered to unit of refractivity N by the equation.

$$N = (n-1) \times 10^6$$

The gas making up dry air do not possesses any permanent dipole moment. In contrast, the molecules of water vapour have a permanent dipole moment of 6.13×10^{-30} coulomb-meter.

In view of the foregoing, and from the fact that the total refractivity (N) is the sum of the refractivities of the parts, the excess refractivities for moist air can be written as Eq. (2-3)

$$N = \frac{CA_d P_d}{T} + \frac{Ce}{T} \left(A_w + \frac{B_w}{T}\right) \qquad \text{-------} \quad (2\text{-}3)$$

Where

A_d = the respective constant of dry air

P_d = atmospheric pressure of dry air in millibars.

e = water vapour pressure in millibars.

A_w and B_w = the constants for water vapour.

In March 1962, B.R. Bean[7] reported that the product CA_d for dry air is 77.6 degrees per millibar. This seems to be true also for the product CA_w for water vapour. The ratio B_w/A_w has been mesured fairly accurately and is equal to 4810. Substituting all the values CA_d, CA_w, and B_w/A_w in the equation (2-3) gives.

$$N = \frac{77.6}{T} \left(P + \frac{4810e}{T}\right) \qquad \text{------------ (2-4)}$$

where the atmospheric pressure, P , in used for $P_d + e$.

The radio wave refractivity of moist air, as expressed by equation (2-4) is valid for the frequency below 50 GHz[(1) - (5)] This equation is used in this thesis to calculate the radio wave refractivity.

II.2 Earth effective radius coefficient.

The radio wave that propagates through the earth's atmosphere encounters variations in the atmosphere, radio wave refractive index, n , always has a value slightly greater than unity near the earth's surface and approaches unity with increasing height.

Dr. Henry R Reed and Carl M Russell[(8)] introduced the Snell's law for spherical, notplane atmosphere of the earth, so the boundaries of constant refractive index are spheres concentric with earth's center. Snell's law for the curved boundaries may be derived with the aid of Fig.1

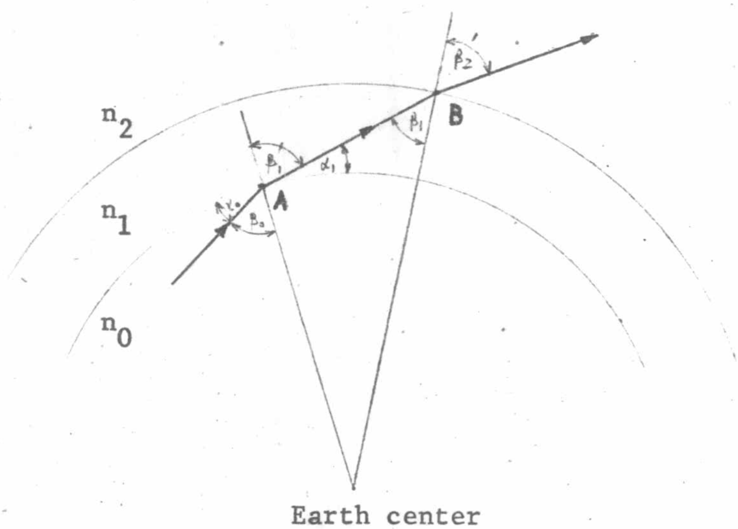

Fig. 1 The derivation of Snell's law for the curved earth.

From the Snell's law.

$$n_0 \sin\beta_0 = n_1 \sin\beta'_1 \quad \text{----------- (2-5)}$$

$$n_1 \sin\beta_1 = n_2 \sin\beta_2 \quad \text{-----------(2-6)}$$

Multiplication of Eq. (2-5) by r_0 and Eq. (2-6) by r_1 gives

$$n_0 r_0 \sin\beta_0 = n_1 r_0 \sin\beta'_1 \quad \text{--------- (2-7)}$$

$$n_1 r_1 \sin\beta_1 = n_2 r_1 \sin\beta'_2 \quad \text{--------- (2-8)}$$

Let us consider the triangle OAB by using trigonometric relation[9]

$$\frac{\sin(180^\circ-\beta_1)}{r_1} = \frac{\sin\beta'_1}{r_1} = \frac{\sin\beta_1}{r_0} \quad \text{---- (2-9)}$$

Substituting (2-9) in (2-7) yields

$$n_0 r_0 \sin\beta_0 = n_1 r_1 \sin\beta_1 \quad \text{---- (2-10)}$$

Then

$$n_0 r_0 \sin\beta_0 = n_1 r_1 \sin\beta_1 = \text{constant}$$

$$= n_2 r_2 \sin\beta_2 \quad \text{----- (2-11)}$$

Practically it is more convenient to measure an angle with the horizontal plane

Let α_0 be the angle between the ray and the plane normal to the radius vector at the boundary.

Then, Eq. (2-11) becomes

$$n_0 r_0 \cos\alpha_0 = n_1 r_1 \cos\alpha_1 \quad \text{-------- (2-12)}$$

To determine the amount of bending, refer to Fig.2, and Fig.3 In Fig.2 BC represents a wave front propagating along a direction of CE.

After a time interval, the two points B and C have moved to D and E respectively.

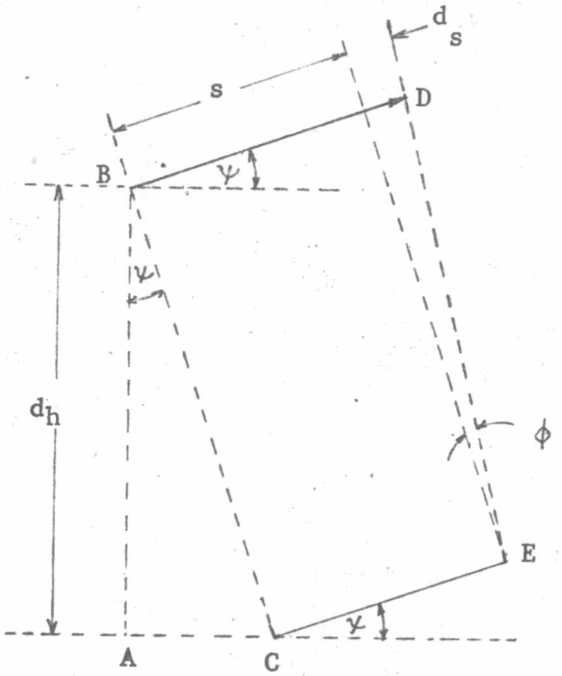

Fig. 2 Geometry for determination of ray bending.

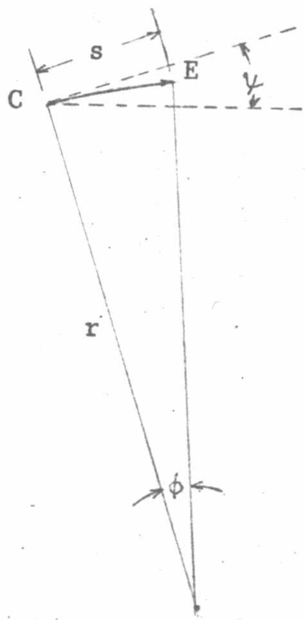

Fig.3 Earth geometry for curvature CE.

Omitting the second order term, dvdn, from Eq. (2-13), selving for the value of v, we obtain

$$v = -\frac{ndv}{dn} \qquad \text{-------- (2-14)}$$

Let s be the distance, we have

$$s = vdt \qquad \text{-------- (2-15)}$$

then

$$s + ds = (v + dv)\, dt \qquad \text{-------- (2-16)}$$

Subtracting Eq. (2-15) from (2-16) gives

$$ds = dvdt \qquad \text{------------ (2-17)}$$

Obviously from Fig.2 and Fig.3

$$\frac{ds \cos \gamma}{dh} = \tan \emptyset \qquad \text{------------ (2-18)}$$

$$= \emptyset \qquad \text{------------ (2-19)}$$

In which we have used $\tan \emptyset \approx \emptyset$ for a small value of $\emptyset$

From Fig.3

$$\frac{s}{r} = \emptyset \qquad \text{------------ (2-20)}$$

then

$$\frac{ds \cos \gamma}{dh} = \frac{s}{r} \qquad \text{------------ (2-21)}$$

Subxtituting the values of ds, s and v from Eqs. (2-14) (2-15) and (2-17) respectively in Eq. (2-21) yields.

$$\frac{dvdt \cos \gamma}{dh} = -\frac{n\, dvdt}{rdn}$$

and

$$r = \frac{n}{\frac{dn}{dh} \cos \gamma} \qquad \text{--------- (2-22)}$$

To related the earth effective radius coefficient, (K), with the radius of the earth, Fig.4 will be helpful

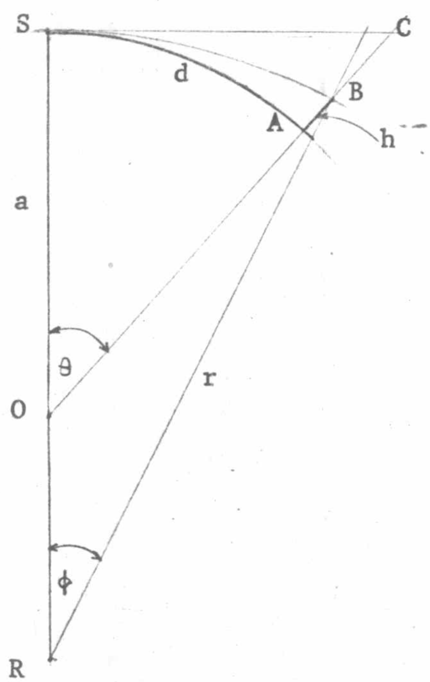

Fig.4 Geometry for curved earth representation for modified earth radius

In this figure, let

O	=	the true center of the earth,
d	=	SA = distance along the surface of the earth,
SC	=	tangential line to the earth at point S,
D	=	SB = path of ray starting horizontally from S,
a	=	OS = true radius of the earth
h	=	radial height AB.

When h is small in comparison to a and d, SB is nearly equal to SA, SB may be considered as an are of the circle whose center is at R and has a radius r. Then for small value of $\emptyset$

$$\phi = \frac{SB}{r}$$

$$= \frac{SA}{r}$$

$$= \frac{a\theta}{r} \qquad \text{-------- (2-23)}$$

Obviously,

$$AC = OC - OA$$

$$= a\,(\sec\theta - 1)$$

$$= \frac{d^2}{2a} \qquad \text{---------(2-24)}$$

where $\sec\theta \approx 1 + \theta^2/2$, when θ is small and $\theta \approx \frac{d}{a}$ are substituted.

Since both θ and ϕ are small angles, BC may be consider as a prolongation of RB then

$$BC = r\,(\sec\phi - 1)$$

$$= \frac{d^2}{2r} \qquad \text{---------(2-25)}$$

The radial height is the difference between AC and BC,

$$h = \frac{d^2}{2a} - \frac{d^2}{2r}$$

$$= \frac{d^2}{2a}\left(1 - \frac{a}{r}\right) \qquad \text{------(2.26)}$$

From Eq. (2-22) for small value of ψ, $\cos\psi \approx 1$, then

$$\frac{dn}{dh} = -\frac{n}{r} \qquad \text{------(2-27)}$$

Substituting Eq. (2-27) in Eq. (2-26) gives

$$h = \frac{d^2}{2a}\left(1 + \frac{a}{n}\frac{dn}{dh}\right)$$

and

$$\frac{d^2}{2ah} = \frac{1}{1 + \frac{a}{n}\frac{dn}{dh}} \qquad \text{--------- (2-28)}$$

The propagation path is refracted in the atmosphere can be considered straight by enlarging the global radius from a to a_e which $a_e = (a+h) = Ka$

Let us consider the are SB, and SA as straight lines. For SB and SA are very short compare with a and Ka, then from trigonometric relation[9] for the right triangle OBS,

$$D^2 = (Ka)^2 - (Ka - h)^2$$

$$= (Ka)^2 - (Ka)^2 + 2(Ka)h + h^2$$

$$= 2(Ka)h + h^2 \qquad \text{---------(2-29)}$$

and from the right triangle SAB,

$$d^2 = D^2 - h^2 \qquad \text{----------(2-30)}$$

Substituting the value of D^2 in Eq. (2-29) in Eq. (2-30) gives

$$d^2 = 2(Ka)h$$

thus,

$$K = \frac{d^2}{2ah} \qquad \text{---------(2-31)}$$

From Eq. (2-28) and (2-31) we get

$$K = \frac{1}{1 + \frac{a}{n}\frac{dh}{dh}} \qquad \text{---------(2-33)}$$

In this thesis Eq.(2-33) is used to calculate the value of K.

As well known to radio engineers, K is not always constant. It depends on atmospheric conditions. In general a question exists how much the variation of K affects the site selection[10] of radio stations. This point must be made clear first before finding a profile[11-15] of radio path by enlarging the global radius with the value of h in the profile which called effective height of the surface, h_k, as shown in Fig.5

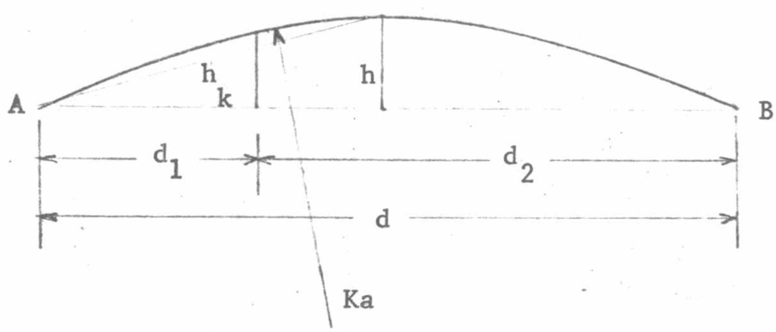

Fig.5 Value of the effective height of thesurface in path profile, h_k

From Eq. (2 - 31), finding the value of h, $h = \frac{d^2}{2Ka}$, by substituting the value of $d = \frac{d}{2}$ from Fig.5 at the middle of the path and solving the value of h_k by similar triangle.[9]

From Fig.5 the height h_k are given by Eq. (2-34) at the middle of the path, and Eq. (2-35) at any point along the path

$$h_k = \frac{d^2}{8\ Ka} \qquad \text{--------(2-34)}$$

$$= \frac{d_1 d_2}{2\ Ka} \qquad \text{-------(2-35)}$$

where a is the earth's radius (6370 km.)

II.3 Variation of the earth effective radius as a function of the surface refractivity, N_s

It has been observed that the variation of the mean values of the refractive index at the atmosphere (1) - (5) may often be well approximated by the following exponential formula (1) - (4)

$$n(h) = 1 + N_s \ \exp(-bh) \times 10^{-6} \qquad \text{------(2-36)}$$

where $N = (n-1) \times 10^6$ and the suffic "s" refers to the values at the surface of the earth, h is the height above the surface which expressed in kilometers and b is determined by the following relation (1) - (4)

$$\exp(-b) = 1 + \frac{\Delta N}{N_s} \qquad \text{------(2-37)}$$

Where

ΔN = the difference in N values at the height of 1 km abobe the surface of the earth.

The formula in Eq. (2-37) may be used for estimating N. In usual cases only the surface meteorological data are available For example if N_s = 289 and a = 6370 Km, K = 4/3 Eq. (2-37) will give b = 0.136, this is the basic reference atmosphere. In temperate climates, the average values of N_s vary from about 310 to 320 and ΔN about - 38 to - 42. One may thus define an average atmosphere as one in which N_s takes the value of 315 and ΔN of -40 so that

$$n(h) = 1 + 315 \times 10^{-10} \exp(-0.136\, h) \quad \text{-------(2-38)}$$

In this atmpsphere, the gradient of the refractive index at the surface of the earth corresponds to a value of K=4/3 If extensive radiosonde data were available, so that a good determination can be made of the average different ΔN between the values at the surface of the earth and height of one kilometre above, these actual average measured values of ΔN and N_s may be used in Eqs. (2-36) and (2-37) for determining the characteristics of the atmosphere.