Chapter II

Theory of starting torque calculation

2.1 <u>Introduction</u> The calculation of the starting torque of a single-phase induction motor can be done by two methods first, from the equivalent circuit of the motor, and second from the physical parameters of the motor. In this thesis , both methods of finding the starting torque of an induction motor will be derived and calculated.

2.2 <u>Calculation of the starting torque from the equivalent circuit.</u> In order to find the starting torque from the equivalent circuit of a given motor, its equivalent circuit must be derived first.

2.2.1 <u>The Equivalent circuit</u> It has been known that the main flux of the poly-phase induction motor is a rotating flux. On the other hand, the main flux of the single-phase motor is an alternating flux, fixed in space. It is posible to set up the fundamental equation of the single-phase motor and derive its operational characteristics on the basis of the alternating flux. However, it is also posible to set up the fundamental equation and derive the characteristic of the single-phase motor on the basis of the operational characteristic of the poly-phase motor. This latter posibility is based on the fact that an alternating m.m.f.(or flux) can be replaced by two rotating m.m.f.'s (or flux)

It has been found that the alternating m.m.f. is given by the equation.

$$\text{m.m.f.} = 1.8 \frac{N}{p} K_{dp} I \sin \omega t \cos \frac{\pi}{\tau} x \quad * \qquad (1)$$

When

N = Number of turn

P = Number of pole

K_{dp} = winding factor

τ = pole pitch

applying eq.(1) by the relation

$$\sin \alpha \cos \beta = \frac{1}{2} \text{ Sin } (\alpha - \beta) + \frac{1}{2} \sin (\alpha + \beta)$$

there result

$$\text{m.m.f.} = 0.9 \frac{N}{p} K_{dp} I \sin (\omega t - \frac{\pi}{\tau} x) + 0.9 \frac{N}{p} K_{dp} I \sin (\omega t + \frac{\pi}{\tau} x)$$

i.e., the alternating m.m.f. can be replaced by two rotating m.m.f.'s traveling in opposite direction and each having an amplitude equal to half of that of the alternating m.m.f. Use will be made of this property for the analysis of the single phase motor. To these two rotating m.m.f's there correspond two fluxes rotating in opposite directions, each with synchronous speed.

The rotating flux which travel in the same direction of rotation as the rotor is called the forward rotating flux while the rotating flux which travel in the opposite direction to the rotor is called backward rotating flux.

Contrary to the polyphase induction motor, while the rotor e.m.f. is induced by only one rotating flux, it is induced

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*Liwschitz, Garik and Whipple, A - C Machines, 2^{hd} edition, D.Van Nostrand co.

here by two rotating fluxes, and the influence of each flux on the rotor is to be considered separately. The effect of each of the two rotating flux on the rotor of the single phase motor is the same as that of the single rotating flux on the rotor of the poly phase motor.

Assume that the rotor has a speed n r.p.m. Then, according to the definition of the slip S, the slip of the rotor with respect to the forward rotating flux is

$$S_f = \frac{N_s - n}{N_s} = 1 - \frac{n}{N_s} = S$$

Where N_s = Synchronous speed r.p.m.

Since the backward rotating flux rotating popposite to the rotor, the slip of the rotor with respect to this backward rotating flux is

$$S_b = \frac{N_s - (- n)}{N_s} = 1 + \frac{n}{N_s} = 2 - S$$

General Equations

It has been known already that in the induction motor, contrary to other electric machines, only one machine part, the stator, is connected to a source of power. The rotor of the induction motor is not connected to any power line, but receives its e.m.f. and current by means of induction. This is the same principle as the transformer and the transformer principle is applicable. The vector diagram of the induction motor is shown in fig 2.1

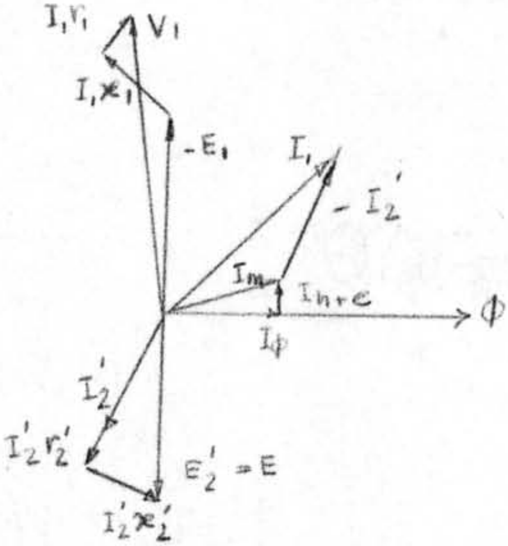

Fig 2.1 Voltage and current diagram of the induction motor.

Kirchhoff's mesh equation for stator at standstill is

$$\dot{V}_1 + \dot{E}_1 - j\,\dot{I}_1\,x_1 = \dot{I}_1\,r_1,$$

and that for the rotor at standstill is

$$E_2' - j\,\dot{I}_2'\,x_2' = \dot{I}_2'\,r_2'.$$

The induced e.m.f. in the rotor by the rotating flux while the rotor is running at slip S can be expressed as

$$\dot{E}_{2s}' = S\,\dot{E}_2'.$$

Hence the Kirchhoff's mesh equation of rotor circuit while rotor is running at slip S becomes,

$$S\,\dot{E}_2' - j\,\dot{I}_2'\,Sx_2' = \dot{I}_2'\,r_2'$$

and $$\dot{E}_2' - j\,\dot{I}_2'\,x_2' = \dot{I}_2'\,\frac{r_2'}{S}. \qquad (3)$$

When the rotor is running at a speed n, the speed of the m.m.f. wave produced by the rotor is $N_s - n$. Since the rotor speed is n, the speed of the stator m.m.f. wave relative to stator is $N_s - n + n$, which equals to N_s. This is the same as the speed of the stator m.m.f. wave relative to the stator, i.e., rotor and stator m.m.f. waves are stationary with respect to one another at any rotor speed. So that the basic equation for transformer is applicable to the induction motor during running, i.e.,

$$\dot{I}_1 + \dot{I}_2' = \dot{I}_m = -\dot{E}_1\,\dot{Y}_m = -\dot{E}_2'\,\dot{Y}_m$$

$$-\dot{E}_1 = -\dot{E}_2' = (\dot{I}_1 + \dot{I}_2)'\,\dot{Z}_m = \dot{I}_m\,\dot{Z}_m \qquad (4)$$

Where $\dot{I}_1$ = primary current

$\dot{I}_2'$ = secondary current referred to the primary current.

$\dot{I}_m$ = magnetizing current.

$\dot{E}_1$ = induce voltage in primary by main flux

$\dot{E}_2'$ = induce voltage in secondary by main flux referred to primary.

$\dot{Y}_m$ = main flux admittance

$\dot{Z}_m$ = main flux impedance

The two rotating flux induce two e.m.f.'s and two current (I_{2f}' and I_{2b}') of the frequencies of Sf_1 and $(2-s)\,f_1$ respectively in the rotor circuit. Since the frequencies are different, the two e.m.f. and their-current must be considered separately, i.e., two Kirchhoff's mesh equations must be set up for the rotor,

From Eq. (3) $$\dot{E}_{2f}' - j\,\dot{I}_{2f}'\,X_2' = \dot{I}_{2f}'\,\frac{r_2'}{S} \qquad (5)$$

and $$\dot{E}_{2b}' - j\,\dot{I}_{2b}'\,X_2' = \dot{I}_{2b}'\,\frac{r_2'}{2-S} \qquad (6)$$

The e.m.f.'s $\dot{E}_{2f}'$ and $\dot{E}_{2b}'$ become,

from Eq.(4) $$\dot{E}_{2f}' = -(\dot{I}_1 + \dot{I}_{2f}')\,\dot{Z}_m \qquad (7)$$

and $$\dot{E}_{2b}' = -(\dot{I}_1 + \dot{I}_{2b}')\,\dot{Z}_m, \qquad (8)$$

where $(I_1 + I_{2f}')$ is the magnetizing current of the forward rotating flux, and $(I_2 + I_{2b}')$ is the magnetizing current of the backward rotating flux.

All quantities mentioned before are refered to the stator, therefore, the e.m.f.'s induced in the stator winding by the two rotating fluxes are also equal to E_{2f}' and E_{2b}', and the Kirchhoff's mesh equations for the stator are

$$\dot{V}_1 = -\dot{E}_1 + \dot{I}_1 r_1 + \dot{I}_1 x_1$$

and $$\dot{E}_1 = \dot{E}_{2f}' + \dot{E}_{2b}'\,.$$

Therefore the equivalent circuit of a single-phase induction motor is based on the equation.

$$\begin{aligned}\dot{V}_1 &= \dot{I}_1(r_1 + j\,x_1) - \dot{E}_{2f}' - \dot{E}_{2b}' \\ &= \dot{I}_1(r_1 + J\,x_1) + (\dot{I}_1 + \dot{I}_{2f}')\dot{Z}_m + (\dot{I}_1 + \dot{I}_{2b})\dot{Z}_m \qquad (9)\end{aligned}$$

Deriviation of Equivalent circuit

Eq. (9) can be derived to give the equivalent circuit of the single phase induction motor as follows:-

From Eq. (5) $$\dot{E}_{2f}' = \dot{I}_{2f}' \frac{r_2'}{S} + j \dot{I}_{2f}' X_2' \quad (10)$$

From Eq. (3) $$\dot{E}_{2f}' = - \dot{I}_1 \dot{Z}_m - \dot{I}_{2f}' \dot{Z}_m \quad (11)$$

Subtracting Eq.(11) from Eq.(10)

$$\dot{I}_{2f}' \left(\frac{r_2'}{S} + j X_2' + \dot{Z}_m\right) = - \dot{I}_1 \dot{Z}_m. \quad (12)$$

Then $$\dot{I}_{2f}' = \frac{-\dot{I}_1 \dot{Z}_m}{\frac{r_2'}{S} + j x_2' + \dot{Z}_m} \quad (13)$$

From Eq. (6) $$\dot{E}_{2b}' = \dot{I}_{2b}' \frac{r_2'}{2-S} + j \dot{I}_{2b}' X_2', \quad (14)$$

and from Eq.(8) $$\dot{E}_{2b}' = - \dot{I}_1 \dot{Z}_m - \dot{I}_{2b}' \dot{Z}_m \quad (15)$$

Subtracting Eq.(15) from Eq.(14);

$$\dot{I}_{2b}' \left(\frac{r_2}{2-S} + j X_2' + \dot{Z}_m\right) = \dot{I}_1 \dot{Z}_m \quad (16)$$

Then $$\dot{I}_{2b}' = \frac{- \dot{I}_1 \dot{Z}_m}{\frac{r_2'}{2-S} + jx_2' + \dot{Z}_m} \quad (17)$$

Put Eqs. (13) & (17) in Eq. (9),

$$\dot{V}_1 = \dot{I}_1(r_1+jx_1) + \left(\dot{I}_1 - \frac{\dot{I}_1 \dot{Z}_m}{\frac{r_2'}{S} + jx_2' + \dot{Z}_m}\right)\dot{Z}_m + \left(\dot{I}_1 - \frac{\dot{I}_1 \dot{Z}_m}{\frac{r_2'}{2-S} + jX_2' + \dot{Z}_m}\right)\dot{Z}_m$$

$$= \dot{I}_1 \left[(r_1+jx_1)+\dot{Z}_m \left(1-\frac{\dot{Z}_m}{r_2'+jx_2'+\dot{Z}_m}\right) +\dot{Z}_m \left(1-\frac{\dot{Z}_m}{\frac{r_2'}{2-S}+jx_2'+\dot{Z}_m}\right)\right]. \quad (18)$$

Let $\dot{Z}_1 = r_1+jx_1$, $\dot{Z}_{2f}' = \frac{r_2'}{S}+jx_2'$,

$\dot{Z}_{2b}' = \frac{r_2'}{2-S}+jx_2'$, and $\dot{Z}_m = r_m+jx_m$, Eq.(18) then becomes

$$\dot{V}_1 = \dot{I}_1 \left[\dot{Z}_1+\dot{Z}_m\left(1-\frac{\dot{Z}_m}{\dot{Z}_{2f}'+\dot{Z}_m}\right)+\dot{Z}_m\left(1-\frac{\dot{Z}_m}{\dot{Z}_{2b}'+\dot{Z}_m}\right)\right]$$

$$= \dot{I}_1\left[\dot{Z}_1+\frac{\dot{Z}_m\,\dot{Z}_{2f}'}{\dot{Z}_m+\dot{Z}_{2f}'}+\frac{\dot{Z}_m\,\dot{Z}_{2b}'}{\dot{Z}_m+\dot{Z}_{2b}'}\right] \quad (19)$$

The second term in the bracket is the impedance of Z_m and Z_{2f}' connected in parallel; the third term in the bracket is the impedance of Z_m and Z_{2b}' connected in parallel. The equivalent circuit of the single-phase induction motor can, therefore, be constructed as shown in Fig. 2.2

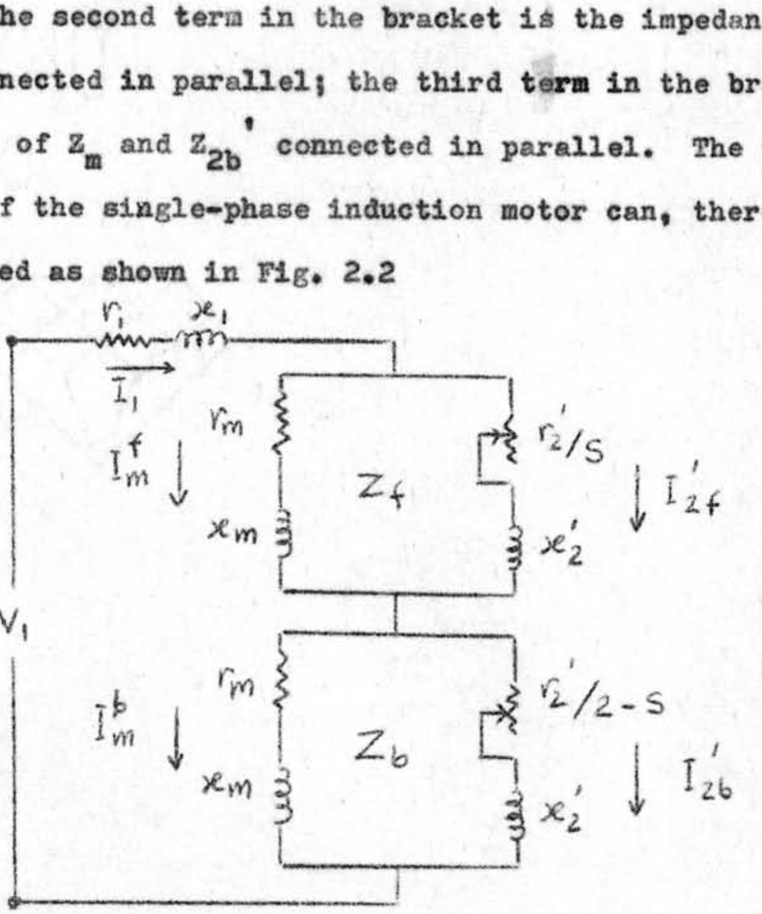

Fig. 2.2 Equivalent circuit of the single-phase induction motor.

2.2.2 Derivation of starting torque from Equivalent circuit Parameters

During the starting period, the capacitor-start motor can be condidered as an unsymmetrical two-phase motor i.e., the two sets of stator winding are displaced in space from each other by an angle of α electrical degrees and the impressed voltage is applied to these two windings instantaneously. Assuming that the machine is not saturated, the superposition of the two fluxes becomes permissible.

First consider the main winding only. Fig 2.3 is the equivalent circuit of the main winding and can be simplified as follow.

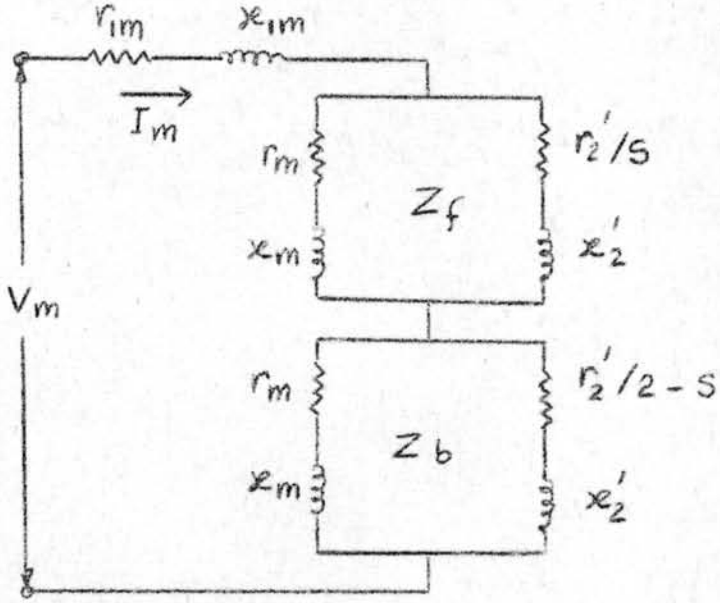

Fig 2.3 Equivalent circuit of the main winding.

From Fig 2.3

$$\dot{Z}_f = R_f + j X_f$$

For which

$$R_f = \frac{A\,(r_m + r'_{2/S}) + B\,K_2\,Xm}{(r_m + r'_{2/S})^2 + (K_2 X_m)^2}$$

and

$$X_f = \frac{B\,(r_m + r'_{2/S}) - AK_2\,X_m}{(r_m + r'_{2/S})^2 + (K_2 X_m)^2}$$

Where

$$K_2 = 1 + (X'_{2/X_m}) = 1 + \frac{T}{2}$$

$$A = r_m\,(r'_{2/S}) - X_2'\,X_m$$

$$B = (r'_{2/S})\,X_m + X_2'\,r_m$$

$$\dot{Z}_b = R_b + J\,X_b$$

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$$R_b = \frac{A\,(r_m + r'_{2/2-S}) + BK_2\,Xm}{(r_m + r'_{2/2-S})^2 + (K_2\,X_m)^2}$$

$$X_b = \frac{B\,(r_m + r'_{2/2-S}) - A\,K_2 X_m}{(r_m + r'_{2/2-S})^2 + (K_2\,X_m)^2}$$

Fig. 2.3 can then be simplipid to Fig. 2.4

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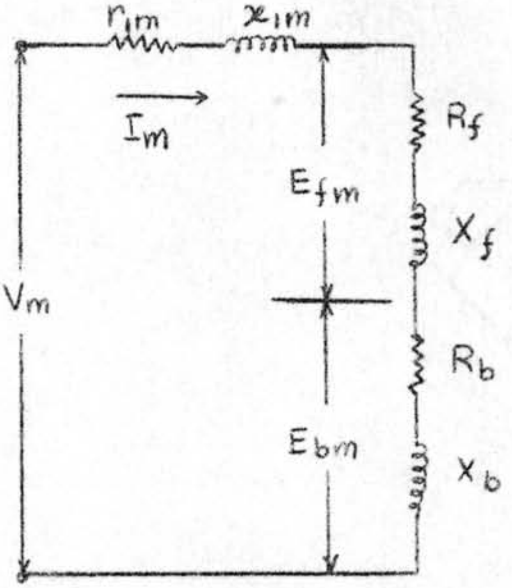


Fig. 2.4 Equivalent circuit of the main winding.

The quantities E_{fm} and E_{bm} shown in Fig. 2.4 are the components of V_m necessary to overcome the impedances of Z_f and Z_b the corresponding induced e.m.f.'s are

$$-\dot{E}_{fm} = -\dot{I}_m (R_f + jX_f) \qquad (20)$$

$$\text{and} \quad -\dot{E}_{bm} = -\dot{I}_m (R_b + jX_b) \qquad (21)$$

Each of these two e.m.f.'s consists of two components, one in phase with I_m, the other 90° ahead of I_m. Therefore, with

$$I_m = \sqrt{2}\, I_m \sin \omega t$$

the instantaneous value of $-E_{fm}$ and $-E_{bm}$ are

$$- e_{fm} = -\sqrt{2}\, I_m \left[R_f \sin \omega t + X_f \sin \left(\omega t + \frac{\pi}{2} \right) \right]$$

$$= -\sqrt{2}\, I_m \left(R_f \sin \omega t + X_f \cos \omega t \right)$$

$$- e_{bm} = -\sqrt{2}\, I_m \left(R_b \sin \omega t + X_b \cos \omega t \right)$$

Current flows in the starting winding only. It is assumed that this winding is placed α electrical degrees ahead of the main winding with respect to a forward rotating flux, i.e., with respect to the direction of the rotor.

Assuming that winding has $N_{s,eff}$ turns,

and the main winding has $N_{m,eff}$ turns,

Let $N_{s,eff} = a\, N_{m,eff}$.

A voltage V_s is impressed on the winding and it carries the current.

$$i_s = \sqrt{2}\, I_s \sin (\omega t + \varphi)$$

Where I_s is the r.m.s. starting current.

The equivalent of this winding is shown in Fig. 2.5

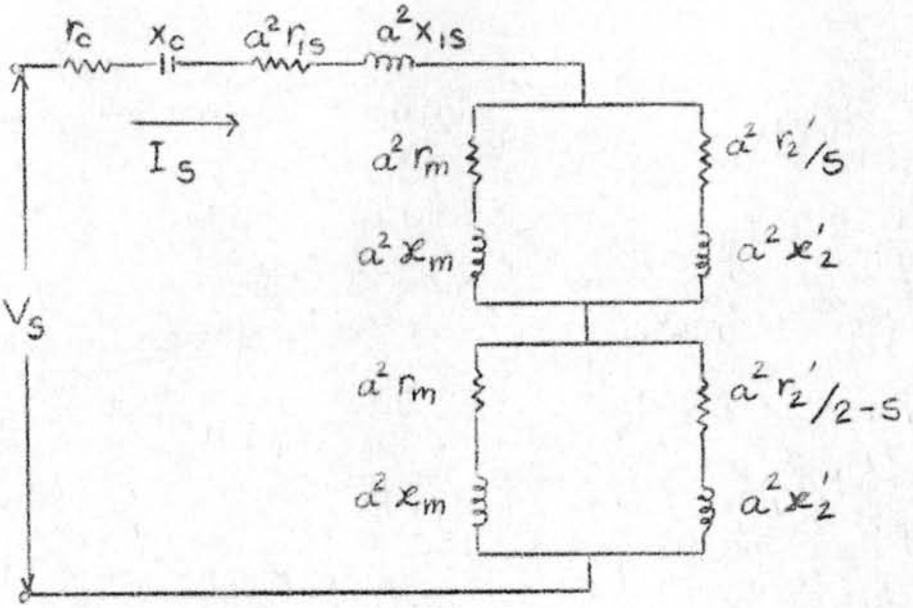


Fig 2.5 Equivalent circiut of the starting winding.

Where r_m, x_m, r_2' and x_2' all refer to main winding. Since the starting winding has a $N_{m,eff}$ turns, these quantities appear in its equivalent circuit multiplied by a^2.r_{1s} and x_{1s} are not the actual values of resistance and leakage reactance of the starting winding but the actual values times $1/a^2$, sothat $a^2 r_{1s}$ and $a^2 x_{1s}$ are the actual values.

The fig 2.5 can be simplified to Fig. 2.6 by the same procedure as for the main winding.

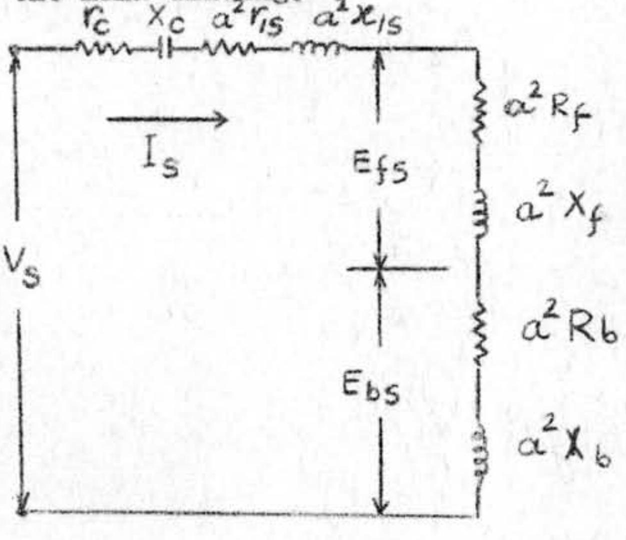


Fig 2.6 Equivalent circuit of the starting winding.

From Fig. 2.6

$$-\dot{E}_{fs} = -\dot{I}_s a^2 (R_f + jX_f) \tag{22}$$

$$-\dot{E}_{bs} = -\dot{I}_s a^2 (R_b + jX_b) \tag{23}$$

$$-e_{fs} = -\sqrt{2}\, I_s a^2 \left[R_f \sin(\omega t + \varphi) + X_f \cos(\omega t + \varphi)\right]$$

$$-e_{bs} = -\sqrt{2}\, I_s a^2 \left[R_b \sin(\omega t + \varphi) + X_b \cos(\omega t + \varphi)\right]$$

__Both windings carry current.__ Since each of the two windings produces a forward and backward rotating flux, the two fluxes of the main winding induce two e.m.f.'s in the starting winding and vice versa, the two fluxes of the starting winding will induce two e.m.f.'s in the main winding. The four fluxes and the e.m.f. induced by them are tabulated below:

flux	Emf's in the M-winding	Emf's in the S-winding
$\emptyset_{fm}$	$-\dot{E}_{fm} = -\dot{I}_m(R_f + jX_f)$	$-\dot{E}_{s,fm} = -a\, \dot{E}_{fm} \epsilon^{j\alpha}$
$\emptyset_{bm}$	$-\dot{E}_{bm} = -\dot{I}_m(R_b + jX_b)$	$-\dot{E}_{s,bm} = -a\, \dot{E}_{bm} \epsilon^{-j\alpha}$
$\emptyset_{fs}$	$-\dot{E}_{m,fs} = -(1/a)\dot{E}_{fm} \epsilon^{-j\alpha}$	$-\dot{E}_{fs} = -\dot{I}_s a^2(R_f + jX_f)$
$\emptyset_{bs}$	$-\dot{E}_{m,bs} = -(1/a)\dot{E}_{bs} \epsilon^{j\alpha}$	$-\dot{E}_{bs} = -\dot{I}_s a^2(R_b + jX_b)$

With respect to the e.m.f. induced by the forward flux of the main winding($\emptyset_{fm}$) in the starting winding, E_{sfm}, it should be remembered that the latter winding has a $N_{m,eff}$ turns and that it is placed α a head of the main winding in the direction of the forward rotating flux, thus

$$-\dot{E}_{s,fm} = -a\,\dot{E}_{fm}\epsilon^{j\alpha}$$

With respect to the backward rotating flux of the main winding, the starting winding is placed behind the main winding by the angle α. Therefore,

$$-\dot{E}_{s,bm} = -a\,E_{bm}\epsilon^{-j\alpha}$$

with the same consideration, the two emf's induced in the main winding by the two rotating fluxes of the starting winding are

$$-\dot{E}_{m,fs} = -\left(\frac{1}{a}\right)\dot{E}_{fm}\epsilon^{-j\alpha}$$

and

$$-\dot{E}_{m,bs} = -\left(\frac{1}{a}\right)\dot{E}_{bs}\epsilon^{j\alpha}$$

rewriting the above equation gives,

$$\begin{aligned} -\dot{E}_{s,fm} &= -a\,\dot{I}_m\,(R_f + jX_f)\epsilon^{j\alpha} \\ &= -a\,\dot{I}_m\Big[(R_f\cos\alpha - X_f\sin\alpha) + \\ &\qquad + j\,(X_f\cos\alpha + R_f\sin\alpha)\Big] \end{aligned} \tag{24}$$

$$\begin{aligned} -\dot{E}_{s,bm} &= -a\,\dot{I}_m\,(R_b + JX_b)\,\epsilon^{-j\alpha} \\ &= a\,\dot{I}_m\,(R_b\cos\alpha + X_b\sin\alpha) + \\ &\qquad + j\,(X_b\cos\alpha - R_b\sin\alpha)\Big] \end{aligned} \tag{25}$$

$$\begin{aligned} -\dot{E}_{m,fs} &= -a\,\dot{I}_s\,(R_f + jX_f)\epsilon^{-j\alpha} \\ &= -a\,\dot{I}_s\Big[(R_f\cos\alpha + X_f\sin\alpha) + \\ &\qquad + j\,(X_f\cos\alpha - R_f\sin\alpha)\Big] \end{aligned} \tag{26}$$

$$- \dot{E}_{m,bs} = - a\, \dot{I}_s\, (R_b+jX_b)\, \epsilon^{j\alpha}$$
$$= -a\, \dot{I}_s\left[(R_b\, \cos\alpha - X_b \sin\alpha\,)+j(X_b \cos\alpha + R_b \sin\alpha)\right] \qquad (27)$$

The instantaneous values of these emf's are

$$- e_{s,fm} = -\sqrt{2}\ a\ I_m\left[(R_f \cos\alpha - X_f \sin\alpha)\ \sin\omega t + (X_f \cos\alpha + R_f \sin\alpha)\ \cos\omega t\,\right]$$

$$- e_{s,bm} = -\sqrt{2}\ a\ I_m\left[(R_b \cos\alpha + X_b \sin\alpha)\ \sin\omega t + (X_b \cos\alpha - R_b\ \sin\alpha)\ \cos\omega t\,\right]$$

$$- e_{m,fs} = -\sqrt{2}\ a\ I_s\left[(R_f \cos\alpha + X_f \sin\alpha)\ \sin(\omega t + \varphi) + (X_f \cos\alpha - R_f \sin\alpha)\ \cos(\omega t + \varphi)\right]$$

$$- e_{m,bs} = -\sqrt{2}\ a\ I_s\left[(R_b \cos\alpha - X_b \sin\alpha) \sin(\omega t + \varphi) + (X_b \cos\alpha + R_b \sin\alpha\,)\ \cos(\,\omega t + \varphi\,)\right]$$

For the main winding,

$$\dot{V}_m - \dot{E}_{fm} - \dot{E}_{bm} - \dot{E}_{m,fs} - \dot{E}_{m,bs} - j\dot{I}_m\, x_{1m} \pm j\,\omega M_{ms}\, \dot{I}_s = \dot{I}_m r_{1m} \quad (28)$$

and for the starting winding

$$\dot{V}_s - \dot{E}_{fs} - \dot{E}_{bs} - \dot{E}_{s,fm} - \dot{E}_{s,bm} - j\, \dot{I}_s\, (X_c + a^2 x_{1s}) \pm j\,\omega M_{ms} \dot{I}_m = \dot{I}_s (r_c + a^2 r_{1s}) \quad (29$$

The terms $\pm\ j\omega M_{ms}\ I_s$ and $\pm\ j\omega M_{ms}\ I_m$ are taken into account for the mutual induction of both windings by their slot-leakage and harmonic fluxes. Inserting Eq (2) to (27) into the Eq. (28) and Eq. (29) gives

$$\dot{V}_m = \dot{I}_m\left[(r_{1m}+jx_{1m})+(R_f+jX_f)+(R_b+jX_b)\right] + \dot{I}_s\Big\{a\left[(R_f \cos\alpha + X_f \sin\alpha) + j\,(X_f \cos\alpha - R_f \sin\alpha\,)\right] + a\,\left[(R_b \cos\alpha - X_b \sin\alpha) + {} + j(X_b\ \cos\alpha\, + R_b\ \sin\alpha\,)\right]\ \pm\ j\omega M_{ms}\Big\}$$

$$\begin{aligned}\dot{V}_s &= \dot{I}_s \Big\{\Big[(r_c+a^2r_{1s})+j(X_c+a^2x_{1s})\Big] +a^2(R_f+jX_f)+a^2(R_b+jX_b)\Big\} \\ &\quad + \dot{I}_m\Big\{a\Big[(R_f\cos\alpha-X_f\sin\alpha)+j(X_f\cos\alpha+R_f\sin\alpha)\Big] \\ &\quad + a\Big[(R_b\cos\alpha+X_b\sin\alpha)+j\ (X_b\cos\alpha-R_b\sin\alpha)\Big]\mp j\omega M_{ms}\Big\}\end{aligned}$$

The emf's $\pm j\omega M_{ms}\ I_s$ and $\pm j\omega M_{ms}\ I_m$ can be best taken care of as part of the slot and differential leakage, i.e., as parts of $j\ I_m\ X_{1m}$ and $j\ I_s\ X_{1s}$

With the abbreviations

$$l_m = r_{1m}+R_f+R_b \qquad l_s = r_c+a^2(r_{1s}+R_f+R_b)$$

$$m_m = X_{1m}+X_f+X_b \qquad m_s = x_c+a^2(x_{1s}+X_f+X_b)$$

$$l_m+jm_m = \dot{Z}_m \qquad l_s+j\ m_s = \dot{Z}_s$$

$$a\Big[(R_f+R_b)\ \cos\alpha+(X_f-X_b)\sin\alpha\Big] = q_1$$

$$a\Big[(X_f+X_b)\ \cos\alpha-(R_f-R_b)\sin\alpha\Big] = q_2$$

$$a\Big[(R_f+R_b)\ \ \cos\alpha-(X_f-X_b)\sin\alpha\Big] = t_1$$

$$a\Big[(X_f+X_b)\ \cos\alpha+(R_f-R_b)\sin\alpha\Big] = t_2$$

$$\dot{Z}_q = q_1+j\ q_2$$

$$\dot{Z}_t = t_1+j\ t_2$$

$$\dot{V}_m = \dot{I}_m\ \dot{Z}_m+\dot{I}_s\ \dot{Z}_q$$

$$\dot{V}_s = \dot{I}_s\ \dot{Z}_s+\dot{I}_m\ \dot{Z}_t$$

$$\dot{I}_m = \frac{\dot{V}_m \dot{Z}_s - \dot{V}_s \dot{Z}_q}{\dot{Z}_m \dot{Z}_s - \dot{Z}_q \dot{Z}_t}$$

$$\dot{I}_s = \frac{\dot{V}_s \dot{Z}_m - \dot{V}_m \dot{Z}_t}{\dot{Z}_m \dot{Z}_s - \dot{Z}_q \dot{Z}_t}$$

For the capacitor start motor at the starting instant, $S = 1$

$$\alpha = \pi/2, \; V_m = V_s = V, R_f = R_b, \; X_f = X_b$$

$$q_1 = a\,(X_f - X_b) = 0, \; t_1 = -a\,(X_f - X_b) = 0$$

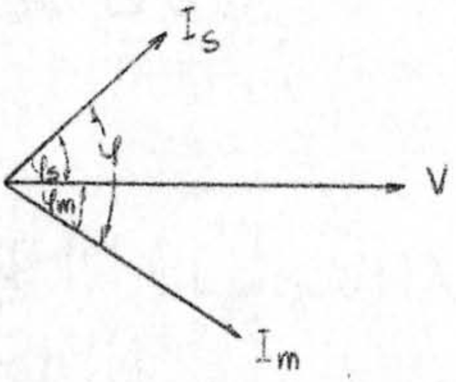
$$q_2 = -\,a(R_f - R_b) = 0, \; t_2 = a\,(R_f - R_b) = 0$$

$$\dot{I}_m = \frac{\dot{V}\,\dot{Z}_s - 0}{\dot{Z}_m\,\dot{Z}_s - 0} = \frac{\dot{V}}{\dot{Z}_m}$$

$$= \frac{\dot{V}}{l_m + jm_m}$$

$$\dot{I}_s = \frac{\dot{V}_s\,\dot{Z}_m - 0}{\dot{Z}_m\,\dot{Z}_s - 0} = \frac{\dot{V}}{\dot{Z}_s}$$

$$= \frac{V}{l_s + jm_s}$$



$$\varphi = \varphi_s + \varphi_m$$

$$\varphi_s = \tan^{-1} \frac{m_s}{l_s}$$

$$\varphi_m = \tan^{-1} \frac{m_m}{l_m}$$

The power of the rotating field and the torque. The power of the rotating field is determined by the sum of the product of e.m.f. induced in each winding and its current.

The instantaneous power of the rotating field is

$$P_{\text{rotating field}} = \left[(e_{fm}-e_{bm})+(e_{m,fs}-e_{m,bs})\right]\sqrt{2}\, I_m \sin \omega t + \\ + \left[(e_{fs}-e_{bs})+(e_{s,fm}-e_{s,bm})\right]\sqrt{2}\, I_s \sin(\omega t+\varphi) \text{ watt.}$$

By substituting $e_{s,fm}$, $e_{s,bm}$, $e_{m,fs}$ and $e_{m,bs}$ in to the above equation, two terms for the instataneous power are obtained: one term is independent of time and represents the average value of the power of the rotating field; the other term depends upon $2\omega t$ and represent a power which alternate with twice the line-frequency.

The first term is

$$P_{\text{rotating field average}} = (I_m{}^2+a^2I_s{}^2)\,(R_f-R_b) \\ -2\, I_m\, I_s a\left[R_f\cos(\alpha+\varphi)+(R_b\cos(\alpha+\varphi)\right] \text{ watt.}$$

and the second term

$$P_{\text{rotating field alternate}} = I_m{}^2\left[-(R_f-R_b)\cos 2\omega t+(X_f-X_b)\sin 2\omega t\right] + \\ +a^2\, I_s{}^2\left[-(R_f-R_b)\cos 2(\omega t+\varphi)+(X_f-X_b) \\ \sin 2\,(\omega t+\varphi)\right] \\ +2\, I_m I_s a\, \cos\alpha\left[-(R_f-R_b)\cos(2\omega t+\varphi)+ \\ +(X_f-X_b)\sin(2\omega t+\varphi)\right] \text{ watt.}$$

The average torque is

$$T_{\text{ave}} = \frac{7.04}{n_s} \cdot P_{\text{rot. f.ave}} \quad \text{lb - ft.}^{*}$$

during start $\alpha = \pi/2$

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* Liwschitz, Garik and Whipple, A-C Machines, 2nd edition, D.Van Nostrand Co.,New York, P.140 - 141.

$$\text{Protating field average} = (I_m^2+a^2I_s^2)\ (R_f-R_b)+2\ I_mI_s a(R_f+R_b)\sin\varphi \text{ watt.}$$

$$\begin{aligned}\text{Protating field alternate} = & I_m^2\left[-(R_f-R_b)\cos 2\omega t+(X_f-X_b)\sin 2\omega t\right] \\ & + a^2\ I_s^2\left[-(R_f-R_b)\cos 2(\omega t+\varphi)+\right. \\ & \left.+(X_f-X_b)\sin 2(\omega t+\varphi)\right] \quad \text{watt.}\end{aligned}$$

At $S = 1$, $R_f = R_b$ and $X_f = X_b$, therefore

$$\text{Protating field average} = 4\ I_m\ I_s\ a\ R_f\ \sin\varphi \text{ watt.}$$

and the starting torque

$$T_{s=1} = \frac{7.04}{n_s}\ 4.I_m\ I_s\ a.R_f\ \sin\varphi \ \text{Lb-ft.}$$

2.2.3 <u>Determination of the Equivalent Circuit Parameters</u>

The parameters of the equivalent circuit of the single-phase induction motor can be determined by no load test and locked rotor test.

1. The No load Test. The given motor is run at a rated voltage V_1. With the starting winding open. I_c and P_o are measured.

At the synchronous speed, there is no difference in speed between the rotor and the forward rotating flux, but there is a difference in speed equal to twice the synchronous speed between the rotor and the backward rotating flux, therefore I_{2b}' flows in the rotor of the single phase motor.

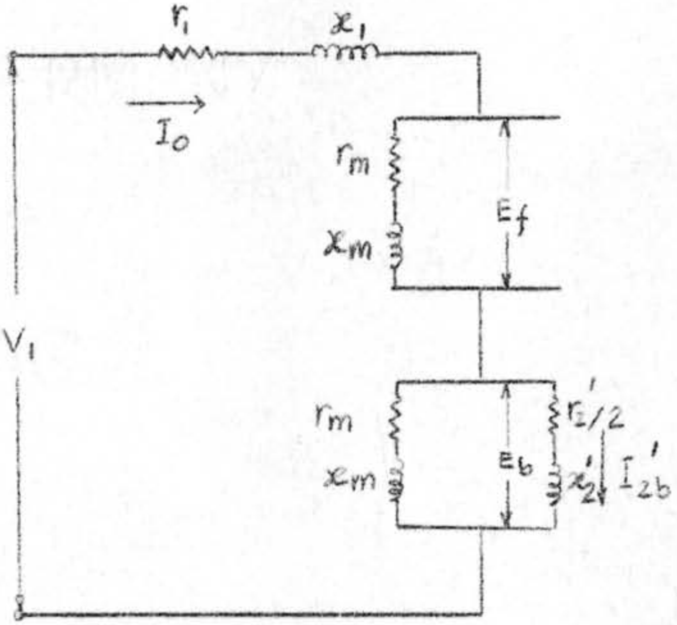

Fig. 2.7 Equivalent circuit of Single phase Motor at No-load. Since $r_m + j x_m$ is large in comparision with $r_2'/2 + + j x_2'$, $I_{2b}' \approx I_o$. It can be assumed that $r_2' = \frac{r_1}{2}$. Hence the rotor copper losses at no load can be assumed equal to $0.5 I_o^2 r_1$.

The power input at no-load P_o consist of the iron losses due to the main flux P_{h+e}, the friction and windage losses P_{F+W}, the rotation iron losses Pir. rot, the copper losses in the stator winding $I_o^2 r_1$, and the copper losses in the rotor winding $0.5 I_o^2 r_1$. Subtracting from P_o by the sum ($P_{F+W} + I_o^2 r_1 + 0.5 I_o^2 r_1$), the remainder is the sum of P_{h+e} + Pir. rot. It can be assumed with fair approximation that the iron losses due to the main flux are equal to $\frac{1}{2}$ (P_{h+e}+Pir.rot).* Thus the iron losses due to the main flux, which are necessary for the

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* Liwschitz, Garik and Whipple, A-C Machines, 2nd edition, D.Van Nostrand Co., New York, P. 128 - 130.

determination of the parameter r_m, are known.

It follows from the equivalent circuit that, for no load ($S \approx 0$)

$$V_1 \approx I_o \cdot \frac{x_m}{K_2} (2 K_2^2 - 1) ,^*$$

Where $K_2 = 1 + \frac{x_2'}{x_m}$

or $x_m \frac{V_1}{I_o} \frac{K_2}{2K_2^2-1}$.

x_m can not be determined from the above equation because K_2 is unknown. However, K_2 varies within narrow limits, usually between 1.03 and 1.07 *, and a certain value for K_2 can be assumed at first. This value can be checked later. A knowledge of x_m is necessary for the determination of r_m.

As for the poly-phase motor,

$$g_m = \frac{P_{h+e}}{E_1^2}$$

$$E_1 \approx V_1 - I_o x_1$$

E_1 consists of two parts: the voltage across the forward branch E_{2f}' , and the voltage across the backward branch E_{2b}'. Since $(r_m + jx_m)$ is large in coparision with $(r_{2/2}' + jx_2')$, the voltage E_{2f}' is large in comparision with the voltage E_{2b}'. It can be assumed that the ratio of the two voltage is the same as that of the two impedances $(r_m + jx_m)$ which is approximately equal jx_m and $(r_{2/2}') + jx_2'$ i.e.,

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* Liwschitz, Garik and Whipple, A-C Machines, 2nd edition, D.Van Nostrand Co., New York, P 128-130.

$$\frac{E_{2f}'}{E_{2b}'} \approx \frac{x_m}{\sqrt{(r_{2/2}')^2 + x_2'^{\,2}}} = C.$$

Since $E_1 = E_{2f}' + E_{2b}'$,

therefore, $E_{2f}' = E_1 \frac{C}{1 + C}$.

Assuming that the iron losses are produced by the forward rotating flux only, because the backward e.m.f. is small and therefore, the backward rotating flux is weak, then

$$g_m \approx \frac{P_{h+e}}{E_{2f}'^{\,2}}$$

Further $r_m \approx g_m x_m^2$

Thus the no load test yields the main flux parameter x_m and r_m, provided that the resistance r_1 and r_2' and the leakage reactance x_1 and x_2' are known.

2. **The Locked Rotor Test.** This test is made with the starting winding open, and V_L , I_L and P_L are measured.

$$Z_L = \frac{V_L}{I_L}, \quad R_L = \frac{P_L}{I_L^2}, \quad X_L = \sqrt{Z_L^2 - R_L^2}$$

The equivalent circuit for the locked roter test is shown in Fig.2.8

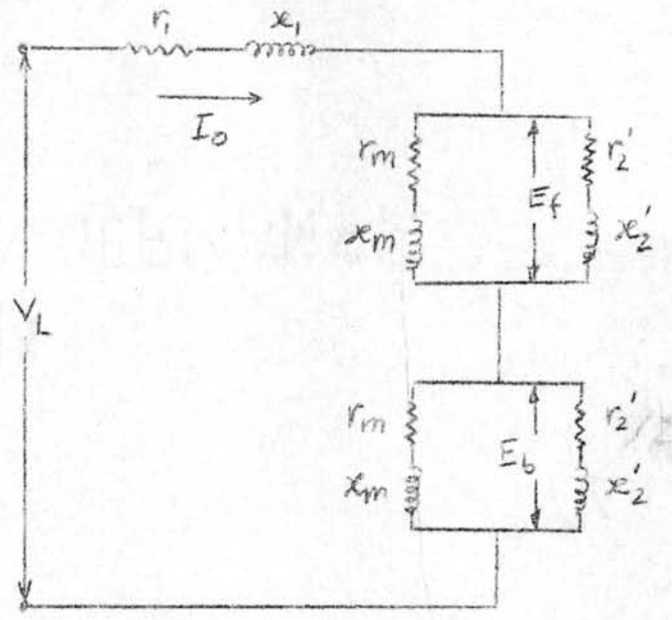

Fig. 2.8 Equivalent circuit of single phase motor at stand still.

From Fig. 2.8,

$$Z_{(S=1)} \approx \left(r_1 + \frac{2 r_2'}{K_2^2} \right) + j\,(x_1 + 2x_2')$$

Yield with fair approximation i.e., at standstill the equivalent resistance of the motor is approximately equal to the primary resistance plus twice the rotor resistance, since $K_2^2 \approx 1$ and the equivalent reactance of the motor is approximately equal to primary reactance plus twice the rotor reactance.

The value of r_1 is measured separately. Then, using the measured locked - rotor resistance R_L ,

$$r_2' = \frac{R_L - r_1}{2} \quad K_2^2$$

For separation of x_1 and x_2', it is usually assumed that

$$x_1 \approx 2\, x_2' = \frac{x_L}{2}$$

With these value of x_1 and x_2', the previously assumed value for K_2 can be checked, and the resistance r_2', the quantity C and the resistance r_m can be determined.

2.3 Calculation of the Starting Torque from the Physical Parameters of the Motor.

The starting torque of the capacitor start induction motor can be calculated from its physical dimensions, the winding diagram and the value of the capacitor. The starting torque equation given by C.R.Boothby, with the correction by R.H. Trickey to take into account the effect of magnetizing current, is given as

$$T_s = \frac{1.88\ p\ E^2 K\ R_{rm}}{f} \ \frac{R_a X_{1m} - R_m\ (X_{1a} - X_c)}{\left[R_m^2 + X_{1m}^2\right]\left[R_a^2 + (X_{1a} - X_c)^2\right]}\ K_r \ ^*$$

Where

K_r = leakage flux factor = $\frac{X_o - X_{1m}}{x_o}$

K = ratio of effective conductors

= $\frac{N_{a.}\ C_{wa}}{N_{m.}\ C_{wm}}$

P = no. of poles.

R_{rm} = rotor resistance in terms of the stator main winding

R_a = sum of resistance in starting winding and rotor resistance in terms of the starting winding.

R_m = Total main winding resistance in term of stator main winding.

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* John H.Kuhlmann, Design of Electrical Apparatus, 3rd edition, John Wiley & Son, Inc., New York, P.356 -380

X_{1m} = Total leakage reactance in term of stator main winding.

X_{1a} = Total leakage reactance in term of the starting winding.

X_c = Capacitive reactance of starting condenser.

In order to find the above resistance and reactance, the following steps have to be followed.

1. The winding distribution factor. The winding distribution factor is a weighted mean chord factor of pitch factor and is calculated by multiplying the chord factor of each coil per pole group by the turn in the coil and divided the sum of these product by the total number of turns.

$$\text{W.D.F.} = \frac{\text{Chord factor X turn in coil}}{\text{Total no. of turns}}$$

2. The length of half mean-turn. For concentric type winding is

$$L_{sm} = \frac{4.2\,(D + d_s)}{S_s} \text{ X slot span} + 1$$

Where D = dia of stator

d_s = depth of stator slot

S_s = number of stator slot

1 = lengh of stator.

3. The resistance of main stator winding.

$$R_{sm} = \frac{L_{sm.}\, N_m \text{ X } 0.692}{a \text{ X } S_{sm} \text{ X } 10^6}$$

Where L_{sm} = length of half mean turn of stator main winding

N_m = number of series conductor

a = number of circuit use.

S_{sm} = cross section area of stator main winding.

4. <u>The rotor resistance per phase in terms of the main winding of the stator</u>.

$$R_{rm} = \frac{N_m{}^2 C_{wm}{}^2 m\, r}{10^6} \left(\frac{l_b}{S_b N_b} + \frac{0.64\, D_{er}}{P^2\, S_{er}} \quad K_{ring} \right)$$

Where m = no of phase use 2 for single phase

r = conductivity of the rotor bar

l_b = length of rotor bar

S_b = section area of each rotor bar

N_b = No of rotor bars.

D_{er} = outside dia of end ring.

S_{er} = Section area of end ring.

Kring = Constant to take into account the effect of non-uniform current distribution in the ring, prepared by P.H. Thickey. found in Fig 2.10

5. <u>The leakage reactance</u>

For fractional - horsepower induction motor,

$$X_s = 2\pi f N_m^2\, C_{wm}{}^2 \times 10^8 \left[\frac{6.381}{S_s}\right]\left[F_{ss} + \frac{S_s}{S_r} F_{sr} \right] \text{ ohms}$$

When N_m = No of series conductor for main winding

l = length of stator

S_s = No of stator slot

S_r = No of rotor slot

F_{ss} = stator slot reactance factor.

F_{sr} = rotor slot reactance factor.

6. <u>The zigzag leakage reactance</u> For stator and rotor in term of main winding.

$$X_z = 2\pi f N_m{}^2 C_{wm}{}^2 \text{ X } 10^8 \left[\frac{2.13\, l}{S_s \delta}\right]\left[\frac{(W_{ts1} + W_{tr1})^2}{4(t_{1s}+t_{1r})}\right]$$

When δ = length of air gap

W_{ts1} = width of stator teeth at air gap surface.

W_{tr1} = width of rotor teeth at air gap surface.

t_{1s} = tooth pitch of stator at armature surface.

t_{1r} = tooth pitch of rotor at armature surface.

7. <u>The end connection leakage reactance</u>

$$X_e = 2\pi f N_m{}^2 C_{wm}{}^2 \text{ X } 10^8 \left[\frac{\pi(D+d_s) \text{ X average coil span}}{S_s\, p}\right]$$

When D = dia. of stator

d_s = depth of stator slot

p = no. of pole.

8. <u>The skew leakage reactance</u>

$$X_{sk} = X_m \frac{\theta_{sk}{}^2}{12} K_p$$

Where X_m = magnetizing reactance.

Θ_{sk} = the rotor bar skew expressed in radian.

K_p = stator leakage flux factor usually = 0.95

9. The magnetizing reactance.

$$X_m = 2\pi f N_m^2 C_{wm}^2 \times 10^8 \frac{0.645\,1}{\delta K_p F_S} \tau$$

When τ = pole pitch of stator gap circumference

K = gap coeficient

F_S = saturation factor usually = 1.2

10. The slot reactance factor F_{ss}, F_{sr}

For slot of this shape

$$F_s = \emptyset \frac{d_1}{W_{s3}} + \frac{d_4}{W_{s1}} + \frac{2\,d_3}{W_{s1} + W_{s2}}$$

$\emptyset$ = constant found from Fig. 2.9

11. The Total leakage reactance of stator main winding plus rotor in terms of the main winding of the stator.

$$X_{1m} = X_s + X_z + X_e + X_{sk}$$

12. The open-circuited reactance, of the main winding with secondary open,

$$X_o = X_m + \frac{X_{1m}}{2}$$

13. The leakage flux factors

$$K_r = \frac{X_o - X_{1m}}{X_o}$$

$$K_p = \sqrt{K_r}$$

14. <u>The ratio of effective conductors</u>

$$K = \frac{N_a \cdot C_{wa}}{N_m \cdot C_{wm}}$$

15. <u>The total main winding resistance</u>

$$R_m = R_{sm} + R_{rm}$$

16. <u>The rotor resistance in term of starting winding</u>

$$R_{ra} = K^2 \cdot R_{rm}$$

R_{rm} = rotor resistance in term of the stator main winding.

17. <u>The total resistance in terms of starting winding.</u>

$$R_a = R_{sa} + R_{ra}$$

R_{sa} = resistance of starting winding

18. <u>The total leakage reactance in term of starting winding.</u>

$$X_{1a} = K^2 \cdot X_{1m}$$

<u>The maximum starting torque</u> The maximum starting torque of the motor is found by setting the derivative of the starting torque with respect to X_c to zero. Solve for X_c to obtain the capacitive reactance required for maximum starting torque are shown bellow.

From starting torque eq. given by P.H. Trickey reference

$$T_s = \frac{1.88\ p\ E^2 K\ R_{rm}}{f} \quad \frac{R_a X_{1m} - R_m (x_{1a} - x_c)}{\left[R_m^2 + X_{1m}^2\right]\left[R_a^2 + (X_{1a} - X_c)^2\right]} K_r \quad *$$

* John H.Kuhlmann, Design of Electrical Apparatus, 3^{rd} edition, John Willy & Sons, Inc., New York, P.390.

$$\text{let} \quad \frac{1.88\; p\, E^2 K\, R_{rm} K_r}{f} = K_1$$

$$R_m^2 + X_{1m}^2 = K_2$$

Then $T_S = K_1 \cdot \dfrac{R_a X_{1m} - R_m(X_{1a} - X_c)}{K_2\left[R_a^2 + (X_{1a} - X_c)^2\right]}$

$$\frac{d\,T_s}{d\,X_c} = \frac{K_1}{K_2}\left\{\left[R_a^2 + (X_{1a} - X_c)^2\right]R_m - \left[R_a X_{1m} - R_m(X_{1a} - X_c)\right] \frac{\left[-(X_{1a} - X_c)^2\right]}{\left[R_a^2 + (X_{1a} - X_c)^2\right]^2}\right\} = 0$$

Therefore

$$R_a^2 R_m + X_{1a}{}^2 R_m - 2X_{1a} X_c R_m + X_c^2 R_m + 2R_a X_{1m} X_{1a} - 2R_m X_{1a}{}^2$$

$$+ 2R_m X_c X_{1a} - 2X_c R_a X_{1m} + 2X_c R_m X_{1a} - 2 X_c{}^2 R_m = 0$$

Rewriting to,

$$- R_m X_c{}^2 + 2X_c(R_m X_{1a} - R_a X_{1m}) + R_a{}^2 R_m - R_m X_{1a}{}^2 + 2R_a X_{1m} X_{1a} = 0$$

$$X_c = \frac{-2\,(R_m X_{1a} - R_a X_{1m}) \pm \sqrt{B^2 - 4\,AC}}{-2\,R_m}$$

Where $\sqrt{B^2 - 4\,AC} = \sqrt{\left[2\,(R_m X_{1a} - R_a X_{1m})\right]^2 + 4R_m\,(R_a^2 R_m - R_m X_{1a}{}^2 + 2\,R_a X_{1m} X_{1a}}$

$$= \sqrt{4\,R_a^2 X_{1m}{}^2 + 4R_m{}^2 R_a{}^2}$$

$$= 2R_a \sqrt{R_m^2 + X_{1m}^2}$$

$$= 2 R_a Z_m$$

Thus for maximum starting torque

$$X_c = \frac{-2 R_m X_{1a}}{- 2 R_m} + \frac{2R_a X_{1m}}{- 2R_m} \pm \frac{2 R_a Z_m}{- 2 R_m}$$

$$= X_{1a} + \frac{R_a}{R_m} (Z_m - X_{1m})$$

Considering,

$\pm \dfrac{R_a Z_m}{- 2R_m}$ if we use $+ \dfrac{R_a Z_m}{- 2 R_m}$ the X_c will have minus value so it is invalid. Then $\dfrac{-R_2 Z_m}{-2 R_m}$

will then be used.

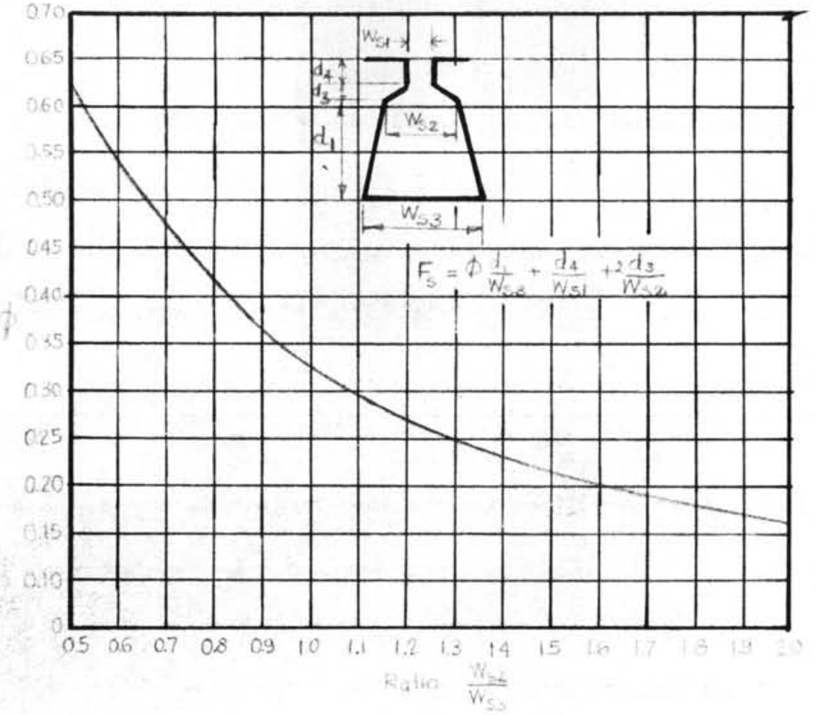

Fig. 2.9 SLOT REACTANCE FACTOR

DESIGN OF ELECTRICAL APPARATUS, BY JOHN H. KUHLMANN THIRD EDITION. JOHN WILEY & SONS, INC. PAGE 369.

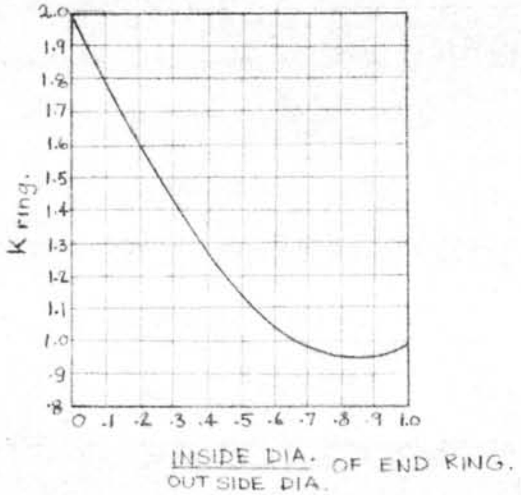

Fig. 2.10 END RING COEFFICIENT

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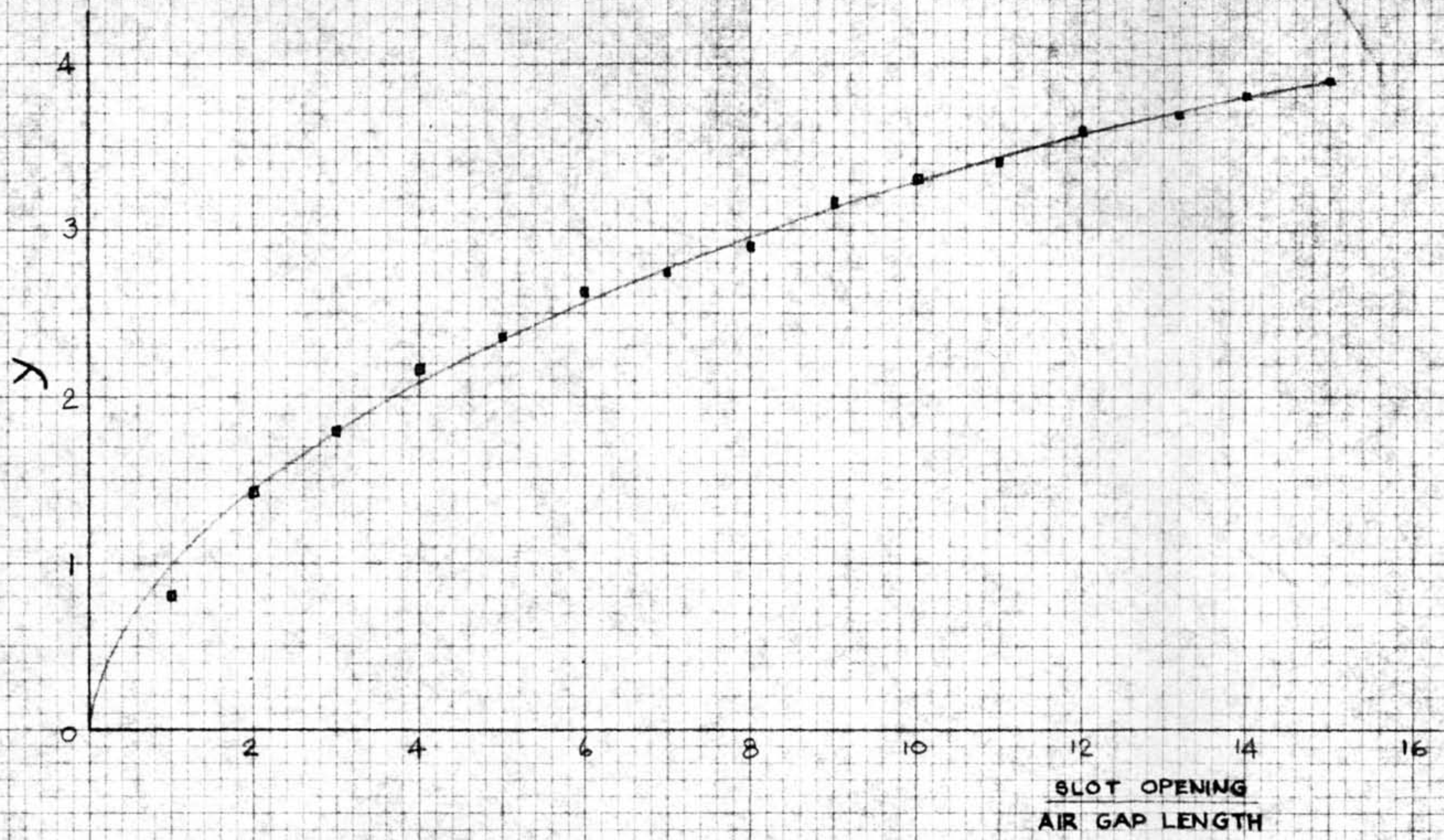

Fig 2.11 AIR GAP COEFFICIENT

$$k = \frac{t_1}{w_{t_1} + y\delta}$$

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