CHAPTER II

THEORY

Analysis of Stiffened Circular Cylindrical Shell.[1]

1 Assumptions.

1. x, y, and z are reference surface coordinates which are orthogonal and along the directions of principal curvatures.

2. The shell is thin.

3. The deflections are small.

4. The rotations about the inplane axes are much larger than that about the normal axis.

5. The normals to the reference surface before deformation remain normal to the reference surface after deformation and they are inextensional. That is $\gamma_{xz} = \gamma_{yz} = \epsilon_{zz} = 0$.

6. Stiffeners are along the principal curvatures and their effects on flexural and extensional stiffness are distributed mathematically over the whole surface of the shell (smeared technique).

7. The connection is monolithic.

8. The stiffeners do not transmit shear force. The shear membrane force is carried entirely by the skin.

9. Stiffeners are in the uniaxial state of stress.

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[1] Ungbhakorn, V., op. cit.

10. Stiffeners are torsionally weak(open section stiffeners).

That is $M_{xy} = M_{yx} = \frac{Eh^3}{12(1+\nu)} \kappa_{xy}$.

11. Skin is in a biaxial state of stress.

2 Stress-strain relations.

Since the skin of the stiffened cylindrical shell is assumed to be in a biaxial stress state. The x-axis is in the longitudinal direction and the y-axis is in the circumferential direction (see Figure 1). Thus, the stress-strain relations in the skin are

$$\begin{aligned} \sigma_{xxsk} &= \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y) \\ \sigma_{yysk} &= \frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x) \\ \sigma_{xysk} &= \frac{E}{2(1+\nu)}\gamma \end{aligned} \qquad (1)$$

The stiffeners are assumed to be in a uniaxial stress state so that the stress-strain relations are

$$\begin{aligned} \sigma_{xxst} &= E_x \cdot \epsilon_x \\ \sigma_{yyr} &= E_y \cdot \epsilon_y \end{aligned} \qquad (2)$$

for the longitudinal and circumferential stiffeners respectively.

3 Strain-displacement relations.

The reference surface of the shell is taken as the midsurface of the skin. The coordinate system is as shown in Figure 1 and u, v, and w being the deformations of material points on the reference surface. The strain-displacement relations are

$$\begin{aligned} \epsilon_x &= \epsilon_{xx} + z\kappa_{xx} \\ \epsilon_y &= \epsilon_{yy} + z\kappa_{yy} \\ \gamma &= \gamma_{xy} + 2z\kappa_{xy} \end{aligned} \qquad (3)$$

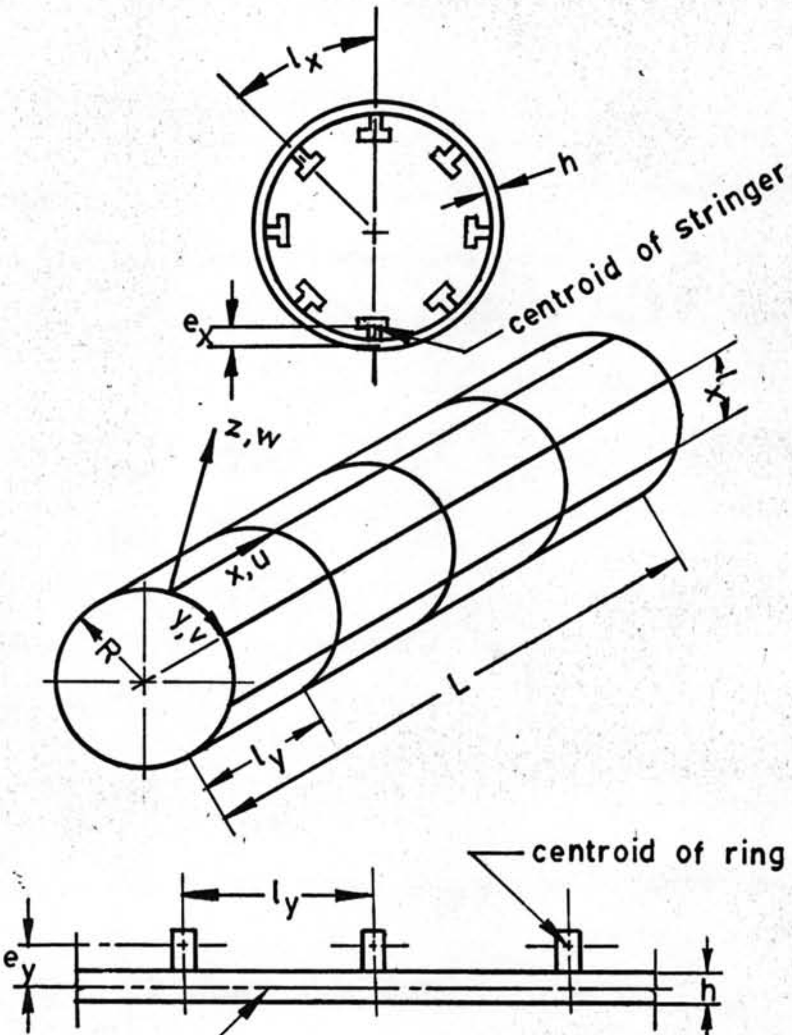

Fig.1 Shell Geometry

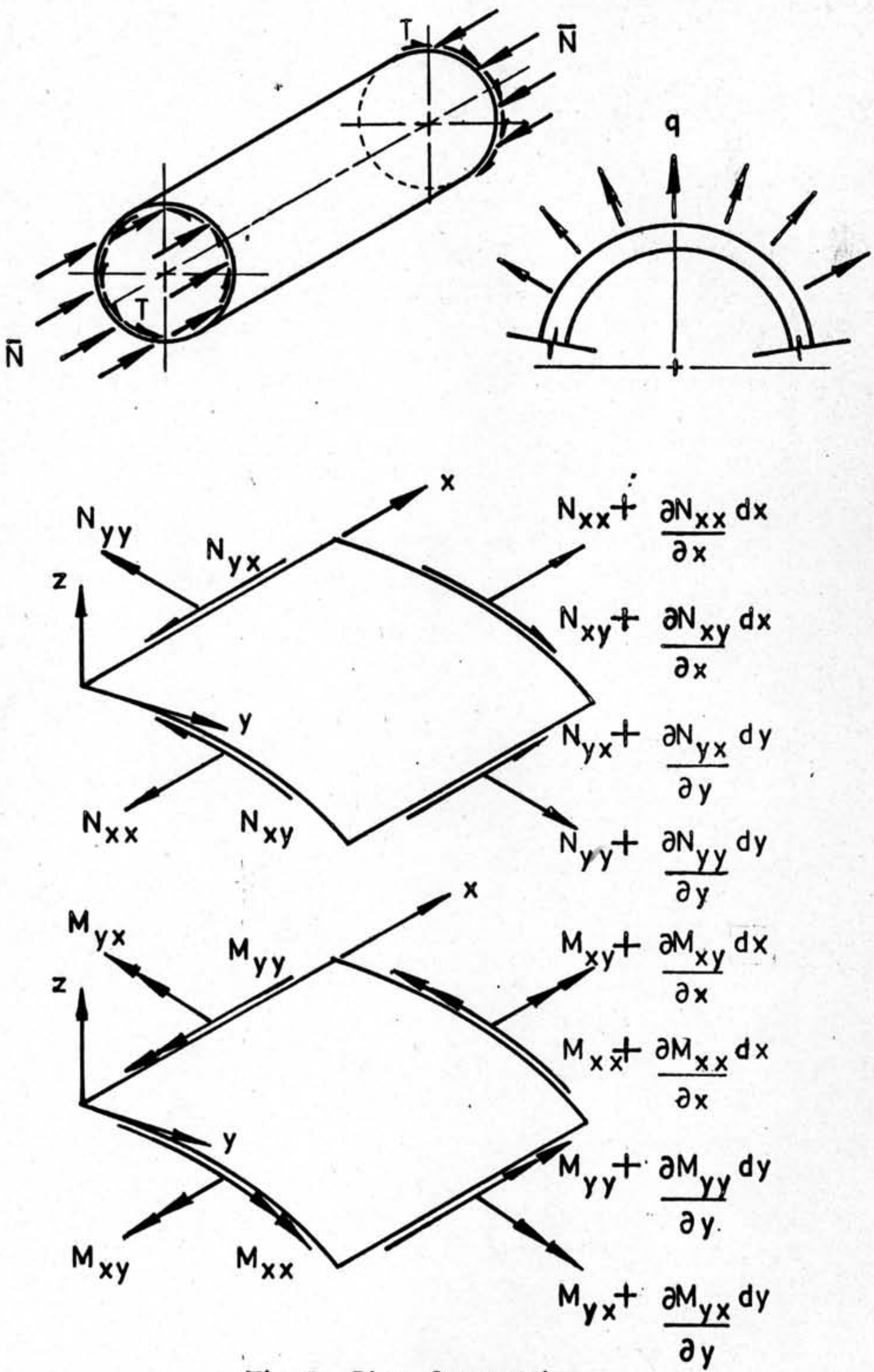

Fig. 2 Sign Convention

$$\kappa_{xx} = -w,_{xx}$$
$$\kappa_{yy} = -w,_{yy}$$
$$\kappa_{xy} = -w,_{xy}$$
$$\epsilon_{xx} = u,_{x}$$
$$\epsilon_{yy} = v,_{y} + \frac{w}{R}$$
$$\gamma_{xy} = u,_{y} + v,_{x}$$

4 <u>**Force and moment resultants.**</u>

The force and moment resultants per unit length are obtained by combining the appropriate integrations of the stresses over the thickness of the shell and then adding to these the corresponding stiffeners contributions. According to assumption 6, the force and moment resultants are

$$N_{xx} = \int_{-h/2}^{+h/2} \sigma_{xxsk}\, dz + \frac{1}{l_x} \int_{A_x} \sigma_{xxst}\, dA_x$$

$$N_{yy} = \int_{-h/2}^{+h/2} \sigma_{yysk}\, dz + \frac{1}{l_y} \int_{A_y} \sigma_{yyr}\, dA_y$$

$$N_{xy} = \int_{-h/2}^{+h/2} \sigma_{xysk}\, dz$$

$$M_{xx} = \int_{-h/2}^{+h/2} z\,\sigma_{xxsk}\, dz + \frac{1}{l_x} \int_{A_x} z\,\sigma_{xxst}\, dA_x \quad \text{---- (4)}$$

$$M_{yy} = \int_{-h/2}^{+h/2} z\,\sigma_{yysk}\, dz + \frac{1}{l_y} \int_{A_y} z\,\sigma_{yyr}\, dA_y$$

$$M_{xy} = \int_{-h/2}^{+h/2} z\,\sigma_{xysk}\, dz + \frac{(GJ)_x}{l_x} \kappa_{xy}$$

$$M_{yx} = \int_{-h/2}^{+h/2} z\,\sigma_{xysk}\, dz + \frac{(GJ)_y}{l_y} \kappa_{yx}$$

Substitution of the stress-strain and kinematic relations, from equations (1) and (3), into equations (4) and performing the indicated integrations. Then combine these results with assumption 10 and a number of new parameters defined in Appendix A, equations (4) become

$$
\begin{aligned}
N_{xx} &= E_{xx}\epsilon_{xx} + \nu E_{xxp}\epsilon_{yy} + e_x E_{xxst}\kappa_{xx} \\
N_{yy} &= \nu E_{yyp}\epsilon_{xx} + E_{yy}\epsilon_{yy} + e_y E_{yyr}\kappa_{yy} \\
N_{xy} &= G_{xy}\gamma_{xy} \qquad\qquad (5) \\
M_{xx} &= (D_{xx} + e_x^2 E_{xxst})\kappa_{xx} + \nu D_{xxp}\kappa_{yy} + e_x E_{xxst}\epsilon_{xx} \\
M_{yy} &= \nu D_{xxp}\kappa_{xx} + (D_{yy} + e_y^2 E_{yyr})\kappa_{yy} + e_y E_{yyr}\epsilon_{yy} \\
M_{xy} &= M_{yx} = D_{xy}\kappa_{xy}
\end{aligned}
$$

5 Prebuckling stresses.

When the cylinder is loaded it is assumed that a membrane state exists, i.e., there is a uniform change in length and radius of the cylinder. Let the superscript "0" denotes the membrane state parameters. Under the membrane state u is a linear function of x only, and v and w are independentof x and y. Therefore

$$
\begin{aligned}
\epsilon_x^o &= \epsilon_{xx}^o = \frac{\partial u}{\partial x} \\
\epsilon_y^o &= \epsilon_{yy}^o = \frac{w}{R} \\
\gamma^o &= 0.
\end{aligned}
$$

The membrane state force resultants become

$$
\begin{aligned}
N_{xx}^o &= E_{xx}\epsilon_{xx}^o + \nu E_{xxp}\epsilon_{yy}^o \\
N_{yy}^o &= \nu E_{yyp}\epsilon_{xx}^o + E_{yy}\epsilon_{yy}^o \qquad\qquad (6) \\
N_{xy}^o &= 0.
\end{aligned}
$$

For a circular cylindrical shell under uniform axial compression

$$
N_{xx}^o = -\bar{N}, \qquad N_{yy}^o = 0.
$$

Hence, equations (6) yield the prebuckling strains

$$\epsilon^{o}_{xx} = \frac{-\bar{N}E_{yy}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}}$$
$$\epsilon^{o}_{yy} = \frac{\nu \bar{N}E_{yyp}}{E_{xx}E_{yy} - \nu^2 E_{xxp}E_{yyp}} \qquad (7)$$

Substitution of equations (7) into equations (1) and (2) yield for the prebuckling stresses of the skin, stringers, and rings in terms of nondimensional parameters, $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ defined in Appendix A, as follows:

$$\sigma_{xxsk} = \frac{-\bar{N}(1+\bar{\lambda}_{yy} - \nu^2)}{h\left[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2\right]}$$
$$\sigma_{yysk} = \frac{-\nu\bar{\lambda}_{yy}\bar{N}}{h\left[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2\right]}$$
$$\sigma_{xxst} = \frac{-E_x(1-\nu^2)(1+\bar{\lambda}_{yy})\bar{N}}{Eh\left[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2\right]} \qquad (8)$$
$$\sigma_{yyr} = \frac{E_y \nu(1-\nu^2)\bar{N}}{Eh\left[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2\right]}$$

6 Buckling equations.

The well-known equilibrium equations of the linear thin shell theory are

$$N_{xx,x} + N_{xy,y} + q^x = 0.$$
$$N_{xy,x} + N_{yy,y} + q^y = 0. \qquad (9a)$$
$$M_{xx,xx} + M_{yy,yy} + 2M_{xy,xy} + (N_{xx}w_{,x})_{,x} + (N_{yy}w_{,y})_{,y} +$$
$$(N_{xy}w_{,x})_{,y} + \frac{N_{yy}}{R} + (N_{xy}w_{,y})_{,x} - q^z = 0.$$

where q^x, q^y, and q^z are the loads in the x, y, and z directions, respectively.

A numbers of authors, Block, Card, and Mikulas,[2] Baruch and Singer,[3] Hedgepeth and Hall,[4] Deluzio and Stuhlman,[5] and Simitses,[6] have investigated the instability of eccentrically stiffened cylinders under the action of single load application by using orthotropic thin shell theory and they have reduced the problem to an eigenvalue problem, with three differential equations. Using the geometry and sign convention shown in Figures (1) and (2), and letting the superscript "1" refer to the additional quantities necessary to bring the membrane state to the adjacent buckled state, These three governing equations are

$$\left[E_{xx}\frac{\partial^2}{\partial x^2}+G_{xy}\frac{\partial^2}{\partial y^2}\right]u^1+\left[(G_{xy}+\nu E_{yyp})\frac{\partial^2}{\partial x\partial y}\right]v^1=\left[\left(q-\frac{\nu}{R}E_{yyp}\right)\frac{\partial}{\partial x}+e_x E_{xxst}\frac{\partial^3}{\partial x^3}\right]w^1$$

$$\left[(G_{xy}+\nu E_{xxp})\frac{\partial^2}{\partial x\partial y}\right]u^1+\left[E_{yy}\frac{\partial^2}{\partial y^2}+G_{xy}\frac{\partial^2}{\partial x^2}\right]v^1=\left[\left(q-\frac{E_{yy}}{R}\right)\frac{\partial}{\partial y}+e_y E_{yyr}\frac{\partial^3}{\partial x^3}\right]w^1 \qquad (9b)$$

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[2] Block, D. L., Card, M. F., and Mikulas, M.M., Jr., Buckling of Eccentrically Stiffened Orthotropic Cylinders (NASA TND-2960, 1965).

[3] Baruch, M. and Singer, J., Effect of Eccentricity of Stiffeners on the General Instability of Cylindrical Shells under Hydrostatic Pressure(J. Mech. Eng. Sci.,5 1963), pp. 23-27.

[4] Hedgepeth, I. M. and Hall, D. B., Stability of Stiffened Cylinders(AIAA J., No. 3, 1965), pp. 2275-2286.

[5] Deluzio, A. and Stuhlman, C., Influence of Stiffener Eccentricity and End Moments on Cylinder Compression Stability (Lockheed Missiles and Space Co., LMSC A-804608, 1964).

[6] Simitses, G. J., A Note on the General Instability of Eccentrically Stiffened Cylinders (J. Aircraft, Vol. 4, No. 5, 1967), pp. 473-475.

$$\left[(D_{xx}+e_x^2E_{xxst})\frac{\partial^4}{\partial x^4}+2(D_{xy}+\frac{\nu}{2}D_{xxp}+\frac{\nu}{2}D_{yyp})\frac{\partial^4}{\partial x^2\partial y^2}+(D_{yy}+e_y^2E_{yyr})\frac{\partial^4}{\partial y^4}\right.$$

$$\left.+\frac{E_{yy}}{R^2}-2\frac{e_y}{R}E_{yyr}\frac{\partial^2}{\partial y^2}\right]w^1+\left[\frac{\nu}{R}E_{xxp}\frac{\partial}{\partial x}-e_xE_{xxst}\frac{\partial^3}{\partial x^3}\right]u^1+$$

$$\left[\frac{E_{yy}}{R}\frac{\partial}{\partial y}-e_yE_{yyr}\frac{\partial^3}{\partial y^3}\right]v^1=N_{xx}^o w^1_{,xx}+N_{yy}^o w^1_{,yy}+2N_{xy}^o w^1_{,xy}$$

Note that the operators in equations (9) are commutative and equations (9) are the buckling equations of the stiffened cylinders subjected to the uniform axial compression, torsion, and hydrostatic pressure and that the pressure load q remains normal to the deflected midsurface during the buckling process. The eigenvalues for the problem are

$$N_{xx}^o=\frac{qR}{2}-\overline{N}$$
$$N_{yy}^o=qR \qquad \text{———————— (10)}$$
$$N_{xy}^o=\frac{T}{2\pi R^2}$$

Hedgepeth and Hall have derived a single higher order Donnell-Batdrof type of an equation by eliminating u^1 and v^1 in equations (9) in terms of the nondimensional groups of parameters defined in Appendix A. Thus, the single buckling equation is

$$(1+\bar{\rho}_{yy})\nabla_D w^1+\nabla_E^{-1}\left[\frac{12Z^2}{1-\nu^2}(1+\bar{\lambda}_{xx})\nabla_C w^1-(\frac{L}{\pi R})^2\,\overline{K}_{yy}\,\nabla_P w^1\right]=$$

$$(\frac{L}{\pi})^2\left[(\tfrac{1}{2}\overline{K}_{yy}-\overline{K}_{xx})\,w^1_{,xx}+\overline{K}_{yy}\,w^1_{,yy}+2\overline{K}_s\,w^1_{,xy}\right] \qquad \text{———— (11)}$$

where

∇^{-1} is an inverse differential operator such that $\nabla\nabla^{-1}=\nabla^{-1}\nabla=1.$

$$\nabla_E=(\frac{L}{\pi})^4\left[\frac{\partial^4}{\partial x^4}+\frac{2}{(1-\nu)(1+\bar{\lambda}_{xx})}\left\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu\right\}\frac{\partial^4}{\partial x^2\partial y^2}+\frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}}\frac{\partial^4}{\partial y^4}\right]$$

$$\nabla_D=(\frac{L}{\pi})^4\left[\frac{1+\bar{\rho}_{xx}}{1+\bar{\rho}_{yy}}\frac{\partial^4}{\partial x^4}+\frac{2}{1+\bar{\rho}_{yy}}\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^4}\right]$$

$$\nabla_P = \left(\frac{L}{\pi}\right)^6 \frac{1}{(1+\bar\lambda_{xx})}\Big[\bar\sigma_x \bar\lambda_{xx} \frac{\partial^6}{\partial x^6} + \frac{(2\bar\lambda_{yy}+1-\nu)}{1-\nu}\bar e_x \bar\lambda_{xx}\frac{\partial^6}{\partial x^4 \partial y^2} +$$

$$\frac{(2\bar\lambda_{xx}+1-\nu)}{1-\nu}\bar e_y \bar\lambda_{yy}\frac{\partial^6}{\partial x^2\partial y^4} + \bar e_y \bar\lambda_{yy}\frac{\partial^6}{\partial y^6} - \nu\left(\frac{\pi}{L}\right)^2\frac{\partial^4}{\partial x^4} + \left(\frac{\pi}{L}\right)^2\frac{1}{1-\nu}\Big\{\nu(1+\nu)$$

$$-(1-\bar\lambda_{yy})(2\bar\lambda_{xx}+1+\nu)\Big\}\frac{\partial^4}{\partial x^2\partial y^2} - \left(\frac{\pi}{L}\right)^2(1+\bar\lambda_{yy})\frac{\partial^4}{\partial y^4}\Big]$$

$$\nabla_C = \frac{(L/\pi)^8}{(1+\bar\lambda_{xx})^2}\Big[\bar e_x^2 \bar\lambda_{xx}\frac{\partial^8}{\partial x^8} + \frac{2\bar e_x^2\bar\lambda_{xx}(\bar\lambda_{yy}+1-\nu)}{1-\nu}\frac{\partial^8}{\partial x^6\partial y^2} +$$

$$\Big\{\bar e_x^2\bar\lambda_{xx}(1+\bar\lambda_{yy}) + 2\bar e_x\bar e_y\bar\lambda_{xx}\bar\lambda_{yy}\left(\frac{1+\nu}{1-\nu}\right) + \bar e_y^2\bar\lambda_{yy}(1+\bar\lambda_{xx})\Big\}\frac{\partial^8}{\partial x^4\partial y^4} +$$

$$\frac{2\bar e_y^2\bar\lambda_{yy}(1+\bar\lambda_{xx}-\nu)}{1-\nu}\frac{\partial^8}{\partial x^2\partial y^6} + \bar e_y^2\bar\lambda_{yy}\frac{\partial^8}{\partial y^8} + 2\nu\left(\frac{\pi}{L}\right)^2\bar e_x\bar\lambda_{xx}\frac{\partial^6}{\partial x^6} -$$

$$2\left(\frac{\pi}{L}\right)^2\Big\{\bar e_x\bar\lambda_{xx}(1+\bar\lambda_{yy}) + \bar e_y\bar\lambda_{yy}(1+\bar\lambda_{xx})\Big\}\frac{\partial^6}{\partial x^4\partial y^2} + 2\nu\left(\frac{\pi}{L}\right)^2\bar e_y\bar\lambda_{yy}\frac{\partial^6}{\partial x^2\partial y^4}$$

$$+\left(\frac{\pi}{L}\right)^2\Big\{(1+\bar\lambda_{xx})(1+\bar\lambda_{yy}) - \nu^2\Big\}\frac{\partial^4}{\partial x^4}\Big]$$

7 Instabilities under uniform axial compression.

General instability. For uniform axial compression the buckling equation (11) becomes

$$(1+\bar\rho_{yy})\nabla_D w^1 + \frac{12Z^2}{1-\nu^2}(1+\bar\lambda_{xx})\nabla_E^{-1}\nabla_C w^1 + \left(\frac{L}{\pi}\right)^2\bar K_{xx} w^1_{,xx} = 0. \qquad (12)$$

The classical simply supported boundary conditions are

$$\begin{aligned} w^1(0,y) &= w^1(L,y) = 0. \\ v^1(0,y) &= v^1(L,y) = 0. \\ M_{xx}(0,y) &= M_{xx}(L,y) = 0. \\ N^1_{xx}(0,y) &= N^1_{xx}(L,y) = 0. \end{aligned} \qquad (13)$$

The displacement function which satisfies all boundary conditions is

$$w^1 = W_{mn}\,\mathrm{Sin}\,\frac{m\pi x}{L}\,\mathrm{Sin}\,\frac{ny}{R}$$

By substituting the assumed displacement function into the buckling equation one can obtain the expression for the buckling load. The resulting expression for the buckling coefficient contains two integer parameters, m and n, representing the mode shape. Let $\beta = \frac{nL}{\pi R}$, then the buckling coefficient is

$$\bar{K}_{xx} = \frac{1}{m^2}\left[(1+\bar{\rho}_{xx})m^4 + 2m^2\beta^2 + (1+\rho_{yy})\beta^4\right] + \frac{12Z^2}{m^2\pi^4(1-\nu^2)}\left\{\bar{e}_x^2\bar{\lambda}_{xx}m^8 + \frac{2}{1-\nu}\bar{e}_x^2\bar{\lambda}_{xx}(1-\nu+\bar{\lambda}_{yy})m^6\beta^2 + \left\{\bar{e}_x^2\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \frac{2(1+\nu)}{1-\nu}\bar{e}_x\bar{e}_y\bar{\lambda}_{xx}\bar{\lambda}_{yy} + \bar{e}_y^2\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\right. m^4\beta^4 + \frac{2}{1-\nu}\bar{e}_y^2\bar{\lambda}_{yy}(1-\nu+\bar{\lambda}_{xx})m^2\beta^6 + \bar{e}_y^2\bar{\lambda}_{yy}\beta^8 - 2\nu\bar{e}_x\bar{\lambda}_{xx}m^6 + 2\left\{\bar{e}_x\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \bar{e}_y\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\right\}m^4\beta^2 - 2\nu\bar{e}_y\bar{\lambda}_{yy}m^2\beta^4 + \left\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2\right\}m^4\Big] \Big/ \left[(1+\bar{\lambda}_{xx})m^4 + \frac{2}{1-\nu}\left\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu\right\}m^2\beta^2 + (1+\bar{\lambda}_{yy})\beta^4\right] \qquad (14)$$

For any given stiffened shell geometry the critical load coefficient, $\bar{K}_{xx_{cr}}$, is obtained through minimization of equation (14) with respect to all integer values of m and n, except m = 0.

Let $\bar{\beta} = \frac{nL}{m\pi R}$, and for an internally stiffened shell $\bar{e}_x$ and $\bar{e}_y$ are negative numbers; therefore, equation (14) can be arranged as

$$\bar{K}_{xx} = Pm^2 + \frac{Q}{m^2} + S \qquad (15)$$

where

$$P = 1+\bar{\rho}_{xx} + 2\bar{\beta}^2 + (1+\bar{\rho}_{yy})\bar{\beta}^4 + \frac{12Z^2}{\pi^4(1-\nu^2)}\left[\bar{e}_x^2\bar{\lambda}_{xx} + \frac{2}{1-\nu}\bar{e}_x^2\bar{\lambda}_{xx}(1-\nu+\bar{\lambda}_{yy})\bar{\beta}^2 + \left\{\bar{e}_x^2\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \frac{2(1+\nu)}{1-\nu}\bar{\lambda}_{xx}\bar{\lambda}_{yy}\bar{e}_x\bar{e}_y + \bar{e}_y^2\bar{\lambda}_{yy}(1+\lambda_{xx})\right\}\bar{\beta}^4 + \frac{2}{1-\nu}\bar{e}_y^2\bar{\lambda}_{yy}(1-\nu+\bar{\lambda}_{xx})\bar{\beta}^6 + \bar{e}_y^2\bar{\lambda}_{yy}\bar{\beta}^8\right] / B$$

$$B = 1+\bar{\lambda}_{xx} + \frac{2}{1-\nu}\left\{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu\right\}\bar{\beta}^2 + (1+\bar{\lambda}_{yy})\bar{\beta}^4$$

$$Q = \frac{12Z^2}{\pi^4(1-\nu^2)}\left[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2\right]/B$$

$$S = \frac{12Z^2}{\pi^4(1-\nu^2)}\left[\nu\bar{e}_x\bar{\lambda}_{xx}-\left\{\bar{e}_x\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \bar{e}_y\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\right\}\bar{\beta}^2 + \nu\bar{e}_y\bar{\lambda}_{yy}\bar{\beta}^4\right]/B$$

For the purpose of the first stage of computer program analysis of the buckling mode, m^2 is first treated as a continuous variable. Minimization of equation (15) with respect to m^2 yields

$$m^2 = \sqrt{\frac{Q}{P}}$$

Hence $$\bar{K}_{xx} = 2\sqrt{PQ} + S \qquad (16)$$

Panel instability. The panel instability is the special case of the general instability when all stringers and skin between two adjacent rings participate. Thus, by setting all rings parameters to zero, the expression for panel instability can be obtained from equation (14). That is substituting $\bar{e}_y = 0.$, $\bar{\lambda}_{yy} = 0.$, $\bar{\rho}_{yy} = 0.$, and $L = l_y$ into equation (14). The resulting expression for panel instability with the sign of $\bar{e}_x$ changed for inside stiffeners is

$$\bar{K}_{xx_P} = (1+\bar{\rho}_{xx})m^2 + 2\beta^2 + \frac{\beta^4}{m^2} + \frac{12Z^2}{\pi^4(1-\nu^2)}\left[\bar{e}_x^2\bar{\lambda}_{xx}(m^2+\beta^2)^2 - 2\bar{e}_x\bar{\lambda}_{xx}(\beta^2-\nu m^2) + 1-\nu^2+\bar{\lambda}_{xx}\right]\Big/\left[(1+\bar{\lambda}_{xx})m^2 + \frac{2}{1-\nu}(1-\nu+\bar{\lambda}_{xx})\beta^2 + \frac{\beta^4}{m^2}\right] \qquad (17)$$

For any given stiffened geometry shell the critical load coefficient for panel buckling is obtained by minimization of equation (17) with respect to all integer values of m and n.

Local stringer and skin buckling. When stiffeners are are closely spaced the local stringer and skin buckling are governed by the equation of a flat plate. The critical stress of a flat plate with various edge conditions is given by Bleich[7] as

$$\delta_{cr} = K\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{a}{b}\right)^2 \qquad (18)$$

where

a = skin thickness, thickness of stiffener web, or thickness of stiffener flange.

b = stringer spacing, height of stiffener web, or width of stiffener flange.

K = 4, for four sides simply supported.

$K = \left(\frac{d}{l_y}\right)^2 + 0.425$, for three sides simply supported and one unloading side free.

In the design analysis of the local buckling, it is assumed that all edges of stiffeners and skin connecting to any part of the cylinder are simply supported . With both rings and stringers inside, the possible buckling failure modes are as follows

Skin wrinkling. The skin wrinkling is considered as the buckling of a flat plate of size l_x by l_y. The critical stress is

$$\delta_{xxsk} = \frac{\pi^2 E}{3(1-\nu^2)}\left(\frac{h}{l_x}\right)^2 \qquad (19)$$

Local stringer buckling. There are two possible cases of the local stringer buckling.

1. When the rings are deepest, the portion of a stringer between any adjacent rings is treated as a flat plate of length l_y. The stringer web is considered as four sides simply supported and the flange portion, a flat plate, is considered as three sides

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[7] Bleich, F., Buckling Strength of Metal Structures (McGraw-Hill Book Company, 1952), pp. 329-331.

simply supported and the unloaded side free.

2. When the stringers are deepest, the material of the stringer web below the ring material is assumed to buckle as a flat plate of length l_y with four sides simply supported and the outstanding portion of the stringer web is considered as a flat plate of length L with four sides simply supported. The stringer flange is also treated as a flat plate of length L with three sides simply supported and the unloaded side free.

It has been discovered that during the design process when the stringers are deepest and in the region where $\bar{\alpha}_x > \bar{\alpha}_y$, either the resulting design configuration will always have the ring and stringer thickness which are too thin to be fabricated or the stringer will buckle. Thus, one can avoid this subcase of the local stringer failure by concentratingonly in the region where $\bar{\alpha}_y > \bar{\alpha}_x$ in the favor of practical limitation on fabrication. Since both rings and stringers are inside and the rings are in tension, hence there is no possible buckling failure of the rings.

The critical stresses of the stringers for several types of stiffening members for the configuration when the rings are deepest, are tabulated in Table 1.

Mathematical Formulation.

1. Phase 1.

By assuming that the eccentricities of the stiffening members are small in comparison to the radius of the stiffened cylindrical shell, then the common stiffener material at the intersection of

TABLE 1 Critical Stresses of Stringers.

Stringer Type	Stringer Web, $\sigma_{xxsw_{cr}}$	Stringer Flange, $\sigma_{xxsf_{cr}}$
RS	$\frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}}\right)^2\left[\left(\frac{d_{wx}}{l_y}\right)^2 + 0.425\right]$	———
TS	$\frac{\pi^2 E_x}{3(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}}\right)^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{2t_{fx}}{b_{fx}-t_{wx}}\right)^2\left[\left(\frac{b_{fx}-t_{wx}}{2l_y}\right)^2 + 0.425\right]$
IAS	$\frac{\pi^2 E_x}{3(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}}\right)^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{t_{fx}}{b_{fx}-t_{wx}}\right)^2\left[\left(\frac{b_{fx}-t_{wx}}{l_y}\right)^2 + 0.425\right]$
CS,ZS	$\frac{\pi^2 E_x}{3(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}-2t_{fx}}\right)^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{t_{fx}}{b_{fx}}\right)^2\left[\left(\frac{b_{fx}}{l_y}\right)^2 + 0.425\right]$
IS	$\frac{\pi^2 E_x}{3(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}-2t_{fx}}\right)^2$ *	$\frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{2t_{fx}}{b_{fx}}\right)^2\left[\left(\frac{b_{fx}}{2l_y}\right)^2 + 0.425\right]$
AS	$\frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}-t_{fx}}\right)^2\left[\left(\frac{d_{wx}-t_{fx}}{l_y}\right)^2 + 0.425\right]$	———

* In the case of the design without geometric constraint one may have short plate, $\frac{l_y}{d_{wx}} < 1$,

then $\sigma_{xxsw_{cr}}$ has the form $\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{a}{b}\right)^2\left(\frac{b}{l_y} + \frac{l_y}{b}\right)^2$.

of the stringers and rings is considered negligible. Thus, the weight of the stiffened shell is given by

$$W = 2\pi RLh\rho_{sk} + \rho_x \int_0^L \int_0^{2\pi R} \frac{A_x}{l_x} dy.dx + \rho_y \int_0^L \int_0^{2\pi R} \frac{A_y}{l_y} dy.dx \qquad (20)$$

The weight of the stiffened cylindrical shell in terms of the nondimensional parameters defined in Appendix A is

$$W = 2\pi RLh\rho_{sk}\left[1 + \frac{1}{1-\nu^2}\left(\frac{E\rho_x}{E_x\rho_{sk}}\bar{\lambda}_{xx} + \frac{E\rho_y}{E_y\rho_{sk}}\bar{\lambda}_{yy}\right)\right] \qquad (21)$$

The classical general instability buckling parameter of the thin stiffened cylindrical shell subject to a uniform axial compression with simply supported boundary conditions is given by equation (15). The requirement for minimum weight against general instability leads to the objective function (composite weight function).

$$W^* = W + \lambda\left|\bar{N}_{xx_{cr}} - \bar{N}\right| \qquad (22)$$

where

W = weight of the stiffened shell,

$\bar{N}$ = applied compressive load,

$\bar{N}_{xx_{cr}}$ = general instability load obtained from minimization of equation (15) with respect to m^2 and $\bar{\beta}^2$, and

λ = lagrange multiplier.

Equation (22) can be put into nondimensional form as

$$\bar{W}^* = \frac{\bar{W}}{Z} + \lambda^*\left|\bar{K}^*_{xx_{cr}} - \bar{N}^*\right| \qquad (23)$$

where

$$\bar{W}^* = \frac{W^*}{2\pi L^3\rho_{sk}(1-\nu^2)^{\frac{1}{2}}}, \quad \bar{K}^*_{xx_{cr}} = \frac{\bar{K}_{xx_{cr}}}{Z^3},$$

$$\bar{N}^* = \frac{12R^3\bar{N}}{\pi^2 EL^4(1-\nu^2)^{\frac{1}{2}}}, \quad \lambda^* = \frac{\pi EL\lambda}{24\rho_{sk}R^3}, \qquad (24)$$

$$\bar{W} = 1 + \frac{1}{1-\nu^2}\left[\frac{E\rho_x}{E_x\rho_{sk}}\bar{\lambda}_{xx} + \frac{E\rho_y}{E_y\rho_{sk}}\bar{\lambda}_{yy}\right]$$

Thus, $\bar{W}^*$ is a function of the following parameters,

$$\bar{W}^* = F(Z, \bar{\lambda}_{xx}, \bar{\lambda}_{yy}, \bar{\rho}_{xx}, \bar{\rho}_{yy}, \bar{e}_x, \bar{e}_y, m^2, \bar{\beta}^2) \text{ ——— } (25)$$

From equations (14) and (23) it can be seen that there is no minimum $\bar{W}^*$ with respect to reasonably finite values of the parameters Z, $\bar{e}_x$, $\bar{e}_y$, $\bar{\rho}_{xx}$, and $\bar{\rho}_{yy}$. It is convenient to introduce four new parameters $\bar{\alpha}_x$, $\bar{\alpha}_y$, C_x, and C_y. The new parameters, $\bar{\alpha}$, denote the ratio of the radius of gyration of the stiffeners to that of the skin of unit width. Their expressions are given in Appendix A. The new parameters, C_x and C_y, called shape parameters, are just numbers characterizing the shapes of the stiffeners. For example, C is equal to one for rectangular stiffeners, greater than one for tee and inverted angle stiffeners, and less than one for channel, zee, I, and angle stiffeners. Using these new parameters one can eliminate the parameters $\bar{e}_x$, $\bar{e}_y$, $\bar{\rho}_{xx}$, and $\bar{\rho}_{yy}$ in equation(25) through the relation of equations in Appendix A. Hence

$$\bar{W}^* = F\left[\bar{\lambda}_{xx}, \bar{\lambda}_{yy}, m^2, \bar{\beta}^2, (Z, \bar{\alpha}_x, \bar{\alpha}_y, C_x, C_y)\right] \text{ ——— } (26)$$

The change of parameters from $\bar{e}_x$, $\bar{e}_y$, $\bar{\rho}_{xx}$, and $\bar{\rho}_{yy}$ to $\bar{\alpha}_x$, $\bar{\alpha}_y$, C_x, and C_y are convenient because the ranges of these new parameters are known. For example, for rectangular rings $\bar{\alpha}_y = \frac{d_{wy}}{h}$. But for the assumption of thin ring theory $\frac{R}{d_{wy}} > 20.$, therefore

$$\bar{\alpha}_y > \frac{R}{20h}.$$

Therefore, it is proposed to generate the design charts and tables in the $\bar{\alpha}_x - \bar{\alpha}_y$ space for each type of the stiffening members. The precise statement of the mathematical formulation in Phase 1 is as follows.

In the $\bar{\alpha}_x - \bar{\alpha}_y$ space, for each type of the stiffeners and for each Z and a given load parameter, $\bar{N}$, minimize the weight parameter of the stiffened circular cylindrical shell, $\bar{W}$, with respect to $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ subject to the equality constraint of the general instability. That is

$$\text{Minimize } \bar{W} \text{ subject to } \bar{K}^{*}_{xx_{cr}} = \bar{N}^{*} \quad \text{" } \bar{\lambda}_{xx} - \bar{\lambda}_{yy} \text{ "} \qquad (27)$$

Courant[8] has shown that if $\overset{*}{\lambda}$ is provided sufficiently large, the solution of the unconstrainted of equation (23) will approach the solution of the constrainted minimization of equation (27). The exact solution will be obtained when λ^{*} approaches infinity.

2 Phase 2.

Considering only the absolute values of these stresses during the design process, the stresses of the local buckling of the skin and stringers given in Table 1 must be greater than the applied stresses given by equations (8). Furthermore, the applied stresses must be less than a certain suitable stress level, for example, the yield stress of the material. Of all the rings spacing l_y, obtained from the constraint of stringer buckling, one must select the one (there are many) which does not yield panel buckling.

Mathematical Search Technique.

1 Selection criteria.

V. Ungbhakorn[9] has selected the irregular simple or flexible

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[8]Courant, R., Calculas of Variations and Supplementary Notes and Exercises (Revised and amended by J. Moses, New York University Institute of Mathematical Sciences, New York, 1956-1957), pp 270-276.

[9]Ungbhakorn, V., op. cit.

polyhedron method of Nelder and Mead[10] in his research for the two dimensional minimization problem. Because the simplex has been designed to adapt itself to the topography of the objective function, hence, high reliability.

2 Search technique of Nelder and Mead.

There are four basic operations in the search technique of Nelder and Mead. The reflection, expansion, contraction, and reduction of the simplex. This method minimized a function of n independent variables using (n+1) vertices of a simplex in the n-dimensional euclidean space. In V. Ungbhakorn's research it is two-dimensional problem then a simplex is a triangle. The vertex which yields the highest value of the objective function is projected through the center of gravity or centroid of the remaining vertices. Improved values of the objective function are found by successively replacing the point with the highest value of the objective function by better points until the minimum is found.

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[10] Nelder, J. A. and Mead, R., A Simplex Method of Function Minimization (Computer J., 7, 1964), pp. 308-313.