CHAPTER II

LITERATURE REVIEW

Several authors have originated and developed models to describe the traffic flow on highway. There are at least two approaches to this problem,- microscopic and macroscopic. In recent years, numerous mathematical theories of traffic flow have been developed and it has been shown that the two approaches are interrelated. Various models have been put forward from time to time. A typical, but by no means complete lists of them are as follows:

1) Empirical - by Greenshields.[6]
2) Analogy - by Lighthill and Whitham,[8] Greenberg.[11]
3) Car-following - by Herman and others,[13,31] Edie.[16]
4) Statistical - by Prigogine.[25]

To date, success has not been achieved in accurately plotting the q-u-k diagram in its entirety. The matter is made more complicated and confusing by disagreement among investigators as to the exact geometry of the curve. Some believe that ends of the arch (q-k diagram) to be nearly linear whitst a substantial body of opinion[5] holds that there is a discontinuity at the point of critical density which seperates free-flow traffic from constrained traffic. A brief description of the evolution of literatures about theoretical relationship between flow, speed, and density may be summarized as follows.

Greenshields[6] plotted speed, u , against density, k , for one lane of traffic, because this plane represented the only one of the three planes in the fundamental surface with a single valued relationship. He then fit a curve to the plotted data using the method of least squares which essential estimates the true parameters of the curve, u = f(k). Greenshields hypothesized that a linear relationship provided the best fit to the speed-density data. The linear model is expressed as

$$u = u_f\left(1 - \frac{k}{k_j} \right) \quad \text{- - - - - - - - - - -} \quad (1)$$

From this linear relationship, the flow-density and speed-flow equation are parabola by substitute in equation q = uk and results in maximum flow at 50 percent of the jam density. Greenshields model usually exhibits the undesirable characteristic of an extremely low jam density.

Wardrop[7] considered normal traffic flow to be a combination of several uniform traffic streams, each traveling at a constant speed. Each traffic stream had its own speed, u , flow, q , and density, k. He then proved that:

$$q = uk \quad \text{- - - - - - - - - - - - - - - - -} \quad (2)$$

Lighthill and Whitham[8] suggested that an effective description of various traffic phenomena could be derived from assumptions of continuity and fluid analogy. They arrived at a qualitative description of the flow-density diagram without deriving a specific speed-denstity relationship. They reported that flow increases with increasing density until a maximum is reached, thence flow decreases to zero with further increase in the density.

Richards[9] made the specific assumption that speed is proportional to a difference in density as follows.

$$u = a(k_j - k) \quad \text{------------} \quad (3)$$

where $a = u_f / k_j$ as a traffic fluid state. With this assumption, the flow-density and speed-flow relation are parabolic. Richards applied his model to flow at signalized intersections.

Helly[10] discussed the shortcomings of the Richards model stating that the results do not seem realistic. Richards' vehicular fluid starts from rest by mean of fluid pressure alone, thus beginning with an infinite acceleration; subsequently, changes in speed occur at diminishing rates so that maximum velocity is never reached. In real life, the leading car at the beginning of a green signal is not primarily motivated by the traffic behind it, but rather by a desire to reach a destination. Drivers generally accelerate rather rapidly, reaching cruising speed in about 20 seconds.

Greenberg,[11] developed a relationship between speed and density by using the equation of continuity and the equation of motion for a real fluid which is expressed mathematically as

$$u = u_o \ln(k_j/k) \quad \text{------------} \quad (4)$$

This equation can be transformed by Wardrop's formula to give the relationship between flow and density as:

$$q = u_o k \ln(k_j/k) \quad \text{------------} \quad (5)$$

This expression results in maximum flow at 37 percent of the jam density. The Greenberg model results in a concave-shaped speed-density relationship and does not have a y - intercept (free-flow speed of infinity). This model has been verified experimentally by fitting it to the data collected by the Port of New York Authority in their study of the Lincoln Tunnel and by Greenberg to data from the Merrit Parkway. Although the model provided a good fit to the data collected in both these cases, uncertainty in the region of high densities arose due to absence of data for this region. The left portion of the curve beyond maximum flow undefined by experiment, as in previous investigations.

From the car-following law based upon the reciprocal-spacing function, Gazis, Herman, and Potts(12) have obtain the traffic equation:

$$q = \alpha_0 k \ln(k_j/k) \quad \text{- - - - - - - - - -} \quad (6)$$

This can be identified as Greenberg' s model by determing the proportionality constant $\alpha_0 = u_0$, where α_0 is the characteristic speed; the latter can be measured from the behavior of a pair of successive cars moving along the highway. The relationship between car-following models and traffic stream models was first examined by Gazis et al.(12)

Continuing examination of the relationship between microscopic and macroscopic models led to a generalized form of the car-following equation:

$$\ddot{x}_{n+1}(t + T) = \alpha \frac{\left[\dot{x}_{n+1}(t + T)\right]^m}{\left[x_n(t) - x_{n+1}(t)\right]^l} \left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right] \quad \text{- - -} \quad (7)$$

where m and l are constants. This equation was first proposed by Gazis, Herman, and Rothery;[13] the general expression has been further examined by May and Keller.[14] The steady-state flow formulation of this equation can be obtained by integrating the above equation; it is given by Gazis et al[13] as

$$f_m(u) = cf_l(s) + c' \quad \text{- - - - - - - - -} \quad (8)$$

By selecting proper combinations of the exponents m and l, known microscopic and macroscopic traffic flow models can be obtained.

Underwood[15] has demonstrated a model of the form

$$u = u_f e^{-k/k_0} \quad \text{- - - - - - - -} \quad (9)$$

at low densities. For certain lower type facilities, there seems to be, for practical purposes, an exponential relationship between u and k of this form. However, this relation does not hold for expressway type facilities. This model has shortcoming in that it does not represent zero speed at high densities.

Edie[16] has described a model that is a composite of Greenberg's model and Underwood's model, where Greenberg's model is at high densities and Underwood's model is at low densities. When normalized speed is plotted against normalized density, the two models become tangent in the mid range of density.

Drew,[17] using the fluid-flow approach proposed by Greenberg, employs a more general derivation of this problem and proposes a family of models of the form

$$u = u_f\left(1 - \frac{k}{k_j}\right)^{\frac{n+1}{2}} \quad \text{- - - - - - - -} \quad (10)$$

where n is a real number. Drew[18] also discusses the case for n=o where the speed-density relation becomes

$$u = u_f \left[1 - \left(\frac{k_j}{k}\right)^{\frac{1}{2}} \right] \quad \text{-------- (11)}$$

This expression is often referred to as the parabolic model. Substituting n = 1 and n = -1 into Drew's general model, results in Greenshields' linear model and Greenberg's exponential model, respectively.

Drake, May, and Schofer[19] have proposed use of the bell-shaped or normal curve which gave satisfactory results when compared to speed-density measurements. This model is expressed mathematically as

$$u = u_f e^{-\frac{1}{2}(k/k_o)^2} \quad \text{-------- (12)}$$

In devoloping a model to represent urban traffic, Dick[20] assumed that there is a fixed upper limit to speed. He combined this assumption with Greenberg's model to produce the results that the relationship between speed and density can be represented by two straight lines, one a linear relationship between speed and the logarithm of the density and the other a line for low densities where the speed is constant, make the statistical analysis considerable easier.

Pipes and Munjal[21,22] have proposed a family of models of the form

$$u = u_f(1-k/k_j)^n \quad \text{-------- (13)}$$

where n is a real number greater than zero. It will be seen that for n=1 the relationship reduces to Greenshields' model.

May and Keller[23,24] have discussed methods for fitting single-and multi-regime models by using the generalized car-following law as stated by Gazis, Herman, and Rothery.[13]

The Boltzmann-like model of traffic has been introduced by Prigogine and Herman.[25,26] This model attempts to intergrate low-density flow (individual vehicles) and high-density flow (platoons) into a single model.