CHAPTER 9

CONCLUSIONS

9.1 Principal Conclusions

The following major conclusions have been obtained from the stochastic analysis of convective diffusional deposition of polydisperse aerosols on a dust loaded fiber.

(1) The average profile of deposited particles on the fiber surface was affected by both the Peclet number Pe and the interception parameter R and to a lesser extent by the geometric standard deviation σ_g. When Pe = 0 (pure interception), the profile was known to be uniform all over the entire surface. As Pe gradually increased, the effect of small convection caused a peak to appear around the front stagnation point. Further increase in Pe then caused the peak to split and shift away from the front stagnation point, because the convective effect became dominate.

(2) The collection efficiency raising factor increased with Pe but decreased as R increased. The effect of σ_g on was inconclusive because the effect of σ_g on was minor.

(3) The size distribution of deposited particles was tentatively concluded to be log-normal at small σ_g and Pe. In case of large σ_g and Pe the sample size distribution seemed slightly different from log-normal.

(4) As long as the Pe number and the interception parameter of a polydisperse aerosols were the same as those of a monodisperse system, the effects of polydispersity, namely σ_g, were slight and hard to detect. Unless a very large σ_g, say, $\sigma_g > 1.4$ or more, was used, it would be all right and more convenient to predict the behavior of a poly disperse system by a corresponding monodisperse one. In any case, a much larger sample size than 50 would be necessary to yield more definite conclusions on the effects of σ_g.

8.2 Recommended Future Work

An interesting extension of this work would be to determine the drag force acting on deposited particles, so that pressure drop across a fibrous filter mat might be obtained as a function of filtration time. To do this, a mathematical model that could estimate the drag force acting on dendrites of various configurations is required. Then all the data obtained in the present work might be fed to the pressure drop model, thus eliminating the necessity of simulating dendritic deposition again.

Pendse et al. (1981) determined experimentally the hydrodynamic drag force in a uniform flow field acting on a spherical body with samll particles attached to its surface. The experimental data were then used to substantiate the approximate force expressions derived from available theories for several types of collector-particle geometries. They also developed procedures for applying these results to the prediction of the pressure drop across a filter bed. Nevertheless, similar work on a cylindrical body has not yet appeared, and some simplified but reliable procedure must still be developed to treat dendrites of infinitely different configurations before the recommended furture work could be attempted.

APPENDICES

APPENDIX A

NOTATION

a	Constant used in eqn (6.6.4)
$a_k(\theta)$	Function defined by eqn (3.1.8)
A	Filtration area of a fibrous filter (cm^2)
$A(t)$	Fluctuating term used eqn (4.3.1) (cm/s)
b	Constant used in eqn (6.6.4)
$b_k(\theta)$	Functions defined by eqn (3.1.9)
B	Mobility of an aerosol particle (s/gm)
c	Particle concentration (g/cm^3)
c_0	Approaching mass concentration of aerosols
c_e	Effluent particle concentration (g/cm^3)
c_i	Influent particle concentration (g/cm^3)
C_s	Cunningham slip factor (-)
d, d_p	Diameter of an aerosol particle (μm, cm)
d_g, d_{pm}	Geometric mean diameter of an aerosol particle (μm)
D	Diffusion coefficient of aerosol particle (cm^2/s)
f_i	Observed frequencies of the i^{th} class
F_i	Theoretical frequencies of the i^{th} class
h	Thickness of filter mat (μm, cm)
H	Half height of generation plane (-)
$j_k^{(s)}$	Particle flux on the surface of a particle in the k^{th} layer of a dendrite due to the s component of flow ($s = r$, or θ)
k	Boltzmann's constant (erg/K)

K	Hydrodynamic factor
Kn_f	Fiber Knudsen coefficient, $= l/R_f$ (-)
Kn_p	Particle Knudsen coefficient, $= l/r_p$ (-)
l	Mean free path of an aerosol particle (μm)
L	Total length of all the fiber in unit volume of filter (μm, m)
L	Sample size (-)
m	Loaded mass of particles in a unit filter volume (gm/cm^3)
m_k	Expected particle number in the K^{th} layer of a dendrite
M(θ)	Integer function of θ defined by eqn (3.2.12)
M^{th}	Order of particle captured
n	Number of samples
n	Particle number concentration
n	Standard normal random vector $= (n_x, n_y, n_z)$
N	Member of a dendrite
N(τ, θ)	Frequency function of number of dendrites per unit length of fiber at time τ and angle θ, taking into account both sides of the fiber
p	Position vector (cm)
P	Dimensionless posion vector (-)
Pe	Peclet number (-)
r	Radial cylindrical coordinate
r	Position vector (cm)
r_p	Radius of an aerosol particle (μm)
R	Interception parameter (-)
R_f	Radius of fiber (μm), also Dimensionless radius of fiber (-)

R_c Radius of Kuwabara cell (μm), also

Dimensinless radius of Kuwabara cell (-)

$R^{(s)}_{i, j}$ Rato of increase of m_i by deposition on particles occupying the i_{th} layer due to the flow component in s direction (s = r, or θ)

s Standard deviation

t Time (s)

t Filtration time (s)

T Dimensionless time (-)

T Temperature (˙c)

u Velocity vector of fluid stream (cm/s)

u_s, U_s Superficial velocity through the filter (cm/s)

U_∞ Approaching velocity of fluid stream (cm/s)

U Dimensionless velocity vector of fluid stream (-)

v Velocity of aerosol particle (cm/s)

v Velocity vector of aerosol particle (cm/s)

w Relative velocity vector of particle to air (cm/s)

w_{pm} Mass of an average serosol particle, $d_p = d_{pm}$ (cm/s)

w_{cap} Accumulated mass of particles captured on the effictive fiber length (gm)

w_M Average mass of captured particle no. M on the effective fiber length (gm)

$w_{M, i}$ Mass of captured particle no. M of the Monte Carlo sample no. i on the effective fiber length (gm)

W Probability function defined by eqn (4.3.3)

W_M Average total mass of incoming particles generated on the entired generation plane between the M^{th} and the (M+1)-st captures on fiber section III (gm)

$W_{M, i}$ — Total mass of incoming particles generaled on the entired generation plane between the M^{th} and (M+1)-st captures on fiber section III of Monte Carlo sample no. i (gm)

$\bar{x}$ — Sample mean

X, Y, Z — Dimensionless spatial coordinates (-)

y — Half distance apart of the limiting trajectories (cm)

Z_1, Z_2, Z_3, Z_4, Z_5 — Dimensionless length of fiber section I, II III, IV, V respectively (-)

Greek Letters

α	Packing density of a filter (cm^3 fiber/cm^3 filter)
β	Reciprocal of relaxation time of a particle (s^{-1})
γ	Rate of particle opproaching a clean fiber per unit length
ε	Small positive number
θ	Angular cylindrical coordinate measured counter clockwise from the down stream stagnation point (deg.)
λ	Collection efficiency raising factor (cm^3/gm)
λ	Filter coeffieient (cm^{-1})
λ_0	Clean filter coefficient (cm^{-1})
η	Collection efficiency of a single fiber (-)
η_0	Collection efficiency of a clean single fiber (-)
μ	Sample mean
μ	Fluid viscosity (gm/cm.s)
ρ	Coordination number, that is, maximum number of particles in the (k + 1)-st layer which can be attached directly to the same particle of the k^{th} layer
ρ_f	Fiber density (gm/cm^3)
ρ_p	Particle density (gm/cm^3)
$\phi_{i,j}^{(s)}$	Fraction of γ colliding with a target particle in the i^{th} layer to become part of the j^{th} layer owing to the s component of flow (s = r, or θ)
σ	Standard deviation
σ	Standard deviation of normal standard random number n (cm)

σ_g	Gemetric standard deviation of particle diameter (cm)
τ	Dimensionless time (-)
τ	Time measure from beginning of aerosols flow through the filter (s)
τ	Relaxation time (s)
τ_f	Duration of filtration time (s)
τ_{ug}	Duration of period of unhindered dendrite growth (s)
ν	Degree of freedom
$\nu_0(\theta)$	Function of θ which depended on collection mechanism defined by Payatakes (1977)
$\chi(t;\tau,\theta)$	Dendritic age distribution function at time τ and angular position θ
χ^2	Chi-square statistic
ψ	Kuwabara stream function of flow around a fiber

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D	Diffusion
f	Fiber
I	Interception
i	i^{th} Monte Carlo sample
i	i^{th} step
g	Geometric
k	K^{th} layer of a dendrite
m	Mean, geometric mean
M	Dust loaded
M	M^{th} captured order
p	Particle
r	r direction
x	x direction
y	y direction
z	z direction
θ	θ direction
O	Initial
1, 2, 3, 4, 5,	Number of sections

APPENDIX B

RANDOM NUMBERS GENERATION

The effectiveness of the stochastic simulation is intimately connected with the quality of the random numbers being used. Appendix B summarizes the methods used in this investigation to generate random numbers, and gives listings of FORTRAN IV subroutines employed for generation of random numbers. Furthermore statistical tests of their properties have already been carried out but they are not reproduced here.

B.1 Methods of Generating Uncorrelated Random Numbers.

When a computer subprogram is used, as in this study, to generate a sequence of random numbers, each particular sequence will be completely defined by its starting value or seed. The term "pseudo-random numbers" is sometimes used to describe such numbers.

The algorithms used to generate the uncorrected uniform and normal random numbers in this study were developed by Pike and Hill (Collected Algorithms from CACM). The general procedure was : (1) select a nonrepetitive starting value, (2) generate a uniformly distributed random number lying between 0 and 1 and with a mean of 0.5, (3) if appropriate, use two uniform random numbers from (2) to generate a normally distributed random number with a mean of 0.0 and a standard deviation of 1.0, (4) repeat steps (2) and (3) until a sequence of random numbers have been obtained.

Pike and Hill recommended that the starting value should be an odd integer between 1 and 67, 108, 863. Subroutine START breaks up a starting value into two integral numbers in order to prevent overflowing without having to use double-precision integers on the IBM 370/138 computer. To generate several starting values for separate sequence of random numbers, the following procedure was adopted : (1) an odd number between 1 and 67, 108, 863 was selected from a random number table, (2) the number was used to generate a sequence of uniform random numbers, (3) subroutine RESTRT was used to transform each uniform random number in the sequence in to a new starting value later used.

To break up the cyclic nature of a very long sequence, a new starting value is used for each record of the Monte Carlo simulation.

B.2 Random Number Generators

B.2.1 Starting Value

```
      SUBROUTINE START(ISTRT)
C
C
C     SUBROUTINE START TRANSFORMS THE STARTING VALUE ISTRT SO THAT IT
C     MAY BE USED BY SUBROUTINE RANDOM OR SUBROUTINE RND WITHOUT
C     CAUSING OVERFLOWING OF THE INTEGER NUMBER
C     THE VALUE OF STRT MUST BE AN ODD NUMBER OBTAINED FROM A TABLE
C     OF RANDOM NUMBERS, PREFERABLY BETWEEN 42758321 AND 67108863
C
C
C     ISTRT = STARTING VALUE
C
```

```
      COMMON/CMN2/ RUNF,RRM,IY1,IY2
C
      I = 2*(ISTRT/2)+1
      IY2 = I/16384
      IY1 = I-16384*IY2
      RETURN
      END
```

```
      SUBROUTINE RESTRT
C
C
C     SUBROUTINE RESTRT YIELDS A NON-REPETITIVE STARTING VALUE FOR
C     THE RANDOM NUMBER GENERATORS
C
C

      COMMON/CMN2/ RUNF,RNRM,IY1,IY2
C
      I = 0
      J = 0
      DO 100 K = 1,3
C
      CALL RANDOM
C
      I = I+IY1
      J = J+IY2
  100 CONTINUE
      IY1 = MOD(123*I,16384)
      IY2 = MOD(789*J,4096)
      RETURN
      END
```

B.2.2 Uniform Random Number Generator

Subroutine RANDOM was used to generate a uniform random numbers, whose values lie between 0 and 1

```
      SUBROUTINE RANDOM
C
C
C     SUBROUTINE RANDOM GENERATES A UNIFORM RANDOM NUMBER WITH A MEAN
C     OF 0.5 AND A RANGE OF 0.0 TO 1.0
C     THE SEQUENCE OF RANDOM NUMBERS HAS A LIMITING CYCLE LENGTH OF
```

```
C
C
C        RUNF = UNIFORMLY DISTRIBUTED RANDOM NUMBER
C

         COMMON/CMN2/ RUNF,RNRM, IY1,IY2

C
         IY = 3125*IY1
         N = IY/16384
         IY1 = IY-16384*N
         IY = 3125*IY2+N
         IY2 = IY-4096*(IY/4096)
         RUNF = DFLOAT(16384*IY2+IY1)/67108864.0D0
         RETURN
         END
```

B.2.3 Normal Random Number Generator

Subroutine RND was used to generate a normal random number with a mean of 0.0 and a standard deviation of 1.0.

```
         SUBROUTINE RND

C
C
C        SUBROUTINE RND GENERATES A STANDARD NORMAL RANDOM NUMBER SUING
C        TWO UNIFORM RANDOM NUMBERS
C
C
C        RNRM = NORMALLY DISTRIBUTED RANDOM NUMBER
C

         COMMON/CMN2/ RUNF,RNRM,IY1,IY2

C
C        CALL RANDOM (FIRST TIME)
C
         IX = 3125*IY1
         N = IX/16384
         IX = IX-16384*N
         IY = 3125*IY2+N
         IY = IY-4096*(IY/4096)
         R = DFLOAT(16384*IY+IX)/67108864.0D0
         X = SQRT(-2.0*ALOG(R))
C
```

```
C
C       CALL RANDOM (SECOUND TIME)
C
        IX = 3125*IX
        N = IX/16384
        IT1 = IX-16384*N
        IY = 3125*IY+N
        IY2 = IY-4096*(IY/4096)
        R = DFLOAT(16384*IY2+IY1)/67108864.0D0
        RNRM = X*COS(6.283185*R)
        RETURN
        END
```