INTRODUCTION

In this thesis we shall assume a basic background of the general notions of algebra and especially the results on fields. One classical problem in algebra is finding solutions of polynomial equations and techniques are known that tell us when the solutions can be expressed in terms of radicals. Consequently, an immediate question presents itself: Can the solution of a differential equation be expressed in terms of integrals, exponentials of integrals and algebraic functions? A special case of this problem is the problem of determining whether the integral of an elementary function (a function involving the variable x together with repeated algebraic operations and the taking of exponentials and logarithms) is an elementary function. The set of elementary functions can be added and multiplied so they form a ring and it also has an operator defined on it, the differentiation operator. We call such an object a differential ring and the subject that studies such rings is called differential algebra.

It is our purpose now to introduce and to study only the basic concepts of differential algebra. Using this thesis as a basic background one can easily pursue more advanced questions in the field of differential algebra.

This thesis is organized into four chapters plus the list of references and appendix.

Chapter I is concerned with some elementary definitions and results in abstract algebra which are mostly well-known and are needed in the sequel.

Chapter II contains the basic concepts forming the foundation of differential algebra.

Chapter III Deals with the basis theorem, a very fundamental result in differential algebra. This theorem describes the structure of finitely generated differential rings and is as important to differential algebra as the Hilbert Basis theorem is to algebraic geometry.

Chapter IV is devoted to solving the problem of determining whether an elementary function has an elementary indefinite integral or not.

Conventions:

$\mathbb{Z}^+$ is the set of all positive integers

$\mathbb{Z}$ is the set of all integers

$\mathbb{R}$ is the set of all real numbers

$\mathbb{C}$ is the set of all complex numbers.