CHAPTER II

LITERATURE REVIEWS

2.1 The Sign Convention [2]

The sign convention for distances appearing in Fig. 2-1 is the same as that used in co-ordinate geometry. The distances l and l' (measured from the pole of the surface) are positive if measured to the right, r (the radius of curvature) is positive if measured to the right, and y, h, and h' (measured from the axis) are positive if measured upwards.

To determine the sign of u or u', we rotate the ray about B or B' through an acute angle u or u' toward the axis. A clockwise rotation defines a negative angle, and a counter-clockwise rotation a positive angle. The sign of α is similarly defined by a rotation from radius to axis. The sign of i or i' is determined by rotating about the point of incidence through the angle from ray to radius. A clockwise rotation defines a positive angle and a counter-clockwise rotation indicates a negative angle. The angles u, u', i, i', and α, are always acute.

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[2] R.S. Longhurst, Geometrical and Physical Optics (Hong Kong: The Continental Printing Co. Ltd., 1973), p. 9.

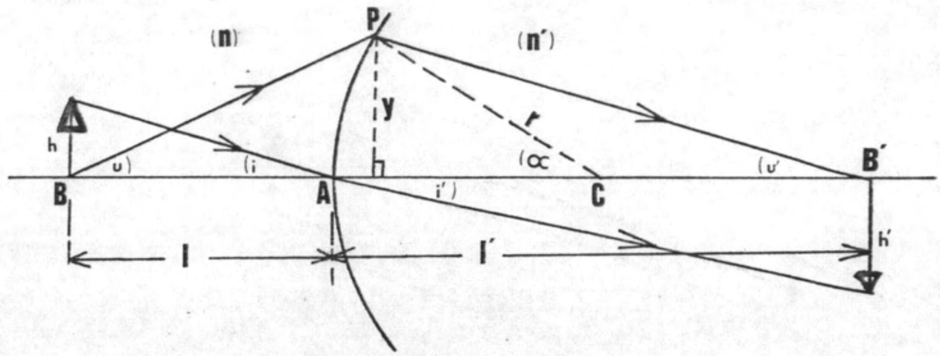

Fig. 2-1. Single refracting surface.

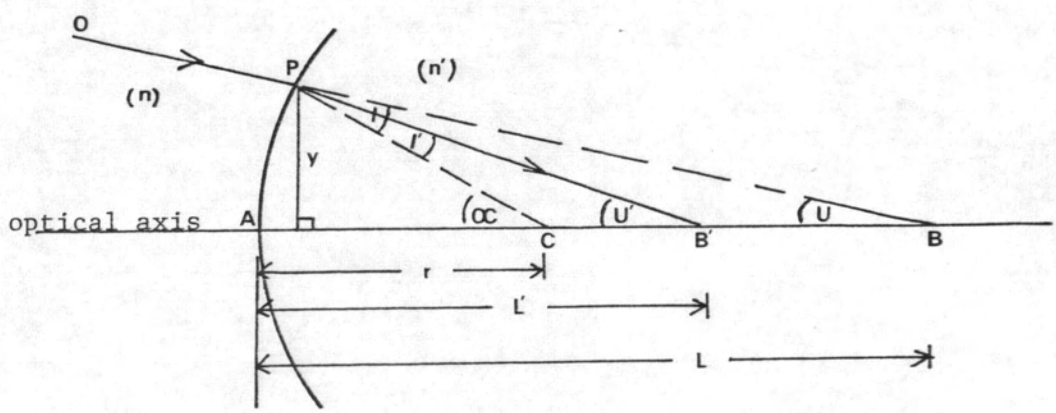

Fig. 2-2. Geometry for ray tracing with an oblique meridional ray.

Definitions of the various symbols in Fig. 2-1 are as follows:

l and l' are the object and image distances;

r is the radius of curvature;

y is the height of the point of incidence;

h and h' are object and image heights;

u and u' are the angles between the ray and the optical axis before and after refraction;

i and i' are angles of incidence and refraction;

and α is the angle subtended at C by the arc AP.

2.2 Principle of Ray Tracing [3,4]

Most of lens equations, e.g., Gaussian equation, Newtonian equation and Lensmaker's equation deal with Gaussian optics. In designing optical instruments it is necessary to determine the path of the light with a greater accuracy than that given by Gaussian optics. One may determine the path of light rays accurately by successive application of the law of refraction (or reflection). This method is known as Ray Tracing and is extensively employed.

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[3] M. Born, and E. Wolf, Principles of Optics (Oxford: Pergamon Press, Ltd., 1975), pp. 190-194.

[4] W.T. Welford, "Gaussian Optics and Aberrations", A Summer School for Non-Specialists in Optics (London: Imperial College Applied Optics Section, 1967), pp. 94-96.

The tracing of an oblique meridional ray is now considered. A meridional ray from an extra-axial object point is shown in Fig. 2-2.

First let A be the pole of the first surface of the system. The surface will be assumed to be a spherical refracting surface of radius r centred at a point C and which separates media of refractive indices n and n'.

An incident ray OP in the meridional plane is specified by the angle U which it makes with the axis at the point B. The distance L = AB. Let I be the angle between the incident ray and the normal, PC. The corresponding quantities relating to the refracted ray are denoted by primed symbols.

When both L and r are finite, and using the sign convention and applying the law of sines to ΔPCB, we have

$$\frac{\sin I}{L-r} = \frac{\mathrm{Sin} U}{r}$$

or $$\sin I = \frac{L-r}{r} . \sin U \qquad (2.1)$$

By Snell's law

$$n \sin I = n' \sin I'$$

or $$\sin I' = \frac{n \sin I}{n'} \qquad (2.2)$$

where, I and I' are angles of incidence and refraction;

n and n' are refractive indices in object and image spaces.

$$\text{Since} \quad \hat{PCB'} + U' + I' = 180^\circ$$

$$\text{and} \quad \hat{PCB} + U + I = 180^\circ$$

$$\text{we get} \quad \hat{PCB'} + U' + I' = \hat{PCB} + U + I$$

$$\text{or} \quad U' = \hat{PCB} + U + I - \hat{PCB'} - I'$$

$$\text{Since} \quad \hat{PCB'} = \hat{PCB}$$

$$\text{we get} \quad U' = U + I - I' \qquad (2.3)$$

where, U and U' are the angles between the ray and the optical axis before and after refraction.

Applying the law of sines to $\Delta PCB'$, we have

$$\frac{\sin I'}{L' - r} = \frac{\sin U'}{r}$$

$$\text{or} \quad L' = \frac{r \sin I'}{\sin U'} + r \qquad (2.4)$$

where, L and L' are the object and image distances.

Equations (2.1), (2.2), (2.3) and (2.4) are called the refraction equations.

In practice, the quantities L and U which specify the incident ray OPB are assumed to be known. It is then necessary to calculate L' and U', which specify the refracted ray PB', by successive application of the refraction equations. The refracted ray PB' now becomes the incident ray for the second surface, so that L' is replaced by L_1' and U' is replaced by U_1'. We now let L_2 be the distance from the pole of the second surface and U_2 be the angle that the incident ray of the second surface makes with the axis.

The transfer equations are

$$L_2 = L_1' - d \tag{2.5}$$

$$U_2 = U_1' \tag{2.6}$$

where, d is the distance between the poles of the two surfaces as shown in Fig. 2-3.

2.3 The Paraxial Region

The paraxial region is a region containing the rays sufficiently close to parallelism with the optical axis of a system that, for the purposes of the computation being made, the angles between the rays and the optical axis, i, are so small that sin i may be taken as equal to i. It is customary to denote the quantities which refer to the paraxial region by small letters. So that equations (2.1), (2.2), (2.3), (2.4), (2.5), and (2.6) become

$$i = \left(\frac{l - r}{r}\right)u \tag{2.7}$$

$$i' = \frac{ni}{n'} \tag{2.8}$$

$$u' = u + i - i' \tag{2.9}$$

$$l' = \left(\frac{l'}{u'}\right) r + r \tag{2.10}$$

$$l_2 = l_1' - d \tag{2.11}$$

$$u_2 = u_1' \tag{2.12}$$

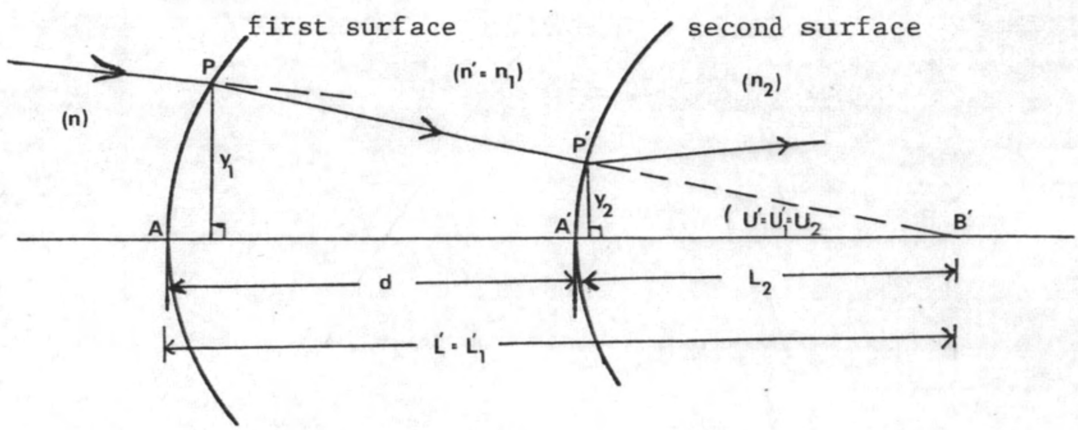

Fig. 2-3. Oblique meridional ray trace through two successive refracting surfaces.

According to Fig. 2-2, we have

$$\alpha = \frac{y}{r} = yc \qquad (2.13)$$

$$u = \frac{y}{l} \qquad (2.14)$$

$$u' = \frac{y}{l'} \qquad (2.15)$$

where, α is the angle subtended at C by the arc AP;

y is the height of the point of incidence;

$c = \frac{1}{r}$ is the curvature of surface.

For a single surface, we have

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n'}{f'} = -\frac{n}{f} = K = (n' - n)c \qquad (2.16)$$

where, K is the power;

f and f' are the first and second focal lengths.

Multiplying (2.16) by y, one has

$$\frac{yn'}{l'} - \frac{yn}{l} = y(n' - n)c$$

$$u'n' - un = \alpha(n' - n)$$

It is clear that, $n(\alpha - u) = n'(\alpha - u') = A \qquad (2.17)$

where, A is called the paraxial refraction invariant.

According to Fig. 2-3, we have

$$y_1 = l_1'u' \qquad (2.18)$$

$$y_2 = l_2u' \qquad (2.19)$$

Multiplying (2.11) by u', we have

$$l_2 u' = l_1' u' - du'$$

or

$$y_2 = y_1 - du' \qquad (2.20)$$

Equation (2.9) can now be expressed in the new form:

$$\begin{aligned} u' &= u + i - i' \\ &= u + \left(\frac{l - r}{r}\right)u - i' \\ &= \frac{lu}{r} - i' \\ &= \frac{lu}{r} - \frac{ni}{n'} \\ &= \frac{lu}{r} - \frac{n}{n'}\left(\frac{l - r}{r}\right)u \\ &= \frac{n'lu - nlu + nur}{n'r} \\ &= \frac{n'y - ny + nur}{n'r} \\ &= \frac{y}{n} - \frac{n}{n'}\left(\frac{y}{r} - u\right) \\ &= \alpha - \frac{n}{n'}(\alpha - u) \end{aligned}$$

Thus

$$u_1' = u_2 = u' = \alpha - \frac{A}{n'} \qquad (2.21)$$

Equations (2.13), (2.17), (2.20) and (2.21) are called the paraxial ray tracing equations.

The central ray of an oblique pencil enters the entrance pupil at the centre, then passes through the centre of the aperture stop and finally emerges from the centre of the exit pupil. In Gaussian approximation; this ray is called a principal ray. The principal ray can be traced in the same manner as the paraxial ray. The symbols for the principal ray are denoted by a bar. The principal ray tracing equations are

$$\bar{\alpha} = \bar{h}c \tag{2.22}$$

$$B = n(\bar{\alpha} - \bar{u}) \tag{2.23}$$

$$\bar{y}_2 = \bar{y}_1 - d\bar{u}' \tag{2.24}$$

$$\bar{u}' = \bar{\alpha} - \frac{B}{n'} \tag{2.25}$$

where, B is called the principal refraction invariant.

2.4 Thick Lens and Thin Lens

A thick lens consisting of surfaces whose radii are r_1 and r_2 and whose poles A and B are distance d apart, n is the refractive index of a lens (Fig. 2-4).

The positions of the principal points of a moderately thick lens in air and its power are

$$AP = -\frac{dr_1}{n(r_2 - r_1)} \tag{2.26}$$

$$BP' = -\frac{dr_2}{n(r_2 - r_1)} \tag{2.27}$$

$$K = \frac{1}{f'} = -\frac{1}{f} = \frac{n(n-1)(r_2 - r_1) + d(n-1)^2}{nr_1 r_2} \tag{2.28}$$

While for thin lens in air, one has

$$K = \frac{1}{f'} = -\frac{1}{f} = \frac{1}{l'} - \frac{1}{l} = (n-1).\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \tag{2.29}$$

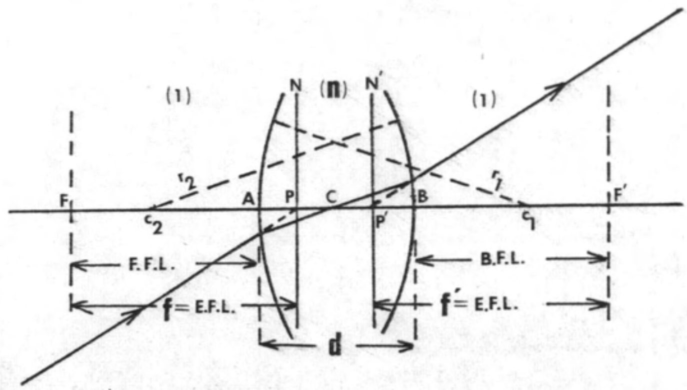

Fig. 2-4. Thick lens.

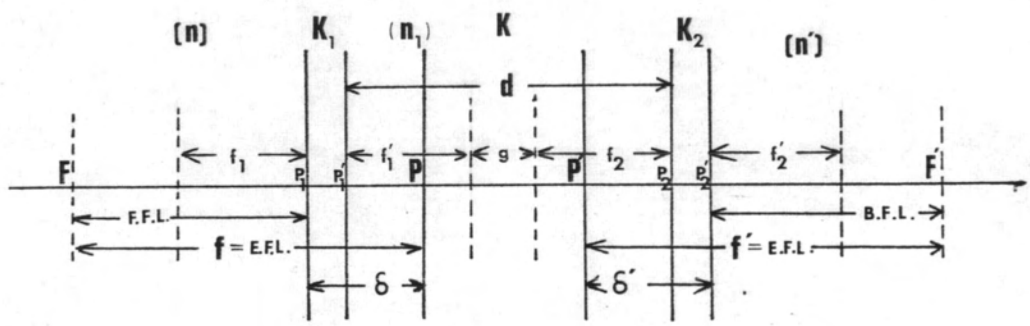

Fig. 2-5. Combination of two systems.

2.5 The Combination of Two Systems

A combined system can be obtained by putting a lens of power K_2 after a lens of power K_1. The distance $P_1'P_2$ between adjacent principal planes is called d. If n is a refractive index in object space, n_1 is a refractive index in the space between the systems of power K_1 and K_2, and n' is a refractive index in image space, the power K of the combined system (Fig. 2-5) is given by

$$K = K_1 + K_2 - \frac{d}{n_1} \cdot K_1 K_2 \qquad (2.30)$$

$$\delta = P_1P = \frac{nn'df_1'}{n'n_1f_1' + n_1^2f_2' - n_1n'd} \qquad (2.31)$$

$$\delta' = P_2'P' = -\frac{n'df_2'}{n'f_1' + n_1f_2' - n'd} \qquad (2.32)$$

$$P_1F = -\frac{nf_1'(n_1f_2' - n'd)}{n_1(n'f_1' + n_1f_2' - n'd)} \qquad (2.33)$$

$$P_2'F' = \frac{n'f_2'(f_1' - d)}{n'f_1' + n_1f_2' - n'd} \qquad (2.34)$$

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Hence $$d = f_1' + g - f_2 \qquad (2.35)$$

where, f, f' are the front and back equivalent focal lengths (E.F.L.s) of the combined system;

f_1, f_1' are the front and back focal lengths of the first system;

f_2, f_2' are the front and back focal lengths of the second system;

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P_1F, $P_2'F'$ are the front and back focal lengths (F.F.L. and B.F.L.) of the compound system;

P_1P is the distance between the first principal plane of the first component and the first principal plane of the system;

$P_2'P'$ is the distance between the second principal plane of the second component and the second principal plane of the system;

g is the optical tube length (measured from the back focus of the objective to the front focus of the eyepiece).

For two thin lenses in contact, equation (2.30) becomes

$$K = K_1 + K_2 \tag{2.36}$$

where, $d = 0$

2.6 The Principle of the Compound Microscope[5]

We now consider the case where the final image is formed at infinity as shown in Fig. 2-6. Then the magnification formula can be interpreted as :

$$M = -\left(\frac{g}{f_o'}\right).\left(\frac{D_v}{f_e'}\right) \tag{2.37}$$

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[5] R.S. Longhurst, Geometrical and Physical Optics (Hong Kong: The Continental Printing Co. Ltd., 1973), pp. 60-61

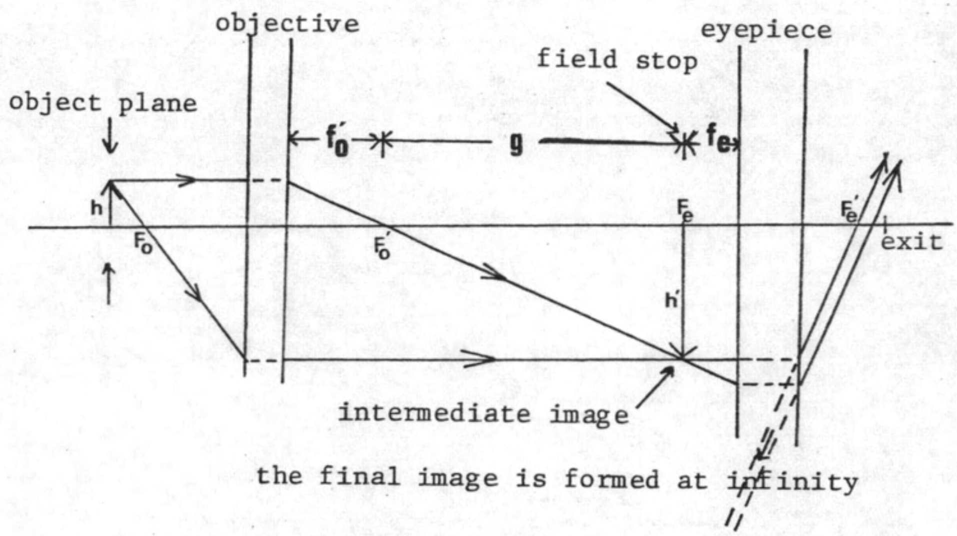

Fig. 2-6. Ray diagram of a compound microscope.

where, M is the magnification of a microscope;

g is the optical tube length and it is usually about 160 mm in modern instruments;

D_v is the least distance of distinct vision = 250 mm;

f'_o, f'_e are the objective focal length and the eyepiece focal length;

$K_o = \frac{1}{f'_o}$ is the power of an objective;

$K_e = \frac{1}{f'_e}$ is the power of an eyepiece;

$(\frac{g}{f'_o})$ is the objective magnification;

$(\frac{D_v}{f'_e})$ is the eyepiece magnification.

2.7 Resolving Power of a Microscope Objective due to Diffraction[6]

We now consider the case of low magnifying power.

We have

$$S = \frac{1.22\ \lambda}{2n \sin u} \tag{2.38}$$

$$N.A. = n \sin u \tag{2.39}$$

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[6] F.A. Jenkins, and H.E. White, Fundamentals of Optics (Tokyo: McGraw-Hill Kogakusha, Ltd., 1976), p. 333.

where, S is the resolving power of an aberration-free objective;

u is the angular semi-aperture of an objective in the object space;

λ is the wavelength of light used ;

n is the refractive index of a medium in the object space;

N.A. is the numerical aperture of an objective.

2.8 The Achromatic Doublet

Fraunhofer type lens is one of the simplest achromatic objective lenses which requires the use of two lenses in contact, one made of flint glass and the other of crown glass (Fig. 2-7). The crown glass lens, which has a large positive power, has the same dispersion as the flint glass lens, for which the power is smaller and negative. The combined power is therefore positive, while the dispersion is neutralized. This therefore causes all colours to focus approximately at the same point. The peak of the visual brightness curve will occurs close to the sodium yellow (D) line.[7]

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[7] F.A. Jenkins, and H.E. White, Fundamentals of Optics (Tokyo: McGraw-Hill Kogakusha, Ltd., 1976), p. 177.

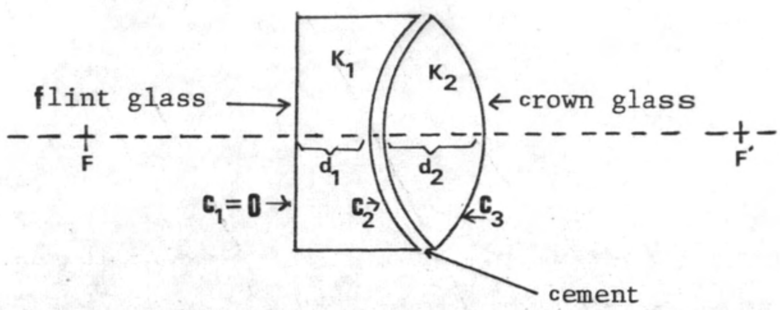

Fig. 2-7. Fraunhofer achromatic doublet.

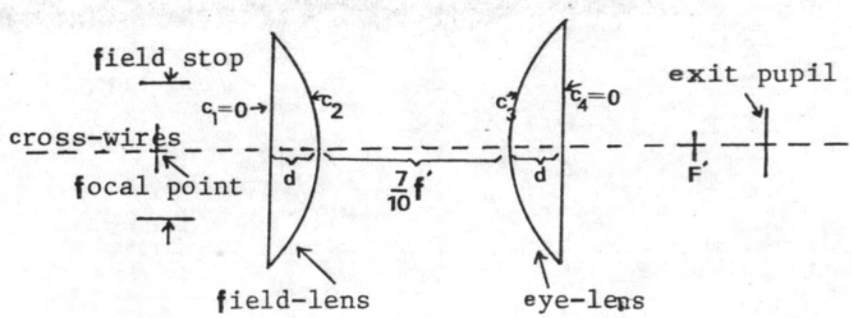

Fig. 2-8. Ramsden eyepiece.

For the case of two thin lenses in contact, the achromatic lens is corrected for only two wavelengths, the hydrogen blue (F) line and the hydrogen red (C) line. The two lines will be in focus at the same point. One then has

$$\nu = \frac{n_D - 1}{n_F - n_C} \tag{2.40}$$

$$\frac{K_1}{K_2} = -\frac{\nu_1}{\nu_2} \tag{2.41}$$

$$\text{total} \quad K = K_1 + K_2 \tag{2.42}$$

$$\text{and} \quad K = (n_D - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{2.43}$$

Applying equations (2.41), (2.42), we have

$$K_1 = -\frac{K\nu_1}{(\nu_2 - \nu_1)} \tag{2.44}$$

$$K_2 = \frac{K\nu_2}{(\nu_2 - \nu_1)} \tag{2.45}$$

where, K_1, K_2 refer to the power of the flint glass and the crown glass components;

n_D is the refractive index for the sodium yellow (D) line;

n_F is the refractive index for the hydrogen blue (F) line;

n_C is the refractive index for the hydrogen red (C) line;

ν_1 is the nu value of flint glass;

ν_2 is the nu value of crown glass.

2.9 Ramsden Eyepiece

This system consists of two plano-convex lenses which are usually made of the same kind of glass and have equal focal lengths. The lens nearer to the eye is called the eye lens, while the lens nearer to the objective is called the field lens (Fig. 2-8). Ramsden type eyepieces are expected to correct for lateral colour and coma,[8] provided the lenses separation is equal to the focal length of either component. Since the first focal plane of the system coincides with the field lens, a reticle or cross hairs must be located there. Under some conditions this is considered desirable. However any dust particles on the lens surface would also be seen in sharp focus. To overcome this difficulty, the lenses are usually moved a little closer together, this moving the focal plane forward causes some sacrifice in lateral achromatism. The separation of $\frac{7}{10}$ of the focal length is selected to design lenses.[9]

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[8] F.A. Jenkins, and H.E. White, Fundamentals of Optics (Tokyo: McGraw-Hill Kogakusha, Ltd., 1976), p. 206.

[9] A.E. Conrady, Applied Optics and Optical Design (London: Oxford University Press , 1929), part one, p. 497.

2.10 Aberration Types[10]

The well-known method in measurement of wavefront aberrations is the method of H.H. Hopkins.[11] We now define $W(\sigma, r', \phi)$ as the wave-aberration function, which can be expressed in a power series in σ^2, r'^2 and $\sigma r' \cos \phi$ where (r', ϕ) are the polar coordinates of a pupil and σ is the size of the field.

Fig. 2-9 shows the reference sphere (be a form of corrected wavefront) in the image space, where E' is the centre of the exit pupil and O' is the Gaussian image of an extra-axial object point at a height h' from the optical axis. B' is the paraxial image on the optical axis.

The wavefront aberration of an element of the wavefront associated with the ray through R' will depend on the aperture ρ, and on the azimuth ϕ as measured from the tangential plane E'B'O'.

ρ_{max} is the radius of the exit pupil and h'_{max} is the off-axis image height at the edge of the field of view of the instrument. We now define a new variable

$$r' = \rho/\rho_{max} \qquad (2.46)$$

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10 W.T. Welford, "Gaussian Optics and Aberrations" A Summer School for Non-Specialists in Optics (London: Imperial College Applied Optics Section, 1967), pp. 99-101.

11 H.H. Hopkins, Wave Theory of Aberrations (New York: Oxford University Press , 1950), pp. 50-55.

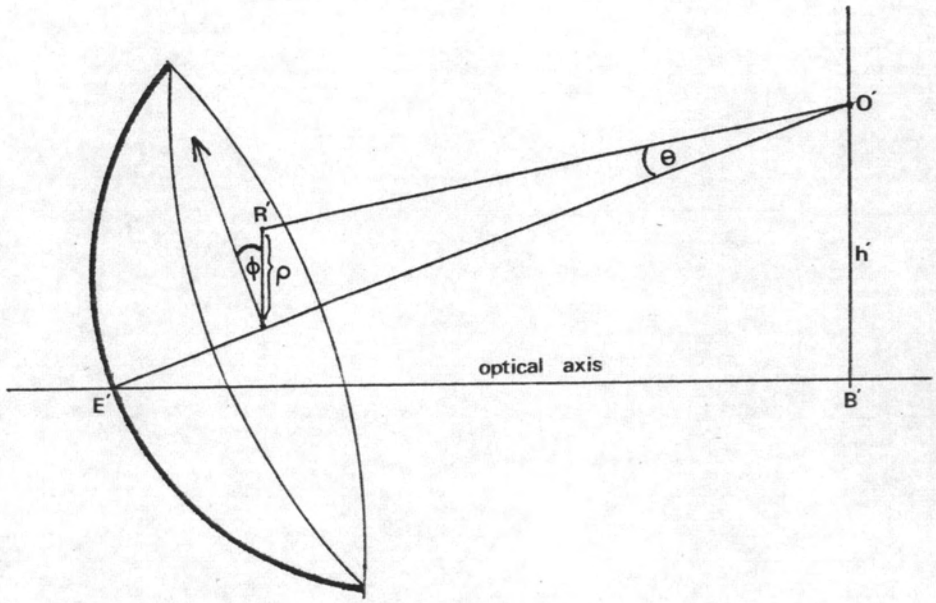

Fig.2-9. Wave in image space.

as being the fractional aperture, and

$$\sigma = h'/h'_{max} \qquad (2.47)$$

as being the fractional field size.

If consider the reference spheres having centres at the Gaussian image points and $W(\sigma, r', \phi)$ being zero for the principal rays, the aberration function becomes

$$W(\sigma, r', \phi) = {}_0W_{40}\, r'^4 + {}_1W_{31}\sigma r'^3\cos\phi + {}_2W_{22}\sigma^2 r'^2\cos^2\phi + {}_2W_{20}\sigma^2 r'^2 + {}_3W_{11}\sigma^3 r'\cos\phi \qquad (2.48)$$

where, ${}_0W_{40}$ is the primary spherical aberration coefficient;

${}_1W_{31}$ is the coma coefficient;

${}_2W_{22}$ is the astigmatism coefficient;

${}_2W_{20}$ is the field curvature coefficient;

${}_3W_{11}$ is the distortion coefficient.

We now consider the point of incidence at the refracting surface of a ray associated with the wavefront proceeding to be the focus in the image plane. The aberrations are obtained by the geometrical method.[12] The total wavefront aberration of a system becomes

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[12] H.H. Hopkins, Wave Theory of Aberrations (New York: Oxford University Press , 1950). pp. 76-86.

$$W = \frac{1}{8} S_I r'^4 + \frac{1}{2} S_{II} r'^3 \cos\phi + \frac{1}{2} S_{III} r'^2 \cos^2\phi$$

$$+ \frac{1}{4}(S_{III} + S_{IV}) r'^2 + \frac{1}{2} S_V r' \cos\phi \qquad (2.49)$$

where, the five Seidel sums are

$$S_I = \sum_1^k A^2 y \Delta\left(\frac{u}{n}\right) \qquad (2.50)$$

$$S_{II} = \sum_1^k AB y \Delta\left(\frac{u}{n}\right) \qquad (2.51)$$

$$S_{III} = \sum_1^k B^2 y \Delta\left(\frac{u}{n}\right) \qquad (2.52)$$

$$S_{IV} = \sum_1^k H^2 P \qquad (2.53)$$

$$S_V = \sum_1^k \frac{B}{A}\left[H^2 P + B^2 y \Delta\left(\frac{u}{n}\right)\right] \qquad (2.54)$$

The various quantities in the sums are defined as

k = number of surfaces;

A = the refraction of the paraxial ray;

$= ni = n(\alpha - u)$ (2.55)

B = the refraction of the principal ray;

$= n\bar{i} = n(\bar{\alpha} - \bar{u})$ (2.56)

H = the transverse invariant;

$= nuh$ (2.57)

$P = -\frac{1}{r}\Delta\left(\frac{1}{n}\right)$ (2.58)

$\alpha = \frac{y}{r}$ (2.59)

$$u = \frac{y}{l} \qquad (2.60)$$

$$\Delta\left(\frac{u}{n}\right) = \frac{u'}{n'} - \frac{u}{n} \qquad (2.61)$$

$$\Delta\left(\frac{1}{n}\right) = \frac{1}{n'} - \frac{1}{n} \qquad (2.62)$$

h = object height;

y = the incidence height at the given surface of a paraxial ray;

r = the radius of curvature of the refracting surface;

n, n' = the refractive indices in the object and image spaces;

$i, \bar{I}$ = the angles of incidence of the paraxial and principal rays;

u, u' = the angles between the ray and the optical axis before and after refraction;

l, l' = the object and image distances.

Therefore if we ignore σ and compare equations (2.48) w.r.t. (2.49), we get

$${}_0W_{40} = \frac{1}{8} S_I \qquad (2.63)$$

$${}_1W_{31} = \frac{1}{2} S_{II} \qquad (2.64)$$

$${}_2W_{22} = \frac{1}{2} S_{III} \qquad (2.65)$$

$${}_2W_{20} = \frac{1}{4} (S_{III} + S_{IV}) \qquad (2.66)$$

$${}_3W_{11} = \frac{1}{2} S_V \qquad (2.67)$$

2.11 Tolerance Conditions for Primary Aberrations[13]

The system can be regarded as being satisfactory, if the aberrations of the optical system are within the tolerance conditions listed in Table 2-1.

Type of Aberration	Tolerance condition for wavelength λ	Tolerance condition for mercury green
Spherical Aberration	$\lvert {}_0W_{40} \rvert \leq 0.94\,\lambda$	.000513 mm.
	$S_I = 8\,{}_0W_{40}$	.004104 mm.
Coma	$\lvert {}_1W_{31} \rvert \leq 0.60\lambda$	.000328 mm.
	$S_{II} = 2\,{}_1W_{31}$	.000656 mm.
Astigmatism	$\lvert {}_2W_{22} \rvert \leq 0.35\lambda$	.000191 mm.
	$S_{III} = 2\,{}_2W_{22}$	.000382 mm.
where, $\lambda = 5{,}461\ \mathring{A}$; $1\mathring{A} = 10^{-7}$ mm.		

Table 2-1. Tolerance conditions.

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[13] M. Born, and E. Wolf, Principles of Optics (Oxford: Pergamon Press, Ltd., 1975), p. 472.

2.12 Calculating the Seidel Aberration Terms

Suppose we now trace a paraxial ray coming from the axial object point and which touches the rim of the aperture stop for a given system and a principal ray, coming from the nominal edge of the field and which passes through the center of the stop.

At each surface, the values of y, u, u' and A can be found for the paraxial ray, and $\bar{y}$, $\bar{u}$, $\bar{u}'$ and B can be found at each surface for the principal ray. The Lagrange invariant H, which connects the two rays, can be determined. The five Seidel sums can be calculated from equations (2.50), (2.51), (2.52), (2.53) and (2.54).

2.13 Bending

After tracing the rays through the optical system, both paraxial ray and principal ray, we obtain only one set of the five Seidel sums. No conclusion can be made about the least aberration of the optical system if there is only one set of the five Seidel sums even though the set satisfies the tolerance condition. The bending technique is used to obtain the other sets of the five Seidel sums, which would make possible comparison among them for the least aberration set.

According to this technique all of the values of α and A are altered while the first or last values of $\bar{h}$, $\bar{u}$, h and u are kept constant.

By tracing the first rays through each surface of the optical system, we have

α_1, A_1 for the first surface

α_2, A_2 for the second surface

α_3, A_3 for the third surface

.................., etc.

If we let x be any suitable constant, the addition or subtraction of $Z = (x \% \text{ of } A_1)$ into the values of α and A of each surface for the second rays tracing gives the following :

$\alpha_1' = \alpha_1 \pm Z$, $A_1' = A_1 \pm Z$ for the first surface

$\alpha_2' = \alpha_2 \pm Z$, $A_2' = A_2 \pm Z$ for the second surface

$\alpha_3' = \alpha_3 \pm Z$, $A_3' = A_3 \pm Z$ for the third surface

.........................., etc.

For the third and subsequent rays tracing, x is replaced by x_1, x_2, x_3,... Z then is replaced by Z_1, Z_2, Z_3,.....

Letting $x_1 - x = x_2 - x_1 = x_3 - x_2 = \ldots$ be any suitable constant, we have for the third rays tracing

$\alpha_1'' = \alpha_1 \pm Z_1$, $A_1'' = A_1 \pm Z_1$ for the first surface

$\alpha_2'' = \alpha_2 \pm Z_1$, $A_2'' = A_2 \pm Z_1$ for the second surface

$\alpha_3'' = \alpha_3 \pm Z_1$, $A_3'' = A_3 \pm Z_1$ for the third surface

.........................., etc.

where, $Z_1 = (x_1\%$ of $A_1)$. The quantity $Z_2 = (x_2\%$ of $A_1)$ is used in the fourth rays tracing and so on. Thus, the data for use in calculating the five Seidel sums and radii of curvature become known. The graph of spherical aberration and coma versus the radii of curvature are constructed in the manner shown in Fig.2-10.

We now choose as the best radius of curvature, the radius in the graph which results in the least aberrations under the tolerance condition. The lens design is thus determined.

2.14 The Testing of Optical Surfaces with Newton's Rings[14]

The surface of an optical part which is being ground to some desired curvature may be compared with that of a correct surface by bringing the two into contact and then observe the interference fringes. For example, if an optical flat surface is placed in contact with a shallow convex spherical surface, a thin air film of varying thickness results (see Fig. 2-11). This film is

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[14] R.S. Longhurst, Geometrical and Physical Optics (Hong Kong: The Continental Printing Co. Ltd., 1973). p. 152.

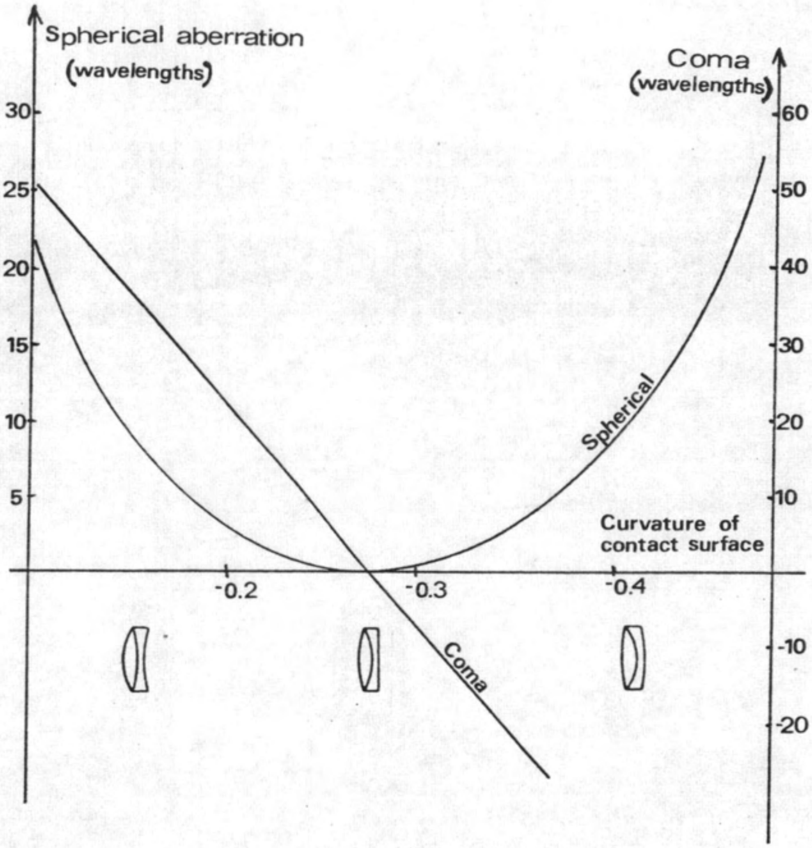

Fig.2-10. Spherical aberration and coma for a cemented doublet.

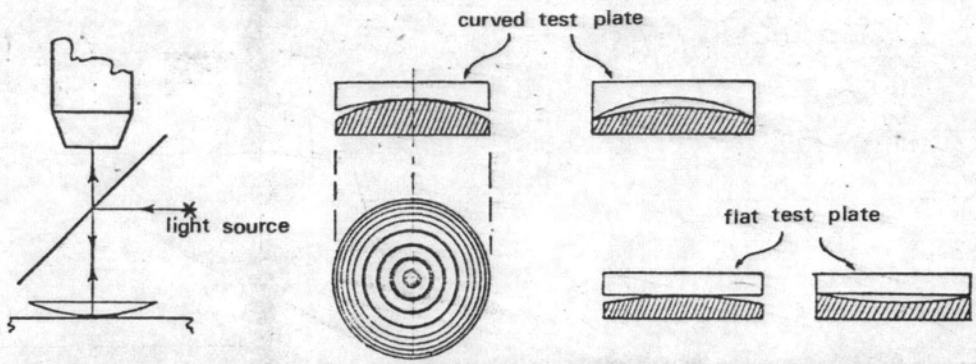

Fig.2-11. Observation of Newton's rings.

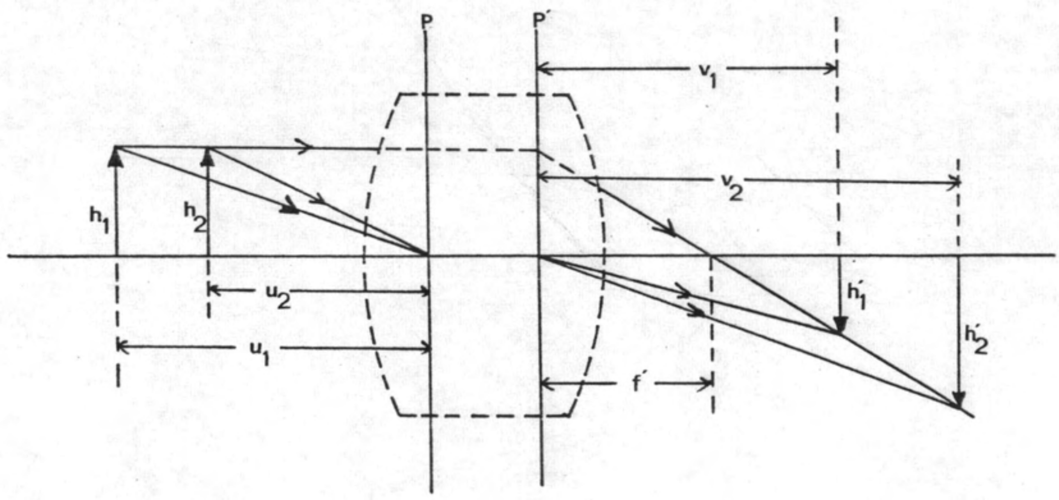

Fig.2-12. The Gaussian diagram of thick lens.

symmetrical about the point of contact. The localized interference fringes, which are the contours of the air film thickness, become concentric circles centred on the point of contact. These fringes were first observed by Newton and are known as Newton's rings. If a plane surface (an "optical flat") is desired, a glass plate whose lower surface is perfectly flat is placed over the surface to be tested. If both surfaces are perfectly flat, the entire area of contact will be dark, or, if contact between them is made at one edge, a series of straight interference fringes, parallel to the line of contact, will be observed. If the surface being ground is not a plane, the interference bands will be curved. By noting the shape and separation of the fringes, the departure of the surface from the desired form can be determined.

We now consider the case of reflected light. If r is the radius of curvature of the spherical surface and d is the thickness of the air film a distance R from the point of contact, we find

$$r = \frac{R^2 + d^2}{2d}$$

If R is large $d^2 \ll R^2$, then we find

$$d = \frac{R^2}{2r} \qquad (2.68)$$

Because the film is thin, any departure from normal incidence can be ignored. Since film is of air, we have $n = 1$. Remembering that there is a phase change of π radian at the lower surface, the conditions for a maximum or minimum are :

for a minimum $2d = p\lambda$,

for a maximum $2d = (p+\frac{1}{2})\lambda$.

Thus the radii of the bright and dark fringes are given [from (2.68)] by

(dark) $$R^2 = pr\lambda \qquad (2.69)$$

(bright) $$R^2 = (p+\frac{1}{2})r\lambda \qquad (2.70)$$

where, $p = 0,1,2,3,...$

λ = the wavelength of light used.

If λ is known, the radius of curvature of the spherical surface can be determined by measuring the radii of the rings.

2.15 Focal Length-Magnification Method[15]

This method can be particularly useful for measurements of focal length on compound lens systems of all kinds. The theory will be made clear from the Gaussian diagram of Fig. 2-12 in which an object placed at h_1 will be imaged at h_1' and the magnification thus produced will be $m_1 = h_1'/h_1 = (v_1-f')/f'$. Similarly, if the object is moved nearer to the lens so that the object distance is now u_2 the image will be formed at a distance v_2 and the magnification will be $m_2 = h_2'/h_1 = (v_2-f')/f'$; by combining the two equations we get

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[15] B.K. Johnson, Optics and Optical Instruments (New York: Dover Publications, Inc., 1960), pp. 31-32.

$$f' = \frac{v_2 - v_1}{m_2 - m_1} \tag{2.71}$$

Utilizing this formula in practice, a millimetre scale on glass is placed on one side of the lens at h_1 and its image is measured by means of a micrometer eyepiece situated at h_1'. The magnification may thus be obtained; this is repeated for a second position of object and image.

2.16 Magnifying Power of a Microscope[16]

The magnifying power of a microscope can be found by measuring the focal lengths of the objective and the eyepiece and applying equation (2.37). It is possible to measure the magnifying power directly by the method illustrated by Fig. 2-13. A transparent scale S is observed through the microscope via a semi-reflecting plate, (a cover glass forms a suitable reflector). Assuming that the image is brought to the least distance of distinct vision, i.e. 25 cm., a sheet of paper AB is placed a similar distance from the eye. The image S' as seen on AB is then traced with a pencil and the sizes of S' and S compared to give the magnifying power.

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[16] R.S. Longhurst, Geometrical and Physical Optics (Hong Kong: The Continental Printing Co. Ltd., 1973), p. 81.

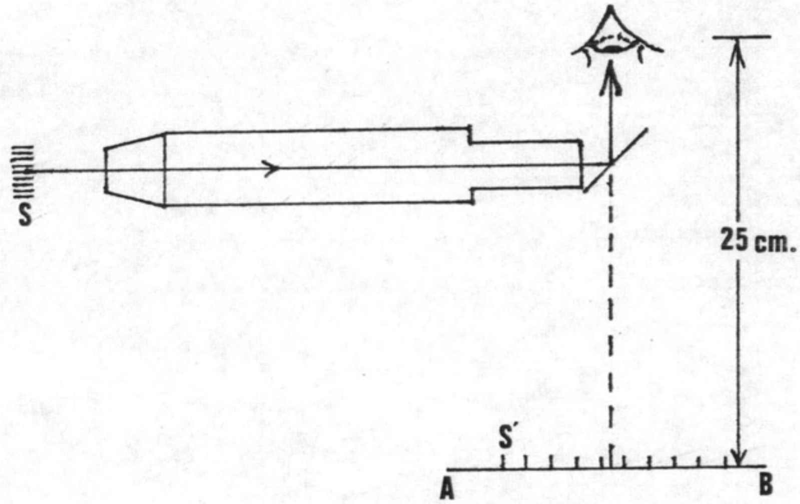

Fig.2-13. Magnifying power of a microscope.

2.17 Measurement of the Actual Resolving Power in Monochromatic Light

A practical method for measuring of the actual resolving power of a microscope objective is the reduced magnification technique. A microscope under test is fitted with a vertical stand on an optical bench. A standard scale with 10 μm division is used as the object and is illuminated with a sodium light source. A low power objective is then introduced between the microscope under test and the standard scale. The objective forms the image of the standard scale at reduced size in the object plane of the microscope under test. The objective and the microscope are shifted on the optical bench until the microscope can just resolve the image. The microscope under test is then replaced by a higher resolving power commercial microscope to see if the reduced image of the standard scale is actually resolved. The eyepiece of the microscope under test only magnifies the resolved image, which is formed by the microscope objective under test, bigger than the resolution limit of eye. At this position the magnification of the objective is found to be M, by using a millimetre scale in place of a standard scale and its image is measured by means of a micrometer eyepiece situated at the object plane of the microscope under test. The resolving power of the microscope objective under test is therefore 10M μm.

2.18 The " Star " Test Method[17]

The " Star " test is a method for evaluating the physical property of objectives and eyepieces. Submicroscopic pinholes in an aluminium film deposited on glass by evaporation act as point objects or " Stars ". A mercury vapor lamp with a green filter and a white light are used alternatively as sources.

The testing microscope which is composed of the objective under test and a very good selected eyepiece of the same design is tested first. A second test involves using a very good selected objective of the same design and the eyepiece under test.

The images of the " Stars " object form the " Airy disc " of the testing microscope under monochromatic light. If at the axial image point the outer fringes at outside focus and inside focus are alike, then lenses are not considered to have any spherical aberration. At the extra-axial image point, coma gives an unsymmetrical side distribution of light at outside and inside focus; while astigmatism gives a radial or tangential line as shown in Fig. 2-24 and Fig. 2-15.

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[17] L.C. Martin, An Introduction to Applied Optics (London: Sir Isaac Pitman & Sons, Ltd., 1932), vol.2, pp. 241-248.

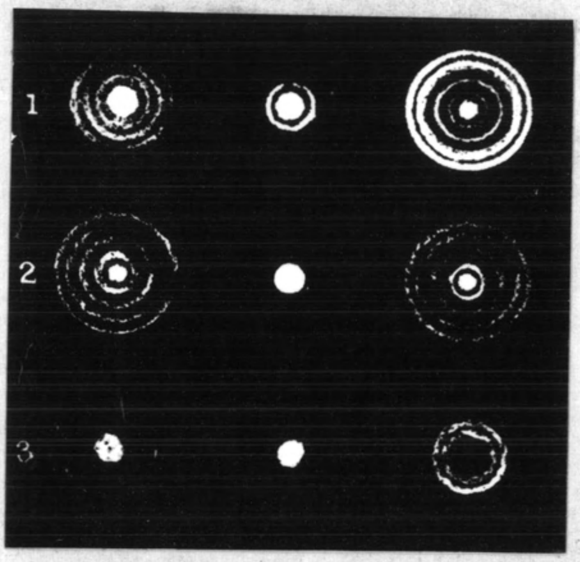

Fig.2-14. Typical star figures for corrected lens and for spherical aberration.
Row 1. Spherical under-correction: Outside focus; at focus; inside focus.
Row 2. Corrected lens: Outside focus; at focus; inside focus.
Row 3. Mid zone with short focus: Outside focus; at focus; inside focus. (Zonal spherical aberration)

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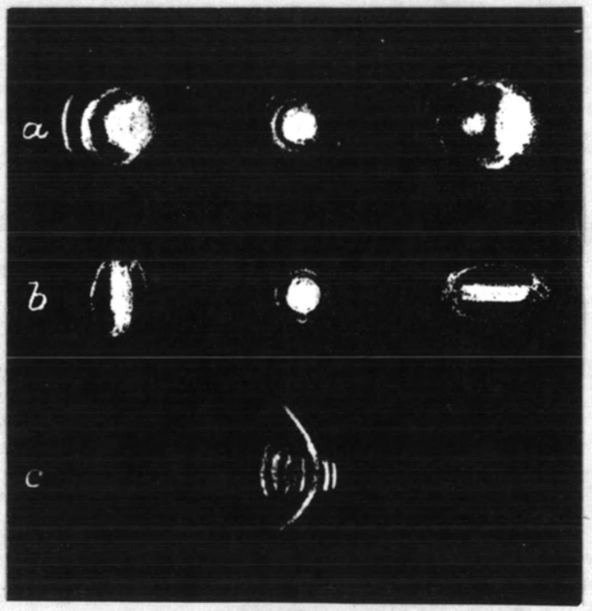

Fig.2-15. Typical star figures for coma and astigmatism.
a = Coma, extra-axial and focal.
b = Astigmatism, extra-axial and focal.
c = Heavy coma and astigmatism together.
The centre image represents the best focus appearance in both (a) and (b).

For a well-corrected achromatic lens, no colour is seen at the best focus under white light, but inside the focus a yellowish disc (greenish at the centre) is fringed with an orange-red border, but the colour is very much less marked than with an un-corrected lens; outside the focus there appears an outer fringe of apple-green surrounding a yellowish disc with a faint reddish-violet centre.