CHAPTER II

ANALYTICAL CALCULATION

The magnetic field

The first of the calculations presented in this chapter is an attempt to calculate the magnetic field (1.3.2) at a fixed point due to a moving charge in the neighbourhood of the point by assuming Feynman's formula for the electric field (1.3.1) and the Lorentz transformation as axioms.

The work is divided into steps as follows

Step 1. Let the motion of the source be given in S. Let there be a test particle at P at t = 0 moving with velocity v in the x-direction. Then the test particle is at rest in S' at P' at $t' = 0$.

The motion of the source is given in S as $x = \xi(t)$, $y = \eta(t)$, $z = \zeta(t)$ (ξ, η, ζ are given, hence the position, the velocity and the acceleration are given)

Step 2 Calculate the motion of the source in S' using the Lorentz transformation.

Step 3 Calculate the electric field on the test particle in S', using Feynman's law in S'.

Step 4 Calculate the acceleration of the test particle in S' by using Newton's Law of motion.

Step 5 Calculate the acceleration of the test particle in S, by the Lorentz transformation.

Step 6 Calculate the force on the test particle in S, using the relativistic law of motion. This is the Lorentz force in S.

$$\vec{F} = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \vec{a}_p = q_1 (\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

Step 7 Calculate the electric field on the test particle in S, using Feynman's formula in S.

Step 8 Substract the electric field in S from the Lorentz's force in S. The difference is the force due to the magnetic field alone in S.

Step 9 Calculate the magnetic field at the origin in S.

Analytical Treatment

Let us consider a test particle and a charge q, both moving with arbitrary velocity in space.

Let S be a frame of reference such that the test particle moves along the x-axis, and at $t = 0$ the test particle has velocity $\vec{v}$ at the origin.

Let S' be the frame of reference in which the test particle is at rest at the origin at time $t' = t = 0$ (at time $t' = t = 0$ the two origins concide)

That is S' has velocity $\vec{v}$ relative to S.

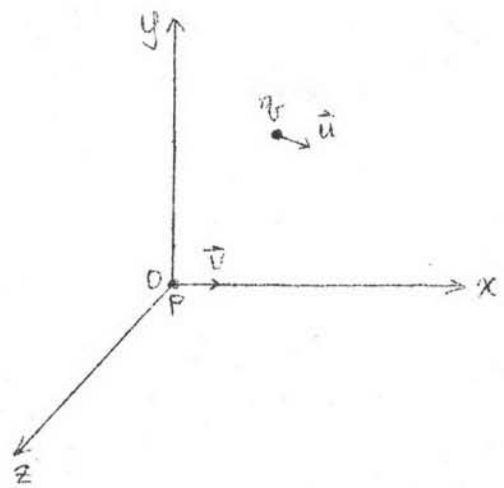

Figure 2.1 A charge q moving in the S frame.

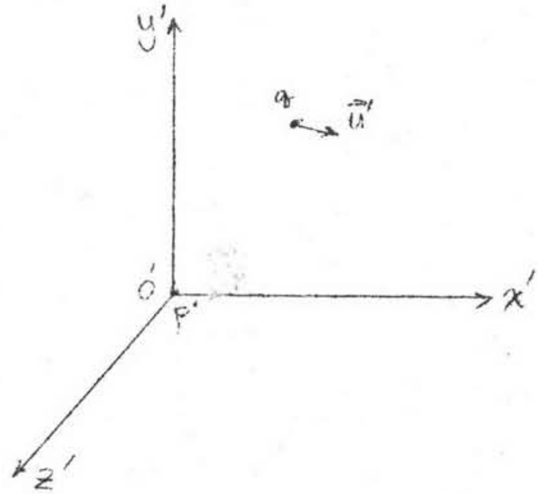

Figure 2.2 The charge q moving in the S' frame

Step 1 Let the motion of the source q be given in S, as

$x = \xi(t), \quad y = \eta(t), \quad z = \zeta(t),$

where ξ, η, ζ are given. Hence the position, velocity and acceleration of the source are known :

$$\text{the position } \vec{r} = (x,y,z) = (\xi(t), \eta(t), \zeta(t))$$

$$\text{the velocity } \vec{u} = (u_x,u_y,u_z) = (\dot{x}, \dot{y}, \dot{z}) = (\dot{\xi}(t), \dot{\eta}(t), \dot{\zeta}(t))$$

$$\text{the acceleration } \vec{a} = (a_x,a_y,a_z) = (\dot{u}_x, \dot{u}_y, \dot{u}_z) = (\ddot{x}, \ddot{y}, \ddot{z}) = (\ddot{\xi}(t), \ddot{\eta}(t), \ddot{\zeta}(t))$$

Step 2 To calculate the motion of the source in S', using the Lorentz's transformation.

Let the charge q have velocity $\vec{u}' = (u'_x, u'_y, u'_z)$ and acceleration $\vec{a}' = (a'_x, a'_y, a'_z) = (\dot{u}'_x, \dot{u}'_y, \dot{u}'_z)$ relative to S'.

The Lorentz's transformation is

$$x' = \gamma(x - vt),\ y' = y,\ z' = z,\ t' = \gamma(t - \frac{v}{c^2}x) \qquad (2.1)$$

where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

By differentiating the first three equations of (2.1) with respect to t' and substituting the value of $\frac{dt}{dt'}$, we obtain the velocity of the source in S' as

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}},\ u'_y = \frac{u_y\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{u_x v}{c^2}},\ u'_z = \frac{u_z\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{u_x v}{c^2}} \qquad \ldots(2.2)$$

where

$$u_x = \dot{x} = \frac{dx}{dt},\ u'_x = \dot{x}' = \frac{dx'}{dt'}$$

$$u_y = \dot{y} = \frac{dy}{dt},\ u'_y = \dot{y}' = \frac{dy'}{dt'}$$

$$u_z = \dot{z} = \frac{dz}{dt},\ u'_z = \dot{z}' = \frac{dz'}{dt'}$$

The reciprocal equations corresponding to (2.2) are

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}},\ u_y = \frac{u'_y\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{u'_x v}{c^2}},\ u_z = \frac{u'_z\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{u'_x v}{c^2}} \qquad \ldots(2.3)$$

Then by differentiating (2.2) with respect to t', we obtain the acceleration in S' :

$$a_x' = \frac{du_x'}{dt'} = \frac{du_x'}{dt} \cdot \frac{dt}{dt'} = \frac{d}{dt}\left\{ \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \right\} \cdot \gamma(1 + \frac{v}{c^2} u_x')$$

$$= \frac{(1 - \frac{u_x v}{c^2}) \frac{d}{dt}(u_x - v) - (u_x - v) \frac{d}{dt}(1 - \frac{u_x v}{c^2})}{\left(1 - \frac{u_x v}{c^2}\right)^2} \cdot \gamma(1 + \frac{v}{c^2} u_x')$$

$$= \frac{(1 - \frac{u_x v}{c^2}) a_x - (u_x - v) \frac{v}{c^2} a_x}{\left(1 - \frac{u_x v}{c^2}\right)^2} \cdot \gamma\left\{1 + \frac{v}{c^2} \frac{u_x - v}{1 - \frac{u_x v}{c^2}}\right\}$$

$$= \frac{a_x \left\{1 - \frac{u_x v}{c^2} + \frac{u_x v}{c^2} - \frac{v^2}{c^2}\right\}}{\left(1 - \frac{u_x v}{c^2}\right)^2} \cdot \frac{\gamma}{1 - \frac{u_x v}{c^2}} \left\{1 - \frac{u_x v}{c^2} + \frac{v}{c^2} u_x - \frac{v^2}{c^2}\right\}$$

$$= \frac{a_x (1 - \frac{v^2}{c^2})}{\left(1 - \frac{u_x v}{c^2}\right)^3} \cdot \gamma (1 - \frac{v^2}{c^2})$$

$$= (1 - \frac{u_x v}{c^2})^{-3} (1 - \frac{v^2}{c^2})^{-3/2} a_x \qquad \text{.........(2.4)}$$

$$a'_y = \frac{d u'_y}{dt'} = \frac{d u'_y}{dt} \cdot \frac{dt}{dt'} = \frac{d}{dt}\left\{\frac{u_y\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{u_x v}{c^2}}\right\} \cdot \gamma\left(1+\frac{v}{c^2}u'_x\right)$$

$$= \frac{\left(1-\frac{u_x v}{c^2}\right)\frac{d}{dt}u_y\sqrt{1-\frac{v^2}{c^2}} - u_y\sqrt{1-\frac{v^2}{c^2}}\,\frac{d}{dt}\left(1-\frac{u_x v}{c^2}\right)}{\left(1-\frac{u_x v}{c^2}\right)^2} \cdot$$

$$\cdot\, \gamma\left\{1+\frac{v}{c^2}\,\frac{u_x - v}{1-\frac{u_x v}{c^2}}\right\}$$

$$= \frac{\left(1-\frac{u_x v}{c^2}\right)\sqrt{1-\frac{v^2}{c^2}}\cdot a_y + u_y\sqrt{1-\frac{v^2}{c^2}}\cdot\frac{v}{c^2}a_x}{\left(1-\frac{u_x v}{c^2}\right)^2} \cdot \frac{\gamma\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_x v}{c^2}\right)}$$

$$= \left(1-\frac{u_x v}{c^2}\right)^{-2}\left(1-\frac{v^2}{c^2}\right)a_y + u_y\frac{v}{c^2}\left(1-\frac{u_x v}{c^2}\right)^{-3}\left(1-\frac{v^2}{c^2}\right)a_x \qquad \text{.....(2.5)}$$

$$a'_z = \frac{du'_z}{dt'} = \frac{du'_z}{dt}\cdot\frac{dt}{dt'} = \frac{d}{dt}\left\{\frac{u_z\sqrt{1-\frac{v^2}{c^2}}}{1-u}\right\}\cdot\gamma\left(1+\frac{v}{c^2}u'_x\right)$$

$$= \frac{\left(1-\frac{u_x v}{c^2}\right)\sqrt{1-\frac{v^2}{c^2}}\cdot a_z + u_z\sqrt{1-\frac{v^2}{c^2}}\,\frac{v}{c^2}a_x}{\left(1-\frac{u_x v}{c^2}\right)^2}\cdot\frac{\gamma\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_x v}{c^2}\right)}$$

$$= \left(1-\frac{u_x v}{c^2}\right)^{-2}\left(1-\frac{v^2}{c^2}\right)a_z + u_z\frac{v}{c^2}\left(1-\frac{u_x v}{c^2}\right)^{-3}\left(1-\frac{v^2}{c^2}\right)a_x \qquad \text{.......(2.6)}$$

The reciprocal equation corresponding to (2.4), (2.5) and (2.6) are

$$a_x = \left(1 + \frac{u'_x v}{c^2}\right)^{-3} \left(1 - \frac{v^2}{c^2}\right)^{3/2} a'_x \qquad \ldots\ldots\ (2.7)$$

$$a_y = \left(1 + \frac{u'_x v}{c^2}\right)^{-2} \left(1 - \frac{v^2}{c^2}\right) a'_y - u'_y \frac{v}{c^2}\left(1 + \frac{u'_x v}{c^2}\right)^{-3}\left(1 - \frac{v^2}{c^2}\right) a'_x \ldots (2.8)$$

$$a_z = \left(1 + \frac{u'_x v}{c^2}\right)^{-2}\left(1 - \frac{v^2}{c^2}\right) a'_z - u'_z \frac{v}{c^2}\left(1 + \frac{u'_x v}{c^2}\right)^{-3}\left(1 - \frac{v^2}{c^2}\right) a'_x \ldots (2.9)$$

Which are obtained by differentiating equation (2.3) with respect to t.

Step 3 To calculate the electric field on the test particle in S' using Feynman's formula in S'

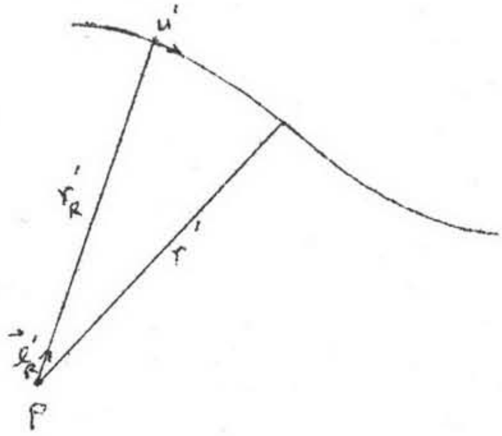

Figure 2.3 Retarded position of source in S' frame.

The Feynman equation for the electric field at the origin in S' is given by

$$\vec{E}' = -q\left[\frac{\vec{e}'_R}{r'^2_R} + \frac{r'_R}{c}\frac{d}{dt'}\left(\frac{\vec{e}'_R}{r'^2_R}\right) + \frac{1}{c^2}\frac{d^2}{dt'^2}(\vec{e}'_R)\right], \quad \ldots\ldots (2.10)$$

where $\vec{e}'_R$ = the unit vector from the position P where E' is measured,

and r'_R = the distance from P to the retarded position in the S' frame = $\sqrt{x_R'^2 + y_R'^2 + z_R'^2}$.

We may write equation (2.10) as

$$\vec{E}' = -q\left[\frac{\vec{r}'_R}{r'^3_R} + \frac{r'_R}{c}\frac{d}{dt'}\left(\frac{\vec{r}'_R}{r'^3_R}\right) + \frac{1}{c^2}\frac{d^2}{dt'^2}\left(\frac{\vec{r}'_R}{r'_R}\right)\right] \quad \ldots.(2.11)$$

The equations for the components are :

$$E'_x = -q\left[\frac{x'_R}{r'^3_R} + \frac{r'_R}{c}\frac{d}{dt'}\left(\frac{x'_R}{r'^3_R}\right) + \frac{1}{c^2}\frac{d^2}{dt'^2}\left(\frac{x'_R}{r'_R}\right)\right], \quad \ldots(2.12)$$

$$E'_y = -q\left[\frac{y'_R}{r'^3_R} + \frac{r'_R}{c}\frac{d}{dt'}\left(\frac{y'_R}{r'^3_R}\right) + \frac{1}{c^2}\frac{d^2}{dt'^2}\left(\frac{y'_R}{r'_R}\right)\right] \quad \ldots.(2.13)$$

$$E'_z = -q\left[\frac{z'_R}{r'^3_R} + \frac{r'_R}{c}\frac{d}{dt'}\left(\frac{z'_R}{r'^3_R}\right) + \frac{1}{c^2}\frac{d^2}{dt'^2}\left(\frac{z'_R}{r'_R}\right)\right] \quad \ldots.(2.14)$$

Step 4 To calculate the acceleration of the test particle in S' by Newton's law of motion

At time $t' = t = 0$ the test particle is at the origin of both systems S and S'.

Since at time $t' = t = 0$, the test particle has zero velocity in S'.

By the relativistic form of Newton second law of motion in S' frame we have

$$\vec{F}' = \frac{m_o}{\sqrt{1 - \frac{(\text{velocity of the particle in } S')^2}{c^2}}} \cdot \vec{a}'_p = m_o \vec{a}'_p$$

But by Coulomb's law $\vec{F}' = q_1 \vec{E}'$,

Therefore $\vec{a}'_p = \frac{q_1}{m_o} \vec{E}'$, and together with equation (2.11), we get

$$\vec{a}'_p = \frac{-q_1 q}{m_o} \left[\frac{\vec{r}'_R}{r'^3_R} + \frac{r'_R}{c} \frac{d}{dt'} \left(\frac{\vec{r}'_R}{r'^3_R} \right) + \frac{1}{c^2} \frac{d^2}{dt'^2} \left(\frac{\vec{r}'_R}{r'_R} \right) \right] \quad \ldots.(2.15)$$

Written in component form this is

$$a'_{p_x} = \frac{-q_1 q}{m_o} \left[\frac{x'_R}{r'^3_R} + \frac{r'_R}{c} \frac{d}{dt'} \left(\frac{x'_R}{r'^3_R} \right) + \frac{1}{c^2} \frac{d^2}{dt'^2} \left(\frac{x'_R}{r'_R} \right) \right], \ldots.(2.16)$$

$$a'_{p_y} = \frac{-q_1 q}{m_o} \left[\frac{y'_R}{r'^3_R} + \frac{r'_R}{c} \frac{d}{dt'} \left(\frac{y'_R}{r'^3_R} \right) + \frac{1}{c^2} \frac{d^2}{dt'^2} \left(\frac{y'_R}{r'_R} \right) \right], \ldots(2.17)$$

$$a'_{p_z} = \frac{-q_1 q}{m_o} \left[\frac{z'_R}{r'^3_R} + \frac{r'_R}{c} \frac{d}{dt'} \left(\frac{z'_R}{r'^3_R} \right) + \frac{1}{c^2} \frac{d^2}{dt'^2} \left(\frac{z'_R}{r'_R} \right) \right] \quad \ldots(2.18)$$

Step 5 To calculate the acceleration of the test particle in S, by the Lorentz transformation.

Since at $t' = t = 0$, the test particle is at rest in S', and has velocity v along x-axis in S, we have

$$v_x = v, \quad v_y = 0, \quad v_z = 0,$$
$$v'_x = 0, \quad v'_y = 0, \quad v'_z = 0.$$

We may use the transformation equations (2.7), (2.8) and (2.9) for the acceleration to obtain

$$a_{p_x} = (1 + \frac{v'_x v}{c^2})^{-3}(1 - \frac{v^2}{c^2})^{3/2} a'_{p_x}$$

$$= (1 - \frac{v^2}{c^2})^{3/2} \frac{(-q_1 q)}{m_o} \left[\frac{x'_R}{(x'^2_R + y'^2_R + z'^2_R)^{3/2}} \right.$$

$$+ \frac{(x'^2_R + y'^2_R + z'^2_R)^{1/2}}{c} \frac{d}{dt'} \left\{ \frac{x'_R}{(x'^2_R + y'^2_R + z'^2_R)^{3/2}} \right\}$$

$$\left. + \frac{1}{c^2} \frac{d^2}{dt'^2} \left\{ \frac{x'_R}{(x'^2_R + y'^2_R + z'^2_R)^{1/2}} \right\} \right], \quad \ldots\ldots\ldots(2.19)$$

$$a_{p_y} = (1 + \frac{v'_x v}{c^2})(1 - \frac{v^2}{c^2}) a'_{p_y} - v'_y \frac{v}{c^2} (1 + \frac{v'_x v}{c^2})^{-3}(1 - \frac{v^2}{c^2}) a'_{p_x}$$

$$= (1 - \frac{v^2}{c^2})(\frac{-q_1 q}{m_o}) \left[\frac{y'_R}{(x'^2_R + y'^2_R + z'^2_R)^{3/2}} + \right.$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}}{c} \frac{d}{dt'} \left\{ \frac{y_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}} \right\}$$

$$+ \frac{1}{c^2} \frac{d^2}{dt'^2} \left\{ \frac{y_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}} \right\} \Bigg] \quad \ldots\ldots(2.20)$$

$$a_{p_z} = (1 + \frac{v_x v}{c^2})^{-3}(1 - \frac{v^2}{c^2}) \; a'_{p_z} - v'_z \frac{v}{c^2}(1 + \frac{v'v}{c^2})^{-3}(1 - \frac{v^2}{c^2}) a'_{p_x}$$

$$= (1 - \frac{v^2}{c^2})(\frac{-q_1 q}{m_o}) \Bigg[\frac{z_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}}$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}}{c} \frac{d}{dt'} \left\{ \frac{z_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}} \right\}$$

$$+ \frac{1}{c^2} \frac{d^2}{dt'^2} \left\{ \frac{z_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}} \right\} \Bigg] \quad \ldots\ldots(2.21)$$

The Relation between $\frac{d}{dt_R}$ and $\frac{d}{dt}$

Consider a light signal from q, emitted at retarded time t_R, reaching P at time t in S.

The motion of the charge q is specified by $\vec{r}_R$ and t_R.

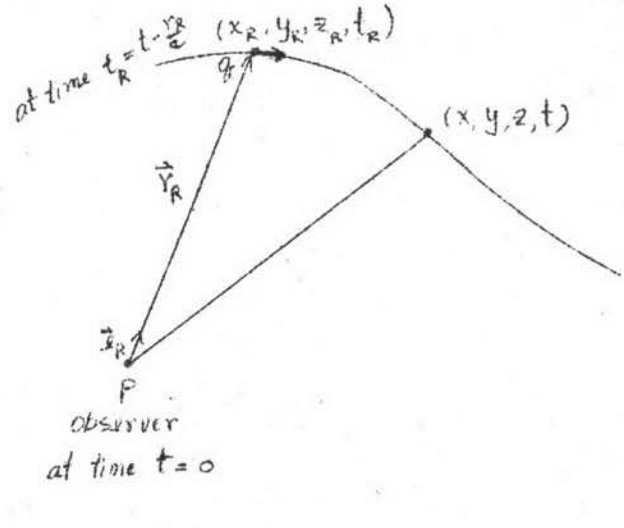

Figure 2.4 Retarded Position of source in S frame.

Suppose $\vec{r}_R$ is given as a function of t_R thus

$$\vec{r}_R = \vec{\rho}(t_R) ,$$

and

$$x_R = \xi(t_R) ,$$

$$y_R = \eta(t_R) ,$$

$$z_R = \varphi(t_R) ,$$

We have

$$r_R = \sqrt{\xi^2(t_R) + \eta^2(t_R) + \varphi^2(t_R)} = c(t - t_R) \quad \ldots(2.22)$$

This is the equation for calculating t for given value of x_R, y_R, z_R and t_R.

Squaring equation (2.22) we have

$$\xi^2(t_R) + \eta^2(t_R) + \varphi^2(t_R) = c^2(t - t_R)^2$$

and by differentiating we obtain

$$2\xi(t_R)\cdot\dot{\xi}(t_R)\,dt_R + 2\eta(t_R)\,\dot{\eta}(t_R)\,dt_R + 2\zeta(t_R)\dot{\zeta}(t_R)\,dt_R$$
$$= 2c^2\,(t - t_R)(dt - dt_R)$$

which gives

$$dt = \left[1 + \frac{\xi(t_R)\,\dot{\xi}(t_R) + \eta(t_R)\,\dot{\eta}(t_R) + \zeta(t_R)\,\dot{\zeta}(t_R)}{c^2(t - t_R)}\right] dt_R \quad \ldots(2.23)$$

where
$$\dot{\xi}(t_R) = \frac{d}{dt_R}\xi(t_R) = u_{R_x},$$
$$\dot{\eta}(t_R) = \frac{d}{dt_R}\eta(t_R) = u_{R_y},$$
$$\dot{\zeta}(t_R) = \frac{d}{dt_R}\zeta(t_R) = u_{R_z},$$

We may write equation (2.23) in short notation as

$$dt = \left[1 + \frac{\vec{r}_R\cdot\vec{u}_R}{cr_R}\right] dt_R \quad \ldots\ldots\ldots\ldots(2.24)$$

or, in another form

$$\frac{d}{dt} = \frac{1}{1 + \dfrac{\vec{r}_R\cdot\vec{u}_R}{cr_R}}\,\frac{d}{dt_R} = \frac{1}{1 + \dfrac{x_R u_{R_x} + y_R u_{R_y} + z_R u_{R_z}}{c\sqrt{x_R^2 + y_R^2 + z_R^2}}}\,\frac{d}{dt_R}$$

We also obtain in a similar fashion the relation between $\frac{d}{dt'}$ and $\frac{d}{dt'_R}$ as follows :

$$\frac{d}{dt'} = \frac{c(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}}{c(x_R'^2 + y_R'^2 + z_R'^2)^{1/2} + x_R' u_{R_x}' + y_R' u_{R_y}' + z_R' u_{R_z}'} \frac{d}{dt_R'} \quad \ldots\ldots.(2.25)$$

Since x_R, y_R, z_R and t_R are the coordinates and time in the S frame, we obtain by the Lorentz transformation the corresponding quanlities in S' as

$$\left.\begin{array}{l} x_R' = \quad (x_R - vt_R)\ ,\ y_R' = y_R,\ z_R' = z_R \\ t_R' = \gamma(t_R - \frac{v}{c^2} x_R) \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{array}\right\} \quad \ldots\ldots\ldots\ldots.(2.26)$$

Substituting (2.25) into (2.19), (2.20) and (2.21) respectively we have

$$a_{p_x} = \frac{-q_1 q}{m_o} (1 - \frac{v^2}{c^2})^{3/2} \Bigg[\frac{x_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}}$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)}{c(x_R'^2 + y_R'^2 + z_R'^2)^{1/2} + x_R' u_{R_x}' + y_R' u_{R_y}' + z_R' u_{R_z}'} \frac{d}{dt'} \left\{ \frac{x_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}} \right\}$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)}{\left\{c(x_R'^2 + y_R'^2 + z_R'^2)^{1/2} + x_R' u_{R_x}' + y_R' u_{R_y}' + z_R' u_{R_z}'\right\}^2} \frac{d^2}{dt_R'^2} \left\{ \frac{x_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}} \right\} \Bigg],$$

$$\ldots\ldots(2.27)$$

$$a_{p_y} = \frac{-q_1 q}{m_o} (1 - \frac{v^2}{c^2}) \Bigg[\frac{y_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}}$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)}{c(x_R'^2+y_R'^2+z_R'^2)^{1/2} + x_R'u_{R_x}' + y_R'u_{R_y}' + z_R'u_{R_z}'} \frac{d}{dt_R'} \left\{ \frac{y_R'}{(x_R'^2+y_R'^2+z_R'^2)^{3/2}} \right\}$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)}{\left\{c(x_R'^2+y_R'^2+z_R'^2)^{1/2} + x_R'u_{R_x}' + y_R'u_{R_y}' + z_R'u_{R_z}'\right\}^2} \frac{d^2}{dt_R'^2} \left\{ \frac{y_R'}{(x_R'^2+y_R'^2+z_R'^2)^{1/2}} \right\} \Bigg] \qquad \ldots\ldots\ldots(2.28)$$

$$a_{p_z} = \frac{-q_1 q}{m_o} \left(1 - \frac{v^2}{c^2}\right) \left[\frac{z_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}} \right.$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)}{c(x_R'^2+y_R'^2+z_R'^2)^{1/2} + x_R'u_{R_x}' + y_R'u_{R_y}' + z_R'u_{R_z}'} \frac{d}{dt_R'} \left\{ \frac{z_R'}{(x_R'^2+y_R'^2+z_R'^2)^{3/2}} \right\}$$

$$+ \frac{(x_R'^2 + y_R'^2 + z_R'^2)}{\left\{c(x_R'^2+y_R'^2+z_R'^2)^{1/2} + x_R'u_{R_x}' + y_R'u_{R_y}' + z_R'u_{R_z}'\right\}^2} \frac{d^2}{dt_R'^2} \left\{ \frac{z_R'}{(x_R'^2+y_R'^2+z_R'^2)^{1/2}} \right\} \Bigg] \qquad \ldots\ldots\ldots(2.29)$$

Consider the expression $\frac{d}{dt_R'} \left\{ \frac{x_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{3/2}} \right\}$ and $\frac{d^2}{dt_R'^2} \left\{ \frac{x_R'}{(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}} \right\}$ in equation (2.27), and put r_R' for $(x_R'^2 + y_R'^2 + z_R'^2)^{1/2}$.

Then

$$\frac{d}{dt'_R}\left\{\frac{x'_R}{(x'^2_R+y'^2_R+z'^2_R)^{3/2}}\right\} = \frac{d}{dt'_R}\left\{x'_R(x'^2_R+y'^2_R+z'^2_R)^{-3/2}\right\}$$

$$= u'_{R_x}(x'^2_R+y'^2_R+z'^2_R)^{-3/2}$$

$$-\frac{3}{2}\, x'_R(x'^2_R+y'^2_R+z'^2_R)^{-5/2}(2x'_Ru'_{R_x}+2y'_Ru'_{R_y}$$

$$+2z'_Ru'_{R_z})$$

$$= \frac{u'_{R_x}}{r'^3_R} - \frac{3x'_R}{r'^5_R}(x'_Ru'_{R_x}+y'_Ru'_{R_y}+z'_Ru'_{R_z}),$$

$$\frac{d}{dt'_R}\left\{\frac{x'_R}{(x'^2_R+y'^2_R+z'^2_R)^{1/2}}\right\} = \frac{d}{dt'_R}\left\{x'_R\cdot(x'^2_R+y'^2_R+z'^2_R)^{-1/2}\right\}$$

$$= u'_{R_x}(x'^2_R+y'^2_R+z'^2_R)^{-\frac{1}{2}}$$

$$-\frac{1}{2}x'_R(x'^2_R+y'^2_R+z'^2_R)^{-\frac{3}{2}}(2x'_Ru'_{R_x}+2y'_Ru'_{R_y}+2z'_Ru'_{R_z})$$

$$= u'_{R_x}(x'^2_R+y'^2_R+z'^2_R)^{-\frac{1}{2}}$$

$$-x'_R(x'^2_R+y'^2_R+z'^2_R)^{-\frac{3}{2}}(x'_Ru'_{R_x}+y'_Ru'_{R_y}+z'_Ru'_{R_z})$$

$$\frac{d^2}{dt_R'^2}\left\{\frac{x_R'}{(x_R'^2+y_R'^2+z_R'^2)^{1/2}}\right\} = \frac{d}{dt_R'}\left\{u_{R_x}'(x_R'^2+y_R'^2+z_R'^2)^{-1/2}\right.$$

$$\left. -x_R'(x_R'^2+y_R'^2+z_R'^2)^{-\frac{3}{2}}(x_R'u_{R_x}'+y_R'u_{R_y}'+z_R'u_{R_z}')\right\}$$

$$= a_{R_x}'(x_R'^2+y_R'^2+z_R'^2)^{-1/2}$$

$$-\frac{1}{2}u_{R_x}'(x_R'^2+y_R'^2+z_R'^2)^{-\frac{3}{2}}(2x_R'u_{R_x}'+2y_R'u_{R_y}'+2z_R'u_{R_z}')$$

$$-3x_R'(x_R'^2+y_R'^2+z_R'^2)^{-5/2}(x_R'u_{R_x}'+y_R'u_{R_y}'+z_R'u_{R_z}')^2$$

$$-x_R'(x_R'^2+y_R'^2+z_R'^2)^{-\frac{3}{2}}(x_R'a_{R_x}'+u_{R_x}'^2+y_R'a_{R_y}'+u_{R_y}'^2$$

$$+z_R'a_{R_z}'+u_{R_z}'^2)$$

$$-u_{R_x}'(x_R'^2+y_R'^2+z_R'^2)^{-\frac{3}{2}}(x_R'u_{R_x}'+y_R'u_{R_y}'+z_R'u_{R_z}')$$

$$= a_{R_x}'r_R'^{-1} - u_{R_x}'r_R'^{-3}(x_R'u_{R_x}'+y_R'u_{R_y}'+z_R'u_{R_z}')$$

$$+3x_R'r_R'^{-5}(x_R'u_{R_x}'+y_R'u_{R_y}'+z_R'u_{R_z}')^2$$

$$-x_R'r_R'^{-3}(u_{R_x}'^2+u_{R_y}'^2+u_{R_z}'^2+x_R'a_{R_x}'+y_R'a_{R_y}'+z_R'a_{R_z}')$$

$$-u_{R_x}'r_R'^{-3}(x_R'u_{R_x}'+y_R'u_{R_y}'+z_R'u_{R_z}')$$

Substitute these expressions into (2.27)

$$
\begin{aligned}
a_{p_x} = \frac{-q_1 q}{m_o}\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} \Bigg[& \frac{x'_R}{r'^3_R} \\
& + \frac{r'^2_R}{c r'_R + x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}} \left\{ \frac{u'_{R_x}}{r'^3_R} - \frac{3x'_R}{r'^5_R}\left(x'_{R_x} u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right) \right\} \\
& + \frac{r'^2_R}{\left(c r'_R + x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)^2} \left\{ \frac{a'_{R_x}}{r'_R} - \frac{u'_{R_x}\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)}{r'^3_R} \right. \\
& + \frac{3x'_R\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)^2}{r'^5_R} - \frac{x'_R\left(u'^2_{R_x} + u'^2_{R_y} + u'^2_{R_z} + x'_R a'_{R_x} + y'_R a'_{R_y} + z'_R a'_{R_z}\right)}{r'^3_R} \\
& \left. - \frac{u'_{R_x}\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)}{r'^3_R} \right\} \Bigg] \quad \ldots(2.27)^*
\end{aligned}
$$

where $a'_{R_x} = \frac{d}{dt'_R} u'_{R_x}$, $a'_{Ry} = \frac{d}{dt'_R} u'_{R_y}$, $a'_{R_z} = \frac{d}{dt'_R} u'_{R_z}$.

Consider the expressions $\frac{d}{dt'_R}\left\{\frac{y'_R}{(x'^2_R + y'^2_R + z'^2_R)^{3/2}}\right\}$ and

$\frac{d^2}{dt'^2_R}\left\{\frac{y'^2_R}{(x'^2_R + y'^2_R + z'^2_R)^{1/2}}\right\}$ in equation (2.28)

We have

$$
\frac{d}{dt'_R}\left\{\frac{y'_R}{(x'^2_R + y'^2_R + z'^2_R)^{3/2}}\right\} = \frac{d}{dt'_R}\left\{ y'_R (x'^2_R + y'^2_R + z'^2_R)^{-3/2} \right\}
$$

$$= u'_{R_y}\left(x'^2_R + y'^2_R + z'^2_R\right)^{-\frac{3}{2}}$$

$$- \frac{3}{2} y'_R \left(x'^2_R + y'^2_R + z'^2_R\right)^{-\frac{5}{2}} \left(2x'_R u'_{R_x} + 2y'_R u'_{R_y} + 2z'_R u'_{R_z}\right)$$

$$= \frac{u'_{R_y}}{r'^3_R} - 3\,\frac{y'_R\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)}{r'^5_R}$$

$$\frac{d}{dt'_R}\left\{\frac{y'_R}{\left(x'^2_R + y'^2_R + z'^2_R\right)^{1/2}}\right\} = \frac{d}{dt'_R}\; y'_R\left(x'^2_R + y'^2_R + z'^2_R\right)^{-1/2}$$

$$= u'_{R_y}\left(x'^2_R + y'^2_R + z'^2_R\right)^{-\frac{1}{2}}$$

$$- y'_R\left(x'^2_R + y'^2_R + z'^2_R\right)^{-\frac{3}{2}}\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)$$

$$\frac{d^2}{dt'^2_R}\left\{\frac{y'_R}{\left(x'^2_R + y'^2_R + z'^2_R\right)^{1/2}}\right\} = a'_{R_y}\left(x'^2_R + y'^2_R + z'^2_R\right)^{-1/2}$$

$$- u'_{R_y}\left(x'^2_R + y'^2_R + z'^2_R\right)^{-\frac{3}{2}}\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)$$

$$+ 3\, y'_R\left(x'^2_R + y'^2_R + z'^2_R\right)^{-5/2}\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)^2$$

$$- y'_R\left(x'^2_R + y'^2_R + z'^2_R\right)^{-\frac{3}{2}}\left(x'_R a'_{R_x} + u'^2_{R_x} + y'_R a'_{R_y} + u'^2_{R_y}\right.$$

$$\left. + z'_R a'_{R_z} + u'^2_{R_z}\right)$$

$$- u'_{R_y}\left(x'^2_R + y'^2_R + z'^2_R\right)^{-\frac{3}{2}}\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)$$

$$
\begin{aligned}
&= a'_{R_y} r'^{-1}_R - u'_{R_y} r'^{-3}_R (x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}) \\
&+ 3y'_R r'^{-5}_R (x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})^2 \\
&- y'_R r'^{-3}_R (u'^2_{R_x} + u'^2_{R_y} + u'^2_{R_z} + x'_R a'_{R_x} + y'_R a'_{R_y} + z'_R a'_{R_z}) \\
&- u'_{R_y} r'^{-3}_R (x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})
\end{aligned}
$$

Substitute these expressions into (2.28) :

$$
\begin{aligned}
a_{p_y} = \frac{-q_1 q}{m_o} \left(1 - \frac{v^2}{c^2}\right) \Bigg[& \frac{y'_R}{r'^3_R} \\
& + \frac{r'^2_R}{cr'_R + x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}} \left\{ \frac{u'_{R_y}}{r'^3_R} - \frac{3y'_R (x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})}{r'^5_R} \right\} \\
& + \frac{r'^2_R}{(cr'_R + x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})^2} \left\{ \frac{a'_{R_y}}{r'_R} - \frac{u'_{R_y} (x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})}{r'^3_R} \right. \\
& + \frac{3y'_R (x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})^2}{r'^5_R} - \frac{y'_R (u'^2_{R_x} + u'^2_{R_y} + u'^2_{R_z} + x'_R a'_{R_x} + y'_R a'_{R_y} + z'_R a'_{R_z})}{r'^3_R} \\
& \left. - \frac{u'_{R_y} (x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})}{r'^3_R} \right\} \Bigg] \qquad \ldots\ldots\ldots\ldots\ldots (2.28)^*
\end{aligned}
$$

Consider the expressions $\frac{d}{dt'_R}\left\{\frac{z'_R}{(x'^2_R+y'^2_R+z'^2_R)^{3/2}}\right\}$ and

$\frac{d^2}{dt'^2_R}\left\{\frac{z'_R}{(x'^2_R+y'^2_R+z'^2_R)^{1/2}}\right\}$ in equation (2.29)

We have

$$\frac{d}{dt'_R}\left\{\frac{z'_R}{(x'^2_R+y'^2_R+z'^2_R)^{3/2}}\right\} = \frac{d}{dt'_R}\left\{ z'_R(x'^2_R+ y'^2_R+ z'^2_R)^{-3/2}\right\}$$

$$= u'_{R_z}(x'^2_R+y'^2_R+z'^2_R)^{-3/2}$$

$$- 3z'_R(x'^2_R+y'^2_R+z'^2_R)^{-\frac{5}{2}}(x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z}) ,$$

$$\frac{d}{dt'_R}\left\{\frac{z'_R}{(x'^2_R+y'^2_R+z'^2_R)^{1/2}}\right\} = \frac{d}{dt'_R}\left\{ z'_R(x'^2_R+ y'^2_R+ z'^2_R)^{-1/2}\right\}$$

$$= u'_{R_z}(x'^2_R+ y'^2_R+ z'^2_R)^{-1/2}$$

$$- z'_R(x'^2_R+y'^2_R+z'^2_R)^{-\frac{3}{2}}(x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z}),$$

$$\frac{d^2}{dt'^2_R}\left\{\frac{z'_R}{(x'^2_R+y'^2_R+z'^2_R)^{1/2}}\right\} = a'_{R_z}(x'^2_R+ y'^2_R+ z'^2_R)^{-1/2}$$

$$-u'_{R_z}(x'^2_R+y'^2_R+z'^2_R)^{-\frac{3}{2}}(x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})$$

$$+ 3z'_R(x'^2_R+y'^2_R+z'^2_R)^{-\frac{5}{2}}(x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})^2$$

$$-z'_R(x'^2_R+y'^2_R+z'^2_R)^{-\frac{3}{2}}(x'_R a'_{R_x}+u'^2_{R_x}+y'_R a'_{R_y}+u'^2_{R_y}+z'_R a'_{R_z}+u'^2_{R_z})$$

$$-u'_{R_z}(x'^2_R+y'^2_R+z'^2_R)^{-\frac{3}{2}}(x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})$$

$$= a'_{R_z} r'^{-1}_R - u'_{R_z} r'^{-3}_R (x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})$$

$$+3z'_R r'^{-5}_R (x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})^2$$

$$-z'_R r'^{-3}_R (u'^2_{R_x}+u'^2_{R_y}+u'^2_{R_z}+x'_R a'_{R_x}+y'_R a'_{R_y}+z'_R a'_{R_z})$$

$$-u'_{R_z} r'^{-3}_R (x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})$$

Substitute the expressions into (2.29) :

$$a_{p_z} = \frac{-q_1 q}{m_o}\left(1-\frac{v^2}{c^2}\right)\Bigg[\frac{z'_R}{r'^3_R} +$$

$$+\frac{r'^2_R}{(cr'_R+x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})}\left\{\frac{u'_{R_z}}{r'^3_R}-\frac{3z'_R(x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})}{r'^5_R}\right\}$$

$$+\frac{r'^2_R}{(cr'_R+x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})^2}\left\{\frac{a'_{R_z}}{r'_R}-\frac{u'_{R_z}(x'_R u'_{R_x}+y'_R u'_{R_y}+z'_R u'_{R_z})}{r'^3_R}\right. +$$

$$+ \frac{3z'_R}{r'^5_R}(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z})^2 - \frac{z'_R}{r'^3_R}(u'^2_{R_x} + u'^2_{R_y} + u'^2_{R_z} + x'_R a'_{R_x} + y'_R a'_{R_y} + z'_R a'_{R_z})$$

$$- \frac{u'_{R_z}}{r'^3_R}\left(x'_R u'_{R_x} + y'_R u'_{R_y} + z'_R u'_{R_z}\right)\Bigg\}\Bigg] \quad \ldots\ldots\ldots\ldots\ldots(2.29)^*$$

The acceleration of the test particle in S is

$$a_p = \sqrt{a_{p_x}^2 + a_{p_y}^2 + a_{p_z}^2}$$

where a_{p_x}, a_{p_y} and a_{p_z} are given in $(2.27)^*$, $(2.28)^*$ and $(2.29)^*$ respectively. This quantity a_p is required for substitution into the equation.

$$\vec{F} = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}\,\vec{a}_p = q_1\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

for the Lorentz force on the test charge, but at this point the calculation was abandoned because the expressions have become too difficult and step 6 to step 9 were not completed.

Instead we studied a special case and compared the electric field which obtained from Feynman's formula with that obtained from Berkeley Physics Course formula.

Field of a stationary source

Consider a system in which the observer is at the origin and the charge is at rest at a distance r from the origin (Figure 2.5).

Berkeley Physics Course formula

The source is stationary, it has no velocity, therefore from (1.3.3) and (1.3.4), we have $\gamma = 1$, $x' = x$, $y' = y$, and

$$E_x = \frac{-qx}{(x^2+y^2)^{3/2}}, \quad E_y = \frac{-qy}{(x^2+y^2)^{3/2}},$$

$$E = \sqrt{E_x^2 + E_y^2} = \frac{-qr}{r^3} = \frac{-q}{r^2}.$$

Hence, $\vec{E} = -q \dfrac{\vec{e}}{r^2}$, which is Coulomb's law.

Feynman's formula

The source is stationary, it has no velocity and no acceleration. Therefore the second and the third terms of the right hand side of equation (1.3.1) disappear.

(Since $\vec{e}_R = \vec{e} = \text{constant}$, $r_R = r = \text{constant}$)

Hence $\vec{E} = -q \dfrac{\vec{e}_R}{r_R^2} = -q \dfrac{\vec{e}}{r^2}$.

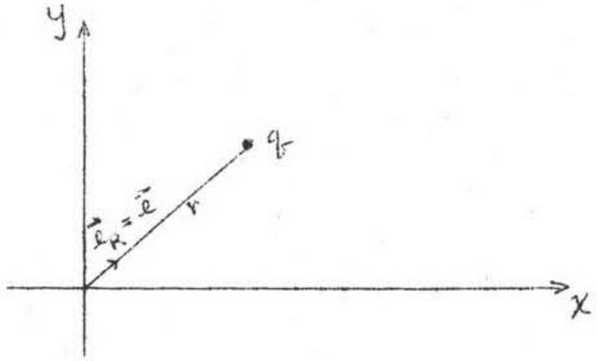

Figure 2.5 The sourse at rest in S.

Field of a uniformuly moving charge

a) Consider a system in which the observer is at rest at the origin and a constant moving charge is at x at time t in the rest frame of the observer.

Berkeley Physics Course formula

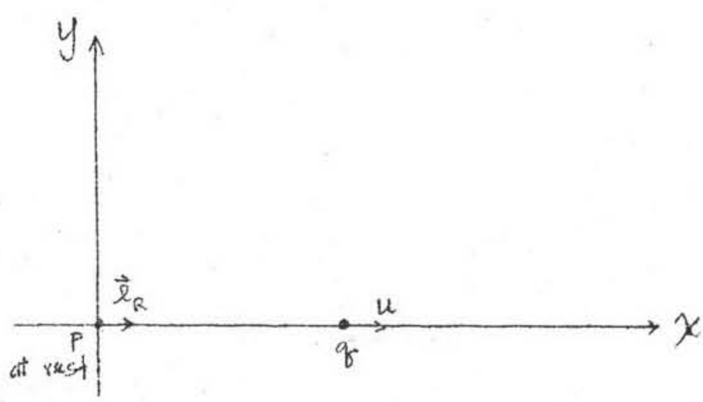

Figure 2.6 A charge q moving along x-axis in S frame.

In S, the observer is at rest at the origin, and the charge moves with constant velocity u along the x-axis.

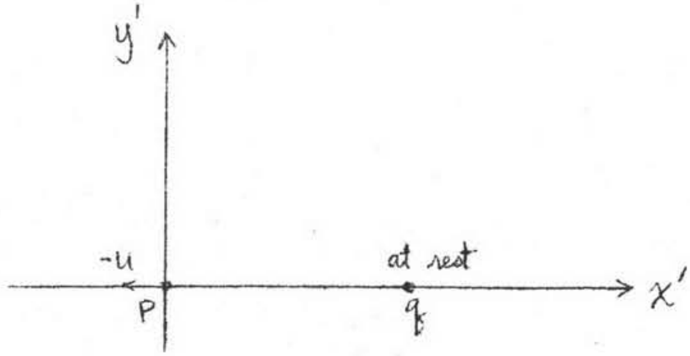

Figure 2.7 The charge q is stationary in S' frame.

S' is the frame of reference with q is at rest. Suppose at time $t' = t = 0$, P is at the origin in S'.

Then the electric field at P due to q is

$$E' = \frac{-q}{x'^2}$$

The relation between the two coordinates are given by the Lorentz transformation

$$x' = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{u}{c^2}x) \qquad \text{where} \qquad \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

But from the Berkeley Physics Course Volume 2 page 158, we have

$E' = E$ (since in this case E' and E are parallel to u).

Therefore, in S, we have

$$E = \frac{-q}{(\gamma x)^2} = \frac{-q}{x^2}\left(1 - \frac{u^2}{c^2}\right) = -\frac{q(c^2 - u^2)}{x^2 c^2} \quad \text{....(2.30)}$$

Feynman's formula

Provided the source remains on the same side of the origin, $\vec{e}_R$ is constant for a small interval of time, and the third term of Feynman's equation (1.3.1) disappears. Therefore we have

$$\vec{E} = -q\left[\frac{\vec{e}_R}{r_R^2} + \frac{r_R}{c}\frac{d}{dt}\left(\frac{\vec{e}_R}{r_R^2}\right)\right]$$

$$= -q\,\vec{e}_R\left[\frac{1}{r_R^2} + \frac{r_R}{c}\frac{d}{dt}\left(\frac{1}{r_R^2}\right)\right]$$

The magnitude of $\vec{E}$ is

$$E = -q\left[\frac{1}{r_R^2} + \frac{r_R}{c}\frac{d}{dt}\left(\frac{1}{r_R{}^2}\right)\right] \quad \text{............(2.31)}$$

But we have from (2.24)

$$\frac{d}{dt} = \frac{1}{1 + \dfrac{\vec{r}_R \cdot \vec{u}_R}{c r_R}}\frac{d}{dt_R},$$

where $\vec{r}_R = (x_R, y_R, z_R)$

$\vec{u}_R = (u_{R_x}, u_{R_y}, u_{R_z})$.

In this case, we have

$$\frac{d}{dt} = \left\{ \frac{c r_R}{c r_R + x_R u_{R_x} + y_R u_{R_y} + z_R u_{R_z}} \right\} \frac{d}{dt_R}$$

$$= \left\{ \frac{c x_R}{c x_R + x_R u} \right\} \frac{d}{dt_R} ,$$

Since we are concerned only with the x-direction and

$u_{R_x} = u_R = u = \frac{dx}{dt}$, $u_{R_y} = u_{R_z} = 0$, $x_R = r_R$.

Hence from (2.31) we may write.

$$E = -q \left[\frac{1}{x_R^2} + \frac{x_R}{c} \left\{ \frac{c x_R}{c x_R + x_R u} \right\} \frac{d}{dt_R} \left(\frac{1}{x_R^2} \right) \right]$$

$$= -q \left[\frac{1}{x_R^2} - \frac{2}{(c + u)\, x_R^2}\, u \right] , \text{ Since } \frac{dx_R}{dt_R} = u_R = u$$

$$= \frac{-q}{x_R^2} \left(\frac{c - u}{c + u} \right) \qquad \text{.....................(2.32)}$$

Now, we shall find x_R in term of x

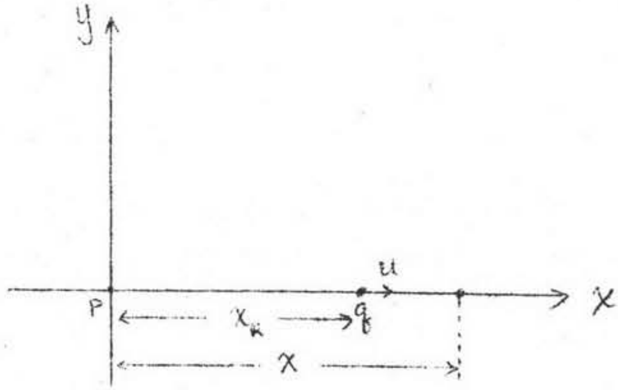

Figure 2.8 Source moving along the x-axis.

Since the field has speed c, the field travels a distance c in a unit time and a distance x_R in a time x_R/c

Therefore the time that the field takes to travel from q to P is $\frac{x_R}{c}$, which is equal to the time the charge q takes to go for a distance x from P.

Therefore $$x - x_R = u \cdot \frac{x_R}{c},$$

or $$x_R = \frac{xc}{u + c}.$$

Substituting this expression for x_R in (2.32), we get

$$E = -q \frac{(c - u)}{(c + u)} \cdot \frac{(c + u)^2}{x^2 c^2}$$

$$= \frac{-q (c^2 - u^2)}{x^2 c^2} \qquad \text{.....................(2.33)}$$

This agrees with (2.30).

b) Consider a system in which the observer is at rest at the origin and a constant moving charge is on the xy plane at (x,y) at time t. The velocity of the charge is a constant u parrallel to the x-axis.

Berkeley Physics formula

Let S' be the frame of reference in which the charge q is at rest. Suppose at time $t' = t = 0$, P is at the origin P' in S'.

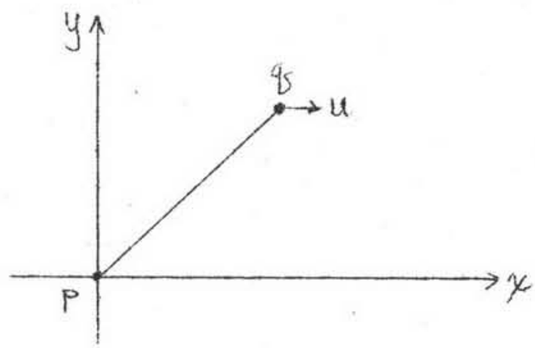

Figure 2.9 Source moving on the xy plane parrallel to the x-axis.

We have from the Berkeley formula that the electric field at P is

$$E_x = \frac{-q\gamma x}{\left\{(\gamma x)^2 + y^2\right\}^{3/2}} , \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.34)$$

$$E_y = \frac{-q\gamma y}{\left\{(\gamma x)^2 + y^2\right\}^{3/2}} , \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.35)$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$.

The relation of x and y interms of x_R & y_R

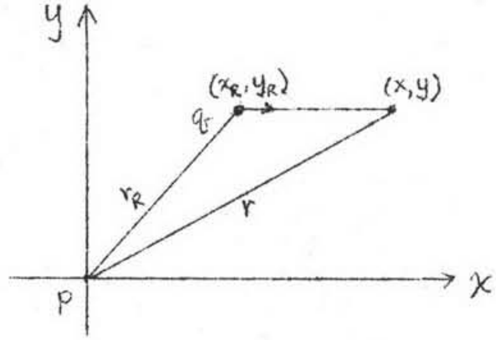

Figure 2.10 Retarded position of the source in S.

Since we have $x = x_R + (t - t_R)\, u$

and $t - t_R = \frac{r_R}{c}$,

it follows that

$$x = x_R + \frac{r_R}{c}\, u ,$$

$$y = y_R .$$

Substituting these expressions for x and y in (2.34) and (2.35), we get

$$E_x = \frac{-\, q\, \gamma\, (x_R + \frac{r_R}{c}\, u)}{\left\{ \gamma^2 (x_R + \frac{r_R}{c}\, u)^2 + y_R^2 \right\}^{3/2}} , \qquad \ldots\ldots\ldots\ldots (2.36)$$

$$E_y = \frac{-\, q\, \gamma\, y_R}{\left\{ \gamma^2 (x_R + \frac{r_R}{c}\, u)^2 + y_R^2 \right\}^{3/2}} , \qquad \ldots\ldots\ldots\ldots (2.37)$$

where $\gamma = \dfrac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$.

Feynman's formula

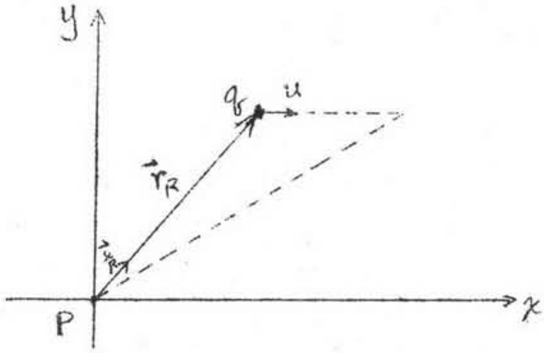

Figure 2.11 Retarded position of the source in S.

The Electric field at P is

$$\vec{E} = -q \left[\frac{\vec{e}_R}{r_R^2} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{\vec{e}_R}{r_R^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \vec{e}_R \right] \ldots\ldots(2.38)$$

From (2.24), $\dfrac{d}{dt} = \dfrac{1}{1 + \dfrac{\vec{r}_R \cdot \vec{u}}{c r_R}} \dfrac{d}{dt_R}$, since u is constant, $\vec{u} = \vec{u}_R$.

That is

$$\frac{d}{dt} = \frac{c r_R}{c r_R + x_R u_x + y_R u_y} \frac{d}{dt_R} ,$$

$$= \frac{c r_R}{c r_R + x_R u} \frac{d}{dt_R} , \quad \text{since } u_x = u, \; u_y = 0.$$

Therefore from (2.38) we may write

$$\vec{E} = -q\left[\frac{\vec{r}_R}{r_R^3} + \frac{r_R^2}{cr_R + x_R u}\frac{d}{dt_R}\left(\frac{\vec{r}_R}{r_R^3}\right) + \frac{r_R^2}{(cr_R + x_R u)^2}\frac{d^2}{dt_R^2}\left(\frac{\vec{r}_R}{r_R}\right)\right]$$

The correspondi~~ng~~ components are

$$E_x = -q\left[\frac{x_R}{r_R^3} + \frac{r_R^2}{cr_R + x_R u}\frac{d}{dt_R}\left(\frac{x_R}{r_R^3}\right) + \frac{r_R^2}{(cr_R + x_R u)^2}\frac{d^2}{dt_R^2}\left(\frac{x_R}{r_R}\right)\right] \quad \ldots(2.39)$$

$$E_y = -q\left[\frac{y_R}{r_R^3} + \frac{r_R^2}{cr_R + x_R u}\frac{d}{dt_R}\left(\frac{y_R}{r_R^3}\right) + \frac{r_R^2}{(cr_R + x_R u)^2}\frac{d^2}{dt_R^2}\left(\frac{y_R}{r_R}\right)\right] \ldots(2.40)$$

Consider $\frac{d}{dt_R}\left(\frac{x_R}{r_R^3}\right)$, $\frac{d^2}{dt_R^2}\left(\frac{x_R}{r_R}\right)$, $\frac{d}{dt_R}\left(\frac{y_R}{r_R^3}\right)$, $\frac{d^2}{dt_R^2}\left(\frac{y_R}{r_R}\right)$ in

(2.39) and (2.40) .

$$\frac{d}{dt_R}\left(\frac{x_R}{r_R^3}\right) = \frac{d}{dt_R}\left\{x_R\ (x_R^2 + y_R^2)^{-3/2}\right\}$$

$$= x_R\left(-\frac{3}{2}\right)(x_R^2+y_R^2)^{-\frac{5}{2}}\left(2x_R\frac{dx_R}{dt_R} + 2y_R\frac{dy_R}{dt_R}\right) + (x_R^2+y_R^2)^{-\frac{3}{2}}\frac{dx_R}{dt_R}$$

$$= -3\,u\,x_R^2\,r_R^{-5} + u\,r_R^{-3}$$

(Since $\frac{dx_R}{dt_R} = u_R = u, \quad \frac{dy_R}{dt_R} = 0$).

$$\frac{d^2}{dt_R^2}\left(\frac{x_R}{r_R}\right) = \frac{d}{dt_R}\left\{\frac{d}{dt_R}\,x_R\,(x_R^2 + y_R^2)^{-\frac{1}{2}}\right\}$$

$$= \frac{d}{dt_R}\left\{-\frac{1}{2}\,x_R(x_R^2+y_R^2)^{-\frac{3}{2}}\left(2x_R\frac{dx_R}{dt_R} + 2y_R\frac{dy_R}{dt_R}\right) + (x_R^2+y_R^2)^{-\frac{1}{2}}\frac{dx_R}{dt_R}\right\}$$

$$\frac{d^2}{dt_R^2}\left(\frac{x_R}{r_R}\right) = \frac{d}{dt_R}\left\{ - x_R^2\, u\, (x_R^2 + y_R^2)^{-\frac{3}{2}} + u\,(x_R^2 + y_R^2)^{-\frac{1}{2}} \right\}$$

$$= \frac{3}{2}\, x_R^2 u\, (x_R^2 + y_R^2)^{-\frac{5}{2}}\left(2x_R \frac{dx_R}{dt_R} + 2y_R \frac{dy_R}{dt_R}\right) - 3x_R u^2 (x_R^2 + y_R^2)^{-\frac{3}{2}}$$

$$= 3x_R^3\, u^2\, (x_R^2 + y_R^2)^{-\frac{5}{2}} - 3x_R u^2\, (x_R^2 + y_R^2)^{-\frac{3}{2}}$$

$$= 3x_R^3\, u^2\, r_R^{-5} - 3x_R\, u^2\, r_R^{-3}.$$

$$\frac{d}{dt_R}\left(\frac{y_R}{r_R^3}\right) = \frac{d}{dt_R}\left\{ y_R\, (x_R^2 + y_R^2)^{-3/2} \right\}$$

$$= -\frac{3}{2}\, y_R\, (x_R^2 + y_R^2)^{-\frac{5}{2}}\left(2x_R \frac{dx_R}{dt_R} + 2y_R \frac{dy_R}{dt_R}\right) + (x_R^2 + y_R^2)^{-\frac{3}{2}} \frac{dy_R}{dt_R}$$

$$= -3\, y_R x_R u\, r_R^{-5}$$

$$\frac{d^2}{dt_R^2}\left(\frac{y_R}{r_R}\right) = \frac{d}{dt_R}\left\{ \frac{d}{dt_R}\quad y_R\, (x_R^2 + y_R^2)^{-\frac{1}{2}} \right\}$$

$$= \frac{d}{dt_R}\left\{ -\frac{1}{2}\, y_R\, (x_R^2 + y_R^2)^{-\frac{3}{2}}\left(2x_R \frac{dx_R}{dt_R} + 2y_R \frac{dy_R}{dt_R}\right) + (x_R^2 + y_R^2)^{-\frac{1}{2}} \frac{dy_R}{dt_R} \right\}$$

$$= \frac{d}{dt_R}\left\{ - x_R y_R u\, (x_R^2 + y_R^2)^{-\frac{3}{2}} \right\}$$

$$= - y_R u \left\{ u\, (x_R^2 + y_R^2)^{-\frac{3}{2}} - \frac{3}{2} x_R (x_R^2 + y_R^2)^{-\frac{5}{2}} (2x_R u) \right\}$$

$$= - y_R u^2 r_R^{-3} + 3\, x_R^2\, y_R u^2\, r_R^{-5}$$

(Since $u = u_R$ = constant. $\dfrac{du_R}{dt_R} = 0$)

Substituting all these values in (2.39) and (2.40) we obtain

$$E_x = - q \left[\frac{x_R}{r_R^3} + \frac{u r_R^{-1} - 3u x_R^2\, r_R^{-3}}{(c r_R + x_R u)} + \frac{3x_R^3 u^2 r_R^{-3} - 3x_R u^2 r_R^{-1}}{(c r_R + x_R u)^2} \right] ,$$

$$E_y = - q \left[\frac{y_R}{r_R^3} - \frac{3x_R y_R\, u\, r_R^{-3}}{(c r_R + x_R u)} + \frac{3x_R^2 y_R u^2 r_R^{-3} - y_R u^2 r_R^{-1}}{(c r_R + x_R u)^2} \right]$$

Note that when $u = 0$ these formulas are the same as (2.36) and (2.37) with $u = 0$, however it is very difficult to compare the formulas for the general case so a new approach to this problem is given below. First we discuss a simpler special case.

A Special case

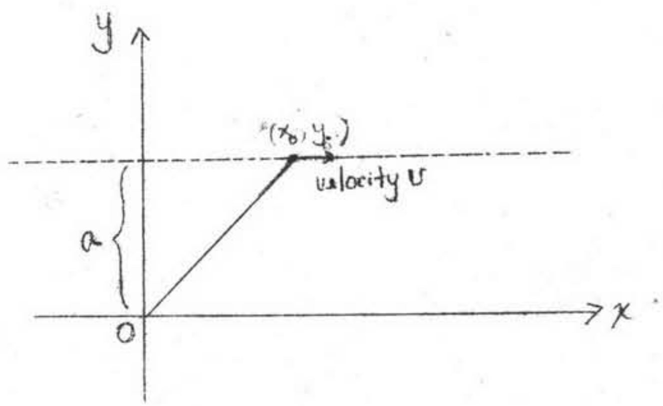

Figure 2.12 Source moving parallel to the x-axis with y = a.

At the moment the charge is at (x_o, y_o) we require the light signal originating from the charge at (0, a) to reach (0,0). The light signal takes a time $\frac{a}{c}$.

Therefore $\quad x_o = \frac{va}{c} = vt_o$

and we have $\quad y_o = a$.

The electric field at (0,0) due to the charge at (x_o,y_o) at time t_o, according to the Berkeley Physics Course formula is

$$E_x = \frac{-\gamma q x_o}{\left\{(\gamma x_o)^2 + y_o^2\right\}^{3/2}} = \frac{-\gamma qva/c}{\left\{(\gamma va/c)^2 + a^2\right\}^{3/2}},$$

$$E_y = \frac{-\gamma q y_o}{\left\{(\gamma x_o)^2 + y_o^2\right\}^{3/2}} = \frac{-\gamma qa}{\left\{(\gamma va/c)^2 + a^2\right\}^{3/2}},$$

where $\quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.

<u>The General case</u>

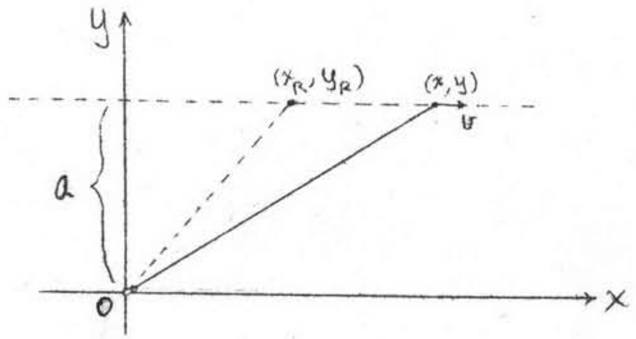

Figure 2.13 Retarded position in S.

At the moment the charge is at (x,y) we require the light signal originating from the charge at (x_R, y_R) to reach (0,0). The light signal takes time $\frac{\sqrt{x_R^2 + y_R^2}}{c}$.

$$\text{Therefore} \quad x = x_R + \frac{v}{c}\sqrt{x_R^2 + a^2} = vt,$$

$$y = a.$$

Berkeley Physics Course formula

The electric field at (0,0) due to charge at (x,y) at time t is

$$E_x = \frac{-\gamma q x}{\left\{(\gamma x)^2 + y^2\right\}^{3/2}} = \frac{-\gamma q v t}{\left\{(\gamma v t)^2 + a^2\right\}^{3/2}} \qquad \ldots\ldots(2.41)$$

$$E_y = \frac{-\gamma q y}{\left\{(\gamma x)^2 + y^2\right\}^{3/2}} = \frac{-\gamma q a}{\left\{(\gamma v t)^2 + a^2\right\}^{3/2}} \qquad \ldots\ldots\ldots(2.42)$$

Retarded Position as a function of time

$$\text{Since} \quad x_R + \frac{v}{c}\sqrt{x_R^2 + a^2} = vt,$$

it follows that

$$x_R = \frac{vt \pm \frac{v}{c}\sqrt{v^2t^2 + a^2\left(1 - \frac{v^2}{c^2}\right)}}{1 - \frac{v^2}{c^2}}$$

Since $x_R < vt$, we need the negative sign. Therefore

$$x_R = \frac{1}{(1 - \frac{v^2}{c^2})} \left\{ vt - \frac{v}{c} \sqrt{v^2 t^2 + a^2 (1 - \frac{v^2}{c^2})} \right\} \qquad \ldots\ldots\ldots(2.43)$$

$$y_R = a$$

and $r_R = \sqrt{x_R^2 + y_R^2}$

$$= \frac{1}{(1 - \frac{v^2}{c^2})} \left[v^2 t^2 + \frac{v^2}{c^2} \left\{ v^2 t^2 + a^2 (1 - \frac{v^2}{c^2}) \right\} - \frac{2v^2 t}{c} \left\{ v^2 t^2 + a^2 (1 - \frac{v^2}{c^2}) \right\}^{\frac{1}{2}} + a^2 (1 - \frac{v^2}{c^2})^2 \right]^{\frac{1}{2}} \quad \ldots(2.44)$$

Feynman's formula

The electric field at (0,0) due to the charge at (x,y) at time t is

$$\vec{E} = - q \left[\frac{\vec{r}_R}{r_R^3} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{\vec{r}_R}{r_R^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{\vec{r}_R}{r_R} \right) \right]$$

The corresponding components are

$$E_x = - q \left[\frac{x_R}{r_R^3} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{x_R}{r_R^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{x_R}{r_R} \right) \right], \quad \ldots\ldots(2.45)$$

$$E_y = - q \left[\frac{y_R}{r_R^3} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{y_R}{r_R^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{y_R}{r_R} \right) \right], \quad \ldots\ldots\ldots(2.46)$$

Consider $\frac{d}{dt}\left(\frac{x_R}{r_R^3}\right)$, $\frac{d^2}{dt^2}\left(\frac{x_R}{r_R}\right)$, $\frac{d}{dt}\left(\frac{y_R}{r_R^3}\right)$, $\frac{d^2}{dt^2}\left(\frac{y_R}{r_R}\right)$

Let $$A = v^2t^2 + \frac{v^2}{c^2}\left\{v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right\} - \frac{2v^2t}{c}\left\{v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right\}^{\frac{1}{2}} + a^2\left(1-\frac{v^2}{c^2}\right)^2 \qquad \ldots\ldots\ldots\ldots(2.47)$$

$$\frac{d}{dt}\left(\frac{x_R}{r_R^3}\right) = \frac{d}{dt}\left[\left(1-\frac{v^2}{c^2}\right)^{-1}\left\{vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}}\right\}\left(1-\frac{v^2}{c^2}\right)^3 A^{-\frac{3}{2}}\right]$$

$$= \left(1-\frac{v^2}{c^2}\right)^2 \frac{d}{dt}\left\{vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}}\right\} A^{-\frac{3}{2}}$$

$$= \left(1-\frac{v^2}{c^2}\right) A^{-\frac{3}{2}}\left\{v - \frac{v^3t}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{-\frac{1}{2}}\right\}$$

$$+ \left(1-\frac{v^2}{c^2}\right)^2\left\{vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}}\left(-\frac{3}{2}A^{-\frac{5}{2}}B\right)\right\} \ldots \qquad \ldots\ldots\ldots(2.48)$$

where $$B = \frac{dA}{dt} = 2v^2t + \frac{2v^4t}{c^2} - \frac{2v^2}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}} - \frac{2v^4t^2}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{-\frac{1}{2}} \qquad \ldots\ldots\ldots(2.49)$$

$$\frac{d}{dt}\left(\frac{x_R}{r_R}\right) = \frac{d}{dt}\left[\left(1-\frac{v^2}{c^2}\right)^{-1}\left\{vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}}\right\}\left(1-\frac{v^2}{c^2}\right)A^{-\frac{1}{2}}\right]$$

$$= \frac{d}{dt}\left\{vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}}\right\} A^{-\frac{1}{2}}$$

$$= A^{-\frac{1}{2}}\left\{v - \frac{v^3t}{c}\left(v^2t^2+a^2\left(1-\frac{v^2}{c^2}\right)\right)^{-\frac{1}{2}}\right\} - \frac{1}{2}A^{-\frac{3}{2}}B\left\{vt - \frac{v}{c}\left(v^2t^2+a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}}\right\}$$

$$\frac{d^2}{dt^2}\left(\frac{x_R}{r_R}\right) = A^{-\frac{1}{2}}\left\{\frac{v^5t^2}{c}(v^2t^2+a^2(1-\frac{v^2}{c^2}))^{-\frac{3}{2}}\right\} -$$

$$- \frac{1}{2}A^{-\frac{3}{2}}B\left\{v - \frac{v^3t}{c}\ (v^2t^2+a^2(1-\frac{v^2}{c^2}))^{-\frac{1}{2}}\right\}$$

$$- \frac{1}{2}A^{-\frac{3}{2}}B\left\{v - \frac{v^3t}{c}\ (v^2t^2+a^2(1-\frac{v^2}{c^2}))^{-\frac{1}{2}}\right\}$$

$$- \frac{1}{2}(A^{-\frac{3}{2}}M - \frac{3}{2}A^{-\frac{5}{2}}B^2)\left\{vt - \frac{v}{c}(v^2t^2+a^2(1-\frac{v^2}{c^2}))^{\frac{1}{2}}\right\}$$

$$= A^{-\frac{1}{2}}\left\{\frac{v^5t^2}{c}(v^2t^2+a^2(1-\frac{v^2}{c^2}))^{-\frac{3}{2}}\right\}$$

$$- A^{-\frac{3}{2}}B\left\{v - \frac{v^3t}{c}\ (v^2t^2+a^2(1-\frac{v^2}{c^2}))^{-\frac{1}{2}}\right\}$$

$$- \frac{1}{2}A^{-\frac{3}{2}}M\left\{vt - \frac{v}{c}\ (v^2t^2+a^2(1-\frac{v^2}{c^2}))^{\frac{1}{2}}\right\}$$

$$+ \frac{3}{4}A^{-\frac{5}{2}}B^2\left\{vt - \frac{v}{c}\ (v^2t^2+a^2(1-\frac{v^2}{c^2}))^{\frac{1}{2}}\right\} \quad \ldots\ldots(2.50)$$

where

$$M = \frac{dB}{dt} = 2v^2 + \frac{2v^4}{c^2} - \frac{6v^4t}{c}\ (v^2t^2+a^2(1-\frac{v^2}{c^2}))^{-\frac{1}{2}}$$

$$+ \frac{2v^6t^3}{c}\ (v^2t^2+a^2\ (1-\frac{v^2}{c^2}))^{-\frac{3}{2}} \quad \ldots\ldots\ldots(2.51)$$

$$\frac{d}{dt}\left(\frac{y_R}{r_R^3}\right) = \frac{d}{dt}\left\{a\quad(1-\frac{v^2}{c^2})^3\quad A^{-\frac{3}{2}}\right\}$$

$$= -\frac{3}{2}a\ (1-\frac{v^2}{c^2})^3\ A^{-\frac{5}{2}}\ B \quad \ldots\ldots\ldots\ldots(2.52)$$

$$\frac{d}{dt}\left(\frac{y_R}{r_R}\right) = \frac{d}{dt}\left\{ a\left(1-\frac{v^2}{c^2}\right) A^{-\frac{1}{2}} \right\}$$

$$= -\frac{1}{2}\, a \left(1-\frac{v^2}{c^2}\right) A^{-\frac{3}{2}} B$$

$$\frac{d^2}{dt^2}\left(\frac{y_R}{r_R}\right) = -\frac{1}{2}\, a\left(1-\frac{v^2}{c^2}\right)\left\{ A^{-\frac{3}{2}} M - \frac{3}{2} A^{-\frac{5}{2}} B^2 \right\}$$

$$= -\frac{1}{2}\, a \left(1-\frac{v^2}{c^2}\right) A^{-\frac{3}{2}} M + \frac{3}{4}\, a\left(1-\frac{v^2}{c^2}\right) A^{-\frac{5}{2}} B^2 \quad \ldots\ldots(2.53)$$

Substituting (2.43), (2.44), (2.48), (2.50), (2.52), (2.53) into (2.45) and (2.46) respectively we obtain

$$\begin{aligned}
E_x = -q \Bigg[& A^{-\frac{3}{2}}\left(1-\frac{v^2}{c^2}\right)^2 \left\{ vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}} \right\} \\
& + \frac{A^{-1}}{c}\left(1-\frac{v^2}{c^2}\right) \left\{ v - \frac{v^3 t}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{-\frac{1}{2}} \right\} \\
& - \frac{3}{2}\,\frac{A^{-2}B}{c}\left(1-\frac{v^2}{c^2}\right) \left\{ vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}} \right\} \\
& - \frac{A^{-\frac{3}{2}}B}{c^2} \left\{ v - \frac{v^3 t}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{-\frac{1}{2}} \right\} \\
& - \frac{1}{2}\,\frac{A^{-\frac{3}{2}}M}{c^2} \left\{ vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}} \right\} \\
& + \frac{3}{4}\,\frac{A^{-\frac{5}{2}}B^2}{c^2} \left\{ vt - \frac{v}{c}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{\frac{1}{2}} \right\} \\
& + A^{-\frac{1}{2}}\,\frac{v^5 t^2}{c^3}\left(v^2t^2 + a^2\left(1-\frac{v^2}{c^2}\right)\right)^{-\frac{3}{2}} \Bigg] \quad \ldots\ldots\ldots(2.54)
\end{aligned}$$

$$E_y = -q\left[A^{-\frac{3}{2}} a\left(1 - \frac{v^2}{c^2}\right)^3 - \frac{3}{2} A^{-2} \frac{Ba}{c} \left(1 - \frac{v^2}{c^2}\right)^2 - \frac{a}{2c^2} A^{-\frac{3}{2}} M \left(1 - \frac{v^2}{c^2}\right) + \frac{3}{4c^2} a A^{-\frac{5}{2}} B^2 \left(1 - \frac{v^2}{c^2}\right)\right] \qquad \text{.........(2.55)}$$

where A, B and M are given by (2.47), (2.49) and (2.51). The expressions on the right hand sides of (2.54) and (2.55) should equal the expression on the right hand sides of (2.41) and (2.42) respectively. The comparison was not completed because the expressions are so complicated and instead a simpler case was investigated as explained below.

For a special case t = 0

The Berkeley Physics Course formula equations (2.41) and (2.42) gives

$$E_x = 0, \qquad \text{..........(2.56)}$$

$$E_y = -\frac{\gamma q}{a^2}, \qquad \text{..........(2.57)}$$

where $$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

From the Feynman formula equations (2.54) and (2.55) we have th value of A,B, M from (2.47), (2.49) and (2.51)

$$A = a^2\left(1 - \frac{v^2}{c^2}\right)$$

$$B = -\frac{2av^2}{c}\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

$$M = 2v^2\left(1 + \frac{v^2}{c^2}\right)$$

Therefore from (2.54)

$$\begin{aligned} E_x = -q \Bigg[& a^{-3}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(1 - \frac{v^2}{c^2}\right)^2\left(-\frac{v}{c}a\right)\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} + \frac{va^{-2}}{c}\left(1-\frac{v^2}{c^2}\right)^{-1}\left(1-\frac{v^2}{c^2}\right) \\ & - \frac{3}{2}\,\frac{a^{-4}}{c}\left(1 - \frac{v^2}{c^2}\right)^{-2}(-2)\,\frac{av^2}{c}\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}\left(1- \frac{v^2}{c^2}\right)\left\{-\frac{v}{c}a\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}\right\} \\ & - \frac{a^{-3}}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}(-2)\,\frac{av^2}{c}\left(1- \frac{v^2}{c^2}\right)^{\frac{1}{2}} v \\ & - \frac{1}{2}\,\frac{a^{-3}}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \cdot 2v^2\left(1 + \frac{v^2}{c^2}\right)\left\{-\frac{v}{c}\,a\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}\right\} \\ & + \frac{3}{4}\,\frac{a^{-5}}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}}\,\frac{4a^2v^4}{c^2}\left(1 - \frac{v^2}{c^2}\right)\left\{-\frac{v}{c}\,a\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}\right\}\Bigg] \end{aligned}$$

which gives

$$E_x = -q\left(1 - \frac{v^2}{c^2}\right)^{-1} a^{-2}\left(\frac{v^3}{c^3}\right)$$

which is not the same as (2.56)

From (2.55) when $t = 0$

$$\begin{aligned} E_y = -q \Bigg[& a \cdot a^{-3}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(1 - \frac{v^2}{c^2}\right)^3 \\ & - \frac{3}{2}\,\frac{a}{c}\,a^{-4}\left(1 - \frac{v^2}{c^2}\right)^{-2}\left(-2a\,\frac{v^2}{c}\right)\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}\left(1 - \frac{v^2}{c^2}\right)^2 \end{aligned}$$

$$- \frac{1}{2} \frac{a}{c^2} a^{-3} (1 - \frac{v^2}{c^2})^{-\frac{3}{2}} . 2v^2(1 + \frac{v^2}{c^2})(1 - \frac{v^2}{c^2})$$

$$+ \frac{3}{4} \frac{a}{c^2} a^{-5} (1 - \frac{v^2}{c^2})^{-\frac{5}{2}} . \frac{4a^2v^4}{c^2} (1 - \frac{v^2}{c^2})^2 \Big]$$

$$= - q \left[a^{-2}(1 - \frac{v^2}{c^2})^{\frac{3}{2}} + \frac{3a^{-2}v^2}{c^2} (1 - \frac{v^2}{c^2})^{\frac{1}{2}} - \frac{a^{-2}v^2}{c^2} (1 + \frac{v^2}{c^2})(1 - \frac{v^2}{c^2})^{-\frac{1}{2}} + 3 \frac{a^{-2}v^4}{c^4} (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} \right]$$

which gives

$$E_y = \frac{- \gamma q}{a^2} \qquad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which is the same as (2.57)

For a special case when a = 0

The Berkeley Physics Course formula, equations (2.41), (2,42) give

$$E_x = \frac{- \gamma q t v}{(\gamma v t)^3} = \frac{- q}{\gamma^2 t^2 v^2} , \qquad \text{..............(2.58)}$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$,

$$E_y = 0 \qquad \text{..............(2.59)}$$

From the Feynman formula equation (2.54) and (2.55) we have the value of A, B, M from (2.47), (2.49), (2.51)

$$A = v^2 t^2 \left(1 - \frac{v}{c}\right)^2$$

$$B = 2v^2 t \left(1 - \frac{v}{c}\right)^2$$

$$M = 2v^2 \left(1 - \frac{v}{c}\right)^2$$

Therefore from (2.54)

$$E_x = - q \Bigg[v^{-3} t^{-3} \left(1 - \frac{v}{c}\right)^{-3} \left(1 - \frac{v^2}{c^2}\right)^2 vt \left(1 - \frac{v}{c}\right)$$

$$+ \frac{v^{-2} t^{-2}}{c} \left(1 - \frac{v}{c}\right)^{-2} \left(1 - \frac{v^2}{c^2}\right) v \left(1 - \frac{v}{c}\right)$$

$$- \frac{3}{2} \frac{v^{-4} t^{-4}}{c} \left(1 - \frac{v}{c}\right)^{-4} 2v^2 t \left(1 - \frac{v}{2}\right)^2 \left(1 - \frac{v^2}{c^2}\right) vt \left(1 - \frac{v}{c}\right)$$

$$- \frac{v^{-3} t^{-3}}{c^2} \left(1 - \frac{v}{c}\right)^{-3} 2 v^2 t \left(1 - \frac{v}{c}\right)^2 v \left(1 - \frac{v}{c}\right)$$

$$- \frac{1}{2} \frac{v^{-3} t^{-3}}{c^2} \left(1 - \frac{v}{c}\right)^{-3} 2v^2 \left(1 - \frac{v}{c}\right)^2 vt \left(1 - \frac{v}{c}\right)$$

$$+ \frac{3}{4} \frac{v^{-5} t^{-5}}{c^2} \left(1 - \frac{v}{c}\right)^{-5} 4 v^4 t^2 \left(1 - \frac{v}{c}\right)^4 vt \left(1 - \frac{v}{c}\right)$$

$$+ v^{-1} t^{-1} \left(1 - \frac{v}{c}\right)^{-1} \frac{v^2 t^{-1}}{c^3} \Bigg]$$

which gives

$$E_x = \frac{-q}{\gamma^2 t^2 v^2} - \frac{q}{\left(1 - \frac{v}{c}\right) t^2 v^2} \cdot \frac{v^3}{c^3} \text{, where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which is not equal to (2.58)

From (2,55) where a = 0, gives

$$E_y = 0$$

which is equal to (2.59)

Thus, the Berkeley Physics Course formula disagrees with the Feynman formula for the above special cases. The reason for this may be that the Feynman formula is not exact, but is correct up to second order in the speed of light. From such text as "Klassische Feldtheorie" by D. Iwanenko and A.Sokolow, one can show that by using retarded potentials the expression for the potentials can be expanded into a power series of higher time derivatives to infinite order.