REFERENCES

1. WIESNER, J.B., and LEE, Y.W.: "Experimental Determination of System Functions by the Method of Correlation". Presented at the Institute of Radio Engineers National Convention, New York, 1950

2. LEE, Y.W.: "Statistical Theory of Communication", Wiley, 1960

3. BALCOMB, J.D., DEMUTH, H.B., and GYFTOPOULOS, E.P.: "A Crosscorrelation Method for Measuring the Impulse Response of Reactor Systems", Nuclear Sci. Engng., 1961, 11, p.159

3. POORTVLIET, D.C.J.: "The Measurement of System Impulse Response by Means of Crosscorrelation with Binary Signals", Technological University, Delft, Netherlands, Electronics Laboratory Report, 1962

5. STERN, T.E., BLAQUIERE, A., and VALVAT, J.: "Reactivity Measurement Using Pseudorandom Source Excitation", Reactor Sci. Technol., 1962, 16, p.499

6. BRIGGS, P.A.N., HAMMOND, P.H., HUGHES, M.T.G., and PLUMB, G.O.: "Correlation Analysis of Process Dynamics Using Pseudorandom Binary Test Perturbations", Proc. Instn. Mech. Engrs., 1964-65, 179, Pt. 3H (to be published)

7. DAVIES, W.D.T.: "System Identification for Self-Adaptive Control", 1970

8. GODFREY, K.R., and MURGATROYD, W.: "Input Transducer Errors in Binary Crosscorrelation Experiments", Proc. IEE, 1965, 112, (3), p.565-573

9. GODFREY, K.R., and EVERETT, D.: "Input Transducer Errors in Binary

Crosscorrelation Experiments-2", Proc. IEE, 1966, 113,(1), p.185-189

10.ROBERTS,P.D.,and DAVIES,R.H.,:"Statistical Properties of Smoothed Maximum-length Linear Binary Sequences", Proc. IEE, 1966, 113, p.190-196

11.STERN,T.E.:"Reactor Impulse Response Using Random Source Excitation and Crosscorrelation", p.203-218

12.BALCOMB,J.D.:"Crosscorrelation Method of Measuring System Dynamic Response", p.183-201

13.ZIERLER,N.:"Linear Recurring Sequences", Vol.7, No.1, 1960, p.31 -48

14.HUFFMAN,D.A.:"The Synthesis of Linear Sequential Coding Networks", 1955, p.77-95

15.HARA,M.,and SUDA,N.:"Some Investigations into the Application of Pseudorandom Binary Signals to Reactor-Dynamics Measurements", p.247-270

16.GODFREY,K.R.:"Theory and Application of Pseudorandom Sequences", CONTROL, 1966, p.305-308

17.EYKHOFF,P.,Van der GRINTEN,P.M.E.M.,KWAKERNAAK,H.,and VELTMAN, B.P.Th.:"System Modelling and Identification", Survey, 1966

18.BRIGGS,P.A.N.,CLARKE,D.W.,and HAMMOND,P.H.:"An Introduction to Statistical Identification Methods in Control Systems"

19.WILSON,H.:"Automatic Elimination of Bias in Binary Crosscorrelation Experiments", ELECTRONICS LETTERS, Vol.3, No.11, 1967, p.514

20. VITERBI, A.J.: "Digital Communications with Space Applications", Prentice-Hall, 1964, Chapter 3

21. ROE, G.M.: "Pseudorandom Sequences for the Determination of System Response Characteristics", Report No.63-RL-3341E, General Electric Research Laboratory, 1963

APPENDIX

APPENDIX A

Operation of the modulo two addition is that the result of addition of two varibles is the remainder of this addition divided by 2. This operation is shown in Table A.1. It is seen that the operation of modulo two addition has some properties of both addition and subtraction and denoted by the symbol $\oplus$.

If the generator can not produce a signal of value 0 and 1, but it produces other values above and below zero level. So we will replace 1 and 0 in Table A.1(a) by +a and -a in Table A.1(b) respectively, where +a and -a are the amplitude of levels produced from the generator.

Properties of modulo two addition are listed below;

(a) $X \oplus X \oplus X \oplus \ldots \oplus X = 0$

where there are even numbers of varible X.

(b) $X \oplus X \oplus X \oplus \ldots \oplus X = X$

where there are odd numbers of varible X.

(c) $X \oplus Y \oplus Z \oplus \ldots \quad = 0$

where there are even numbles of the varible X,Y,Z,...to have the value 1.

(d) $X \oplus Y \oplus Z \oplus \ldots \quad = 1$

if an odd number of the varibles X,Y,Z,...have the value 1.

(e) commutative property of modulo two addition

$$X \oplus Y = Y \oplus X$$

(f) associative property of modulo two addition

$\oplus$	0	1
0	0	1
1	1	0

(a)

$\oplus$	$-a$	$+a$
$-a$	$-a$	$+a$
$+a$	$+a$	$-a$

(b)

Table A.1

The operation of modulo-two addition

$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$$

(g) distributive property of modulo two addition

$$XY \oplus XZ = X(Y \oplus Z)$$

(h) if $X \oplus Y = Z$, then it implies

$$X \oplus Z = Y$$

and $$Y \oplus Z = X$$

and $$X \oplus Y \oplus Z = 0$$

Note

$$(X \oplus Y)+(X \oplus Z) \neq X \oplus (Y+Z)$$

$$(X+Y) \oplus (X+Z) \neq X+(Y \oplus Z)$$

$$(X \oplus Y)\cdot(X \oplus Z) \neq X\cdot(Y \oplus Z)$$

APPENDIX B

The delta function is defined by equation;

$$\delta(\omega-\omega_0) = 0$$
$$\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\,d\omega = 1$$

where ω is not equal to ω_0. Its some properties are

(a) $$\int_{-\infty}^{\infty} \delta(\omega-\omega_0) f(\omega)\,d\omega = f(\omega_0) \quad \text{(B.1)}$$

(b) $$\delta(\omega) = \delta(-\omega) \quad \text{(B.2)}$$

(c) $$\int_{-\infty}^{\infty} e^{-j\omega t}\,dt = \delta(\omega) \quad \text{(B.3)}$$

(d) $$F\{\cos\omega_0 t\} = \tfrac{1}{2}(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)) \quad \text{(B.4)}$$

where $$\cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

and the Fourier Transform of function f(t) is expressed by

$$F\{f(t)\} = \int_{-\infty}^{\infty} f(t)\,e^{-j\omega t}\,dt$$

APPENDIX C

The power density spectrum of any signal x(t) will relate to the autocorrelation function of signal x(t) by using the Fourier Transform,

$$\Phi_{xx}(\omega) = \int_{-\infty}^{\infty} \phi_{xx}(\tau)\, e^{-j\omega\tau}\, d\tau \qquad (C.1)$$

$$= 2\int_{0}^{\infty} \phi_{xx}(\tau) \cos\omega\tau\, d\tau \qquad (C.2)$$

$$= 2\int_{0}^{T} \phi_{xx}(\tau) \cos\omega\tau\, d\tau \qquad (C.3)$$

Equation (C.1) can be changed into equation (C.2), since the autocorrelation function is an even function of and is the periodic function of period T.

Substituting for $\phi_{xx}(\tau)$, when x(t) is m-sequence signal, it gives

$$\Phi_{xx}(\omega) = 2\Big(\int_{0}^{\Delta t} a^2\big(1-\frac{2^n\tau}{(2^n-1)\Delta t}\big)\cos\omega\tau\, d\tau + \int_{\Delta t}^{(2^n-2)\Delta t} \frac{-a^2}{(2^n-1)}\cos\omega\tau\, d\tau\Big)$$

$$= \frac{a^2 2^{n+2}}{(2^n-1)\Delta t\,\omega^2}\sin^2\frac{\omega\Delta t}{2} - \frac{2a^2}{(2^n-1)\,\omega}\sin\omega(2^n-1)\Delta t \qquad (C.4)$$

Because the fundamental frequency is $\frac{2\pi}{T} = \frac{2\pi}{(2^n-1)\Delta t}$, so the r<u>th</u> harmonic is $\frac{2\pi r}{(2^n-1)\Delta t}$. Hence the second term of equation (C.4) is always zero, then

$$\Phi_{xx}(\omega) = \frac{a^2 2^n \Delta t}{(2^n-1)}\left(\frac{\sin\frac{\omega\Delta t}{2}}{\frac{\omega\Delta t}{2}}\right)^2$$

$$= \frac{a^2 2^n \Delta t}{(2^n-1)}\left(\frac{\sin\frac{r\pi}{2^n-1}}{\frac{r\pi}{2^n-1}}\right)^2 \qquad (C.5)$$

where r is positive integer, 1,2,3,.... .

APPENDIX D

From property of m-sequence $x(t)$, the sequence $\bar{x}$ and x' are defined by

$$\bar{x}(t) = \frac{1}{2}(x(t) + a) \qquad (D.1)$$

$$x'(t) = (I \oplus D)\,\bar{x}(t) \qquad (D.2)$$

$$= D^{k_s}\,\bar{x}(t)$$

$$= \bar{x}(t - k_s\Delta t) \qquad (D.3)$$

The sequence $x'(t)$ has only positive non zero parts and their position is corresponded to all positive non zero parts of $w(t)$. Then $w(t)$ is related to $x'(t)$ by

$$w(t) = \frac{\frac{\gamma-\lambda}{2}}{a\Delta t}\,x'(t) \qquad (D.4)$$

$$= \frac{\gamma-\lambda}{2a\Delta t}\,\bar{x}(t-k_s\Delta t) \qquad (D.5)$$

$$= \frac{\gamma-\lambda}{2a\Delta t}\cdot\frac{1}{2}(x(t-k_s\Delta t) + a) \qquad (D.6)$$

The crosscorrelation between input $x(t)$ and error $w(t)$ is

$$\phi_{xw}(\tau) = \frac{1}{T}\int_0^T x(t)\,w(t+\tau)\,dt \qquad (D.7)$$

$$= \frac{1}{T}\int_0^T x(t)\left(\frac{\gamma-\lambda}{4a\Delta t}\right)(x(t+\tau-k_s\Delta t)+a)\,dt$$

$$= \frac{\gamma-\lambda}{4a\Delta t}\left(\frac{1}{T}\int_0^T x(t)\,x(t+\tau-k_s\Delta t)\,dt + \frac{1}{T}\int_0^T x(t)\,dt\right)$$

$$\phi_{xw}(\tau) = \frac{\gamma-\lambda}{4a\Delta t}\left(\phi_{xx}(\tau - k_s\Delta t) + \frac{a^2}{2^n-1}\right) \qquad (D.8)$$

where $T = (2^n-1)\Delta t$, and

$$\int_0^T x(t)\,dt = a\Delta t$$

When the nonreversible transition error is considered. It is seen that, from Figure 3.8, each non zero part of $w_1(t)$ leads non zero part of sequence $x'(t)$ one unit time Δt. Then

$$w_1(t) = \frac{\epsilon(t)-\mu(t)}{2a\Delta t}\,x'(t+\Delta t) \qquad (D.9)$$

$$= \frac{\epsilon(t)-\mu(t)}{2a\Delta t}\,\bar{x}(t-k_s\Delta t+\Delta t) \qquad (D.10)$$

$$= \frac{\epsilon(t)-\mu(t)}{4a\Delta t}\left(x(t-(k_s-1)\Delta t)+a\right) \qquad (D.11)$$

so,

$$\phi_{xw_1}(\tau) = \frac{1}{T}\int_0^T x(t)\,w_1(t+\tau)\,dt \qquad (D.12)$$

$$= \frac{1}{T}\int_0^T x(t)\,\frac{\epsilon(t)-\mu(t)}{4a\Delta t}\left(x(t+\tau-(k_s-1)\Delta t)+a\right)dt$$

$$= \frac{\epsilon(t)-\mu(t)}{4a\Delta t}\left(\phi_{xx}(\tau-(k_s-1)\Delta t) + \frac{a^2}{2^n-1}\right) \qquad (D.13)$$

But each non zero part of $w_2(t)$ is still in phase with non zero part of $x(t)$, then

$$w_2(t) = \frac{(\gamma-\epsilon(t))-(\lambda-\mu(t))}{2a\Delta t}\,x'(t) \qquad (D.14)$$

$$= \frac{(\gamma-\epsilon(t))-(\lambda-\mu(t))}{2a\Delta t}\,\bar{x}(t-k_s\Delta t) \qquad (D.15)$$

$$= \frac{(\gamma-\epsilon(t))-(\lambda-\mu(t))}{4a\Delta t}\left(x(t-k_s\Delta t)+a\right) \qquad (D.16)$$

and

$$\phi_{xw_2}(\tau) = \frac{1}{T}\int_0^T x(t)\,w_2(t+\tau)\,dt \qquad (D.17)$$

$$\phi_{xw_2}(\tau) = \frac{1}{T}\int_0^T x(t)\,\frac{(\gamma-\epsilon(t))-(\lambda-\mu(t))}{4a\Delta t}\left(x(t+\tau-k_s\Delta t)+a\right)$$

$$= \frac{(\gamma-\epsilon(t))-(\lambda-\mu(t))}{4a\Delta t}\left(\phi_{xx}(\tau-k_s\Delta t)+\frac{a^2}{2^n-1}\right) \qquad \text{(D.18)}$$

APPENDIX E

From eqn. (4.5), we have

$$\sum_{i=0}^{m} b_j (t+\tau)^i = b_0 + b_1(t+\tau) + \ldots + b_m(t+\tau)^m \qquad (E.1)$$

$$= (b_0 + \tau b_1 + \tau^2 b_2 + \ldots + \tau^m b_m)$$

$$+ (b_1 + 2\tau b_2 + 3\tau^2 b_3 + \ldots + m\tau^{m-1} b_m)t$$

$$+ (b_2 + 3\tau b_3 + 6\tau^2 b_4 + \ldots + \frac{m(m+1)}{2}\tau^{m-2} b_m)t^2$$

$$+ \ldots$$

$$= \sum_{j=0}^{m} b_j \tau^j + t\sum_{j=1}^{m} j b_j \tau^{j-1} + t^2 \sum_{j=2}^{m} \frac{j(j+1)}{2} b_j \tau^{j-2} + \ldots$$

$$= t^0 \sum_{j=0}^{m} \frac{j!}{(j-0)!0!} b_j \tau^{j-0} + t^1 \sum_{j=1}^{m} \frac{j!}{(j-1)!1!} b_j \tau^{j-1} + \ldots$$

$$= \sum_{i=0}^{m} t^i \sum_{j=i}^{m} \frac{j!}{(j-i)!\,i!} b_j \tau^{j-i}$$

$$= \sum_{i=0}^{m} t^i \sum_{j=i}^{m} {}^{j}C_i \, b_j \tau^{j-i} \qquad (E.2)$$

From eqn. (4.11), we have

$$\sum_{i=0}^{m} M_i \sum_{j=i}^{m} b_j \, {}^{j}C_i \tau^{j-i} = \frac{1}{T}\int_0^T x(t)\,dt \left[b_0 + b_1\tau + b_2\tau^2 + \ldots\right]$$

$$+ \frac{1}{T}\int_0^T x(t)\,t\,dt \left[b_1 + 2b_2\tau + 3b_3\tau^2 + \ldots\right]$$

$$+ \frac{1}{T}\int_0^T x(t)\,t^2\,dt \left[b_2 + 3b_3\tau + 6b_4\tau^2 + \ldots\right]$$

$$+ \ldots$$

$$\sum_{i=0}^{m} M_i \sum_{j=i}^{m} b_j \, {}^jC_i \, \tau^{j-i} = \tau^0\left\{ b_0 \frac{1}{T}\int_0^T x(t)dt + b_1 \frac{1}{T}\int_0^T x(t)t\,dt + \ldots \right\}$$

$$+ \tau^1\left\{ b_1 \frac{1}{T}\int_0^T x(t)dt + 2b_2 \frac{1}{T}\int_0^T x(t)t\,dt + \ldots \right\}$$

$$+ \tau^2\left\{ b_2 \frac{1}{T}\int_0^T x(t)dt + 3b_3 \frac{1}{T}\int_0^T x(t)t\,dt + \ldots \right\}$$

$$+ \ldots$$

$$= \tau^0 \sum_{j=0}^{m} b_j \frac{1}{T}\int_0^T x(t)t^j dt$$

$$+ \tau^1 \sum_{j=1}^{m} b_j \frac{j!}{(j-1)!\,1!} \frac{1}{T}\int_0^T x(t)t^j dt$$

$$+ \tau^2 \sum_{j=2}^{m} b_j \frac{j!}{(j-2)!\,2!} \frac{1}{T}\int_0^T x(t)t^j dt$$

$$+ \ldots$$

$$= \sum_{i=0}^{m} \tau^i \sum_{j=i}^{m} b_j \frac{j!}{(j-i)!\,i!} \frac{1}{T}\int_0^T x(t)t^j dt \qquad \text{(E.3)}$$

where $\quad {}^jC_i = \frac{j!}{(j-i)!\,i!}$

$$M_j = \frac{1}{T}\int_0^T x(t)t^j dt$$

thus eqn.(E.3) becomes

$$= \sum_{i=0}^{m} \tau^i \sum_{j=i}^{m} {}^jC_i \, b_j M_j \qquad \text{(E.4)}$$

APPENDIX F

From eqn.(4.16), we have

$$\phi_{xn_i}^{2}(\tau) = \left\{ \frac{1}{T}\int_0^T x(t)\, n_i(t+\tau)\, dt \right\}^2 \tag{F.1}$$

$$= \frac{1}{T^2}\int_0^T\int_0^T x(t_1)\, x(t_2)\, n_i(t_1+\tau)\, n_i(t_2+\tau)\, dt_1 dt_2 \tag{F.2}$$

Let $t_2 = t_1 - \tau$, we obtain

$$\phi_{xn_i}^{2}(\tau) = \frac{1}{T^2}\int_0^T\int_0^T x(t_1)\, x(t_1-\tau)\, n_i(t_1+\tau)\, n_i(t_1)\, dt_1 dt_1 \tag{F.3}$$

$$= \frac{1}{T}\int_0^T x(t_1)\, x(t_1-\tau)\, dt_1 \; \frac{1}{T}\int_0^T n_i(t_1+\tau)\, n_i(t_1)\, dt_1 \tag{F.4}$$

$$= \phi_{xx}(\tau)\, \phi_{n_i n_i}(\tau) \tag{F.5}$$

APPENDIX G

By Taylor series expansion, the convolution integral $\rho(\tau)$ in eqn. (5.86) can be rewritten as

$$\rho(\tau) = h(\tau)\int_0^T \phi_{xn_i}(\tau-t)dt + h'(\tau)\int_0^T (t-\tau)\phi_{xn_i}(\tau-t)dt + \ldots \qquad (G.1)$$

When the external noise signal is the time polynomial function. The crosscorrelation function $\phi_{xn_i}(\tau-t)$ is

$$\phi_{xn_i}(\tau-t) = A_0 + A_1(\tau-t) + \ldots + A_m(\tau-t)^m \qquad (G.2)$$

From eqn. (G.1) and eqn. (G.2), we obtain

$$\begin{aligned}\rho(\tau) &= h(\tau)\left\{A_0\int_0^T dt + A_1\int_0^T(\tau-t)dt + \ldots + A_m\int_0^T(\tau-t)^m dt\right\} \\ &+ \ldots\ldots\ldots \\ &+ \frac{h^r(\tau)}{r!}\left\{A_0\int_0^T(t-\tau)^r dt + \ldots + A_m\int_0^T(t-\tau)^r(\tau-t)^m dt\right. \\ &+ \ldots \\ &= B_0 h(\tau) + B_1 h(\tau) + \ldots \end{aligned} \qquad (G.3)$$

where

$$\begin{aligned} B_r &= \frac{1}{r!}\left\{A_0\int_0^T(t-\tau)^r dt + \ldots + (-1)^m A_m\int_0^T(t-\tau)^{r+m}dt\right\} \\ &= \frac{1}{r!}\left\{\frac{A_0}{r+1}\left[\langle T-\tau\rangle^{r+1} - \langle-\tau\rangle^{r+1}\right] - \frac{A_1}{r+2}\left[\langle T-\tau\rangle^{r+2} - \langle-\tau\rangle^{r+2}\right]\right. \\ &\qquad \left. + \ldots + \frac{(-1)^m A_m}{r+m+1}\left[\langle T-\tau\rangle^{r+m+1} - (-\tau)^{r+m+1}\right]\right\} \\ &= \frac{1}{r!}\sum_{i=0}^{m}\frac{(-1)^i A_i}{r+i+1}\left\{(T-\tau)^{r+i+1} - (-\tau)^{r+i+1}\right\} \end{aligned}$$

When the external noise signal is the white noise. The crosscorrelation function $\phi_{xn_i}(\tau - t)$ is

$$\phi_{xn_i}(\tau - t) = a\sqrt{K} \tag{G.4}$$

From the eqn. (G.1) and eqn. (G.4), we have

$$\rho(\tau) = h(\tau)\int_0^T a\sqrt{K}dt + h'(\tau)\int_0^T (t-\tau)a\sqrt{K}\,dt + \dots$$

$$= D_0 h(\tau) + D_1 h(\tau) + \dots \tag{G.5}$$

where
$$D_r = \frac{1}{r!}\int_0^T a\sqrt{K}(t-\tau)^r dt$$

$$= a\sqrt{K}\left(\frac{1}{r!}\right)\left(\frac{1}{r+1}\right)\left\{(T-\tau)^{r+1} - (-\tau)^{r+1}\right\}$$

$$= a\sqrt{K}\left(\frac{(-1)^{r+1}}{(r+1)!}\right)\left\{(\tau - T)^{r+1} - (\tau)^{r+1}\right\}$$

APPENDIX H

For example, the binary m-sequence with period $T = 7\Delta t$ is shown in the figure H.1(a). This m-sequence starts with the run of 3 ones. Its time moment can be determined by

$$M_i = \frac{1}{T}\int_0^T x(t)t^i dt \qquad (H.1)$$

when $i = 0$

$$M_0 = \frac{1}{7\Delta t}\left\{\int_0^{3\Delta t}(+a)dt + \int_{3\Delta t}^{4\Delta t}(-a)dt + \int_{4\Delta t}^{5\Delta t}(+a)dt + \int_{5\Delta t}^{7\Delta t}(-a)dt\right\}$$

$$= 0.142857a$$

when $i = 1$

$$M_1 = \frac{1}{7\Delta t}\left\{\int_0^{3\Delta t}(+a)tdt + \int_{3\Delta t}^{4\Delta t}(-a)tdt + \int_{4\Delta t}^{5\Delta t}(+a)tdt + \int_{5\Delta t}^{7\Delta t}(-a)tdt\right\}$$

$$= -0.928571a\Delta t$$

when $i = 2$

$$M_2 = \frac{1}{7\Delta t}\left\{\int_0^{3\Delta t}(+a)t^2dt + \int_{3\Delta t}^{4\Delta t}(-a)t^2dt + \int_{4\Delta t}^{5\Delta t}(+a)t^2dt + \int_{5\Delta t}^{7\Delta t}(-a)t^2dt\right\}$$

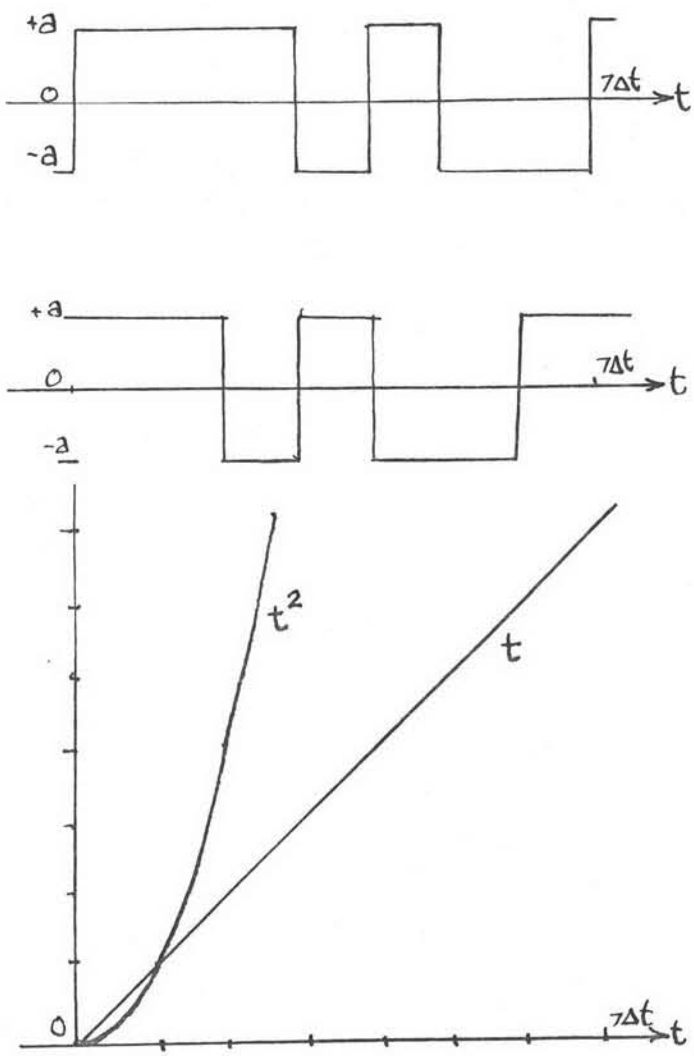
$$= -7.952380a\Delta t^2$$

and so on ...

Let $a = 1$, and $t = 0.1$, we have

$$\left.\begin{aligned} M_0 &= 0.142857 \\ M_1 &= -0.092857 \\ M_2 &= -0.079524 \end{aligned}\right\} \qquad (H.2)$$

and so on ...



(a) The m-sequence x(t) of period $7\Delta t$

(b) The same m-sequence as in (a), but different in starting point

(c) The time polynomial function of external noise.

Figure H-1

If this m-sequence has a new starting point as shown in the figure H.1(b) which advances the original m-sequence by Δt. Thus, the time moments are

$$M_0 = \frac{1}{7\Delta t}\left\{\int_0^{2\Delta t}(+a)dt + \int_{2\Delta t}^{3\Delta t}(-a)dt + \int_{3\Delta t}^{4\Delta t}(+a)dt + \int_{4\Delta t}^{6\Delta t}(-a)dt + \int_{6\Delta t}^{7\Delta t}(+a)dt\right\}$$

$$= 0.142857a$$

$$M_1 = \frac{1}{7\Delta t}\left\{\int_0^{2\Delta t}(+a)tdt + \int_{2\Delta t}^{3\Delta t}(-a)tdt + \int_{3\Delta t}^{4\Delta t}(+a)tdt + \int_{4\Delta t}^{6\Delta t}(-a)tdt + \int_{6\Delta t}^{7\Delta t}(+a)tdt\right\}$$

$$= -0.071428a\Delta t$$

$$M_2 = \frac{1}{7\Delta t}\left\{\int_0^{2\Delta t}(+a)t^2dt + \int_{2\Delta t}^{3\Delta t}(-a)t^2dt + \int_{3\Delta t}^{4\Delta t}(+a)t^2dt + \int_{4\Delta t}^{6\Delta t}(-a)t^2dt + \int_{6\Delta t}^{7\Delta t}(+a)t^2dt\right\}$$

$$= -0.00476a\Delta t^2$$

and so on ...

Let $a = 1$, and $\Delta t = 0.1$, we have

$$\left.\begin{aligned} M_0 &= 0.142857 \\ M_1 &= -0.007143 \\ M_2 &= -0.000048 \end{aligned}\right\} \quad (H.3)$$

and so on ...

It can be seen that the values of the time moment in the eqn. (H.3) are less than the values of the time moment in eqn. (H.1)

VITA

The writer of this thesis, Somchai Jitapunkul, was born on November 15,1950 in Bangkok, Thailand.He received a Bachelor's - Degree of Engineering(Honour) in Electrical Engineering from Chulalongkorn University in 1972. He is now a member of the Faculty staff at Chulalongkorn University.