CHAPTER 3

IMPERFECT TRANSDUCER

3.1 Introduction

Consider the input signal X(t) passes through a transducer into a linear system as shown in figure 3.1. The response of the transducer, in general, can not be identical to the input signal due to the imperfect of the components used in the transducer device. This will cause an error in the output Y(t) of the linear system. When the crosscorrelation between input X(t) and output Y(t) is performed, the value of the impulse response obtained will be deviated. The cause of the deviation in the process will be described in the following sections.

3.2 Transition Error

The distortion occurring in the transition region of the input signal produced by the imperfect transducer may be called the "transition error"[8] or "zero crossing error"[7,3] and denoted by e(t). This error may be considered as a signal.[7,8]

The input signal X(t), the error signal e(t) and the response signal $\dot{X}_e(t)$ are illustrated in figure 3.2(a), (b), and (c) respectively. From figure 3.2, it can be seen that the

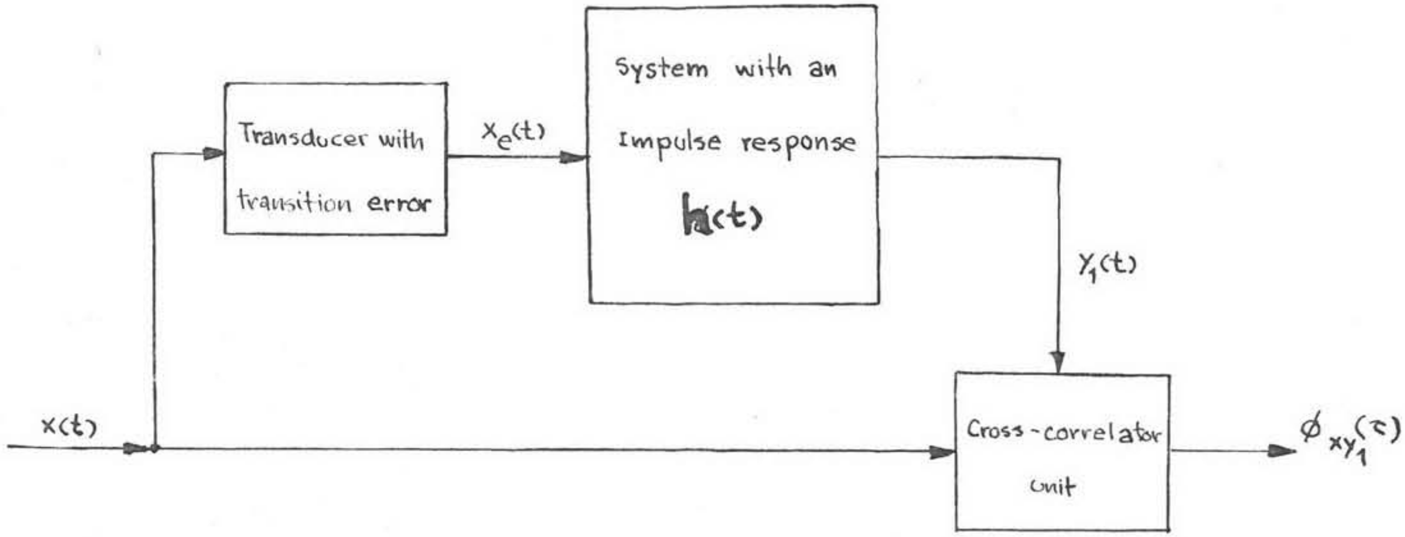


Figure 3.1

Block diagram shows the cross-correlation function when the input signal x(t) passes the transducer response signal $X_e(t)$ can be expressed as.

$$x_e(t) = x(t) + e(t) \tag{3.1}$$

The output response $Y_e(t)$ of the linear system due to response,signal $X_e(t)$ is

$$y_e(t) = \int_0^\infty h(u)\, x_e(t-u)\, du \tag{3.2}$$

The crosscorrelation between the input x(t) and the output response $Y_e(t)$ becomes

$$\phi_{xy_e}(\tau) = \frac{1}{T}\int_0^T x(t)\, y_e(t+\tau)\, dt \tag{3.3}$$

From eqn.(3.2) and eqn.(3.3), we have

$$\phi_{xy_e}(\tau) = \frac{1}{T}\int_0^T \int_0^\infty h(u)\, x(t)\, x_e(t+\tau-u)\, du\, dt \tag{3.4}$$

Substituting for $x_e(t+\tau-u)$ using eqn.(3.1), eqn.(3.4) becomes

$$\phi_{xy_e}(\tau) = \int_0^\infty h(u)\, \frac{1}{T}\int_0^T x(t)\left\{ x(t+\tau-u) + e(t+\tau-u)\right\} dt\, du$$

$$= \int_0^\infty h(u)\, \phi_{xx}(\tau-u)\, du + \int_0^\infty h(u)\, \phi_{xe}(\tau-u)\, du \tag{3.5}$$

The form of the first integral on the right-hand side of eqn.(3.5) is known (see eqn.(1.3)), so that the second integral must be evaluated, and for this the crosscorrelation function $\phi_{xe}(\tau)$ is required.

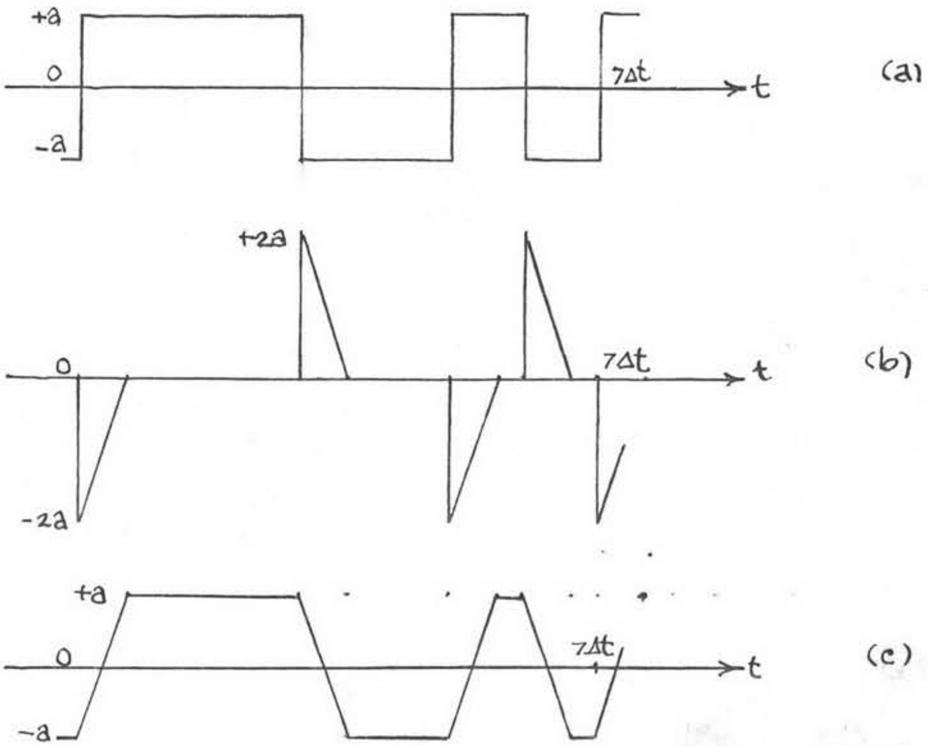


Figure 3.2

(a) m-sequence X of period $7\Delta t$ with states +a and -a

(b) error signal produced by transducer

(c) The input response $x_e(t) = x(t) + e(t)$

3.3 Determination of $\phi_{xe}(\tau)$ for Reversible Transition Error

The transition error $e(t)$ may be considered as non zero discrete areas at each zero crossing of the input signal $x(t)$ as shown in figure 3.2. These non zero parts have both positive and negative direction. The numbers of the positive non zero part are equal to the numbers of negative non zero part, and each has 2^{n-2} numbers[8].

If we let $e_+(t)$ represents the positive non zero part of $e(t)$, and θ_+ be the transition time of the transducer from the +a to the -a state of the input $x(t)$. Similarly, $e_-(t)$ represents the negative non zero part of $e(t)$, and θ_- be the transition time of the transducer from the -a to the +a state of the input $x(t)$. It is assumed for this analysis that both θ_+ and θ_- are less than Δt From figure 3.2, it can be seen that, the error signal, $e(t)$ will be equal to zero except at the time regions θ_+ and θ_-

When the positive non zero part is the same as the negative non zero part, this error signal, $e(t)$ is called "the reversible transition error signal".[7,8,9] Then, we have

$$\theta_+ = \theta_- = \theta \tag{3.6}$$

and

$$e_+(t) = -e_-(t) \tag{3.7}$$

In this case the area of each non zero part of the reversible transition error $e(t)$ is

$$\alpha = \int_0^{\theta} e_+(t)\,dt = \left| \int_0^{\theta} e_-(t)\,dt \right| \tag{3.8}$$

Since the input $x(t)$ and the error $e(t)$ are periodic with period T, the crosscorrelation between the input signal and the error signal is also periodic with the same period[7,20]. Hence it is

$$\phi_{xe}(\tau) = \frac{1}{T}\int_0^T x(t)\,e(t+\tau)\,dt \qquad (3.9)$$

When the value of τ is increasing from 0 to T, the error signal $e(t)$ is relatively shifting to the left of the input signal. It is convenient to divide τ into a series of regions as follows.

3.3.1 Nonoverlapping regions

The nonoverlapping region may be defined as the region when the non zero parts of the error signal $e(t)$ do not overlap the zero crossings of the input signal $x(t)$. These regions exist for

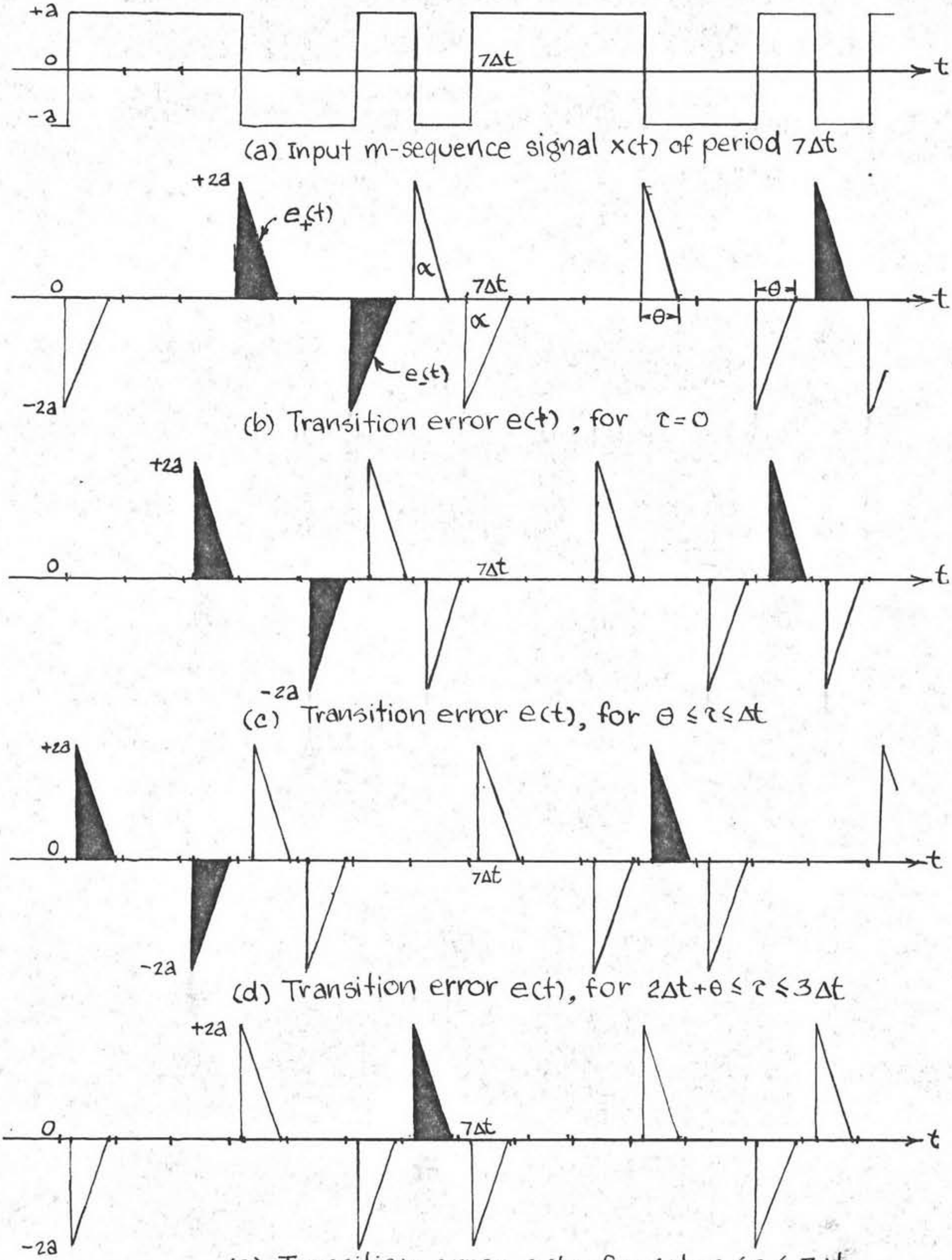
$$k\,\Delta t + \theta \leq \tau \leq (k+1)\Delta t,$$

where k is the positive integer in the range $0 \leq k \leq 2^n-2$. Thus, the crosscorrelation function $\phi_{xe}(\tau)$ can be determined in separate regions of the time shift τ.

(a) For the region $\theta \leq \tau \leq \Delta t$. It is seen that all positive non zero parts of the error signal $e(t)$ move into the positive regions of the input signal $x(t)$, and all negative non zero parts of the error signal also move into the negative regions of the input signal. The crosscorrelation function $\phi_{xe}(\tau)$ is

$$\phi_{xe}(\tau) = \frac{1}{T}\int_0^T x(t)\,e(t+\tau)\,dt$$

$$= \frac{1}{(2^n-1)\Delta t}\sum^{2^{n-1}}\left\{(\text{magnitude of } x(t))(\text{area of each non-zero part of } e(t))\right\}$$



(a) Input m-sequence signal $x(t)$ of period $7\Delta t$

(b) Transition error $e(t)$, for $\tau = 0$

(c) Transition error $e(t)$, for $\theta \leq \tau \leq \Delta t$

(d) Transition error $e(t)$, for $2\Delta t + \theta \leq \tau \leq 3\Delta t$

(e) Transition error $e(t)$, for $6\Delta t + \theta \leq \tau \leq 7\Delta t$

Figure 3.3

The shifting of reversible transition error $e(t)$

$$\phi_{xe}(\tau)= \frac{1}{(2^n-1)\Delta t}\Big\{(+a)(+\alpha)(\text{number of all positive non zero parts of } e(t)) + (-a)(-\alpha)(\text{number of all negative non zero parts of } e(t))\Big\}$$

$$= \frac{a\alpha 2^{n-1}}{(2^n-1)\Delta t} \tag{3.10}$$

(b) For the region $k_1\Delta t+\theta \leq \tau \leq (k_1+1)\Delta t$, where k_1 is positive integer in the range $1 \leq k_1 \leq 2^n-3$. It is seen that one half of the positive non zero parts of the error signal will move into the positive regions of the input signal, and the other half of the positive non zero parts of the error signal will move into the negative regions of the input signal. This is similarly happened for the negative non zero parts of the error signal. Thus, we have

$$\phi_{xe}(\tau)= \frac{1}{(2^n-1)\Delta t}\Big\{(+a)(+\alpha)(\frac{2^{n-2}}{2}) + (-a)(+\alpha)(\frac{2^{n-2}}{2}) + (+a)(-\alpha)(\frac{2^{n-2}}{2}) + (-a)(-\alpha)(\frac{2^{n-2}}{2})\Big\}$$

$$=0 \tag{3.11}$$

(c) For the region $(2^n-2)\Delta t+\theta \leq \tau \leq (2^n-1)\Delta t$, or $0 \leq -\tau \leq \Delta t - \theta$. In this region, all of non zero parts of the error signal will be im the opposite direction of the input signal. Then, we have

$$\phi_{xe}(\tau) = \frac{1}{(2^n-1)\Delta t}\Big\{(-a)(+\alpha)(2^{n-2})+(+a)(-\alpha)(2^{n-2})\Big\}$$

$$= \frac{-a\alpha 2^{n-1}}{(2^n-1)\Delta t} \tag{3.12}$$

3.3.2 Overlapping regions

The overlapping region may be defined as the region when the non zero parts of the error signal e(t) will overlap the zero crossing of the input signal x(t). These regions exist for

$$k\Delta t \leq \tau \leq k\Delta t + \theta .$$

Let the overlapping time be ν . Then each overlapping area is expressed as

$$\beta(\nu) = \int_0^{\nu} e_+(t)dt = \left| \int_0^{\nu} e_-(t)dt \right| \qquad (3.13)$$

It can be seen that the value of $\beta(\nu)$ depends on the overlapping time ν .

In the determination of $\phi_{xe}(\tau)$, the error signal e(t) may be parted into two components of reversible error signal $e_1(t)$ and $e_2(t)$. Let

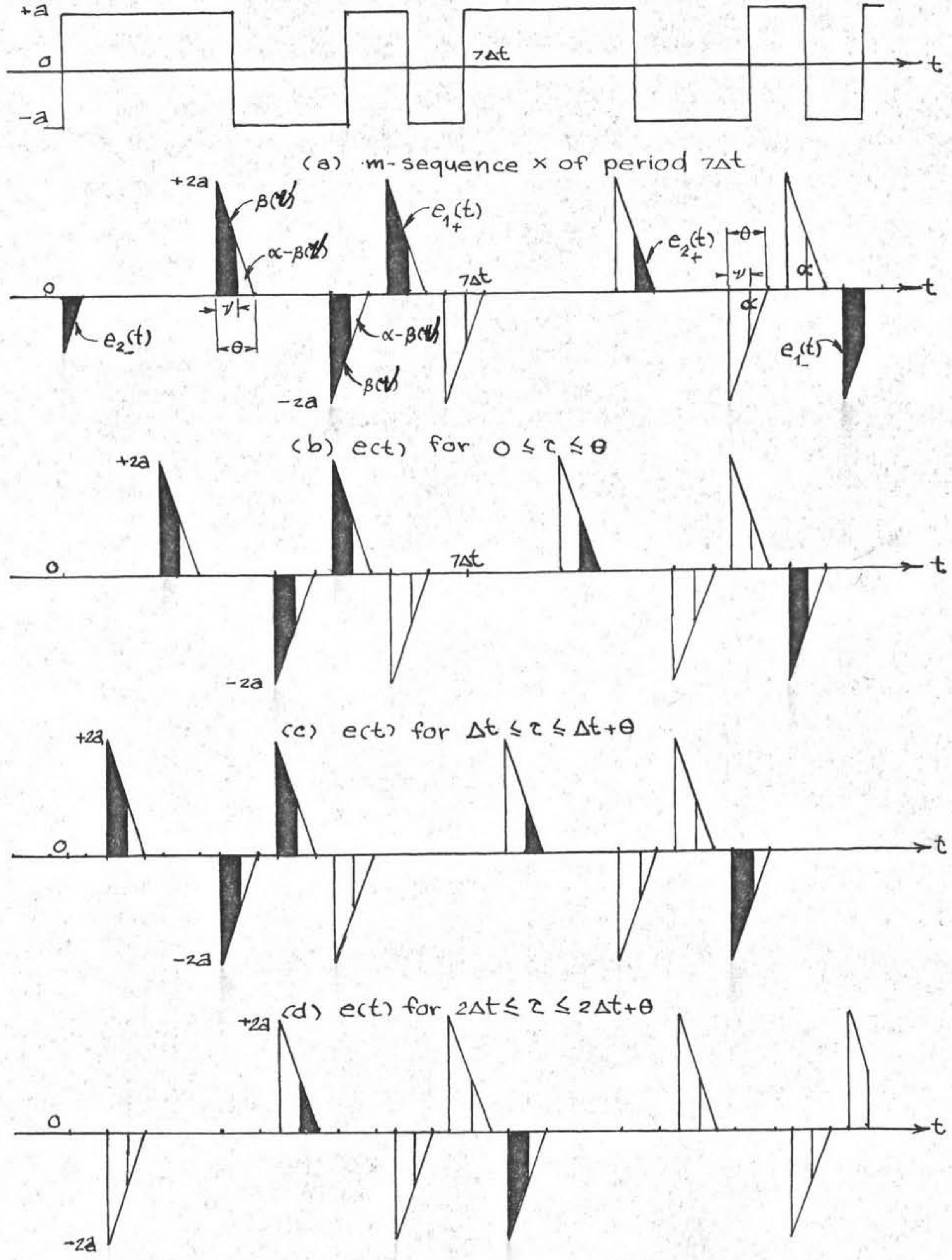
$$\int_0^{\nu} e_{1+}(t)dt = \left| \int_0^{\nu} e_{1-}(t)dt \right| = \beta(\nu) \qquad (3.14)$$

and
$$\int_{\nu}^{\theta} e_{2+}(t)dt = \left| \int_{\nu}^{\theta} e_{2-}(t)dt \right| = \alpha - \beta(\nu) \qquad (3.15)$$

where
$$e(t) = e_1(t) + e_2(t) \qquad (3.16)$$

and
$$\phi_{xe}(\tau) = \phi_{xe_1}(\tau) + \phi_{xe_2}(\tau) \qquad (3.17)$$

Since the error signals $e_1(t)$ and $e_2(t)$ do not overlap the zero crossing of the input signal x(t). The case of non overlapping region can be applied directly to determine the values of $\phi_{xe_1}(\tau)$ and $\phi_{xe_2}(\tau)$. However, the time shift τ of the error signal $e_1(t)$ will lead the error signal $e_2(t)$ by Δt as shown in figure3.4.



(a) m-sequence x of period $7\Delta t$

(b) e(t) for $0 \leq \tau \leq \theta$

(c) e(t) for $\Delta t \leq \tau \leq \Delta t+\theta$

(d) e(t) for $2\Delta t \leq \tau \leq 2\Delta t+\theta$

(e) e(t) for $6\Delta t \leq \tau \leq 6\Delta t+\theta$

Figure 3.4 The shifting of reversible error for overlapping case

(a) For the region $0 \leq \tau \leq \theta$

Applying the similar technique described in the section 3.3.1(a), for $e_1(t)$, and the similar technique described in the section 3.3.1(c), for $e_2(t)$, we obtain

$$\phi_{xe_1}(\tau) = \frac{a\,\beta(\nu)2^{n-1}}{(2^n-1)\,\Delta t} \tag{3.18}$$

and $$\phi_{xe_2}(\tau) = \frac{-a(\alpha - \beta(\nu))2^{n-1}}{(2^n-1)\,\Delta t} \tag{3.19}$$

Therefore

$$\phi_{xe}(\tau) = \frac{-a(\alpha - 2\beta(\nu))2^{n-1}}{(2^n-1)\,\Delta t} \quad , \nu = \tau \tag{3.20}$$

(b) For the region $\Delta t \leq \tau \leq \Delta t + \theta$

Applying the similar technique described in the section 3.3.1(b), for $e_1(t)$, and the similar technique described in the section 3.3.1(a), for $e_2(t)$, we have

$$\phi_{xe_1}(\tau) = 0 \tag{3.21}$$

$$\phi_{xe_2}(\tau) = \frac{a(\alpha - \beta(\nu))2^{n-1}}{(2^n-1)\,\Delta t} \tag{3.22}$$

Therefore

$$\phi_{xe}(\tau) = \frac{a(\alpha - \beta(\nu))2^{n-1}}{(2^n-1)\,\Delta t} \quad , \nu = \tau - \Delta t \tag{3.23}$$

(c) For the region $k_2\Delta t \leq \tau \leq k_2\Delta t + \theta$, where k_2 is the positive integer in the range $2 \leq k_2 \leq 2^n-3$.

Applying the similar technique described in the section 3.3.1(b), for both $e_1(t)$ and $e_2(t)$, we obtain

$$\phi_{xe_1}(\tau) = 0$$

$$\phi_{xe_2}(\tau) \quad = \quad 0$$

Therefore

$$\phi_{xe}(\tau) \quad = \quad 0 \tag{3.24}$$

(d)For the region $(2^n-2)\Delta t \leq \tau \leq (2^n-2)\Delta t+\Theta$

Applying the similar technique described in the section 3.3.1(c), for $e_1(t)$, and the similar technique described in the section 3.3.1(b), for $e_2(t)$, we have

$$\phi_{xe_1}(\tau) \quad = \frac{-a\ \beta(\nu)2^{n-1}}{(2^n-1)\Delta t} \tag{3.25}$$

$$\phi_{xe_2}(\tau) \quad = \quad 0 \tag{3.26}$$

Therefore

$$\phi_{xe}(\tau) \quad = \frac{-a\ \beta(\nu)2^{n-1}}{(2^n-1)\Delta t} \quad , \ \nu = \tau-(2^n-2)\Delta t \tag{3.27}$$

The complete results of the crosscorrelation between the input signal x(t) and the reversible transition error signal e(t) are retabulated in the table 3.1.

3.4 Determination of $\phi_{xe}(\tau)$ for Non-reversible Transition

Error

The error signal, e(t) is called the non-reversible transition error signal[7,8]. When the positive non zero parts are different from the negative non zero parts. It may be assumed that $\Theta_+ > \Theta_-$. The area of each positive non zero part of the error signal e(t) will be

Table 3.1

The cross-correlation function of m-sequence $x(t)$ and reversible transition error

Range of τ	Values of $\phi_{xe}(\tau)$	Note
$0 \leq \tau \leq \theta$	$-\dfrac{a(\alpha-2\beta(\theta))\,2^{n-1}}{(2^n-1)\,\Delta t}$	
$\theta \leq \tau \leq \Delta t$	$\dfrac{a\alpha\,2^{n-1}}{(2^n-1)\,\Delta t}$	
$\Delta t \leq \tau \leq \Delta t+\theta$	$\dfrac{a(\alpha-\beta(\theta))\,2^{n-1}}{(2^n-1)\,\Delta t}$	
$\Delta t+\theta \leq \tau \leq 2\Delta t$	0	
$k_2\Delta t \leq \tau \leq k_2\Delta t+\theta$	0	$2 \leq k_2 \leq 2^n-3$
$(2^n-3)\Delta t+\theta \leq \tau \leq (2^n-2)\Delta t$	0	
$(2^n-2)\Delta t \leq \tau \leq (2^n-2)\Delta t+\theta$	$-\dfrac{a\beta(\theta)\,2^{n-1}}{(2^n-1)\,\Delta t}$	
$(2^n-2)\Delta t+\theta \leq \tau \leq (2^n-1)\Delta t$	$-\dfrac{a\alpha\,2^{n-1}}{(2^n-1)\,\Delta t}$	

greater than the area of each negative non zero part of the error signal e(t), we have

$$\gamma = \int_0^{\theta_+} e_+(t)dt \tag{3.28}$$

$$\lambda = \left| \int_0^{\theta_-} e_-(t)dt \right| \tag{3.29}$$

where $\gamma > \lambda$.

Since

$$\left| \int_{\theta_-}^{\theta_+} e_-(t)dt \right| = 0 \tag{3.30}$$

Thus

$$\lambda = \left| \int_0^{\theta_+} e_-(t)dt \right| \tag{3.31}$$

In the determination of crosscorrelation function $\phi_{xe}(\tau)$, the non-reversible transition error e(t) may be separated into two types of reversible error $e'(t)$ and non-reversible error $e''(t)$. Let

$$\int_0^{\theta_-} e'_+(t)dt = \left| \int_0^{\theta_-} e'_-(t)dt \right| = \lambda \tag{3.32}$$

and

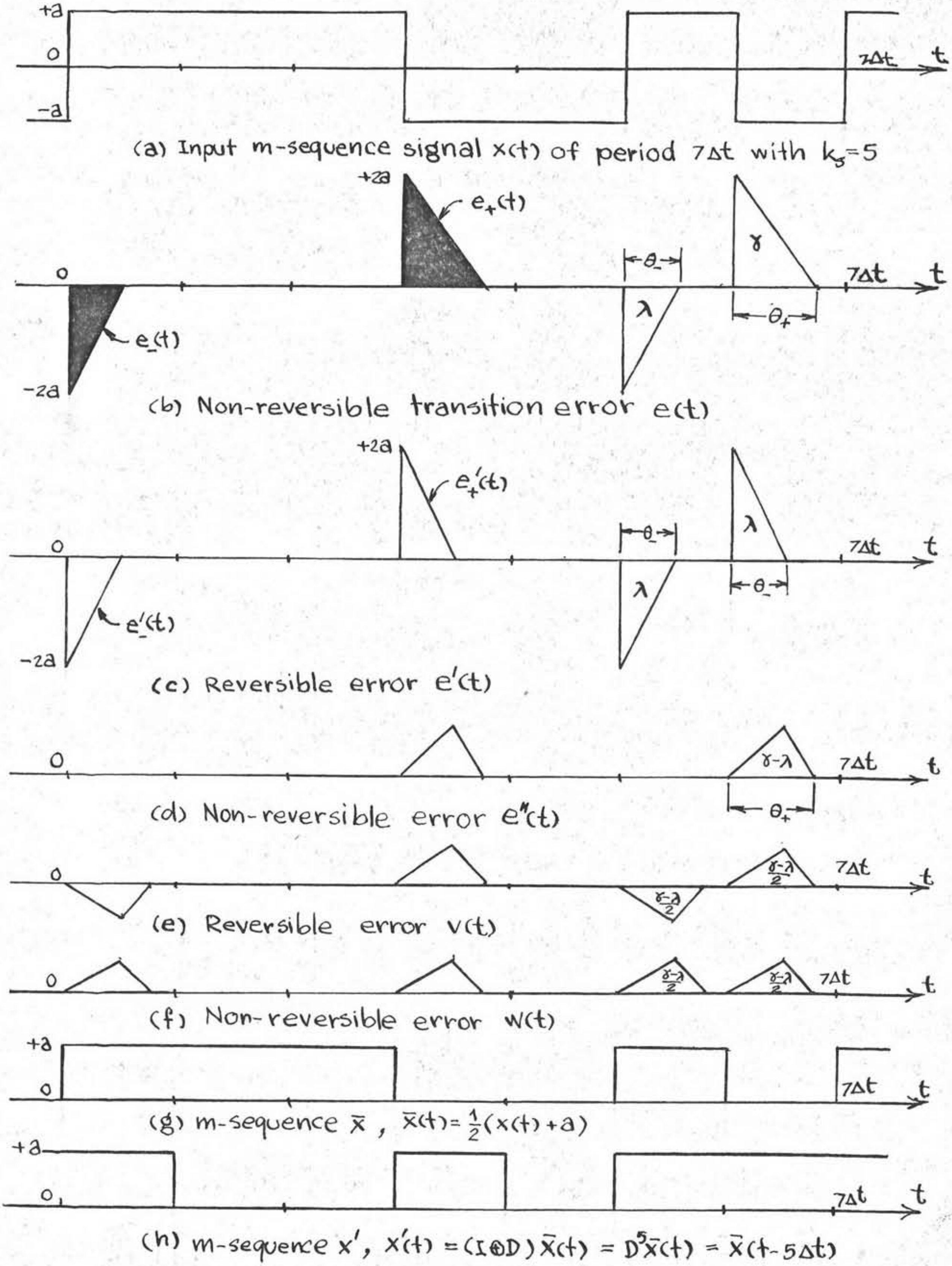
$$\left.\begin{aligned} \int_0^{\theta_+} e''_+(t)dt &= \gamma - \lambda \\ \left| \int_0^{\theta_+} e''_-(t)dt \right| &= 0 \end{aligned}\right\} \tag{3.33}$$

The non-reversible error $e''(t)$ is also divided into two error signals, the reversible error signal v(t) and the non-reversible error signal w(t) which are expressed as

$$\int_0^{\theta_+} v_+(t)dt = \left| \int_0^{\theta_+} v_-(t)dt \right| = \frac{\gamma - \lambda}{2} \tag{3.34}$$

and

$$\left.\begin{aligned} \int_0^{\theta_+} w_+(t)dt &= \frac{\gamma - \lambda}{2} \\ \left| \int_0^{\theta_+} w_-(t)dt \right| &= 0 \end{aligned}\right\} \tag{3.35}$$



(a) Input m-sequence signal $x(t)$ of period $7\Delta t$ with $k_s=5$

(b) Non-reversible transition error $e(t)$

(c) Reversible error $e'(t)$

(d) Non-reversible error $e''(t)$

(e) Reversible error $v(t)$

(f) Non-reversible error $w(t)$

(g) m-sequence $\bar{x}$, $\bar{x}(t) = \frac{1}{2}(x(t)+a)$

(h) m-sequence x', $x'(t) = (I \oplus D)\bar{x}(t) = D^5\bar{x}(t) = \bar{x}(t-5\Delta t)$

Figure 3.5

The numbers of non zero part of the error signal w(t) are equal to the numbers of non zero part of the error signal e(t), but they are all positive direction. From Appendix D, the error signal w(t) has a relation to the input signal x(t) as

$$w(t) = \frac{\gamma - \lambda}{4a\Delta t}\left\{x(t-k_s\Delta t) + a\right\} \tag{3.36}$$

Then
$$e(t) = e'(t) + e''(t) \tag{3.37}$$

$$= e'(t) + v(t) + w(t) \tag{3.38}$$

$$= e'(t) + v(t) + \frac{\gamma - \lambda}{4a\Delta t}\left\{x(t-k_s\Delta t) + a\right\} \tag{3.39}$$

and
$$\phi_{xe}(\tau) = \phi_{xe'}(\tau) + \phi_{xv}(\tau) + \frac{\gamma - \lambda}{4a\Delta t}\left\{\phi_{xx}(\tau - k_s\Delta t) + \frac{a^2}{2^n-1}\right\} \tag{3.40}$$

The waveform of $\left\{\phi_{xx}(\tau - k_s\Delta t) + \frac{a^2}{2^n-1}\right\}$ is illustrated in figure 3.6 for easy reference.

3.4.1 Nonoverlapping regions

The crosscorrelation function $\phi_{xe}(\tau)$ can be obtained in separate regions of the time shift τ.

(a) For the region $\Theta_{+} \leq \tau \leq \Delta t$.

Applying the similar technique described in the section 3.3.1(a), we obtain

$$\phi_{xe'}(\tau) = \frac{a\lambda 2^{n-1}}{(2^n-1)\Delta t} \tag{3.41}$$

$$\phi_{xv}(\tau) = \frac{a\left(\frac{\gamma - \lambda}{2}\right)2^{n-1}}{(2^n-1)\Delta t} \tag{3.42}$$

From figure 3.6, the value of $\phi_{xw}(\tau)$ which relates to $\phi_{xx}(\tau - k_s\Delta t)$

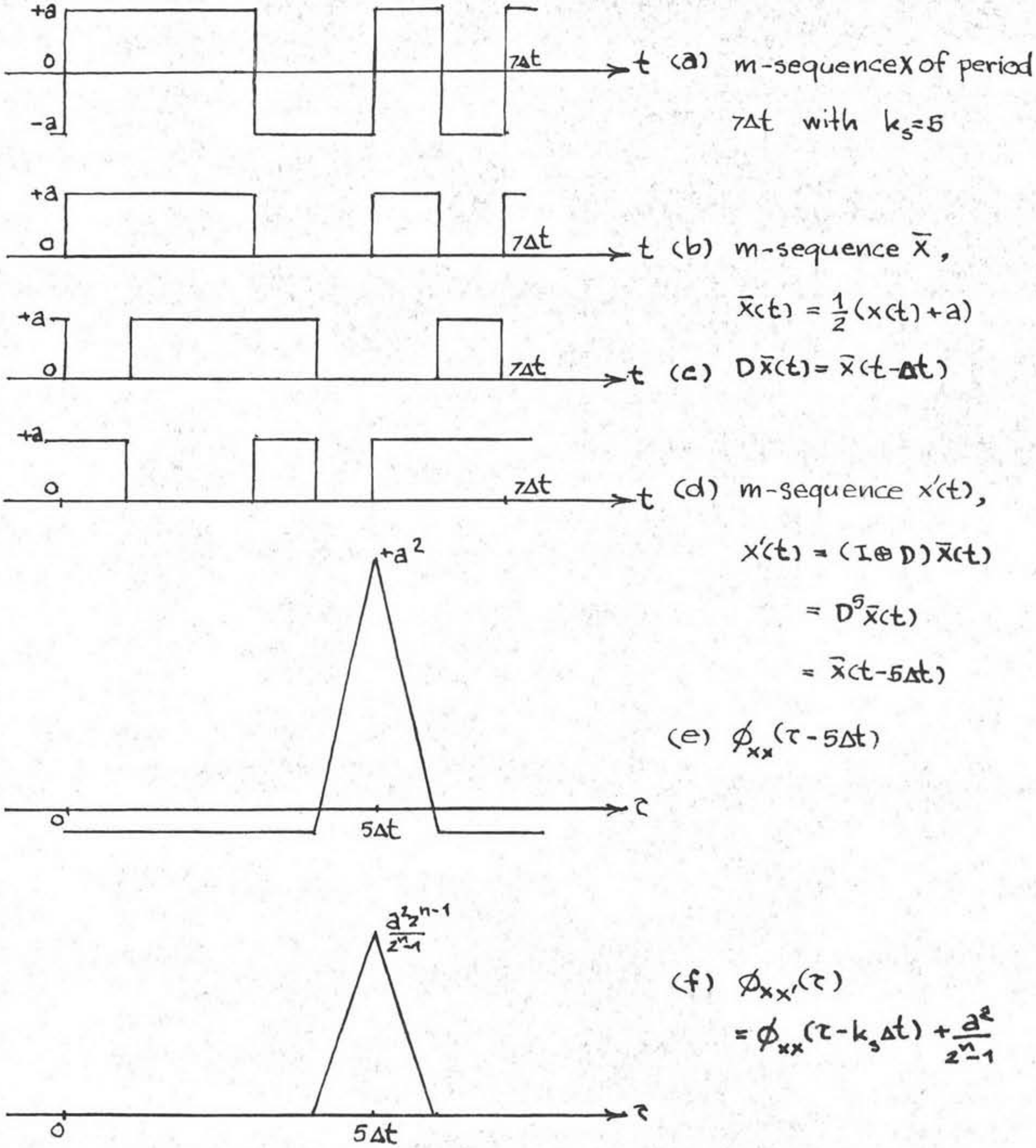

+a
0
-a
7Δt
t (a) m-sequence x of period
7Δt with $k_s=5$
+a
0
7Δt
t (b) m-sequence $\bar{x}$,
$\bar{x}(t) = \frac{1}{2}(x(t)+a)$
+a
0
7Δt
t (c) $D\bar{x}(t) = \bar{x}(t-\Delta t)$
+a
0
7Δt
t (d) m-sequence $x'(t)$,
$x'(t) = (I \oplus D)\bar{x}(t)$
$= D^5\bar{x}(t)$
$= \bar{x}(t-5\Delta t)$
$+a^2$
(e) $\phi_{xx}(\tau-5\Delta t)$
0
5Δt
τ
$\frac{a^2 2^{n-1}}{2^n-1}$
(f) $\phi_{xx'}(\tau)$
$= \phi_{xx}(\tau-k_s\Delta t) + \frac{a^2}{2^n-1}$
0
5Δt
τ

Figure 3.6

depends on particular integer time shift $\tau = \Delta t,\ 2\Delta t,\ 3\Delta t, \ldots$.

Therefore,

$$\phi_{xx}(\tau - k_s \Delta t) + \frac{a^2}{2^n - 1} = 0 \qquad \text{for } \tau = \Delta t \tag{3.43}$$

From eqn.(3.40), eqn.(3.41), eqn.(3.42), and eqn.(3.43), we have

$$\phi_{xe}(\tau) = \frac{a(\gamma + \lambda)2^{n-2}}{(2^n - 1)\Delta t} \tag{3.44}$$

(b) For the region $k_3\Delta t + \Theta_+ \leq \tau \leq (k_3+1)\Delta t$, where k_3 is the positive integer in the range $1 \leq k_3 \leq (k_s - 2)$ and $k_s \leq k_3 \leq 2^n - 3$.

Applying the similar technique described in the section 3.3.1(b), we have

$$\phi_{xe'}(\tau) = 0 \tag{3.45}$$

$$\phi_{xv}(\tau) = 0 \tag{3.46}$$

From figure 3.6, and the value of $\tau = (k_3+1)\Delta t$, we obtain

$$\phi_{xx}(\tau - k_s \Delta t) + \frac{a^2}{2^n - 1} = 0 \tag{3.47}$$

Thus

$$\phi_{xe}(\tau) = 0 \tag{3.48}$$

(c) For the region $(2^n - 2)\Delta t + \Theta_+ \leq \tau \leq (2^n - 1)\Delta t$

Applying the similar technique described in the section 3.3.1(c), we have

$$\phi_{xe'}(\tau) = \frac{-a\lambda 2^{n-1}}{(2^n - 1)\Delta t} \tag{3.49}$$

$$\phi_{xv}(\tau) = \frac{-a\left(\frac{\gamma - \lambda}{2}\right)2^{n-1}}{(2^n - 1)\Delta t} \tag{3.50}$$

From figure 3.6, and the value of $\tau = (2^n-1)\Delta t$, we have

$$\phi_{xx}(\tau - k_s\Delta t) + \frac{a^2}{2^n-1} = 0 \quad (3.51)$$

Therefore,

$$\phi_{xe}(\tau) = \frac{-a(\gamma+\lambda)2^{n-2}}{(2^n-1)\Delta t} \quad (3.52)$$

(d) For the region $(k_s-1)\Delta t + \Theta_+ \leq \tau \leq k_s\Delta t$.

Applying the similar technique described in the section 3.3.1(b), we obtain

$$\phi_{x\acute{e}}(\tau) = 0 \quad (3.53)$$

$$\phi_{xv}(\tau) = 0 \quad (3.54)$$

From the figure 3.6, and the value of $\tau = k_s\Delta t$, we have

$$\phi_{xx}(\tau - k_s\Delta t) + \frac{a^2}{2^n-1} = \frac{a^2 2^{n-1}}{2^n-1} \quad (3.55)$$

Therefore,

$$\phi_{xe}(\tau) = \frac{a(\gamma-\lambda)2^{n-2}}{(2^n-1)\Delta t} \quad (3.56)$$

3.4.2 Overlapping regions

For overlapping regions case, the overlapping time ν is assumed to be less than both Θ_- and Θ_+. The positive overlapping area $\epsilon(\nu)$ and the negative overlapping area $\mu(\nu)$ are expressed as

$$\epsilon(\nu) = \int_0^{\nu} e_+(t)dt \quad (3.57)$$

and $$\mu(\nu) = \left| \int_0^{\nu} e_-(t)dt \right| \quad (3.58)$$

where both $\epsilon(\nu)$ and $\mu(\nu)$ depend on the overlapping time ν. By separating the error signal e(t) into two non-reversible error signals $e_1(t)$ and $e_2(t)$, we have

$$\int_0^{\nu} e_{1+}(t)dt = \epsilon(\nu) \tag{3.59}$$

$$\left|\int_0^{\nu} e_{1-}(t)dt\right| = \mu(\nu) \tag{3.60}$$

$$\int_{\nu}^{\theta+} e_{2+}(t)dt = \gamma - \epsilon(\nu) \tag{3.61}$$

$$\left|\int_{\nu}^{\theta-} e_{2-}(t)dt\right| = \left|\int_{\nu}^{\theta+} e_{2-}(t)dt\right| = \lambda - \mu(\nu) \tag{3.62}$$

Applying the similar technique described in section 3.4.1, we have

$$\begin{aligned} e_1(t) &= e_1'(t) + e_1''(t) \\ &= e_1'(t) + v_1(t) + w_1(t) \end{aligned} \tag{3.63}$$

and

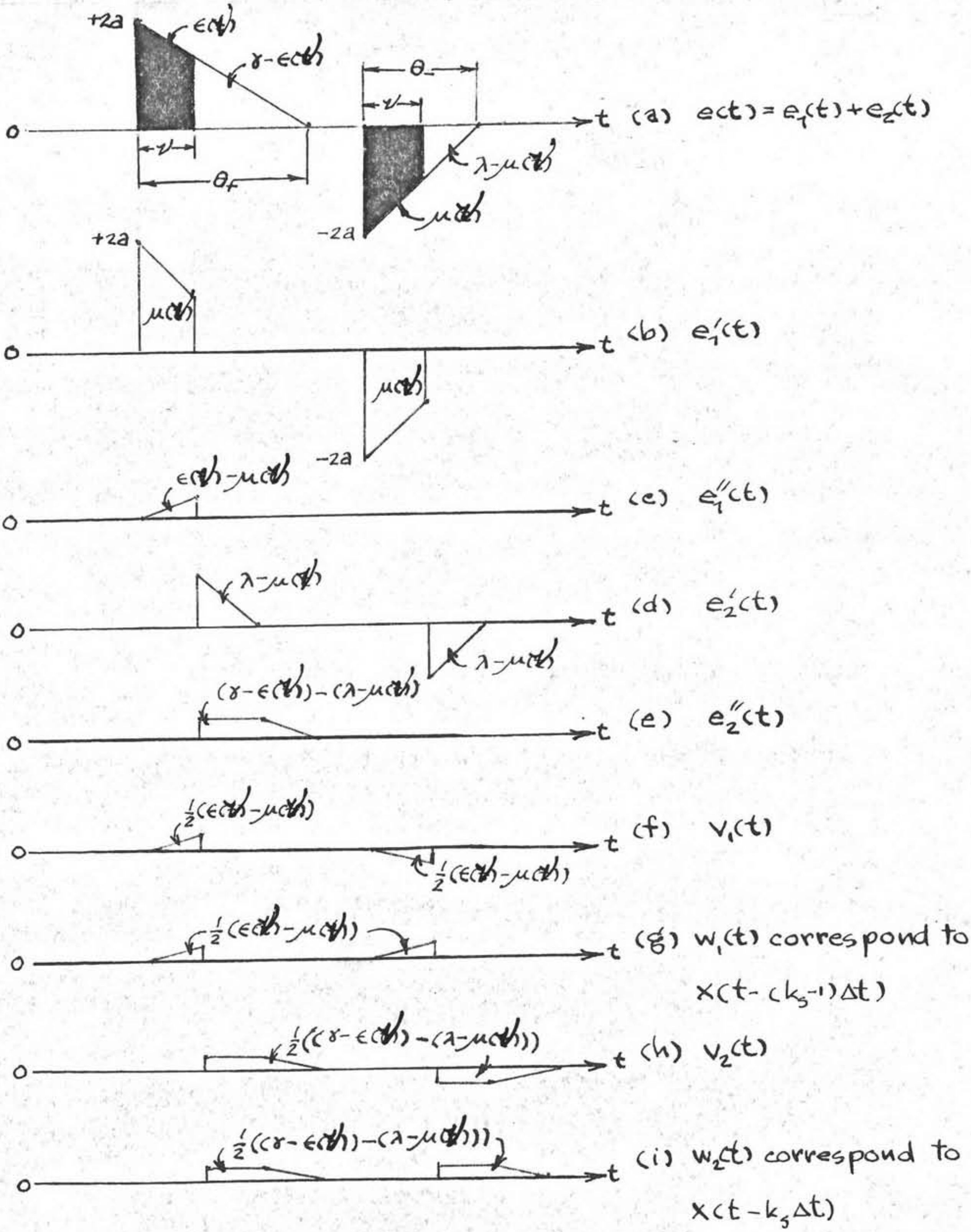
$$\begin{aligned} e_2(t) &= e_2'(t) + e_2''(t) \\ &= e_2'(t) + v_2(t) + w_2(t) \end{aligned} \tag{3.64}$$

where the error signals $e_1'(t)$, $e_2'(t)$, $v_1(t)$, and $v_2(t)$ are reversible error and the error signals $w_1(t)$ and $w_2(t)$ are non-reversible error which can be expressed as

$$\int_0^{\nu} e_{1+}(t)dt = \left|\int_0^{\nu} e_{1-}(t)dt\right| = \mu(\nu) \tag{3.65}$$

$$\int_{\nu}^{\theta-} e_{2+}(t)dt = \left|\int_{\nu}^{\theta-} e_{2-}(t)dt\right| = \lambda - \mu(\nu) \tag{3.66}$$

$$\int_0^{\nu} v_{1+}(t)dt = \left|\int_0^{\nu} v_{1-}(t)dt\right| = \frac{\epsilon(\nu) - \mu(\nu)}{2} \tag{3.67}$$



+2a
ε(t)
γ-ε(t)
θ₋
ν
t (a) e(t) = e₁(t) + e₂(t)
0
ν
θ₊
λ-μ(t)
μ(t)
-2a
+2a
μ(t)
0
t (b) e₁′(t)
μ(t)
-2a
ε(t)-μ(t)
0
t (c) e₁″(t)
λ-μ(t)
0
t (d) e₂′(t)
λ-μ(t)
(γ-ε(t))-(λ-μ(t))
0
t (e) e₂″(t)
½(ε(t)-μ(t))
0
t (f) V₁(t)
½(ε(t)-μ(t))
½(ε(t)-μ(t))
0
t (g) W₁(t) correspond to x(t-(k₅-1)Δt)
½((γ-ε(t))-(λ-μ(t)))
0
t (h) V₂(t)
½((γ-ε(t))-(λ-μ(t)))
0
t (i) W₂(t) correspond to x(t-k₅Δt)

Figure 3.7

$$\int_{\nu}^{\theta_+} v_{2+}(t)dt = \left| \int_{\nu}^{\theta_+} v_{2-}(t)dt \right| = \frac{(\gamma - \epsilon(\nu)) - (\lambda - \mu(\nu))}{2} \quad (3.68)$$

$$\left.\begin{aligned} \int_0^{\nu} w_{1+}(t)dt &= \frac{\epsilon(\nu) - \mu(\nu)}{2} \\ \left| \int_0^{\nu} w_{1-}(t)dt \right| &= 0 \end{aligned}\right\} \quad (3.69)$$

$$\left.\begin{aligned} \int_{\nu}^{\theta_+} w_{2+}(t)dt &= \frac{(\gamma - \epsilon(\nu) - (\lambda - \mu(\nu))}{2} \\ \left| \int_{\nu}^{\theta_+} w_{2-}(t)dt \right| &= 0 \end{aligned}\right\} \quad (3.70)$$

The error signal $w_1(t)$ and $w_2(t)$ are related to the input $x(t)$ (see Appendix D) as,

$$w_1(t) = \frac{\epsilon(\nu) - \mu(\nu)}{4a\Delta t}(x(t-k_s\Delta t+\Delta t)+a) \quad (3.71)$$

$$w_2(t) = \frac{(\gamma - \epsilon(\nu)) - (\lambda - \mu(\nu))}{4a\Delta t}(x(t-k_s\Delta t)+a) \quad (3.72)$$

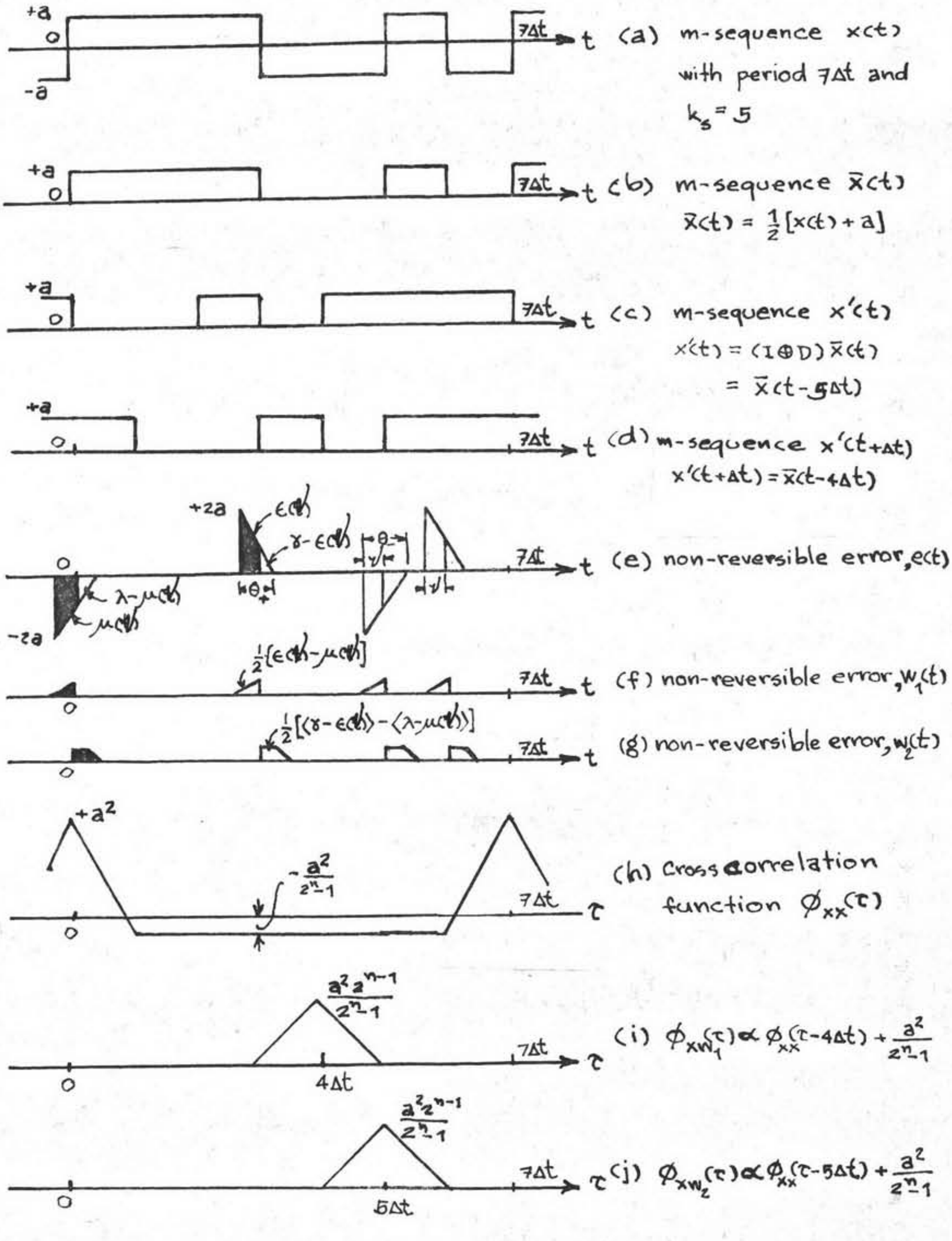
Therefore

$$\begin{aligned} \phi_{xe}(\tau) &= \phi_{xe_1'}(\tau) + \phi_{xv_1}(\tau) + \phi_{xw_1}(\tau) + \phi_{xe_2'}(\tau) + \phi_{xv_2}(\tau) + \phi_{xw_2}(\tau) \\ &= \phi_{xe_1'}(\tau) + \phi_{xe_2'}(\tau) + \phi_{xv_1}(\tau) + \phi_{xv_2}(\tau) \\ &\quad + \frac{\gamma - \lambda}{4a\Delta t}\left\{\phi_{xx}(\tau - k_s\Delta t + \Delta t) + \frac{a^2}{2^n - 1}\right\} \\ &\quad + \frac{(\gamma - \epsilon(\nu)) - (\lambda - \mu(\nu))}{4a\Delta t}\left\{\phi_{xx}(\tau - k_s\Delta t) + \frac{a^2}{2^n - 1}\right\} \end{aligned} \quad (3.73)$$

(a) For the region $0 \leq \tau \leq \theta_+$

Applying the similar technique described in the section 3.3.1(a) for $e_1'(t)$ and $v_1(t)$, and the similar technique described in the section 3.3.1 (c), for $e_2'(t)$ and $v_2(t)$, we obtain

Figure 3.8



+a
0
-a
7Δt
t
(a) m-sequence x(t) with period 7Δt and $k_s = 5$
(b) m-sequence $\bar{x}(t)$
$\bar{x}(t) = \frac{1}{2}[x(t) + a]$
(c) m-sequence $x'(t)$
$x'(t) = (I \oplus D)\bar{x}(t) = \bar{x}(t - 5\Delta t)$
(d) m-sequence $x'(t+\Delta t)$
$x'(t+\Delta t) = \bar{x}(t - 4\Delta t)$
+2a
-2a
(e) non-reversible error, e(t)
(f) non-reversible error, $w_1(t)$
(g) non-reversible error, $w_2(t)$
$+a^2$
$-\frac{a^2}{2^n-1}$
τ
(h) Cross correlation function $\phi_{xx}(\tau)$
$\frac{a^2 2^{n-1}}{2^n-1}$
4Δt
(i) $\phi_{xw_1}(\tau) \propto \phi_{xx}(\tau - 4\Delta t) + \frac{a^2}{2^n-1}$
5Δt
(j) $\phi_{xw_2}(\tau) \propto \phi_{xx}(\tau - 5\Delta t) + \frac{a^2}{2^n-1}$

$$\phi_{xe_1'}(\tau) = \frac{a\mu(\psi)2^{n-1}}{(2^n-1)\Delta t} \qquad (3.74)$$

$$\phi_{xv_1}(\tau) = \frac{a\left(\frac{\epsilon(\psi)-\mu(\psi)}{2}\right)2^{n-1}}{(2^n-1)\Delta t} \qquad (3.75)$$

$$\phi_{xe_2'}(\tau) = \frac{-a(\lambda-\mu(\psi))2^{n-1}}{(2^n-1)\Delta t} \qquad (3.76)$$

$$\phi_{xv_2}(\tau) = \frac{-a\left(\frac{(\gamma-\epsilon(\psi))-(\lambda-\mu(\psi))}{2}\right)2^{n-1}}{(2^n-1)\Delta t} \qquad (3.77)$$

From figure 3.8, and the value of $\tau = 0$, we have

$$\phi_{xx}(\tau-k_s\Delta t+\Delta t) + \frac{a^2}{2^n-1} = 0 \qquad (3.78)$$

$$\phi_{xx}(\tau-k_s\Delta t) + \frac{a^2}{2^n-1} = 0 \qquad (3.79)$$

Therefore

$$\phi_{xe}(\tau) = \frac{-a2^{n-1}}{(2^n-1)\Delta t}\left\{\frac{(\gamma-2\epsilon(\psi)) + (\lambda-2\mu(\psi))}{2}\right\}, \psi=\tau \qquad (3.80)$$

(b) For the region $\Delta t \leq \tau \leq \Delta t+\Theta_+$

Applying the similar technique described in the section 3.3.1(b), for $e_1'(t)$ and $v_1(t)$, and the similar technique described in the section 3.3.1(a), for $e_2'(t)$ and $v_2(t)$, we obtain

$$\phi_{xe_1'}(\tau) = 0 \qquad (3.81)$$

$$\phi_{xv_1}(\tau) = 0 \qquad (3.82)$$

$$\phi_{xe_2'}(\tau) = \frac{a(\lambda-\mu(\psi))2^{n-1}}{(2^n-1)\Delta t} \qquad (3.83)$$

$$\phi_{xv_2}(\tau) = \frac{a\left\{\frac{(\gamma-\epsilon(\psi)) - (\lambda-\mu(\psi))}{2}\right\}2^{n-1}}{(2^n-1)\Delta t} \qquad (3.84)$$

From figure 3.8, and the value of $\tau=\Delta t$, we have

$$\phi_{xx}(\tau-k_s\Delta t+\Delta t) + \frac{a^2}{2^n-1} = 0 \tag{3.85}$$

$$\phi_{xx}(\tau-k_s\Delta t) + \frac{a^2}{2^n-1} = 0 \tag{3.86}$$

Thus

$$\phi_{xe}(\tau) = \frac{a2^{n-1}}{(2^n-1)\Delta t}\left\{\frac{(\gamma-\epsilon(\nu)) + (\lambda-\mu(\nu))}{2}\right\}, \nu=\tau-\Delta t \tag{3.87}$$

(c) For the region $(2^m-2)\Delta t \leq \tau \leq (2^n-2)\Delta t+0_+$

Applying the similar technique described in the section 3.3.1 (c), for $e_1'(t)$ and $v_1(t)$, and the similar technique described in the section 3.3.1(b), for $e_2'(t)$ and $v_2(t)$, we obtain

$$\phi_{xe_1'}(\tau) = \frac{-a\mu(\nu)2^{n-1}}{(2^n-1)\Delta t} \tag{3.88}$$

$$\phi_{xv_1}(\tau) = \frac{-a\left\{\frac{\epsilon(\nu)-\mu(\nu)}{2}\right\}2^{n-1}}{(2^n-1)\Delta t} \tag{3.89}$$

$$\phi_{xe_2'}(\tau) = 0 \tag{3.90}$$

$$\phi_{xv_2}(\tau) = 0 \tag{3.91}$$

From the figure 3.8, and the value of $\tau=(2^n-2)\Delta t$, we have

$$\phi_{xx}(\tau-k_s\Delta t+\Delta t) + \frac{a^2}{2^n-1} = 0 \tag{3.92}$$

$$\phi_{xx}(\tau-k_s\Delta t) + \frac{a^2}{2^n-1} = 0 \tag{3.93}$$

Therefore $$\phi_{xe}(\tau) = \frac{-a2^{n-1}}{(2^n-1)\Delta t}\left\{\frac{\epsilon(\nu)-\mu(\nu)}{2}\right\}, \nu=(2^n-2)\Delta t \tag{3.94}$$

(d) For the region $(k_s-1)\Delta t \leq \tau \leq (k_s-1)\Delta t+\theta_+$

Applying the similar technique described in the section 3.3.1(b) for $e_1'(t)$, $e_2'(t)$, $v_1(t)$, and $v_2(t)$, we obtain

$$\phi_{xe_1'}(\tau) = 0 \tag{3.95}$$

$$\phi_{xe_2'}(\tau) = 0 \tag{3.96}$$

$$\phi_{xv_1}(\tau) = 0 \tag{3.97}$$

$$\phi_{xv_2}(\tau) = 0 \tag{3.98}$$

From figure 3.8, and the value of $\tau = (k_s-1)\Delta t$, we have

$$\phi_{xx}(\tau-k_s\Delta t+\Delta t)+\frac{a^2}{2^n-1} = \frac{a^2 2^{n-1}}{2^n-1} \tag{3.99}$$

$$\phi_{xx}(\tau-k_s\Delta t) + \frac{a^2}{2^n-1} = 0 \tag{3.100}$$

Thus $$\phi_{xe}(\tau) = \frac{a2^{n-1}}{(2^n-1)\Delta t}\left\{\frac{\epsilon(\vartheta)-\mu(\vartheta)}{2}\right\}, \quad \vartheta = \tau-(k_s-1)\Delta t \tag{3.101}$$

(e) For the region $k_s\Delta t \leq \tau \leq k_s\Delta t+\theta_+$

Applying the similar technique described in section 3.3.1(b), the values of $\phi_{xe_1'}(\tau)$, $\phi_{xe_2'}(\tau)$, $\phi_{xv_1}(\tau)$, and $\phi_{xv_2}(\tau)$ are identically to zero.

From figure 3.8, and the value of $\tau = k_s\Delta t$, we obtain

$$\phi_{xx}(\tau-k_s\Delta t+\Delta t) + \frac{a^2}{2^n-1} = 0 \tag{3.102}$$

$$\phi_{xx}(\tau-k_s\Delta t) + \frac{a^2}{2^n-1} = \frac{a^2 2^{n-1}}{2^n-1} \tag{3.103}$$

Therefore

$$\phi_{xe}(\tau) = \frac{a2^{n-1}}{(2^n-1)\Delta t}\left\{\frac{(\gamma-\epsilon(\nu))-(\lambda-\mu(\nu))}{2}\right\}, \quad \nu=\tau-k_s\Delta t \qquad (3.104)$$

(f) For the region $k_4\Delta t \leq \tau \leq k_4\Delta t+\theta_+$, where k_4 is positive integer in the range $2 \leq k_4 \leq k_s-2$ and $k_s+1 \leq k_4 \leq 2^n-3$.

The value of cross-correlation function $\phi_{xe}(\tau)$ is identically to zero. Thus,

$$\phi_{xe}(\tau) = 0 \quad , \nu=\tau-k_4\Delta t \qquad (3.105)$$

The complete results of the crosscorrelation between the input $x(t)$ and non-reversible transition error $e(t)$ are retabulated in table 3.2.

Table 3.2

The crosscorrelation between the input x(t) and non-reversible error e(t)

Range of τ	Values of $\phi_{xe}(\tau)$	Note
$0 \leq \tau \leq \theta_+$	$\frac{-a2^{n-2}}{(2^n-1)\Delta t}\left[(\gamma-2\epsilon(t))+(\lambda-2\mu(t))\right]$	
$\theta_+ \leq \tau \leq \Delta t$	$\frac{a2^{n-2}}{(2^n-1)\Delta t}\left[\gamma+\lambda\right]$	
$\Delta t \leq \tau \leq \Delta t+\theta_+$	$\frac{a2^{n-2}}{(2^n-1)\Delta t}\left[(\gamma-\epsilon(t))+(\lambda-\mu(t))\right]$	
$(k_5-1)\Delta t \leq \tau \leq (k_5-1)\Delta t+\theta_+$	$\frac{a2^{n-2}}{(2^n-1)\Delta t}\left[\epsilon(t)-\mu(t)\right]$	
$(k_5-1)\Delta t+\theta_+ \leq \tau \leq k_5\Delta t$	$\frac{a2^{n-2}}{(2^n-1)\Delta t}\left[\gamma-\lambda\right]$	
$k_5\Delta t \leq \tau \leq k_5\Delta t+\theta_+$	$\frac{a2^{n-2}}{(2^n-1)\Delta t}\left[(\gamma-\epsilon(t))-(\lambda-\mu(t))\right]$	
$(2^n-2)\Delta t \leq \tau \leq (2^n-2)\Delta t+\theta_+$	$\frac{-a2^{n-2}}{(2^n-1)\Delta t}\left[\epsilon(t)-\mu(t)\right]$	
$(2^n-2)\Delta t+\theta_+ \leq \tau \leq (2^n-1)\Delta t$	$\frac{-a2^{n-2}}{(2^n-1)\Delta t}\left[\gamma+\lambda\right]$	
$k_4\Delta t \leq \tau \leq k_4\Delta t+\theta_+$	0	$2 \leq k_4 \leq k_5-2$, $k_5+1 \leq k_4 \leq 2^n-3$
$k_3\Delta t+\theta_+ \leq \tau \leq (k_3+1)\Delta t$	0	$1 \leq k_3 \leq (k_5-2)$, $k_5 \leq k_3 \leq 2^n-3$