CHAPTER 2

LITERATURE REVIEW

2.1 Fundamentals

The perception of colour is a sensation. At (5) the same time, colour measurement can only determine the physical properties of a dyeing. With the help of colorimetry, it is possible, however, to convert physical data in such a manner, so that statements concerning the perception of colour are possible. It is sufficient to know the relative distribution of the radiation, e.g. how much more blue light than red light is present. The relative energy distributions of the most commonly used light sources are known.

The radiation of light source falling on the viewed object is partly reflected by it, partly absorbed and partly transmitted. In color measurement, it is usually assumed that the objects to be measured are opaque, In this case, only the reflected or, conversely, the absorbed portion of the radiation is of interest. The reflected portion is called reflectance. Spectral reflectance is defined as the ratio of light reflected by an object at a certain wavelegth to the light reflected by an ideally white surface(5).The ideally white surface, which must be observed under the same conditions of illumination as the sample, reflects all the incident light diffusely into the semi-space. Since spectral reflectance is the ratio of the

light reflected at the same wavelength from sample and from a white standard, it is, for nonfluorescing samples, independent of the spectral distribution of the light source which is used in the measurement.

By plotting the spectral reflectance as a function of the wavelength, (5,6) one obtains the reflectance curve. This is the true physical characteristic of a dyeing. When color measurement is mentioned, often the determination of the reflectance curve is meant. Because a simple relationship exists between reflectace and dyestuff concentration, formula calculations are possible with the help of reflectance measurements.

2.2 C.I.E. Colour Order System

This system is the most important system which are usually used in connection with instruments for color measurement. C.I.E. system (Commission International de l'Eclairage or International Commission on Illumination) is based on the concept of additive color mixing as derived from experiments in which colors are matched by mixing colored lights. (7)

The C.I.E. system works on formulas, not on color samples. (8)

The C.I.E. system makes it possible to describe color samples in mathematical terms and to represent the dominant wavelength and purity of the sample on a diagram.

Just as color invariably involves three components- a source of light, (7) an illuminated object, and an observer- the CIE system requires and provides both standard light sources

and a standard observer to complete the description of the color of objects. The CIE standard observer is a numerical description of the response to color of the normal human eye, as expressed by the color-matching functions $\bar{x}, \bar{y}$, and $\bar{z}$. The CIE Standard Sources A, B, C and D_{65} simulating incandescent light, noon sunlight, overcast-sky daylight, and phase of average daylight, respectively-exist in physical form as lamp-filter combinations, providing standard light sources for viewing colors. The CIE system describes the colors of objects illuminated by these sources and viewed by observers with normal color vision.

The standard coordinates of the CIE system are the CIE tristimulus values X, Y and Z. These coordinates can be either determined directly in color matching of measuring experiments with colored lights or calculated from spectral reflectance curves with the aid of tables giving the tristimulus values of the pure spectrum color at each wavelength, that is $\bar{x}, \bar{y}$, and $\bar{z}$ in Figure 2.1, These numbers are multiplied, wavelength by wavelength, by the reflectance of the sample and the relative energy of the light source, and the products are added up for all the wavelengths in the visible spectrum. The sums are the tristimulus values of the sample.
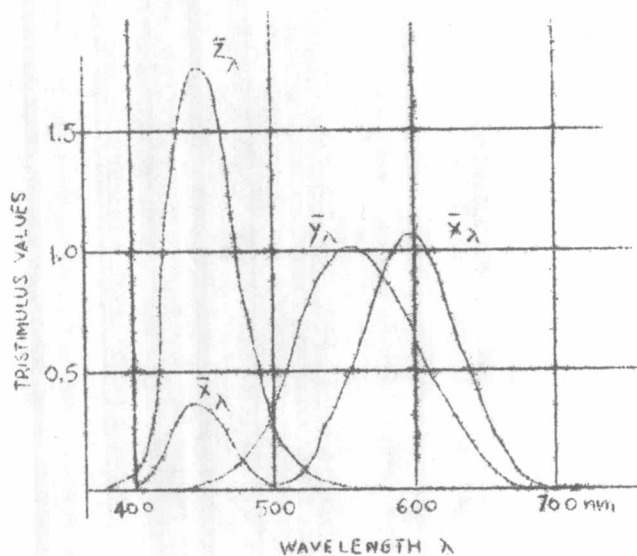
Fig. 2.I 1931 CIE color-matching functions, $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$

In Figure 2.2, the reflectance curve of a green dyeing is plotted. (5) When light of the energy E (λ) falls on a sample with the reflectance R (λ), the sample reflects at each wavelength the portion R of the incident energy E. The eye receives the radiant energy R (λ). E (λ) which is reflected by the object. It has been determined by extensive tests that the colour vision of the human eye can be interpreted correctly with the assumption that three centres of stimulation, each with a different spectral sensitivity, exist in the retina. The sensitivity distribution of the three centres of stimulation of the described eye model have been determined-experimentally and plotted in the third row of Figure 2.2(5). The curves are known as the colour matching functions $\bar{x}$, $\bar{y}$, $\bar{z}$ of the standard observer. It can be seen that one centre of stimulation is primarily sensitive to blue light ($\bar{z}$ - curve), another primarily

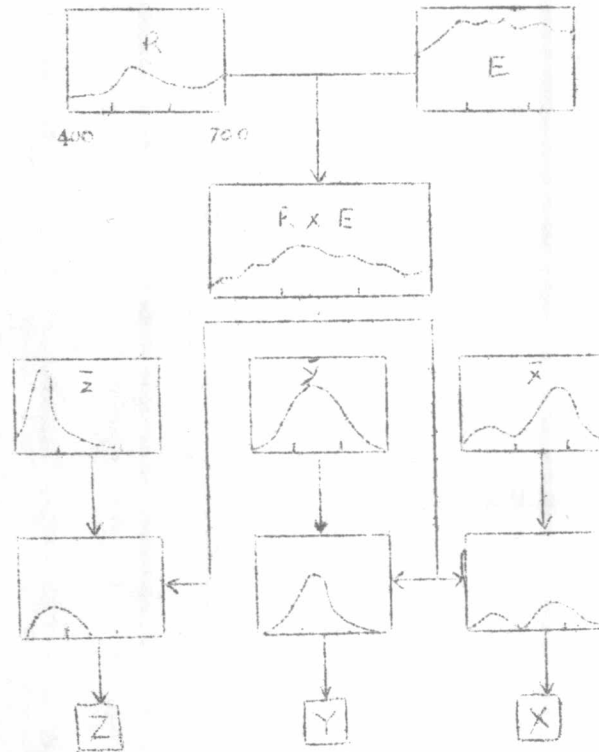to green ($\bar{y}$-curve) and the third primarily to red light ($\bar{x}$-curve),

Fig. 2.2 The diagram shows how the CIE tristimulus values x, y, and z of a color are obtained.

The luminous energy reflected from the observed object and absorbed by the centres of stimulation is summed at each wavelength according to its sensitivity and transmitted as nerve signals to brain. The size of the transmitted signals is proportional to the areas under the three curves in the fourth row of Figure 2.2.

It is possible to calculate the size of these areas by summing the products E.R.$\bar{x}$, E.R.$\bar{y}$ and E.R.$\bar{z}$ for each wavelength:

$$X =$$

$$E.\bar{x}.R \qquad \text{at} \quad 400 \text{ nm}$$

$$+E.\bar{x}.R \qquad \text{at} \quad 420 \text{ nm}$$

$$+\ldots\ldots$$

$$+E.\bar{x}.R \qquad \text{at} \quad 700 \text{ nm}$$

$$= \sum_{400}^{700} E(\lambda).\bar{x}(\lambda).R(\lambda) \qquad\qquad (2.1)$$

and correspondingly :

$$Y = \sum_{400}^{700} E(\lambda).R(\lambda).\bar{y}(\lambda) \qquad\qquad (2.2)$$

$$Z = \sum_{400}^{700} E(\lambda).R(\lambda).\bar{z}(\lambda) \qquad (4, 5) \qquad (2.3)$$

The calculated values are termed tristimulus values X,Y,Z. They are determined not only by the sample (R), but **(5)** also by the illuminant and by the standard observer. The calculated tristimulus values have to be identified always by the illuminant and standard observer for which they were calculated.

Wavelength, nm	R, %	$E_c\bar{x}$	$E_cR\bar{x}$	$E_c\bar{y}$	$E_cR\bar{y}$	$E_c\bar{z}$	$E_cR\bar{z}$
400	23.3	0.00170	0.04	0.00004	0	0.00807	0.19
420	33.0	0.02474	0.82	0.00073	0.02	0.11889	3.93
440	41.7	0.07944	3.31	0.00524	0.22	0.39846	16.62
460	50.0	0.06719	3.36	0.01387	0.69	0.38569	19.29
480	47.2	0.02222	1.05	0.03234	1.53	0.18908	8.93
500	36.5	0.00104	0.04	0.06797	2.48	0.05724	2.09
520	24.0	0.01151	0.27	0.12914	3.10	0.01423	0.34
540	13.5	0.05566	0.75	0.18285	2.47	0.00390	0.05
560	7.9	0.11751	0.93	0.19668	1.55	0.00078	0.01
580	6.0	0.16822	1.01	0.15972	0.96	0.00032	0
600	5.5	0.17885	0.99	0.10624	0.58	0.00014	0
620	6.0	0.14130	0.85	0.06301	0.38	0.00004	0
640	7.2	0.07381	0.53	0.02884	0.21	0	0
660	8.2	0.02720	0.22	0.01007	0.08	0	0
680	7.4	0.00737	0.05	0.00268	0.02	0	0
700	7.0	0.00164	0.01	0.00058	0	0	0
		Sum = X = 14.23		Sum = Y = 14.29		Sum = Z = 51.45	

Table 2.I The tristimulus values calculation from spectral distribution data.

For a graphical representation of colours it would
be possible to plot the tristimulus values X,Y,Z directly in a
colour space. The representation is more easily understandable
if the chromaticity coordinates are calculated as follows:

$$x = \frac{X}{X+Y+Z} \qquad (2.4)$$

$$y = \frac{Y}{X+Y+Z} \qquad (2.5)$$

$$z = \frac{Z}{X+Y+Z} \qquad (2.6)$$

$$\text{and} \qquad x+y+z = 1 \qquad (2.7)$$

By this definition, x represents the amount of the
red tristimulus value X in the sum of the values X+Y+Z, while
y represents the green tristimulus value, in Figure 2.3 .
The chromaticity coordinates identify, therefore, the hue as
well as the saturation, which are both part of the "chromaticness".
The lightness of the dyeing is not expressed in the term
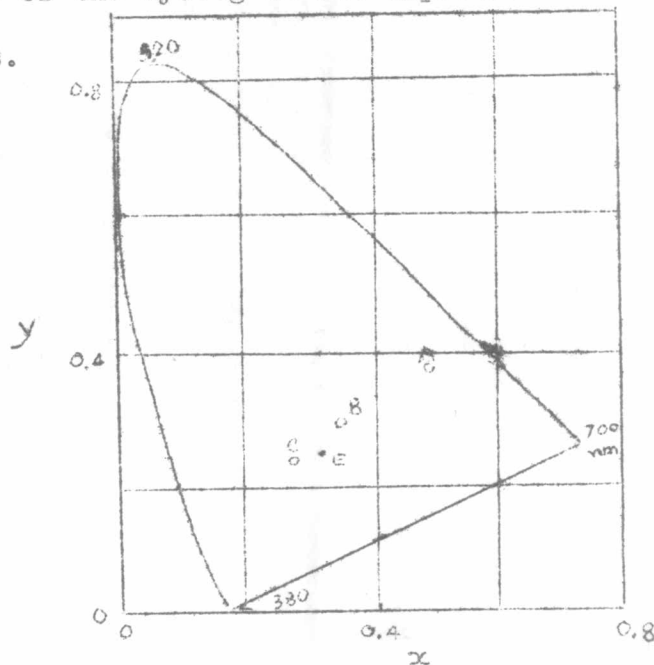chromaticness.

Fig.2.3 1931
CIE (x,y)
chromaticity
diagram

The X,Y,Z primaries are nonreal or imaginary primaries, (9) that is, they cannot be realized by actual color stimuli. In the tristimulus space they are represented by vectors outside the domain of vectors representing real colors; in the chromaticity diagram their representative points are not included among the chromaticities of real colors; they lie outside the chromaticity region bounded by the spectrum locus and the purple line connecting the two ends of the spectrum locus. The triangle formed by the chromaticity points of X,Y.Z completely encloses the spectrum locus and the purple line. This ensures that the chromaticity coordinates, x,y,z and the corresponding tristimulus values X,Y,Z of any real color (monochromatic or otherwise) are never negative.

In the centre of the chromaticity diagram is the neutral point, (7,10) which represents the chromaticness of white, grey or black colours. Colours of equal hue lie on a line between the neutral point and the borderline of the diagram. Their saturation is indicated by the distance from the neutral point. The farther the colour is located from the neutral point, the more saturated it is. An alternative set of chromaticity coordinates, dominant wavelength and purity, correlate more nearly with the visual aspects of hue and chroma, although their steps and spacing are not visually uniform

The dominant wavelength of a color is the wavelength of the spectrum band, which, (9) when mixed with some specified achromatic stimulus, matches the given color. Not all colors

have a dominant wavelength. The achromatic stimulus is usually
provided by a standard source, such as CIE standard source A,
B,C or D$_{65}$ any of which may be considered in this connection
as having a neutral (or achromatic) color, devoid of hue.
The dominant wavelength of a color correlates in an appropriate
way with what would be called in ordinary language the hue of
the color as observed under everyday conditions. Thus, in
general, colors of constant dominant wavelength perceived
under similar conditions would be said to have the same hue.

The excitation purity of any color possessing a
dominant wavelength is an exactly defined ratio of distances
in the chromaticity diagram indicating how far the given color
is displaced from the achromatic color towards the spectrum
color. Excitation purity correlates loosely with saturation
of the color perceived under ordinary observing conditions,
Figure 2.4
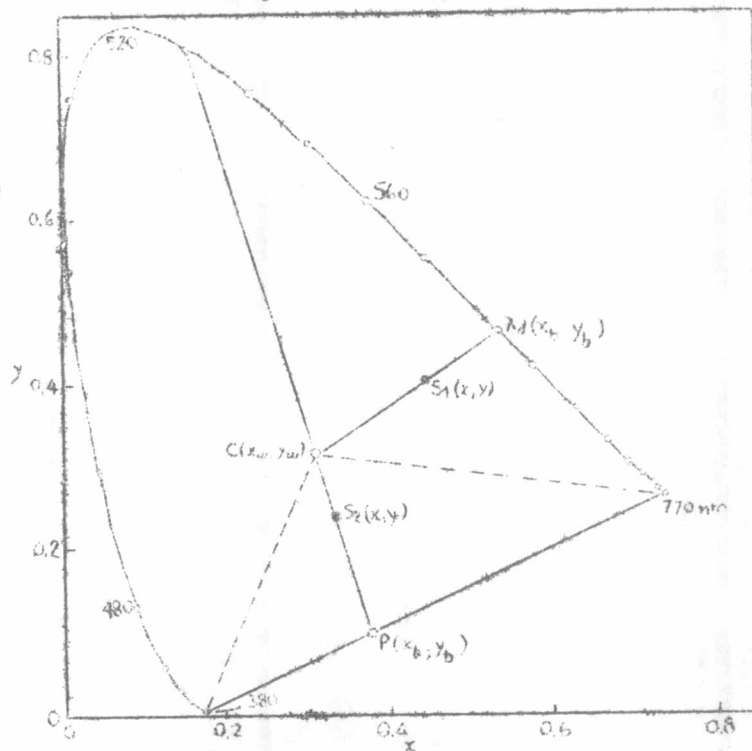
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Fig. 2.4 Determination
of dominant and com-
plementary wavelengths
of stimuli of
chromaticity

For colors not possessing a dominant wavelength it is always possible to match the specified achromatic color with a mixture of the given color and a spectrum color. The wavelength of the latter is termed the complementary wavelength, and it plays a role analogous to dominant wavelength in color specifications and in the definition of excitation purity.
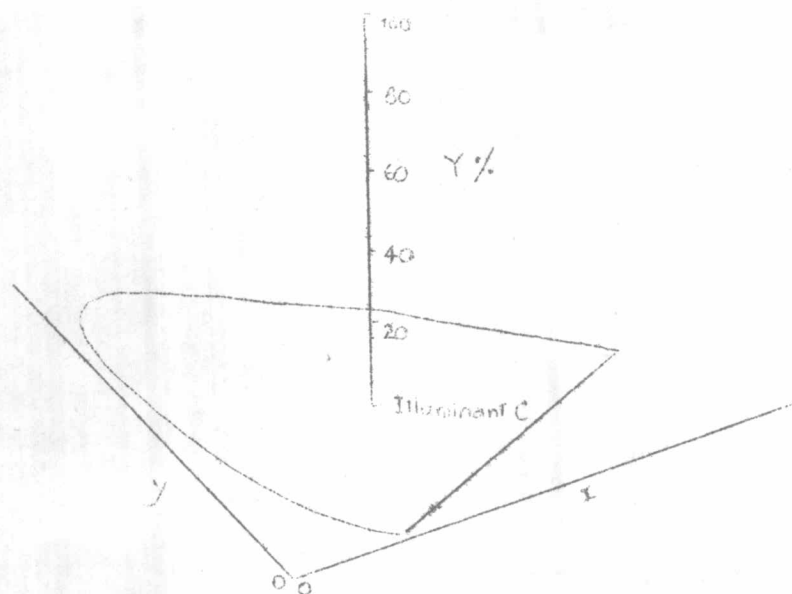
Fig. 2.5 The lightness axis of Chromaticity diagram.

The third dimension of color is conveniently added to the CIE chromaticity diagram by thinking of the luminance or lightness axis rising up from it. Lighter colors lie in space at their proper level of Y and above the point representing their chromaticity x,y, Figure 2.5

The construction of such a color order system is possible in different ways and is usually realized with a colour atlas in which each color is defined by three easily visualized values (x,y,Y). (5)

2.3 The use of Anlab Color Space as an aid to dyers and colorists.

Because ANLAB space quantifies every characteristic relating to the perceived colour of a textile (12,17) dyed with a single dye or dye mixture at any depth of shade, it can be used in the selection of the best combinations for specific shades. The situation is complicated by the fact that all of the shades which can be produced by a two-dye mixture will lie on a curved surface in ANLAB space - that surface which is bounded by the strength lines of the individual dyes. When three dyes are involved, as is customary in textile dyeing, the shades produced do not merely lie on a curved surface, they occupy a 3 - dimensional volume in ANLAB space.
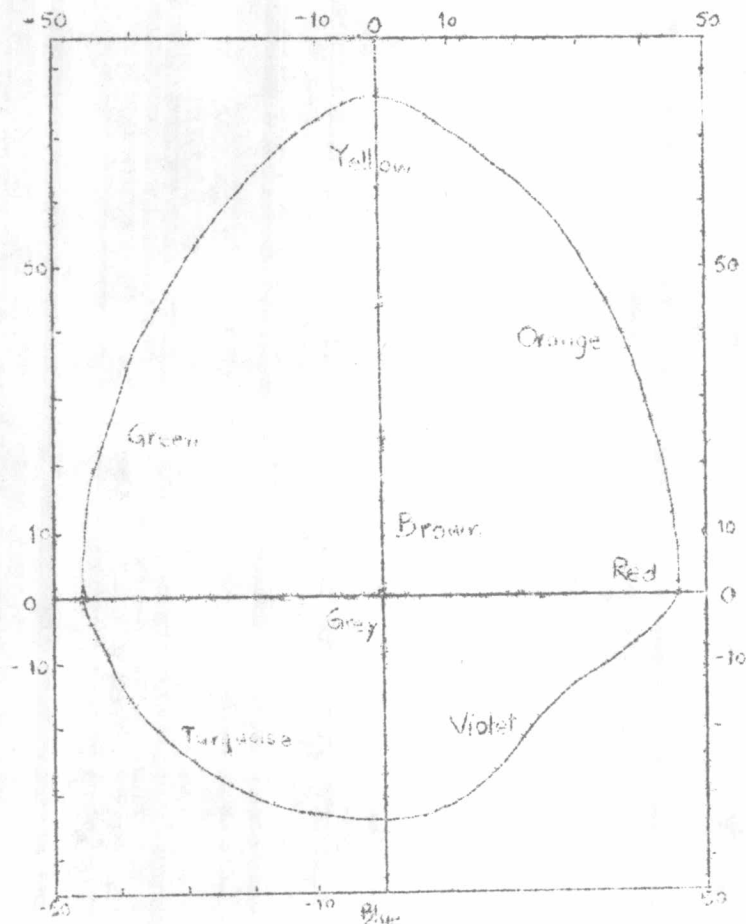

Fig. 2.6 ANLAB Color Space.

An example of such that a colour diagram is given in Figure 2.6. This illustrates selected loci and the position of Procion MX dyes at a medium depth shade (1/2 standard depth) dyed on bleached unmercerized cotton plus a few supporting colours. The lines joining any two colours represents the colours obtained when these are mixed, and, as described earlier, the area enclosed by the closed loop formed by the loci of the binary mixtures of three colours includes all the colours obtainable with that particular combination of three colours. In this colour diagram it would, of course, be possible to join all the colours to all other colours by their appropriate loci and it is possible to obtain a very large number of shades using binary mixtures. However many of these loci and the resulting three colour combinations can be eliminated by the choice of an alternative that is more suitable on technical or commercial considerations. This diagram includes the more important binary mixtures and the derived three colour mixtures. A number of important uses follow:-

1. Selection of the most appropriate three colour mixture.

2. To show the effect of substituting one colour for another on the shade range covered.

3. Illustrating the fit of new colours.

4. Dyes selected for specific fastness requirements.

5. Diagrams of this nature are also particularly useful as teaching aids in practical laboratory

shade matching or shades on the machine as the
loci calibrated to show the amount of dye required
to alter the shade by a given amount and how this
varies according to the position in colour space.

The particular example shown is based on reactive
dyes but it is equally suitable for displaying similar situations
with other dye classes.

2.4 Colour Difference Computation

A colour difference formula (13) which correctly
predicts the visual judgement of an average observer is of
considerable interest, particularly in regard to the small
colour differences encountered in quality control evaluations
of coloured goods. The manufacture (14) of most coloured goods
involves at some stage a decision as to whether the product
will be acceptable against the required standard for the
color. This tolerance may have been set in a variety of ways;
perhaps by long customer usage, perhaps by a committee of
interested parties or just by " they didn't complain last time
when it was worse than this".

The tristimulus values X,Y,and Z can be regarded as
the axes of a three-dimensional (15) Euclidean space in which
every perceivable color can be located, not only the color of
objects such as dyed textiles but also the colors produced by
monochromatic light, the purest color sensations perceivable.
This space concept is considerably simplified when only the

former colors-the so-called surface colors-are of interest,
because the extent of the Y axis is then limited by definition
to 100 for the perfect reflecting diffuser; X and Z are
consequentially limited also, to ca 98 and 118 respectively
for Illuminant C. Such a space is termed a 'color solid'.

Two colors that are not a perfect match according
to the CIE Standard Observer must differ in at least one of
the tristimulus values. They must therefore be located in
different positions in XYZ space and it is natural to assume
that the greater the perceived colour difference between them
the further apart in XYZ space they will be. Unfortunately,
however, XYZ space is non-uniform to a disqualifying degree;
for example, the distance between a green standard and a sample
that was just perceptibly weaker could be as much as thirty
times greater than the distance between a blue standard and
a sample that was just perceptibly redder, yet the two perceived
differences are of equal magnitude. The non-uniformity also
applies to large color differences; the grey that appears
mid-way between black (Y=0) and white (Y=100) does not have a
Y value of ca 50 but ca 20. The need to measure color
differences objectively is as great as the need to measure
color itself-and, since the non-uniformity of XYZ space was
recognised about 35 years ago, attempts have been made to
transform it mathematically into a more uniform color space so
that the distance between sample and standard would be proportional
to the perceived difference between them. The equations for

calculating such distances are known as color-difference equations or formulae, the symbol ΔE (change in sensation, German Empfindung) being universally used for calculated colour differences.

Most of the transformations of XYZ space yield spaces that are also Euclidean and if the axes of such a space are F, G and H then the distance between any two points is given by the simple Pythagorean equation:

$$(\Delta E)^2 = (\Delta F)^2 + (\Delta G)^2 + (\Delta H)^2 \tag{2.8}$$

Where ΔF is the difference between the two F values, etc.

David MacAdam (14) studied his observer concerning visually perceptual limits by chosing twenty-five colors and used one observer, to specify sets of equiluminous colors, observed to be equally noticeably different from each of the twenty-five colors. The field observed was 2° in diameter and luminance of 15 mL surrounded by a 21° field source C and luminance of 7.5 mL. He plotted the results on the CIE chromaticity diagram, and found that they were adequately represented by ellipses drawn around each point representing each of the twenty-five fixed colors, Figure 2.7.
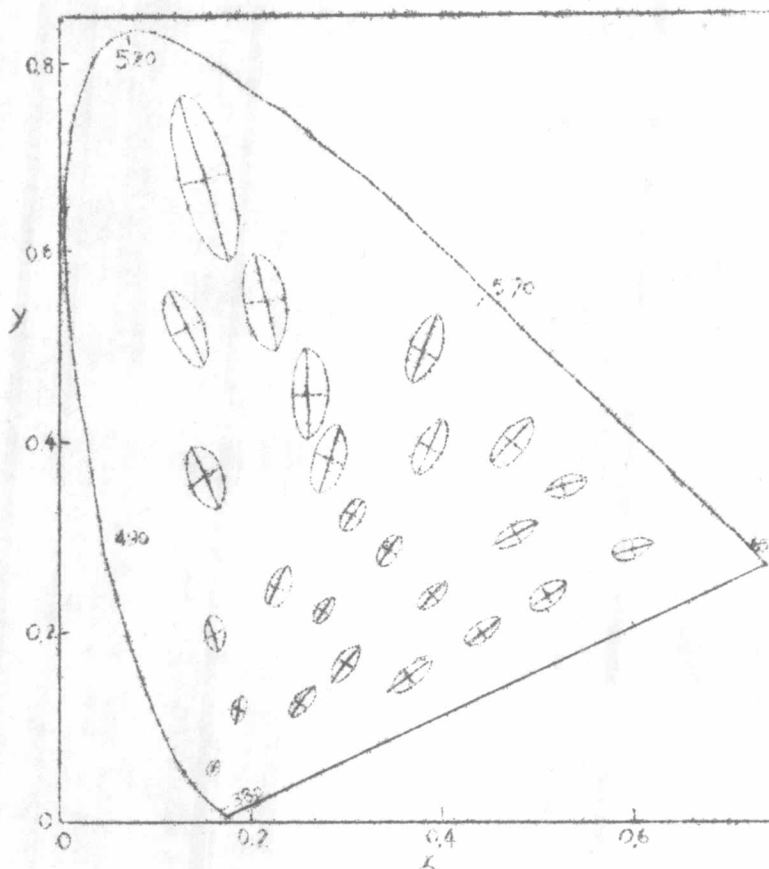
Fig. 2.7 I93I CIE chromaticity diagram showing MacAdam's ellipses
(ten times enlarged).

MacAdam's equation was devised empirically to permit
one to interpolate among the 25 MacAdam ellipses. Friele,
however, inductively devised an equation based on a theory
of the mechanism of colour perception which, when the constants
were suitably optimised by Friele, MacAdam and Chickering,
predicted the size, shape and orientation of MacAdam ellipses
remarkably well.

Friele – MacAdam – Chickering Equation No. 2 (FMC2) (14)

$$P = 0.724 \, X + 0.382 \, Y - 0.098 \, Z \qquad (2.9)$$

$$Q = -0.48 \, X + 1.37 \, Y - 0.1276 \, Z \qquad (2.10)$$

$$S = 0.686\,Z \qquad (2.11)$$

$$a = 17.3*10^{-6}\,(P^2+Q^2)/(1+2.73P^2Q^2/(P^4+Q^4)) \qquad (2.12)$$

$$b^2 = 3.098*10^{-4}\,(S^2+0.2015Y^2) \qquad (2.13)$$

(X,Y,Z are in decimal fractions)

$$K_1 = 0.55669 + 0.04934\,Y - 0.82575*10^{-3}\,Y^2 + 0.79172*10^{-5}$$
$$Y^3 - 0.30087*10^{-7}Y^4 \qquad (2.14)$$

$$K_2 = 0.17548 + 0.027556\,Y - 0.57262*10^{-3}\,Y^2 + 0.63893*10^{-5}$$
$$Y^3 - 0.26731*10^{-7}Y^4 \qquad (2.15)$$

(Where Y values are in percentages)

$$\Delta L_1 = (P\Delta P + Q\Delta Q)/(P^2+Q^2)^{\frac{1}{2}} \qquad (2.16)$$

$$\Delta L_2 = 0.279\,\Delta L_1 /a \qquad (2.17)$$

$$\Delta C_{rg} = (Q\Delta P - P\Delta Q)/(P^2+Q^2)^{\frac{1}{2}} \qquad (2.18)$$

$$\Delta C_{yb} = S\Delta L_1 /(P^2+Q^2)^{\frac{1}{2}} - \Delta S \qquad (2.19)$$

$$\Delta C_1 = ((\Delta C_{rg}/a)^2 + (\Delta C_{yb}/b)^2)^{\frac{1}{2}} \qquad (2.20)$$

$$\Delta C = K_1\Delta C_1 \qquad (2.21)$$

$$\Delta L = K_2\Delta L_2 \qquad (2.22)$$

$$\Delta E = ((\Delta C)^2 + (\Delta L)^2)^{\frac{1}{2}} \qquad (2.23)$$

X,Y,Z are the tristimulus values for either of the two colours constituting the color difference of the pair to be evaluated, and ΔP, ΔQ, ΔS are the difference between the values of P,Q and S for these two colors. In the equations for the factor K_1 and K_2, Y is taken as the average of the Y's for the two colors.

2.5 The Turbid Media Theory

All colorant layers are turbid: (19) that is, all

of them scatter some light. The blue color of the sky and of
the tropic sea comes from scattered light. Only a vacuum is
strictly nonlight - scattering. The media we refer rather
to the strongly light - scattering layers (paper, paint,
vitreous enamel) of commercial interest. Many of these media
have light - scattering and light - absorbing properties
which permit us to use the Kubelka - Munk theory[19] for analysis.

The properties (9) of light - scattering colorant
layers - embracing their reflectance, transmittance, absorptance,
hiding power, and so forth are described approximately by the
Kubelka - Munk Theory. The theoretical model of Kubelka and
Munk (1931) assumes that the colorant layer can be divided up
into a large number of elementary layers - with boundaries
parallel to those of the complete layer - which have identical
optical properties, in Figure 2.8 . The thickness (dx) of
an elementary layer is small compared with the thickness (X)
of the colorant layer as a whole but is large compared
with the diameters of the pigment particles comprising the
layer. Thus the actions of the individual particles need not
be considered but only their average effect in producing on
elementary layer of particular absorbing and scattering
properties. The theoretical model further envisages just two
completely diffuse light fluxes, one proceeding downward
throughout the layer, the other simultaneously proceeding
upward. The downward - proceeding flux (i) in its passage
through an elementary layer is decreased by absorption by an
amount $K_i d_x$, and is also decreased by an amount $- S_i d_x$ by

scattering which reverses the direction of same of the light rays. These rays then belong to the upward instead of the downward - proceeding flux. The quantity K is the fraction of the downward - proceeding flux lost by absorption in the elementary layer, and the quantity S is the fraction lost by having its direction reversed. Quantities K and S are called the absorption coefficient and the scattering coefficient, respectively, of the material forming the colorant layer.

During its passage through the same elementary layer the upward - proceeding flux (j) is reduced by the amounts $Kjdx$ and $Sjdx$ by absorption and scattering in exactly the same way as is the downward-proceeding flux; as the flux $S_i dx$, reversed in direction by scattering from the downward - proceeding flux, is added to the upward - proceeding flux the total change (dj) of the upward - proceeding flux becomes

$$d_j = -(S+K)_j dx + S_i dx \qquad (2.24)$$

The total change (d_i) of the downward - proceeding flux is similarly

$$-d_i = -(S+K)_i dx + S_j dx \qquad (2.25)$$
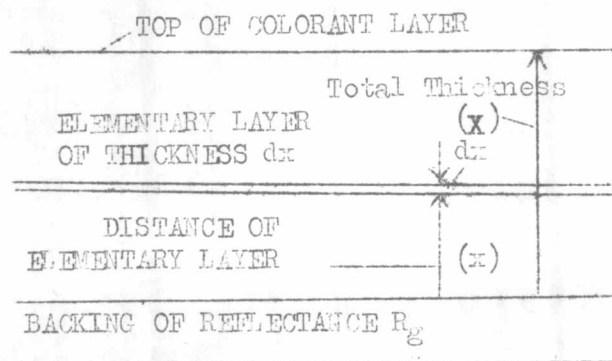
TOP OF COLORANT LAYER

Fig. 2.8 Schematic diagram of a cross-section through a colorant layer.

Different algebraic signs appear in front of the differentials d_i and d_j in the above equations because upward -proceeding flux and distance x are measured in the same direction, whereas downward - proceeding flux and x are measured in opposite directions. The thickness dx of the elementary layer refers to an increase of X , where X = 0 corresponds to the unilluminated side of the colorant layers.

The above differential equations can be integrated to give explicit expressions for the reflectance R and for other properties of the colorant layer.

If the absorption and scattering (19) coefficients of the unknown specimen are thus evaluated as functions of wavelength, the task of the colorant - layer technologist is then to find a known colorant or a combination of known colorants whose absorption and scattering coefficients have the same wavelength dependence as the unknown. If this can be done, the identification of the unknown colorant has been completed. The scattering coefficient of the colorant produced by a combination of constituents is, like the absorption coefficient , merely the sum of the coefficients of the constituents (Duncan, 1940; Persoz, 1945)(19).This simple additivity of the coefficients is what makes the Kubelka - Munk analysis applicable to colorant-layer formulation. This additivity was made use to   solve paint and plastic problems also. It is so important that we shall write it down explicitly:

$$K = (cK_1)_p + (cK_1)_q + (cK_1)_r + \dots \qquad (2.26)$$

$$S = (cS_1)_p + (cS_1)_q + (cS_1)_r + \dots \qquad (2.27)$$

The technologist will thus try to find known colorants such that the weighted sum of the absorption and scattering coefficients is equal for each wavelength to that of the unknown. This is the procedure to be used if opacity and hiding power are of interest, or if the scattering coefficient of the mixture varies importantly with the proportions of the components.

If the colorant layer is completely opaque and has a known fixed amount of light - scattering material in it , a considerable simplication is possible. It is then necessary only to deal with the ratio, K/S of the absorption to the scattering coefficient. This condition is satisfied by textiles colored by the application of dye; and it is sufficiently well approximated by paints consisting of white paint tinted by addition of toners of low scattering coefficient, that is, so low as to be negligible compared to the scattering coefficient of the white paint.

One such (14) function that very roughly works is the simple reciprocal of reflectance. $1/R$, where R is a decimal fraction and not a percentage ie. 100% for the perfect diffuser is rated as 1. This function can be made to work a little better if a constant k, which can be derived empirically is inserted

$$K.F = 1/R \quad \text{(for a narrow wavelength band)}$$

where F is the function factor of the reflectance

Most modern match prediction techniques work upon the principles established by Paul Kubelka and Franz Munk (1931) who studied the behaviour of light in turbid media and set up differential equations which they solved by the application of calculus methods to give an exponential solution which enabled graphical solutions for opacity of pigment layers to be made. The TAPPI opacity /reflectivity chart is one such application.

The often used relationship between absorption / scatter and reflectance also evolved from this study:-

$$K/S = (1-R)^2 / 2R \quad \text{(for a narrow wavelength band,}$$
$$\text{say 0.5 to 2nm.)} \quad (2.28)$$

where K = absorption

S = scatter

R = reflectance as a decimal fraction

Possibility of using Equation 2.28 in color matching prediction and color control is illustrated graphically by a practical example in Figure 2.9

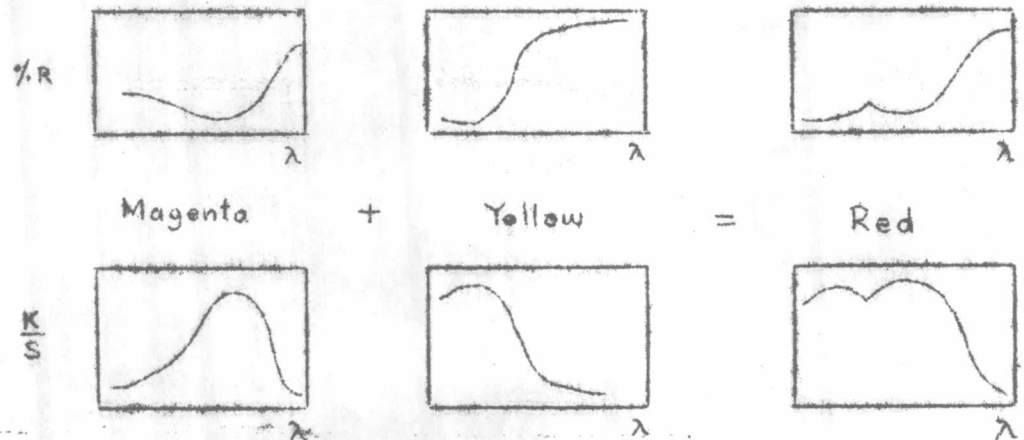Magenta   +   Yellow   =   Red

Fig. 2.9 Additivity of K/S functions of reflectance

The reflectance curves and their corresponding K/S curves in Figure 2.9 show a yellow paint and a magenta paint together with the results of mixing the two paints together in equal parts. It can be seen that the resulting red from this mixture has a reflectance curve which cannot readily be seen as a combination of the two reflectance curves. On the other hand the red K/S curve is formed very simply by adding wavelength by wavelength the heights of the magenta and yellow K/S curves together

2.6 Colour Match Prediction

The (14) ratio K/S is adequate for match prediction when most of the scatter at each wavelength is contributed by the substrate as is the case with textiles, or by the white content in the case of paints and plastics.

The full colour matching equation is

$$\frac{aK_A + bK_B + \ldots + wK_W}{aS_A + bS_B + \ldots + wS_W} = K/S_M \qquad (2.29)$$

Where a, b are concentrations of pigments A, B etc.

   "   w   is   concentration   of white ,

K_A , K_B ,...., K_W are absorption coefficients of pigments and white.

S_A , S_B ,...., S_W are scatter coefficients of pigments and white.

If in the above equation the scatter of the white

and the concentration of the white (or substrate in textiles) is large compared with the scatter contributed by the coloured components whether they be pigments or dyes, then the equation may be simplified to:-

$$a \; K/S_A \; + b \; K/S_B \; + \ldots + w \; K/S_W \;\;\; = \; K/S_M \qquad\qquad (2.30)$$

In this equation the K/S values of the pigments or dyes, are obtained by:-

$\big[(K/S \;\; \text{substrate} + \text{colorant} - K/S \;\; \text{substrate}) \, / \, \text{Concentration}$ colorant.$\big]$, which is known as the calibration factor.

That is, the K/S of the pigment or dye is diminished by that of the white or substrate with which it is mixed and the result is divided by the concentration of the colorant to give the K/S of unit concentration.

(14,20) For simplicity, it is assumed that all measurements are made with the same thickness of fabric. Suppose a fabric containing two coloured substances, A and B, present in concentrations a and b respectively, is measured at two wavelengths, λ_1 and λ_2. Let the reflectances of the substances measured alone, at the same concentrations as they are present in the fabric be respectively-

At λ_1 .... K/S_{A1} . a and K/S_{B1} . b

At λ_2 .... K/S_{A2} . c and K/S_{B2} . b

If K/S_{M1} and K/S_{M2} are the observed K/S functions (from reflectances) of the fabric at λ_1 and λ_2 respectively,

then since the K/S functions of the individual dyes are additive, so it can be solved for a and b gives the expressions-

$$a = \alpha_1 K/S_{M1} + \alpha_2 K/S_{M2} \qquad (2.31)$$

$$b = \beta_1 K/S_{M1} + \beta_2 K/S_{M2} \qquad (2.32)$$

The coefficients α, β are in terms of the calibration factors (K/S_A and K/S_B).

The method is theoretically applicable to more than two colors, and is limited only by the separation of the reflectance bands and the sensitivity of the instrument. For three - colour mixtures the three appropriate wavebands are chosen and measurements of known concentrations of the individual colouring matters and of the unknown concentration made with each, as described for a two - colour mixture. Similarly, If a,b,c are the concentrations of dyes A,B and C respectively,

Then, At λ_1 $K/S_{M1} = K/S_{A1} \cdot a + K/S_{B1} \cdot b + K/S_{C1} \cdot c$ (2.33)

At λ_2 $K/S_{M2} = K/S_{A2} \cdot a + K/S_{B2} \cdot b + K/S_{C2} \cdot c$ (2.34)

At λ_3 $K/S_{M3} = K/S_{A3} \cdot a + K/S_{B3} \cdot b + K/S_{C3} \cdot c$ (2.35)

The solution of three such simultaneous equations is readily obtained by the use of determinants, Thus -

$$\begin{bmatrix} K/S_{A1} & K/S_{B1} & K/S_{C1} \\ K/S_{A2} & K/S_{B2} & K/S_{C2} \\ K/S_{A3} & K/S_{B3} & K/S_{C3} \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} K/S_{M1} \\ K/S_{M2} \\ K/S_{M3} \end{bmatrix} \qquad (2.36)$$

$$\text{or } P * Q = R \qquad (2.37)$$

Where P is the calibration factor array, Q is the
concentration array, and R is the array of K/S values of
mixture.

$$then \quad Q = P^{-1} * R \qquad (2.38)$$

So we need the inverse of P into which R is multiplied
to give Q. This inverse form of the normalised K/S array is
the pre - solved equation system into which any unknown
standard K/S values (diminished by those of the white base)
may be multiplied to give by a very rapid operation the concentrations
required for a match.