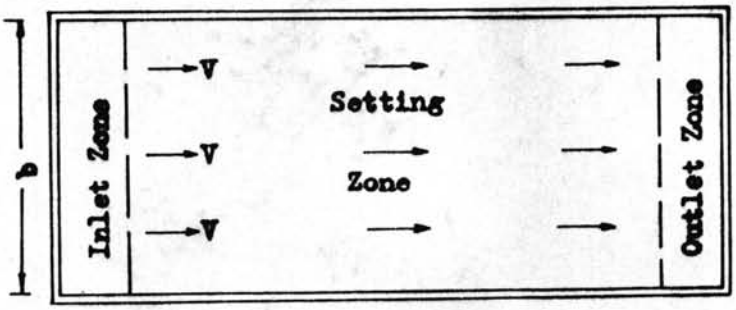
II LITERATURE REVIEW

Historical Development

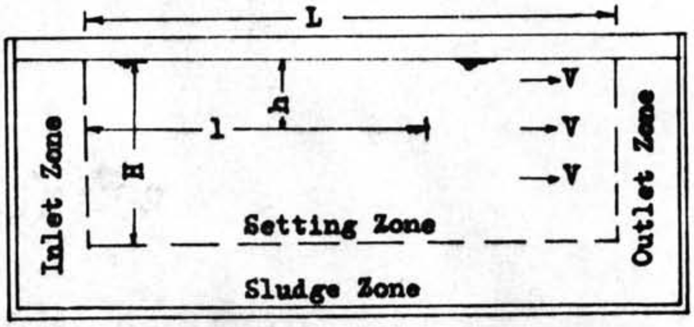
Background of Ideal Sedimentation Tank

HAZEN (1904) suggested that an ideal sedimentation basin should be as shallow as possible. HAZEN proposed that the proportion of sediment removed was a function of the area of the basin and of the hydraulic value of the sediment, and of the quantity of the water treated in a unit of time, and was entirely independent of depth of the basin. The best removal efficiencies were obtained when the basin was arranged so that the incoming water containing the maximum quantity of sediment was kept from mixing with water which was partially clarified. It was stated that as the action of a sedimentation basin was dependent upon its area and not upon its depth, one horizontal subdivision would provide two surface to receive sediment instead of one and would double the amount of incoming flow rate. Two such divisions would treble it, and so on. He then suggested that if the basin was inserted by a series of horizontal plate into a large number of shallow passages, the increase in efficiency would be very great. However the most serious practical difficulty that would be met in carrying out this idea is the method of cleaning (cleaning has to be done very frequently).

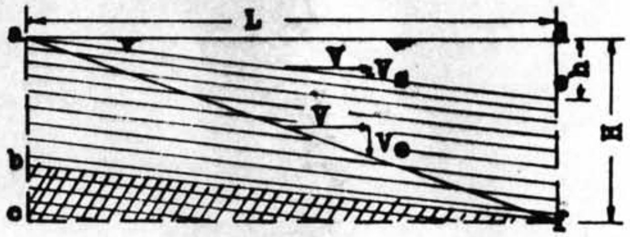
In an ideal settling basin as defined by CAMP (1946) any particle settling in a moving liquid would move in a direction and at a velocity with referenced to the basin, which was the vector sum of its own settling velocity and the velocity of the surrounding liquid. In such a



(a) Plane



(b) Longitudinal Section



(c) Idealized Settling Paths of Discrete Particles

Fig 1. Ideal Rectangular Settling Basin (After CAMP 1946) basin, the paths of all discrete particles will be straight lines and all particles with the same velocity will move in parrallel paths. Moreover, the settling pattern will be the same in all longitudinal sections of the settling zone (Fig 1.). All particles having settling velocities, V_s greater than V_o will fall through the entire depth, H, and be removed. The portion of particles with settling velocities V_s less than V_o which will be removed is equal to the ratio of velocities, V_s/V_o. It may be seen from Fig 1. that particles with V_s less than V_o could be completely removed if false bottoms or tray were inserted at intervals, h, with such particles being trapped in a basin of length L rather than requiring a basin of much greater length. It is clear from Fig 1. that as the height interval "h" is reduced, the length of basin required to remove a given percentage of incoming settleable material decreases. This shows conclusively that for an ideal settling basin, for any given discharge, the removal is a function of the surface area and is independent of the depth of basin or the removal is a function of the overflow rate and for a given discharge, is independent of the detention period. This was infact, suggested by HAZEN (1904) when he stated that the proportion of sediment removed.was a function of surface area of the basin.

Early Attempt at Shallow Depth Sedimentation

A detailed literature review by HANSEN and CULP (1967) described that, as reported by BRAHAM et al (1956), the first attempt at practical application of the tray-settling principal was made in the year 1915. In that patent several shallow settling compartments were formed by a series of conical, circular trays placed one above the other, and mechanical equipment was used for sludge removal.

FREI (1941) added three circular steel, radial flow to an existing primary sewage clarifier. He reported that the suspended solids removed increased from 41 to 61 percent even though the flow through the tank was tripled following the addition of the trays. ELIASSEN (1946) reported that difficulties arose from the inadequate removal of floating sludge and scrum. The digestion of sludge resulted in gas release, which could not escape and thus one of the trays was ruptured.

CAMP (1946) presented a design for a settling basin with horizontal trays spaced at 6 in. distances and felt that 6 in. was the minimum distances permissible for sludge removal. Out let orifices were employed for distributing the flow over the width of the trays. The basin had a detention time of 10.8 min., a velocity of 9.3 ft. per min. and an overflow rate of 667 gpd/sq.ft.

The practical application of shallow tray sedimentation was limited by problems associated with distribution of flow to multiple trays units and method in removing sludge from the closely spaced trays.

SCHMITT and VOIGT (1949) reported the use of a two-storey settling basin in a water treatment plant. The trays, spaced at 15 ft., were in series and were cleaned by draining and hand-hosing. DRESSER (1951) reported a similar use of series trays in the Cambridge, Massa., water treatment plant. Sludge was removed from the trays, which were spaced at 5 ft. Each tray was drained by gravity with nozzles mounted at the end to aid in sludge removal.

CAMP (1953) again stressed the enormous advantages that could be gained by tray settling, once the problem of sludge removal could be satisfactorily taken care of, so that trays with very small vertical spacing could be used.

The earlies attempts in 1915, the commercial marketing of tray settling tanks in 1940's and sporadic instances in the 1940's and 1950's met with only limited success in view of two major problems:

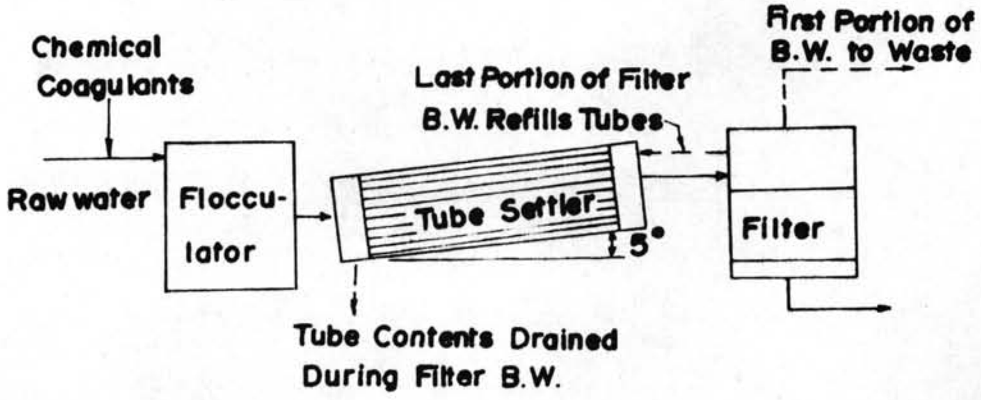
1. The difficulties encountered in proper distribution of flow toa large number of shallow trays.
2. The minimum tray spacing was limited by the vertical clearance required for mechanical sludge removal equipment.

HANSEN and CULP (1967) described some successful works of FISCHERSTROM (1955) in applying tray-settling theory. The latter pointed out the neccessity of maintaining proper hydraulic conditions for efficient sedimentation and desirable overflow rate. Reports of other experiences tend to confirm his feeling that the earlier to apply tray settling using radial-flow circular trays recognized only the importance of overflow rate, and neglected proper hydraulic conditions. It was felt that a Reynold number of 500 (limit of Laminar flow at 32°F) in the settling basin would be most benefit to the settling process. Most settling basins are now operated at Reynold number of 1,000 to 25,000. For a given basin, the Reynold number can be reduced by increasing wetted perimeter, or inserting horizontal or vertical longitudinal baffles in the basin. Horizontal baffles not only decreased the Reynold number but also reduce the overflow rate and the vertical distance, settling particles must fall before striking a bottom surface, accomplishing the same goals as theory of HAZEN (1904). The use of vertical baffles in conjuction with horizontal baffles would reduce the Reynold number even further. HAZEN felt that the minimum baffle spacing was limitted by the sludge removal problem and the difficulty of equally distribution the flow to a large number of trays or rectangular channels. He listed several cases where he had applied his design theory in water treatment plants. The design enabled much shorter settling times than normally used. The reinforced concrete tray were generally spaced at 5 - 6 ft. intervals. It was conduced that application of the tray settling concept was in no way experimental. Cost analysis showed the tray settling basins to be much less expensive than conventional basins.

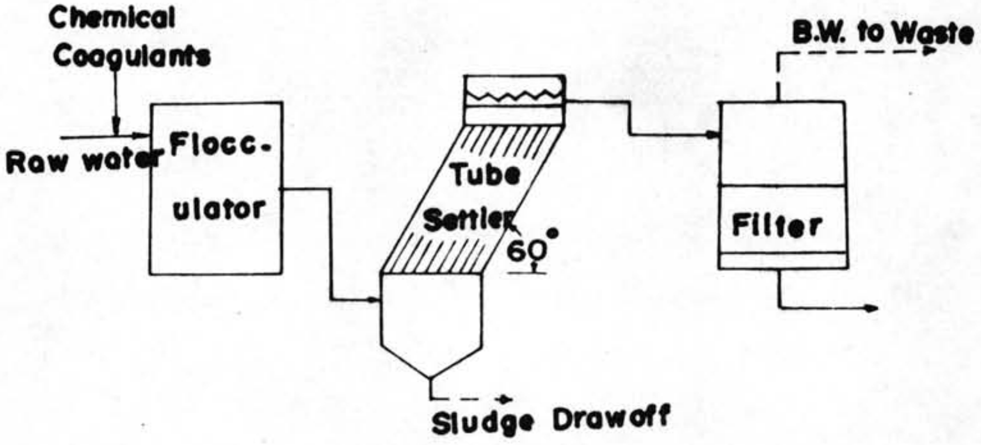
In 1967 HANSEN and CULP reported that the two major problems which had limitted the use of shallow depth sedimentation had been overcame by using very small diameter rather than wide, shallow trays. It was reported that logitudinal flow through tubes with a diameter of few inches offer theorectically optimum hydraulic condition for sedimentation and overcame the hydraulic problems associated with tray settling basins. Such tubes had a large wetted perimeter relative to the wetted area and there by provide laminar flow conditions, as evidence by very low Reynold number. Their report showed that even with the largest tube (4 in. diameter) and highest flow rate (10 gpm./sq.ft. of tube end area), the Reynold number was only 96. It was summarized that the tube configurations would meet the requirements of the shallow depth, laminar flow conditions and reasonable overflow rates. The short detention time made the space saving potential of such configurations readily apparent. The test was begun in 1964 to evaluate mean of sludge removal. Preliminary test with single tubes of various size demonstrated that they were efficient sedimentation devices. During these preliminary tests, it was found that accumulated sludge would be readily removed by periodically draining the tubes if they were inclined slightly to the direction of flow. It was found in HANSEN and CULP's study that an angle of inclination of 5 deg. was adequate for sludge removal by gravity. They successfully demonstrated that, by using circular tubes of 0.5 to 4 inches in diameter and up to 8 ft. in length., the flowrates were varied from 2 to 8 gpm/sq.ft. of tube end area. The test results showed that the turbidity removal efficiency increased as the tube length increased and decreased as the tube diameter and flowrate increased. It was also shown that the influent turbidity and polyelectrolyte dosage had an effect on settling performance. The decrease in detention time to only 3 minute was reported and this made the cost and space saving potential apparent. CULP et al (1968) reported that they had, at that time, tube settling devices in operation in many water treatment plants that were providing excellent clarification with settling detention time of less than 10 min.

Basic Tube Configurations

CULP et al (1968) had described two basic tube configurations shown in Fig2 (a) and (b) . In this study, it was described that operation of the essentially horizontal tube settlers was co-ordinate with that of the filter used to clarify the tube settler effluent. Each time the filter backwashes, the tube settler was drained completely. The falling water surface scoured the sludge deposits from the tubes and carried them to waste. The water drained from the tubes was replaced with the last portion of the filter backwash water. The tubes were inclined only slightly in direction of flow (5 deg.) to promote the



(a) essentially horizontal tube settler



(b) steeply inclined tube settler

Fig 2 Basic Tube Settler Configurations

(After CULP et al 1968)

drainage of sludge during the backwash cycle. If the inclination of the tubes was increased sharply (45 - 60 deg.) continuous gravity drainage. of the settleable materials from the tubes could be achived. The incoming solids settled to the tube bottom and then exited from the tubes by sliding downwards along the tube bottom. A flow pattern was established in which the solids settling to the tube bottom were trapped in downwards flowing stream of concentrated solids. This counter current flow of solids aided in agglomerating particles into larger, heavier particles that settled against the velocity of the upward flowing liquid. The continuous sludge removal achived in these steeply inclined tubes eliminated the need for drainage of black flushing of the tubes for sludge removal.

Horizontal Tube Settler

CULP et al (1968) reported that essentially horizontal tube settler had been used primarily in small (15 000 gpd.) to medium sized (10 mgd.) water treatment plants, where the elimination of operator attention for sludge removal from the clarifier was a significant benefit. By draining the tubes each time the filter backwashed, positive sludge withdrawal from the clarifier was achived. The entire backwash tube drainage cycle was automated. Thus, no operator judgement on when or how much sludge to withdraw from the clarifier was required. They also reported the successful use of package water treatment plant in which the tubes were hexagonal in shape. The hexagonal tubes nested together to form a honeycomb pattern, the detention time with in the tube was 6 min. Tube diameters of 1 - 2 in. and lengths of 2 - 4 ft.

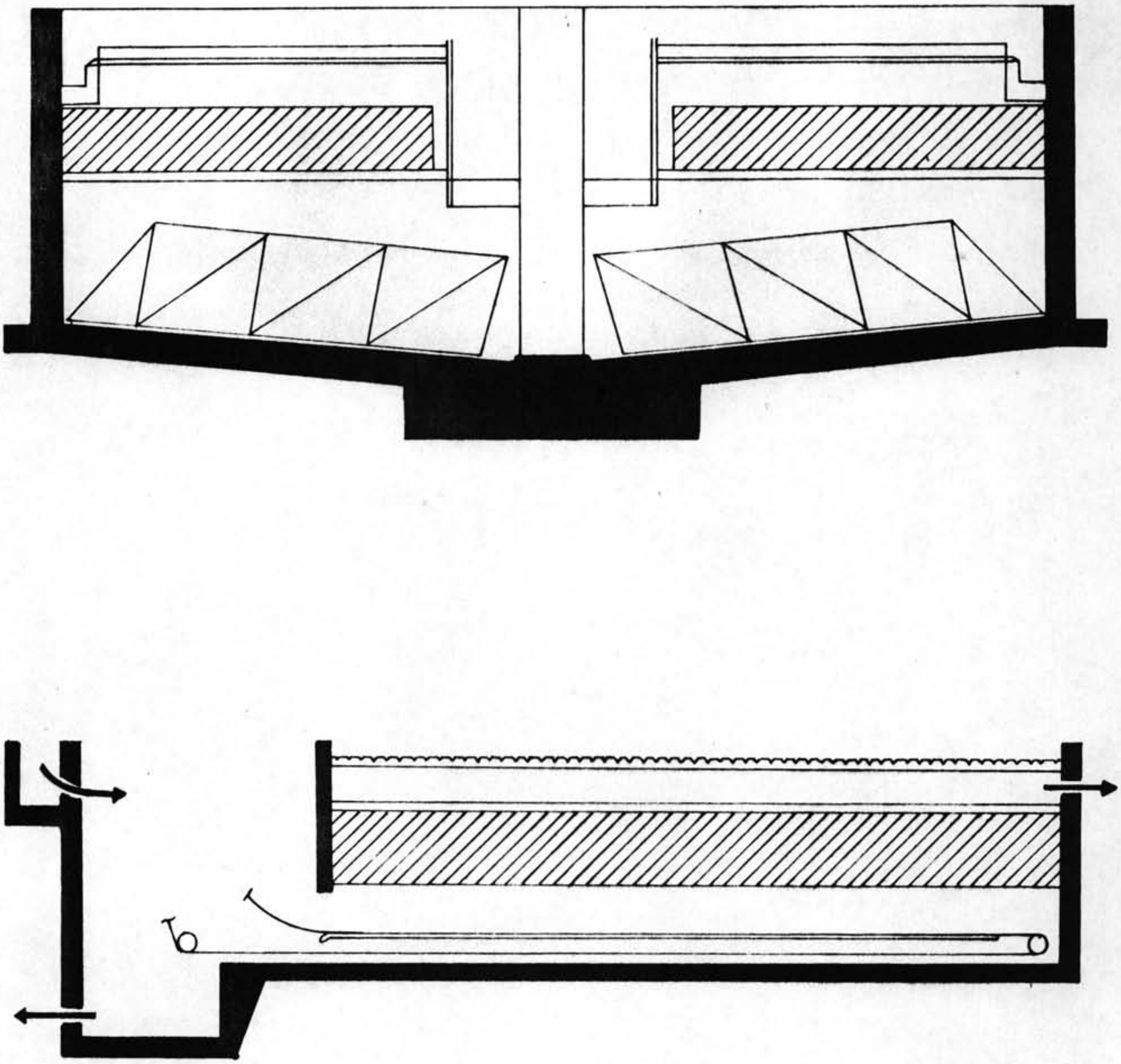


Fig 3. Installation of Tube Modules in Circular and Rectangular Tanks

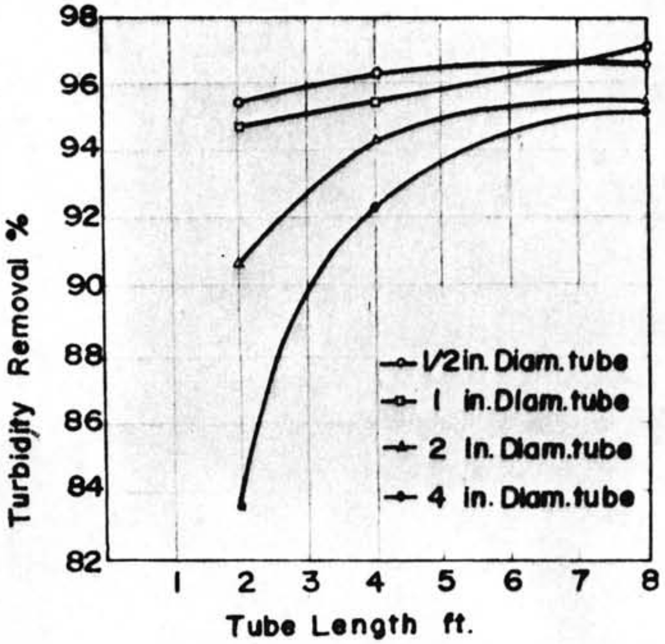
were used in most water treatment applications. Hydraulic loading rates of 3 - 5 gpm./sq.ft. of tube entrance areas were generally used.

Steeply Inclined Tube Settler

In this system, CULP et al. described that the solids that settled to the bottom of a tube inclined at a steep angle (greater than 45 deg.) would slide down the tube bottom continuously. This enabled sludge removal to be achieved without draining or blackflushing the tubes. They also reported that the path traced by a particle settling in a tube was the resultant of two vectors; the velocity of flow through the tube and the settling velocity of the particles. Fig 3 shows the installation of steeply inclined tube modules in an existing conventional rectangular and circular-clarifier.

Physical Variables on Settling Performance

HANSEN and CULP (1967) had performed the study to evaluate the relationship between tube diameter and length, flowrate, level of applied turbidity, polyelectrolyte dosage and the performances of various sized tubes. Tube lengths of 2, 4 and 8 ft. with the diameter of 0.5, 1, 2, and 4 in. were studied. Tube flowrate of 2, 5 and 8 gpm./sq.ft. were selected in their study. Polyelectrolyte of 0, 0.2 and 0.5 mg./l. were used. Tube flowrate were checked every hour and samples of the raw water and tube settler effluent were taken at half-hour intervals. The data collected during these laboratory evaluation of tube settling parameters are graphed on Fig 4 to 8 . Fig 4 and 5 show the effect of tube length and tube diameter on settling performance. Fig 6 to 7 show the



Flow rate 2 gpm/sq. ft.

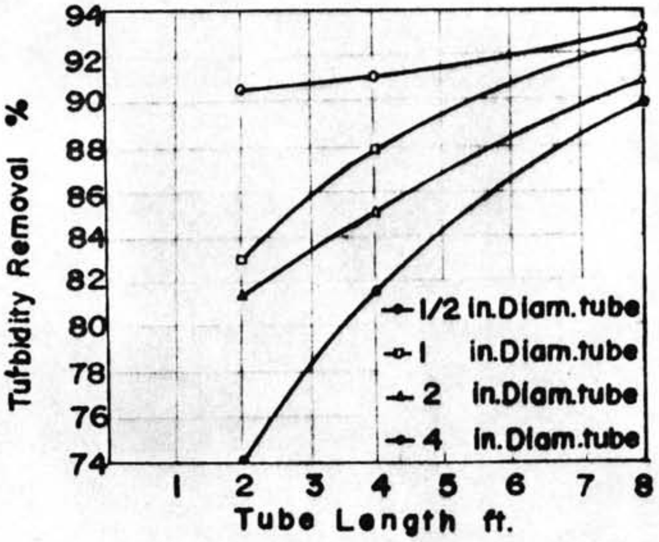
Alum dosage 100 mg/l

no Polyelectolyte

Flocculation time 20 min.

Initial Turbidity 450 JTU

Fig. 4 Effect of Tube Length on Settling Performance
(After HANSEN and CULP 1967)



Flow rate 5 gpm/sq. ft

Alum dosage 100 mg/l

Polyelectrolyte dosage 0.5 mg/l

Flocculation time 20 min.

Initial Turbidity 450 JTU

Fig 5 Effect of Tube Length on Settling Performance
(After HANSEN and CULP 1967)

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effect of flowrate and influent turbidity.

CULP et.al. (1968) reported the study of the effects of tube inclination on settling efficiency. Initial tests were carried out with five individual tubes inclined at angles of 0, 5, 20, 45 and 90 deg. The tube was 1 in. in diameter and 4 ft. long. They observed that at an angle of 45 deg. inclination the settling sludge at the bottom of the tube moved continuously downward and eventually falling into the inlet plenum. Fig 9 shows their study on the effect of tube inclination on settling performance.

YAO (1973) reported that CHEN (1970) conducted an extensive experimental study of circular upflow high-rate settlers. Four tube sizes 0.5, 1, 2 cnd 3 in., three tube lengths 2, 3 and 5 ft., and four flow velocities 0.27, 0.54, 0.80 and 1.07 fpm. were used in his study, the angle of inclination was varied from 0 to 75 deg., the raw water turbidity raried from 15 to 90 mg./l. as SiO_2. The results were shown by Fig 10 to 13 . Fig 10 presents the computer results of all the runs showing the turbidity removal efficiency at various overflow rates. Fig 11, 12 and 13 show the effect of raw water turbidity and flow velocity on efficiency of circular settler.

The results from the above studies can be concluded as follow.

Effect of Tube length

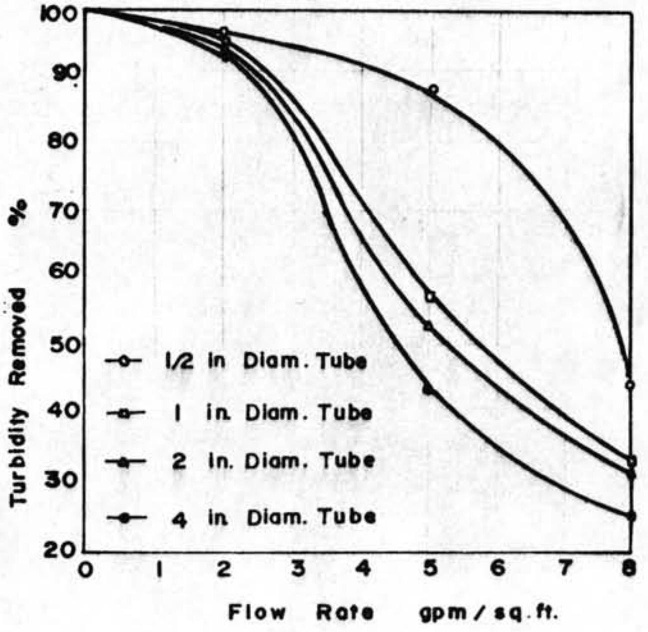
From Fig 4 and 5 it can be seen that the turbidity removal efficiency increase as the tube length increase. The different in removal efficiency is very high in a large tube and low in a small. For example at flowrate of 2 gpm./sq.ft., tube $\frac{1}{2}$ in. diameter., percentage of turbidity removal increase from 95.8 percent at tube 2 ft. long to 96.5 at tube 8 ft. long, while for 4 in. tube increase in percentage is 82.8 to 95.8. The results revealed that at higher flow rates there was a significant difference in turbidity removal with different lengths. For instance, at a flow rate of 5 gpm./sq.ft. and with no polyelectrolyte dosage, the removal for tube ½ in. diameter, 2, 4 and 8 ft. long were 55.6, 69.0 and 82.6 respectively. Similar results were obtained for tubes of larger sizes such as 1, 2, and 4 in. diameter.

Effect of Tube Diameter

From Fig 4 and 5 it can be seen that the percentage of turbidity removal decreases as the tube diameter increases. From the data obtained from HANSEN and CULP (1967) the difference was more significant at higher flowrate than that at lower flow rate and at longer tube length than that at shorter tube length. For instance, from Fig 6 at tube length of 4 ft., tube diameter of ½ in. Turbidity removal efficiency was 95.8 and 41.0 percent at flowrate of 2 and 8 gpm./sq.ft. respectively.

Effect of Flow Rate

The effect of flowrate on the tube settler efficiency can be seen from Fig 6 and 7 that the percentage of turbidity removal decrease as the flow rate increase. HANSEN and CULP (1967) reported that the effect of flow rate was much more pronounced in the absence of polyelectrolyte. It was also noted that the use of polyelectrolyte at flow rate of 5 and 8 gpm./sq.ft., turbidity removals comparable to those obtained at lower flow rate, without the polyelectrolyte, were achived.



Tube length 4 ft.

Alum dosage 100 mg / l

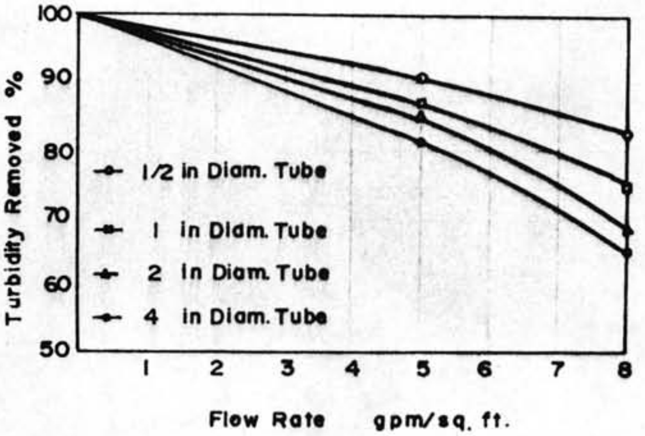
no Polyelectrolyte

Flocculation time 20 min.

Influent turbidity 450 JTU

Fig 6 Effect of Flow Rate on Settling Efficiency

(After HANSEN and CULP 1967)



Tube length 4 ft.

Alum dosage 100 mg/l

Polyelectrolyte dosage 0.5 mg/l

Flocculation time 20 min.

Influent turbidity 450 JTU

Fig 7 Effect of Flow Rate on Settling Efficiency

(After HANSEN and CULP 1967)

HANSEN et.al. (1969) carried out the tests at Pilot-Plant scale to evaluate the effect of overflow rate on the inclined tube-settler. The tube module comprised of 2 in. square and 2 ft. in length, tubes inclined at an angle of 60 deg.. The results indicated that effluent turbidity increase from 7 to 20 JTU., with increase of flowrate from 4 to 7 gpm./sq.ft.

HERNANDEZ and WRIGHT (1970) pointed out that flow rate was the most important parameter effecting tube settler performance.

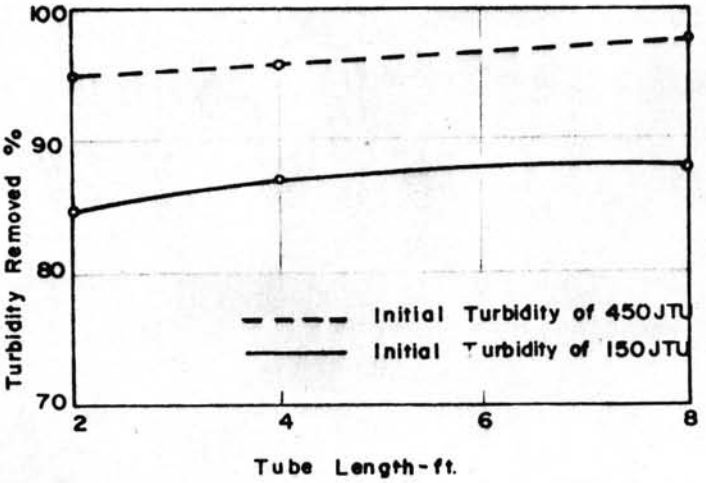
From Fig 12 and 13 show that circular settler with flowrate of less than 4 gpm./sq.ft. tend to perform better than those with flow velocity greater than 6 gpm./sq.ft/

Effect of Turbidity

From Fig 8, 9 and 11 show that the percentage of turbidity removal increases as the raw water turbidity increase. For example tube diameter of 1 in. flowrate of 2 gpm./sq.ft., the percentage of turbidity removal decrease from 95.1 to 85 percent for raw turbidity of 450 and 150 JTU. repectively.

Effect of Angle of Inclination

It was noted by Fig 9 that tube efficiency showed an increased as the angle of inclination was increased to 35 - 45 deg. and then began to decrease as the angle of inclination was increased further. The optimum inclination angle also depends upon the tube diameter, rate of flow, raw water characteristic and coagulation process. This is because all these parameters affect the settling velocity of solid particles and so the



Flow rate 2 gpm/sqft.

Álum dosage 100mg/l at 450 JTU
80mg/l at 150 JTU

Tube diameter 1 in

no Polyelectrolyte dosage

Flocculation time 20 min.

Fig 8 Effect of Influent Turbidity on Settling Performance

(After HANSEN and CULP 1967)

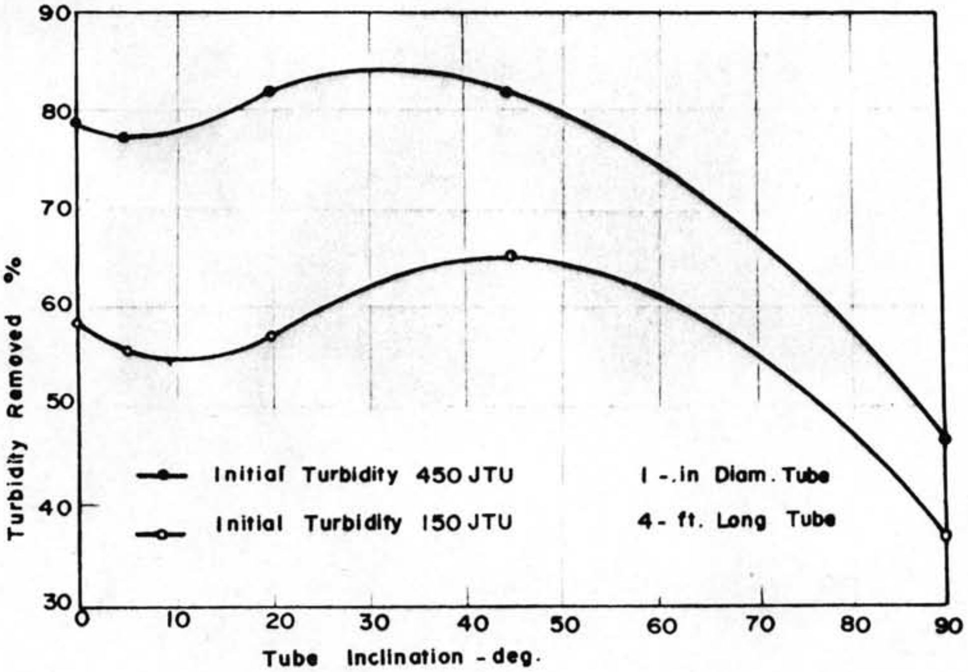


Fig 9 Effect of Tube Inclination on Settling Performance

(After CULP et al 1968)

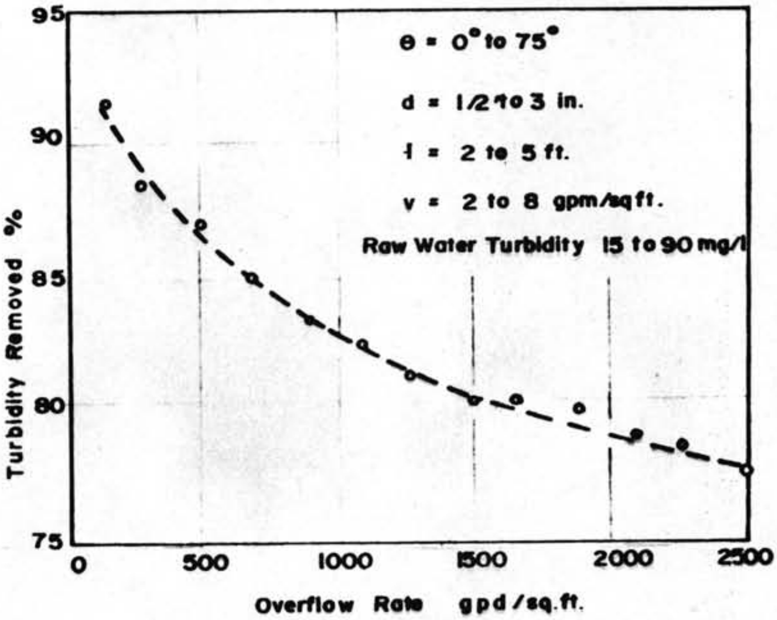


Fig 10 Experimental Efficiency Curve of Circular Settlers

(After YAO 1973)

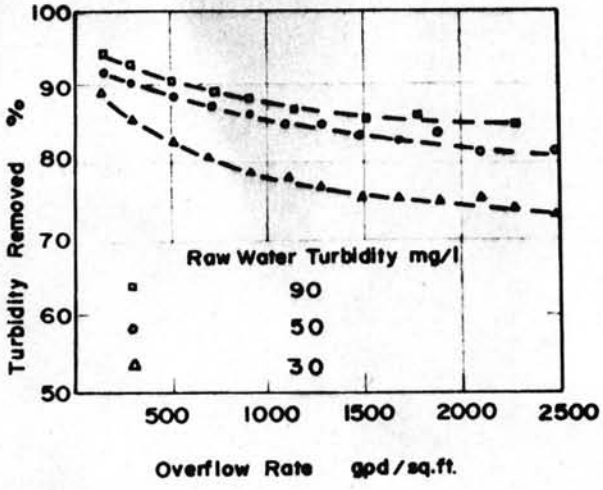


Fig 11 Effect of Raw Turbidity on Efficency of Circular Settlers

(After YAO 1973)

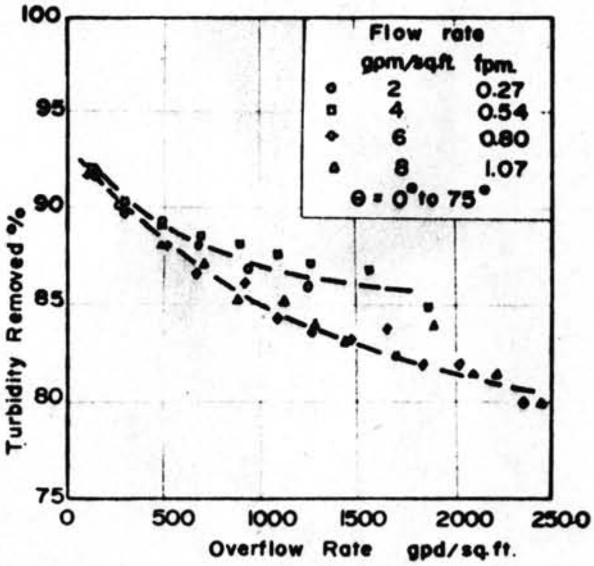


Fig 12 Effect of Flow Rate on Efficency of Circular Settlers

(Raw Water Turbidity 50 mg/l) (After YAO 1973)

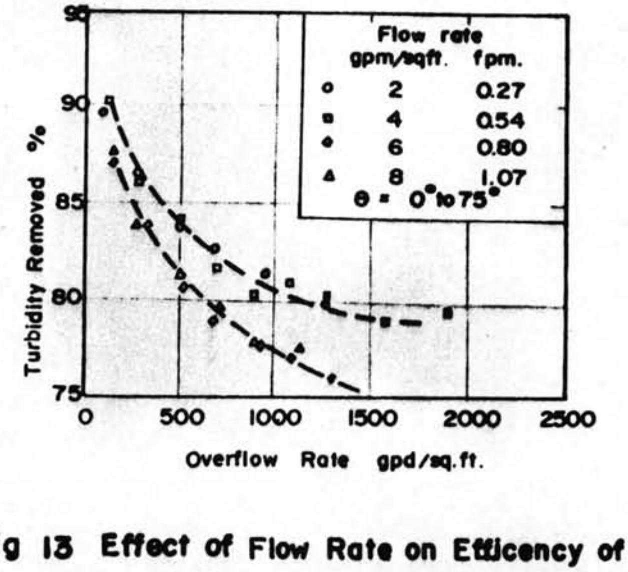


Fig 13 Effect of Flow Rate on Efficency of Circular Settlers

(Raw Water Turbidity 30 mg/l) (After YAO 1973)

removal performance.

Effect of Flow rate, Hydraulic Radius and Length of Tube Settler.

HERNANDEZ and WRIGHT (1970) conducted the research to identify the relationship between the design variables and developed generalized design criteria by mean of curve fitting. The study indicated that the best graphical fitting was obtained when the settler effluent turbidity, or turbidity removal was plotted on semilog paper against the ratio V^2R/L where: R is hydraulic radius of tube in ft., V is the velocity of flow in fps. (as computed from the face velocity of the tube net in gpm./sq/ft. of the tube entrance area) and L is the length of the tube in ft. Significant change in settler efficiency with small change in the factor V^2R/L was indicated by the study. The maximum value for the ratio V^2R/L that yielded good performance depended upon the angle of inclination, the flocculant nature and the density of suspended solid being removed. They recommenced that the value of V^2R/L were on the order 4×10^{-7} for 5 deg. unit, for alum coagulated water, 40×10^{-7} for 60 deg. unit, alum coagulated water and 2×10^{-7} for 60 deg. unit, for activated sludge mixed liquor.

Theory of Ideal Sedimentation Basin

HAZEN (1904) proposed a mathematical analysis to show that the portion of the sediment removed is a function of surface area of the basin and is entirely independent of the basin depth. In his analysis, he assumed a very shallow sedimentation basin.

Let t = the time required for a particle of sediment to fall from the surface to the bottom of the water in the basin, the water mean while being absolutly still.

a = the quotient obtained by dividing the capacity of the basin by the quantity of the water entering or leaving it during each unit of time.

b = the area of basin.

c = the capacity of the basin.

e = the quantity of water treated in a unit of time.

v = the hydraulic value of the sediment, or, in otherword, the velocity at which it settle in still water.

d = the depth of a basin.

Then $$a = \frac{c}{e} = \frac{bd}{e}$$

and $$t = \frac{d}{v}$$

combining $$\frac{a}{t} = \frac{bd/e}{d/v} = \frac{bv}{e} \qquad (1)$$

In other words, the proportion of sediment removed is a function of the area of the basin and of the hydraulic value of the sediment, and of the quantity of water treated in a unit of time, and is entirely independent of the depth of the basin.

CAMP (1946) analyzed the settling of discrete particles in a so-called ideal basin (Fig 1) for which the following assumptions were applicable.

(a) the direction of flow is horizontal, and the velocity is uniform in all parts of the settling zone.

(b) the concentration of suspended particles of each size is uniform over the depth at the inlet end of the settling zone.

(c) particles reaching the bottom remain fixed.

He considered any continuous flow basin to be comprised of four zones (see Fig 1 (a)) according to function as follow.

(1) an inlet zone in which the suspension is dispersed uniformly over the cross section of the basin.

(2) the settling zone in which all settling takes places.

(3) an outlet zone in which the clarified liquid is collected uniformly over the cross section of the basin and directed to an outlet conduit.

(4) a sludge zone at the bottom.

Any particles settling in a moving liquid will move in a direction and at a velocity, with reference to the basin, which is the vector sum of its own settling velocity and the velocity of the surrounding liquid. In an ideal rectangular basin the paths of all discrete particles will be strainght line, and all particles with the same settling velocity will move in parallel paths as illustrated by lines parallel to ae in Fig 1 (c) . All particles having settling velocities equal to or greater than V_o will settled out, but the removal of particles having any settling velocity V_s which is less than V_o will be equal to the ratio $\overline{bc}/\overline{ac}$. From similar traingle this removal ratio is

$$R_r = \frac{V_s}{V_o} = \frac{V_s}{Q/A} = \frac{bLV_s}{Q} \qquad (2)$$

in which bL = the surface area of the settling zone of width, b and length L, Q = the rate of discharge; and $V_o = \frac{Q}{bL}$ = the "over-

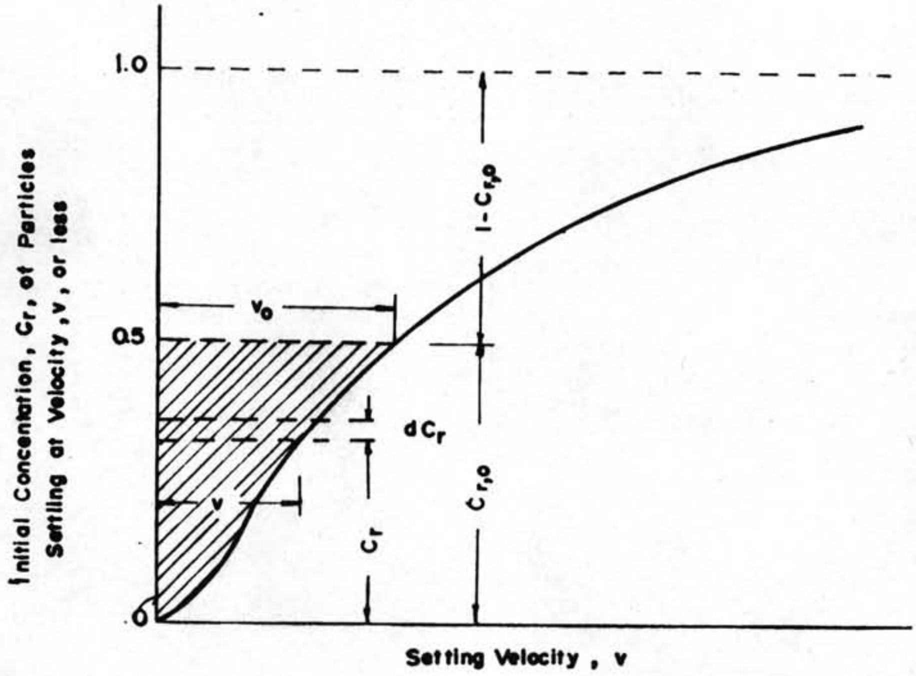


Fig 14 Typical Settling Velocity Analysis Curve of Suspension for Discrete Particles (After CAMP 1946)

flowrate" or the discharge perunits of surface area of the settling zone. Since all particles which settler faster than V_o will be 100 percent removed, the removal of such particles in terms of the total suspension will be $1-Cr,o$ where as Cr,o is the concentration of particles in the original suspension. The removal of particles having any settling velocity V_s which is less than V_o is $\int_o^{Cr} \frac{V_s}{V_o} dCr$. The total removal of all particles is therefore.

$$R = 1 - Cr,o + \frac{1}{V_o} \int_o^{Cr,o} V_s dCr \qquad (3)$$

From the curve (Fig 14) $\frac{1}{V_o} \int_o^{Cr,o} V_s dCr$ is the average vertical distance from the curve to the horizontal line for $Cr = Cr,o$. It is the shaded area in Fig 14 divided by V_o.

Theoretical Study of High Rate Settler

YAO (1970) conducted theoretical research on the characteristics of high-rate settlers and the governing physical properties of high-rate settling systems. He stated that in the small conduits used as high-rate settlers, laminar flow developed and the velocity distribution can be quite difference from CAMP's (1946) model, uniform flow across the tank cross section and that the particle paths were not straight line. With CAMP's model; the overflow rate of a settling tank expressed in rate of flow per unit tank area represented the critical fall velocity of the suspended particles. Theorectically, suspended particles with fall velocity greater than or equal to this critical value will be completely removed in the tank. He stated that the difficulties had arisen:

(1) no information is available whether the parameter overflowrate still retained the same physical significance for settler other than those rectangular in shape and (2) nothing is known as to how to calculate the overflowrate for settler such as inclined circular tube. These facts indicated CAMP's model is in need of extensive generalization if it is to be applied to high-rate settler. In this theorectical study, Fig 15 showed the coordinate system used. The x-axis is parallel to the axis of the high rate settler. Term Θ is the angle of inclination of the settler. Term P represents a particle which is subjected to the drag force of the flow with local velocity, u, in the x-direction and settling velocity, V_s, in the vertical direction. He derived the following equation:

$$\frac{dy}{dx} = \frac{-V_s \cos\Theta}{u - V_s \sin\Theta} \qquad (4)$$

which is the differential equation for the particle trajectory. He gave the general equation for particle trajectory obtained from Eq. 4 that:

$$\int \frac{u}{V_o} dY - \frac{V_s}{V_o} Y \cdot \sin\Theta + \frac{V_s}{V_o} X \cos\Theta = C_1 \qquad (5)$$

where C_1 is the adjusted intregation constant, V_o is the average flow velocity, $Y = y/d$, $X = x/d$ and is the spacing or size of the settler. C_1 and $\int \frac{u}{V_o} dY$ can be evaluated for a particular particle trajectory in a given high-rate settling system.

YAO (1970) also stated that the performance of a high-rate settling system can be characterized by a parameter, S, with

$$Sc = \frac{V_{sc}}{V_o} (\sin\Theta + L\cos\Theta) \qquad (6)$$

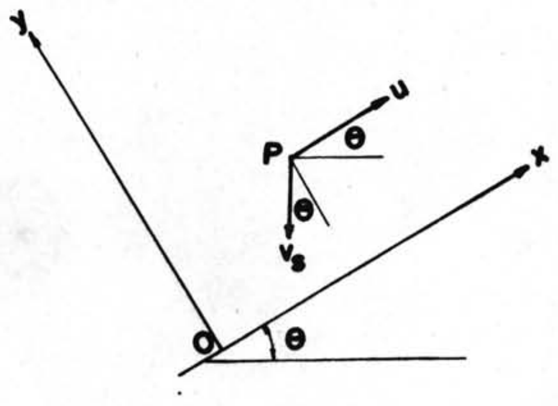


Fig 15 Coordinate system
(After YAO 1970)

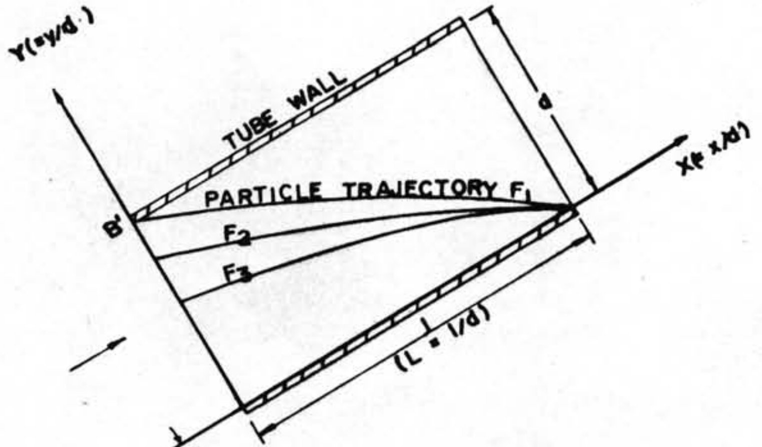


Fig 16 High rate settling system using a circular tube settler
(After YAO 1970)

in which L = relative length (1/d) and $Sc = \left(\frac{u}{V_o} dY \right) \quad Y = 1$. He stated that among the family of trajectories in Fig 16, there was a limiting trajectory which started at B' and represented the upper most trajectory in the family. The physical significance of the limiting trajectory was that it defined the critical particle fall velocity, V_{sc}, for a given system. Any suspended particles with its fall velocity greater than or equal to V_{sc} would be completely removed in the settler. If V_s in the Eq.5 is substituted by V_{sc} the critical S value, Sc, will be obtained. YAO reported that any suspended particle with its S-value greater than or equal to Sc would be completely removed, in Theory at least, from the flow without the need of knowing the critical fall velocity of the system at all. The critical S-value of various type of settlers were reported as follow:

Circular Tube $\quad Sc = \frac{V_{sc}}{V_o} (\sin\theta + L\cos\theta) = \frac{4}{3}$ (7)

Parallel Plates $\quad Sc = \frac{V_{sc}}{V_o} (\sin\theta + L\cos\theta) = 1$ (8)

Square Conduits $\quad Sc = \frac{V_{sc}}{V_o} (\sin\theta + L\cos\theta) = \frac{11}{8}$ (9)

Shallow Open Trays $\quad Sc = \frac{V_{sc}}{V_o} (\sin\theta + L\cos\theta) = 1$ (10)

Uniform Flow $\quad Sc = \frac{V_{sc}}{V_o} (\sin\theta + L\cos\theta) = 1$ (11)