CHAPTER III

DESIGN OF REINFORCEMENT

3.1 Specifications

Specified compressive strength of concrete, f_c' = 240 kg/cm²

Specified yield strength of reinforcement, f_y = 2,400 kg/cm²

Modulus of elasticity of concrete, E = 234,000 kg/cm²

Modulus of elasticity of steel, E_s = 2.039×10^6 kg/cm²

Modular ratio, n = 8.7

$$c = \frac{0.003}{0.003 + f_y / E_s} = 0.718\ d$$

$$a = 0.85\ C = 0.61d$$

$$P_b = 0.85\ \frac{f_c'}{f_y} \cdot \frac{a}{d} = 0.052$$

$$P_{max} = 0.75\ P_b = 0.039$$

$$q = P_{max}\ \frac{f_y}{f_c'} = 0.39$$

3.2 Design of Prototype

The torsional moments, bending moments and shears used in the design were analysed and shown in Table 4. Since the depth of the helical stair was 20 cm. at mid span and increased linearly along its length to be 30 cm. towards both the upper and lower support, the helical stair was designed at every 60 degrees. Example design at the upper support was shown below and followed ACI 318 - 71 Code.

3.2.1 Design for shear and torsion

Torsional moment at the upper support , $M_t = 8573.54$ kg-m.

$$v_{tu} = \frac{3M_t}{\phi \sum x^2 y}$$

$$= \frac{3 \times 8573.54 \times 100}{0.85 \times 30 \times 30 \times 80}$$

$$= 42 \quad kg/cm^2$$

Vertical shear at the upper support , $Q_s = 9345.87$ kgs.

$$v_u = \frac{V}{\phi\ bd}$$

$$= \frac{9345.87}{0.85 \times 80 \times 27.5}$$

$$= 5 \quad kg/cm^2$$

Horizontal shear at the upper support , $Q_r = 986.29$ kgs.

$$v = \frac{V}{\phi bd}$$

$$= \frac{986.29}{0.85 \times 30 \times 77.5}$$

$$= 0.5 \quad kg/cm^2$$

since shear stress caused by the horizontal force in the radial direction is low . Therefore the horizontal shear Q_r is neglected in the design and the section is designed for vertical shear and torsion only.

norminal permissible torsion stress carried by concrete

$$v_{tc} = \frac{0.636\sqrt{f'_c}}{\sqrt{1 + \left(1.2\ \frac{vu}{v_{tu}}\right)^2}}$$

$$= \frac{0.636\sqrt{240}}{\sqrt{1 + \left(\frac{1.2 \times 5}{42}\right)^2}}$$

$$= 9.8 \quad kg/cm^2$$

Maximum torsional stress for each section

$$= \frac{3.18\sqrt{f'_c}}{\sqrt{1 + \left(\frac{1.2\ v_u}{v_{tu}}\right)^2}}$$

$$= \frac{3.18\sqrt{240}}{\sqrt{1 + \left(\frac{1.2 \times 5}{42}\right)^2}}$$

$$= 49.2 \qquad kg/cm^2.$$

$$\alpha_t = 0.66 + 0.33\ (Y_1/X_1)$$

Assuming 2.5 cm. from the face of concrete to the center of stirrup steel

$$X_1 = 30 - 5 = 25 \qquad cm.$$

$$Y_1 = 80 - 5 = 75 \qquad cm.$$

$$\alpha_t = 0.66 + 0.33(75/25)$$

$$= 1.65 > 1.5$$

$$\text{use } \alpha_t = 1.5$$

$$A_t = \frac{(v_{tu} - v_{tc})S \sum x^2 y}{3\alpha_t X_1 Y_1 f_y}$$

$$= \frac{(42 - 9.8) \times 30 \times 30 \times 80S}{3 \times 1.5 \times 25 \times 75 \times 2400}$$

$$= 0.115\ S$$

Spacing of stirrup S $= 14$ cm.

$$A_t = 0.115 \times 14 \qquad cm^2.$$

$$= 1.61 \qquad cm^2.$$

$$A_l = 2\ A_t\left(\frac{X_1 + Y_1}{S}\right)$$

$$= 2 \times 1.61 \times \frac{(25 + 75)}{14}$$

$$= 23 \qquad cm^2.$$

$$\frac{3.52\, b_w S}{f_y} = \frac{3.52 \times 30 \times 14}{2400}$$

$$= 0.615 \quad cm.^2$$

$$2A_t = 2 \times 1.61 \quad cm.^2$$

$$= 3.22 \quad cm.^2 > 0.615$$

$$\therefore \quad A_1 = \left[\frac{28.1\, xS}{f_y}\left(\frac{v_{tu}}{v_{tu} + v_u}\right) - 2A_t\right]\left(\frac{X_1 + Y_1}{S}\right)$$

$$= \left[\frac{28.1 \times 30 \times 14}{2400}\left(\frac{42}{42 + 5}\right) - 3.22\right]\left(\frac{25 + 75}{14}\right)$$

$$= 8.45 \quad cm.^2$$

Longitudinal reinforcement to resist torsion

$$A_1 = 23 \quad cm.^2$$

Norminal permissible shear stress carried by concrete

$$v_c = \frac{0.53\sqrt{f'_c}}{\sqrt{1 + \left(\frac{v_{tu}}{1.2 v_u}\right)^2}}$$

$$= \frac{0.53\sqrt{240}}{\sqrt{1 + \left(\frac{42}{1.2 \times 5}\right)^2}}$$

$$= 1.16 \quad kg/cm.^2$$

$$A_v = \frac{(v_u - v_c)\, bS}{f_y}$$

$$= \frac{(5 - 1.16) \times 80 \times S}{2400}$$

$$= 0.128\ S$$

use spacing of stirrup $S = 14$ cm.

$$A_v = 0.128 \times 14 = 1.8 \quad cm^2$$

$$A_v + 2A_t = 1.8 + 3.22 \quad cm^2$$

$$= 5.02 \quad cm^2$$

3.2.2 Design for flexure and axial loads

Longitudinal force at the upper support, Q_t

$$= 3551.28 \quad kgs.$$

$$\text{Direct stress} = \frac{3551.28}{80 \times 30}$$

$$= 1.48 \quad kg/cm^2$$

Since the direct stress caused by axial load is low. Therefore the axial load, Q_t is neglected in the design and the section is designed for the vertical and lateral moment only.

Vertical moment at the upper support, M_r

$$= 3379.41 \quad kg\text{-}m.$$

Ultimate moment of the section, M_u

$$= \phi \left[bd^2 f'_c \, q(1 - 0.59q) \right]$$

$$b = 80$$

$$d = 30 - 2.5 = 27.5 \quad cm.$$

capacity reduction for flexure, $\phi = 0.9$

$$M_u = 0.9 \left[80 \times 27.5^2 \times 240 \times .389(1 - 0.59 \times 0.389) \right]$$

$$= 39{,}200 \quad kg\text{-}m.$$

Longitudinal reinforcement to resist vertical moment

$$A_s = \frac{M}{\phi \, f_y \, (d - a/2)}$$

$$= \frac{3379.41 \times 100}{0.9 \times 2400 \times 27.5(1 - 0.305)}$$

$$= 8.2 \qquad \text{cm}^2 \text{ (top)}$$

Lateral moment at the upper support , M_s

$$= 3257.88 \qquad \text{kg-m.}$$

Ultimate moment of the section

$$M_u = \phi \left[bd^2 f'_c \, q(1 - 0.59q) \right]$$

$$= 0.9 \left[30 \times 77.5^2 \times 240 \times 0.39(1 - 0.59 \times 0.39) \right]$$

$$= 117{,}000 \qquad \text{kg-m.}$$

∴ Longitudinal reinforcement to resist lateral moment

$$A_s = \frac{3257.88 \times 100}{0.9 \times 2400 \times 77.5(1 - 0.305)}$$

$$= 2.8 \qquad \text{cm}^2$$

3.3 Design of Model

In this work, a half scale reinforced concrete model was constructed for testing . From dimensional analysis, the model was scaled down in proportion so that

$$L_m = \frac{L_p}{2}$$

$$A_m = \frac{A_p}{4}$$

$$W_m = \frac{W_p}{4}$$

where L_m , L_p = Linear scale of the model and prototype respectively

A_m , A_p = Area of steel in the model and prototype respectively

W_m , W_p = Load of the model and prototype respectively

The cross section and reinforcement of the model in table 10 were scaled down from the prototype by using the above equations.

Table 9: Tabulated Values of reinforcement of the prototype

Angle (degrees)	Section (cm.Xcm.)	Area of reinforcement ($cm.^2$)			Stirrups ($cm.^2$)	Spacing (cm.)
		Torsion	Radial moment	Lateral moment		
0	30 X 80	23.0	8.16	2.8	5.02	14
60	28.3 X 80	18.3	1.64	3.2	4.15	14
120	26.7 X 80	20.5	1.03*	3.2	3.67	14
180	25 X 80	22.8	7.8	0.5	4.5	14
240	23.3 X 80	19.8	6.7	0.9	2.2	8
300	21.7 X 80	12.2	10.7	0.1	1.41	10
360	20 X 80	0	12.9	0	0	10
420	21.7 X 80	12.2	10.7	0.1**	1.41	10
480	23.3 X 80	19.8	6.7	0.9**	2.2	8
540	25 X 80	22.8	7.8	0.5**	4.5	14
600	26.7 X 80	20.5	1.03*	3.2**	3.67	14
660	28.3 X 80	18.3	1.64	3.2**	4.15	14
720	30 X 80	23.0	8.16	2.8**	5.02	14

Remark: * longitudinal reinforcement at the bottom face

** longitudinal reinforcement at the internal line radius

Table 10 : Tabulated values of reinforcement of the model

Angle (degrees)	Section (cm.Xcm.)	Area of reinforcement (cm^2)			Stirrups (cm^2)	Spacing (cm.)
		Torsion	Radial moment	lateral moment		
0	15 X 40	5.75	2.04	0.7	1.26	7
60	14.15 X 40	4.56	0.41	0.8	1.04	7
120	13.35 X 40	5.13	0.26 *	0.8	0.92	7
180	12.25 X 40	5.70	1.95	0.13	1.13	7
240	11.65 X 40	4.95	1.68	0.23	0.55	4
300	10.85 X 40	3.05	2.68	0.03	0.35	5
360	10 X 40	0	3.23	0	0	5
420	10.85 X 40	3.05	2.68	0.03 **	0.35	5
480	11.65 X 40	4.95	1.68	0.23 **	0.55	4
540	12.25 X 40	5.70	1.95	0.13 **	1.13	7
600	13.35 X 40	5.13	0.26 *	0.8 **	0.92	7
660	14.15 X 40	4.56	0.41	0.8 **	1.04	7
720	15 X 40	5.75	2.04	0.7 **	1.26	7

Remark : * longitudinal reinforcement at the bottom face

** longitudinal reinforcement at the internal line radius

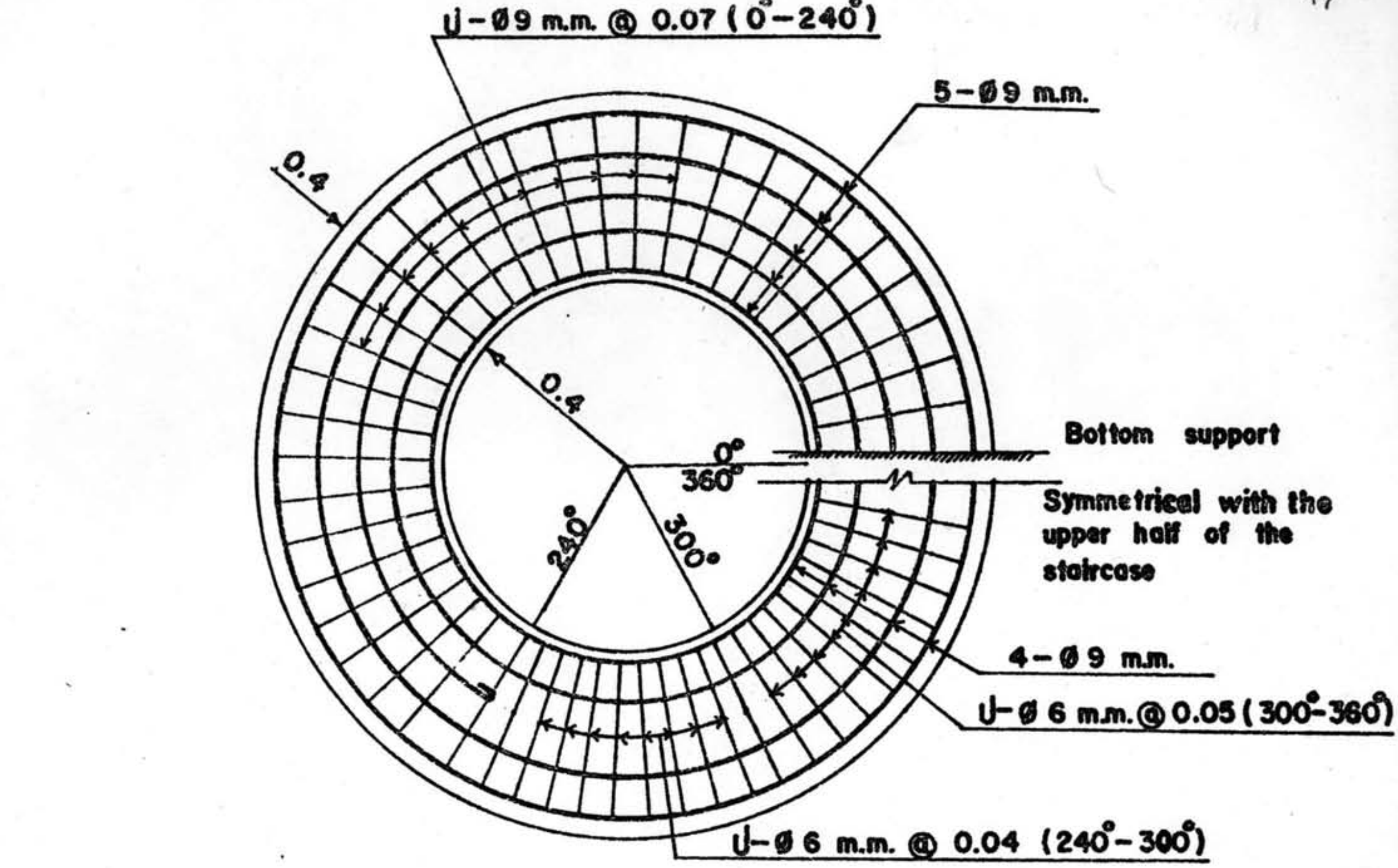


Fig 10 : Detail of reinforcement at the bottom face for lower half of the staircase

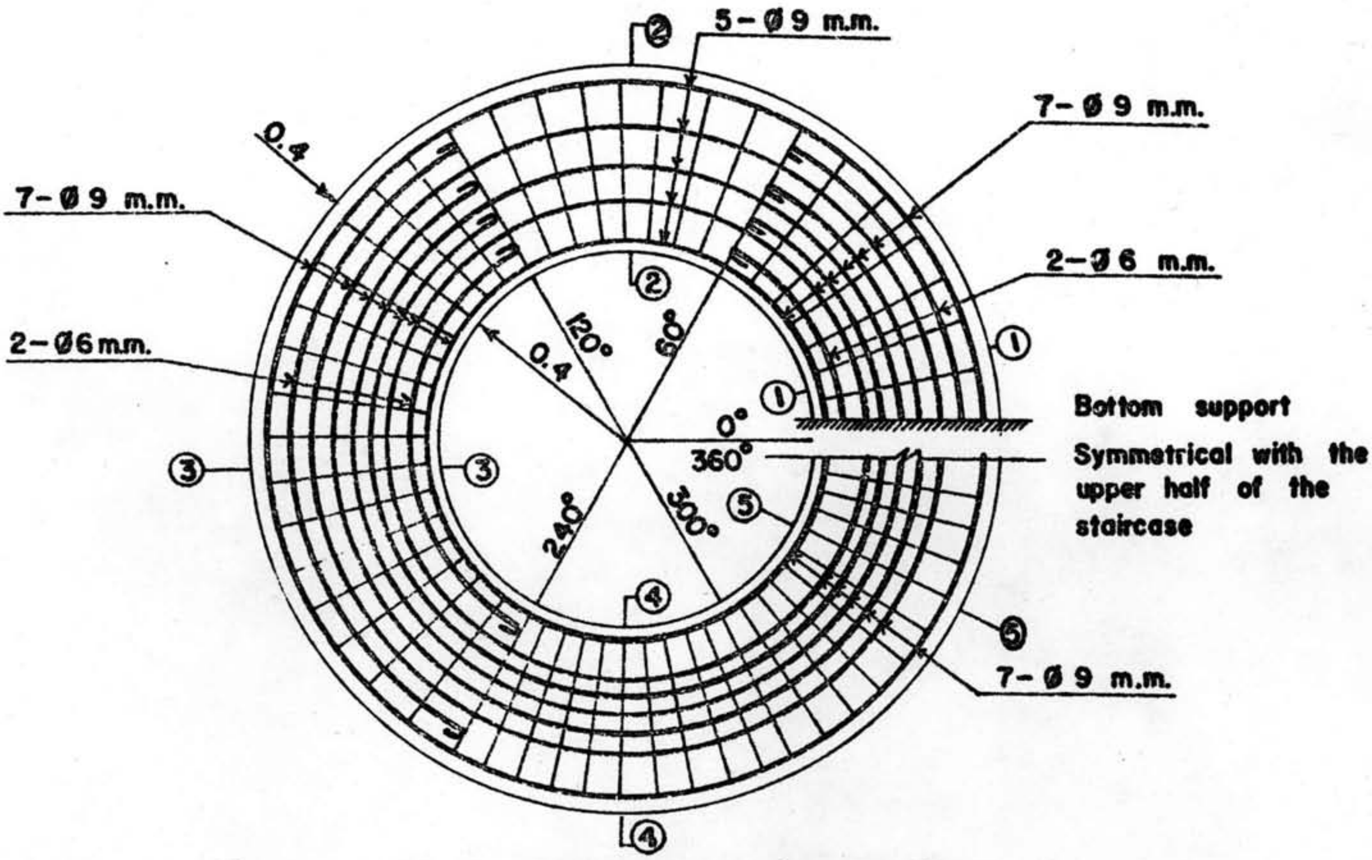
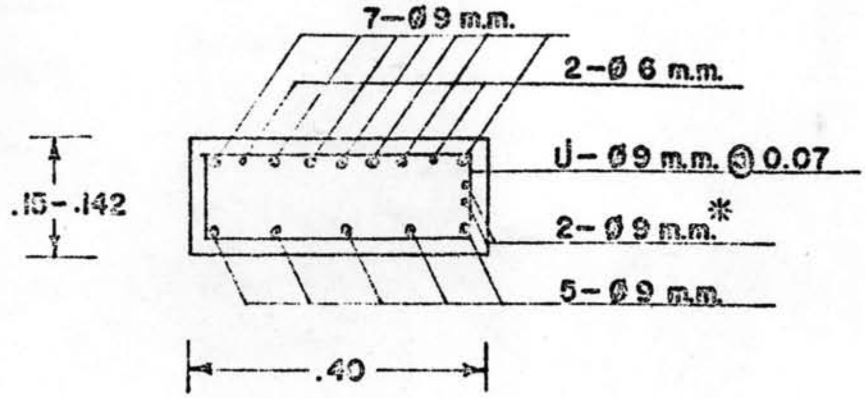
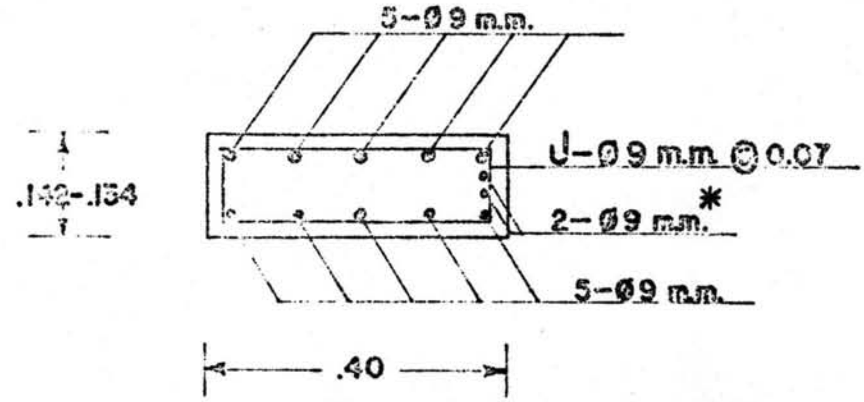


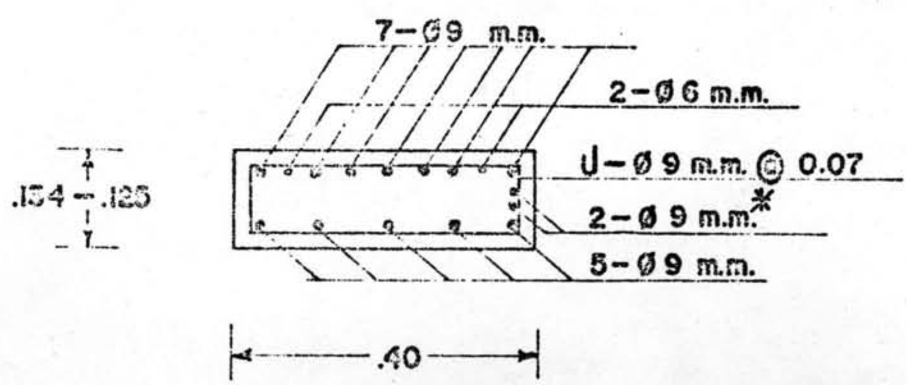
Fig 11 : Detial of reinforcement at the top face for lower half of the stair case



SECTION 1-1 (0-60 degrees)



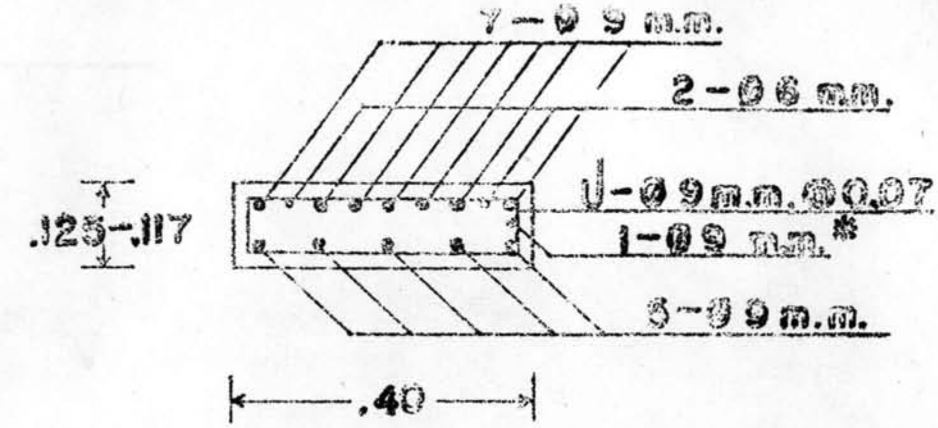
SECTION 2-2 (60-120 degrees)



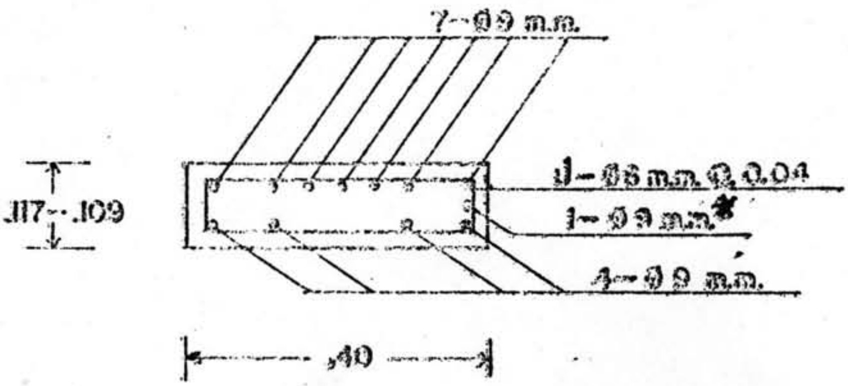
SECTION 3-3 (120-180 degrees)

Fig 12: Section showing reinforcement

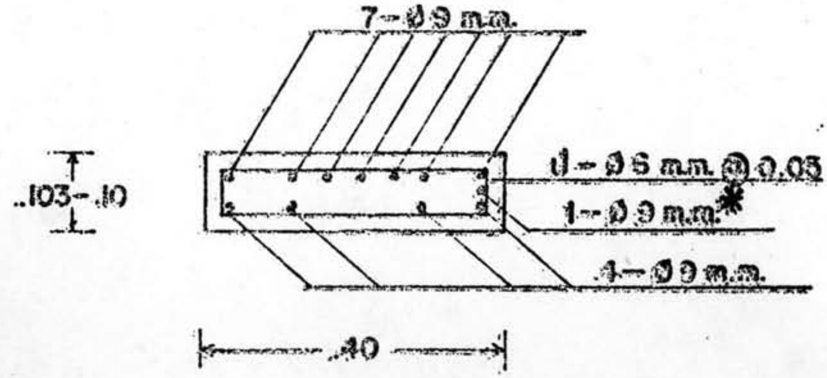
* Reinforced at outer circle for lower half and at inner circle for upper half of the staircase



SECTION 3-3 (180 - 240 degrees)



SECTION 4-4 (240 - 300 degrees)



SECTION 5-5 (300 - 360 degrees)

Fig 13: Section showing reinforcement

* Reinforced at outer circle for lower half and at inner circle for upper half of the staircase