CHAPTER II

ANALYSIS

2.1 Assumptions

In the analysis of the helical stair, the following assumptions are made:

1. The material of the structure is elastic, homogeneous and isotropic.

2. The structure is considered as an elastic linear member defined by the longitudinal centroidal axis of the girder thereby neglecting the slab action.

3. The bending and torsional stiffness of the section of a warped girder having helicoidal shape may be defined by those of a straight prismatic member.

4. Deformations due to shear and direct force are neglected being small in comparison to the deformations caused by bending and torsional moment

5. The moment of inertia and polar moment of inertia of the structure at any cross section may be assumed to be that of the cross section of the structure.

2.2 Geometry of the staircase

The geometry of a helical stair can be defined physically, as shown in Fig 5 in terms of the internal line radius R_i, external line radius R_o, the width b, the depth h, horizontal angle ϕ, an angle of slope α.

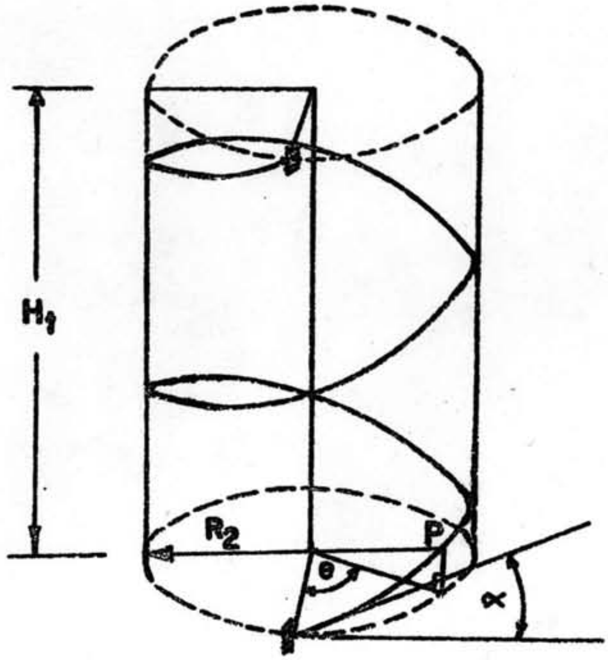


Fig I : Perspective skecth of helicoidal girder

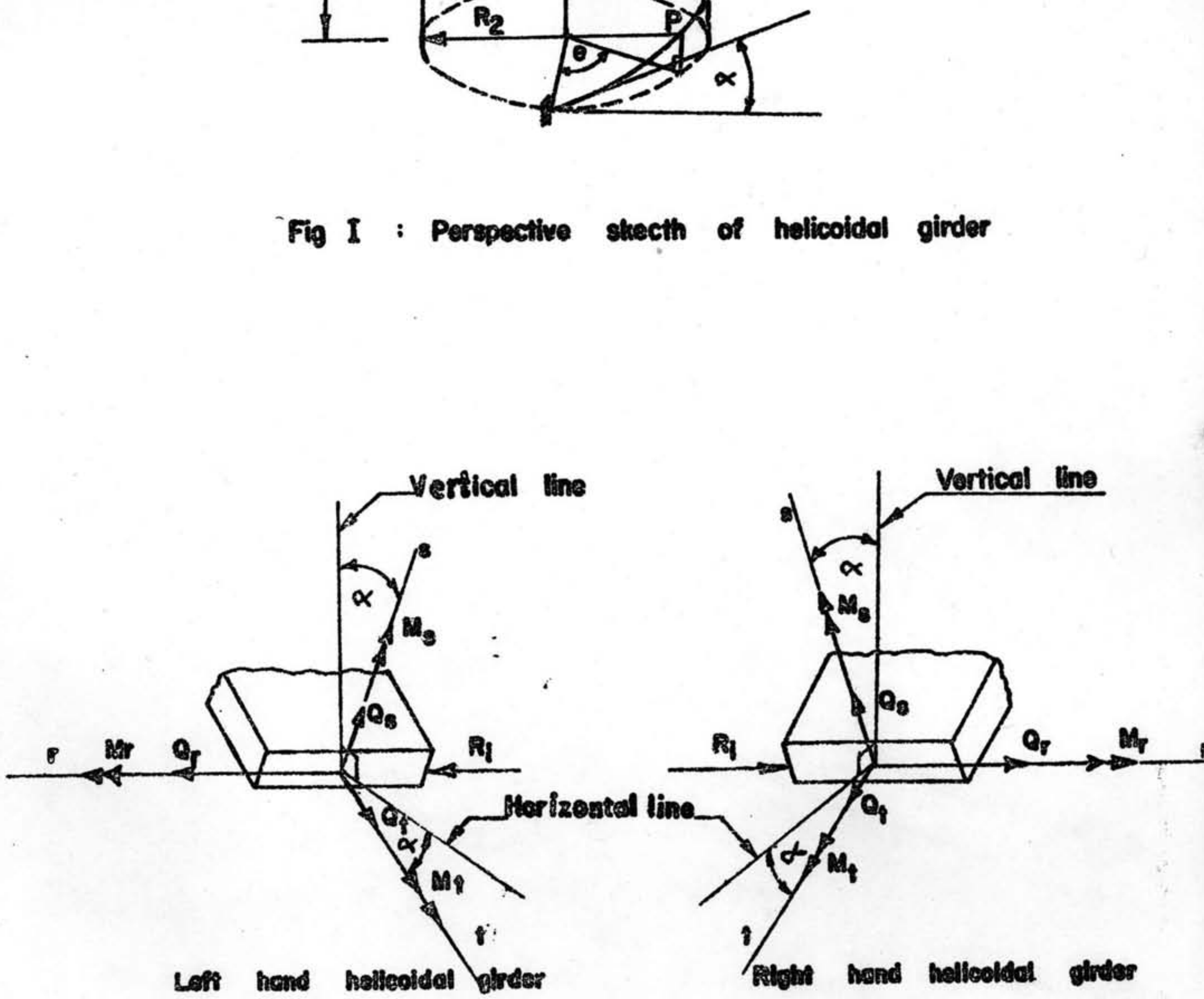


Fig II : Positive direction for internal forces

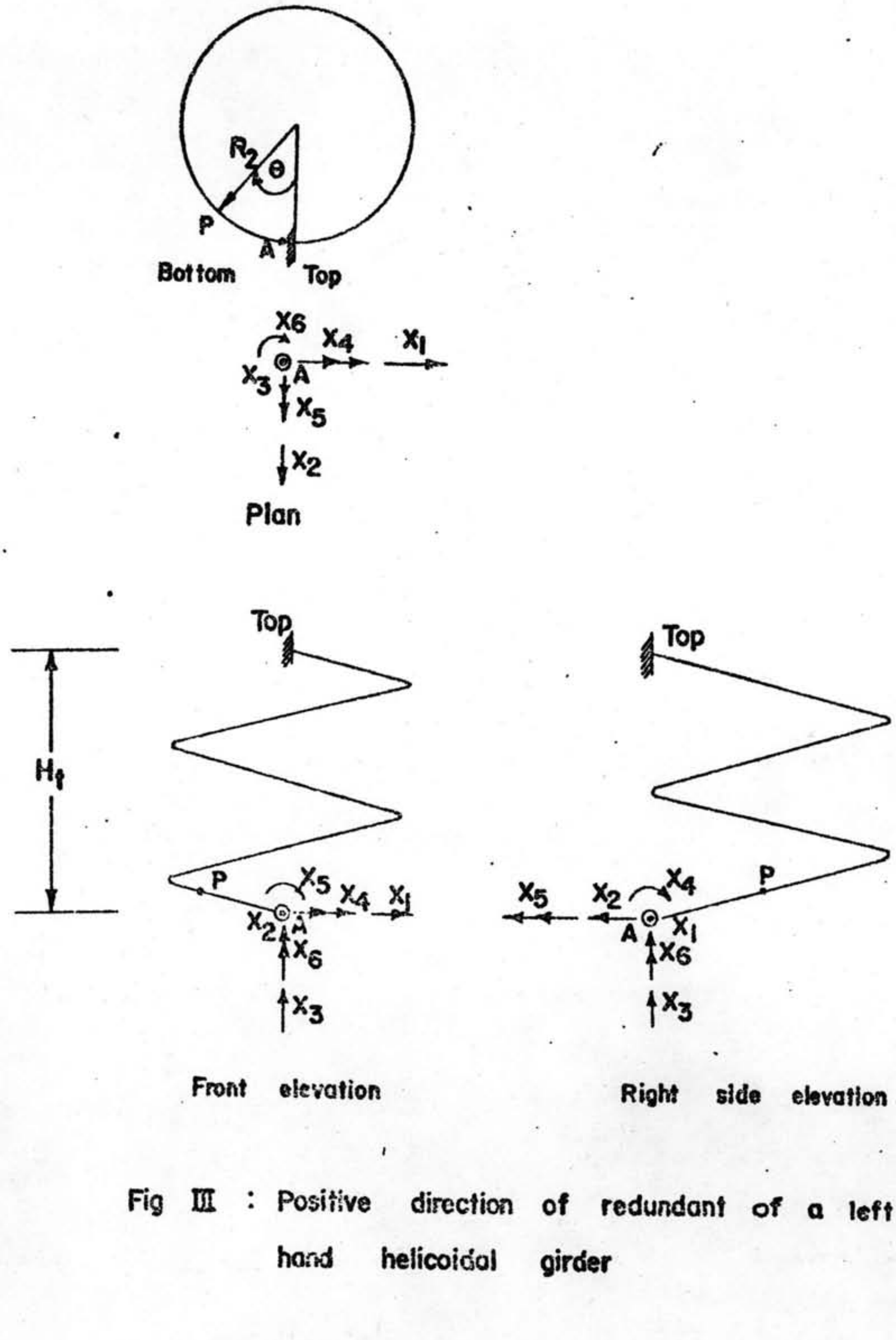
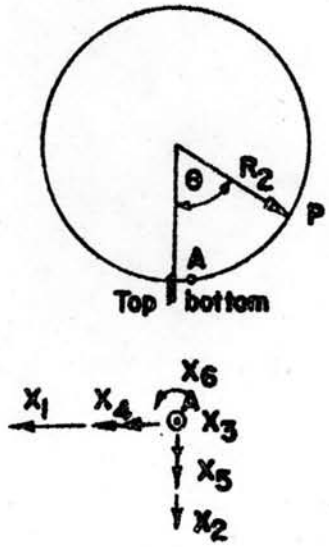
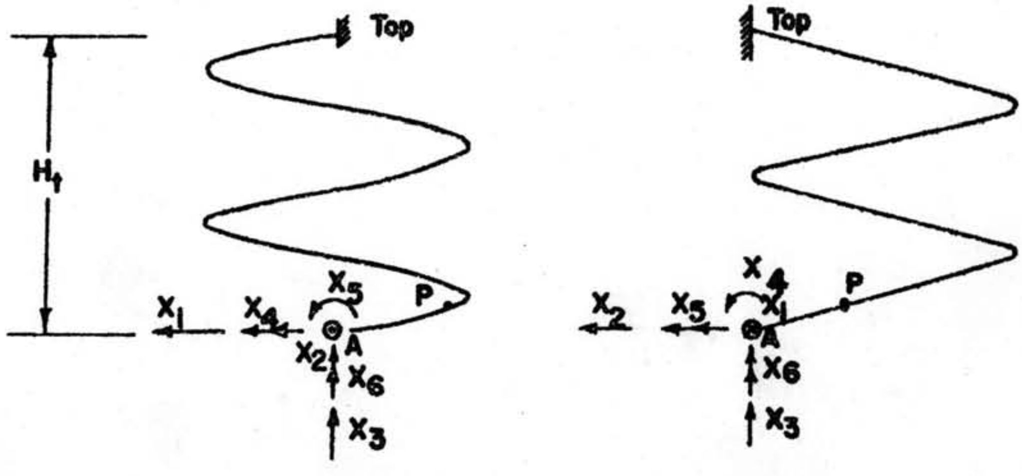


Fig III : Positive direction of redundant of a left hand helicoidal girder



Plan



Front elevation

Right side elevation

Fig 4 : Positive direction of redundants of a right hand helicoildal girder

2.3 Sign Convention

The forces and moments at any cross section of the helical stair are represented by vectors and double arrow head vectors respectively. The double arrow head vectors indicate the axis about which the moments act and the right hand rule is used for the right hand helicoidal girder. In the same manner, the left hand rule is used for the left hand helicoidal girder. The positive direction of the internal forces of the left and right hand helicoidal girder are shown in Fig II.

2.4 Compatibility equations

The helical stair fixed at both ends is a three dimensional structure. For the analysis , the helical stair is simplified as a helicoidal girder which is statically indeterminate to the sixth degrees as shown in Fig I. Then the six reactions at the lower support are selected as the redundants. The six redundants ie three forces X_1, X_2, X_3 and three moments X_4, X_5, X_6 act along three mutually perpendicular directions as shown in Fig 3 and 4 with their positive directions. The redundants X_1, X_4, and X_2, X_5 are horizontal in tangential and radial direction respectively where as the redundants X_3, X_6 are in vertically upward direction. The direction of the redundant moments of the left and right hand helicoidal girder are also established by the left and right hand rule respectively. By using the principle of superposition , the following compatibility equation may be written:

$$D_{11}X_1 + D_{12}X_2 + D_{13}X_3 + D_{14}X_4 + D_{15}X_5 + D_{16}X_6 + D_{1w} = D_1 \text{-----} (1)$$

$$D_{21}X_1 + D_{22}X_2 + D_{23}X_3 + D_{24}X_4 + D_{25}X_5 + D_{26}X_6 + D_{2w} = D_2 \text{-----} (2)$$

$$D_{31}X_1 + D_{32}X_2 + D_{33}X_3 + D_{34}X_4 + D_{35}X_5 + D_{36}X_6 + D_{3w} = D_3 \text{------} (3)$$

$$D_{41}X_1 + D_{42}X_2 + D_{43}X_3 + D_{44}X_4 + D_{45}X_5 + D_{46}X_6 + D_{4w} = D_4 \text{------} (4)$$

$$D_{51}X_1 + D_{52}X_2 + D_{53}X_3 + D_{54}X_4 + D_{55}X_5 + D_{56}X_6 + D_{5w} = D_5 \text{------} (5)$$

$$D_{61}X_1 + D_{62}X_2 + D_{63}X_3 + D_{64}X_4 + D_{65}X_5 + D_{66}X_6 + D_{6w} = D_6 \text{------} (6)$$

where D_{ij} = the displacements corresponding to X_i due to unit value of the redundant X_j only

D_{iw} = the displacements corresponding to X_i due to exter -nal load only

X_i = the redundants at the lower support

D_i = the actual displacements corresponding to redundants X_i

In order to obtain the solution for the redundants the flexibility coefficients D_{ij} and D_{iw} should be known. In this case of study, the external load is the uniform load on whole the stair -case. Therefore the flexibility D_{iw} are the flexibility coefficients of a unit uniform load. By using the method of virtual work, the flexibility D_{ij} and D_{iw} are determined as follow:

$$D_{ij} = \int_0^{\phi} \frac{m_{ri} m_{rj}}{EI_r} ds + \int_0^{\phi} \frac{m_{si} m_{sj}}{EI_s} ds + \int_0^{\phi} \frac{m_{ti} m_{tj}}{GJ} ds$$

$$D_{iw} = \int_0^{\phi} \frac{m_{ri} m_{rw}}{EI_r} ds + \int_0^{\phi} \frac{m_{si} m_{sw}}{EI_s} ds + \int_0^{\phi} \frac{m_{ti} m_{tw}}{GJ} ds$$

where m_{ri}, m_{rj} = moment about the r-axis due to unit value of X_i and X_j respectively

m_{si}, m_{sj} = moment about the s-axis due to unit value of X_i and X_j respectively

m_{ti}, m_{tj} = torsional moment about the t-axis due to unit value of X_i and X_j respectively

m_{rw} = moment about the r-axis due to external load

m_{sw} = moment about the s-axis due to external load

m_{tw} = torsional moment about the t-axis due to external load

I_r = second moment of area of waist section about the r-axis = $\frac{1}{12}bh^3$

I_s = second moment of area of waist section about the s-axis = $\frac{1}{12}hb^3$

$J^{(1)}$ = torsional constant = $\frac{bh^3}{16}\left[\frac{16}{3} - 3.36\,\frac{h}{b}\left(1 - \frac{h^4}{12d^4}\right)\right]$

E = modulus of elasticity

G = molulus of shear

ds = $\frac{R_2}{\cos\alpha}\,d\theta$

θ = angle measured from the lower support

R_2 = radius of center line of step

R_1 = radius of center line of load

(1) Roark , R.J. , Formulas for Stresses AND Strain , 2nd edition Mc GRAW-HILL BOOK COMPANY , NEW YORK and LONDON INC., 1943 , P 163

Table I : Internal forces due to external load

Internal forces	expression of internal forces due to unit uniform load
m_r	$R_1R_2(1 - \cos\theta)$
m_s	$R_2^2 \sin\alpha\,(\theta - \frac{R_1}{R_2}\sin\theta)$
m_t	$R_2^2 \cos\alpha\,(\theta - \frac{R_1}{R_2}\sin\theta)$
q_r	-----
q_s	$-R_2\theta\cos\alpha$
q_t	$R_2\theta\sin\alpha$

Internal Force	Expression of internal forces due to unit values of following redundants					
	X_I	X_2	X_3	X_4	X_5	X_6
m_r	$-R_2\theta \tan\alpha \cos\theta$	$-R_2\theta \sin\theta \tan\alpha$	$-R_2 \sin\theta$	$-\sin\theta$	$\cos\theta$	-
m_s	$R_2\cos\alpha(\cos\theta - 1 - \theta\sin\theta\tan^2\alpha)$	$R_2\cos\alpha(\theta\cos\theta \tan^2\alpha + \sin\theta)$	$R_2\sin\alpha(\cos\theta - 1)$	$\sin\alpha\cos\theta$	$\sin\alpha\sin\theta$	$\cos\alpha$
m_t	$R_2\sin\alpha(1-\theta\sin\theta - \cos\theta)$	$R_2\sin\alpha(\theta\cos\theta - \sin\theta)$	$R_2\cos\alpha(\cos\theta - 1)$	$\cos\alpha\cos\theta$	$\cos\alpha\sin\theta$	$-\sin\alpha$
q_r	$-\sin\theta$	$\cos\theta$	-	-	-	-
q_s	$\sin\alpha\cos\theta$	$\sin\alpha\sin\theta$	$\cos\alpha$	-	-	-
q_t	$\cos\alpha\cos\theta$	$\cos\alpha\sin\theta$	$-\sin\alpha$	-	-	-

Table 2 : Internal Force due to unit value of redundants

Flexibility Coefficients

The flexibility coefficient of the redundants at lower support of a helical stair with central angle ϕ, are expressed as follow :

$$D_{11} = \frac{R_2^3}{EI_r}\,\frac{\tan^2\alpha}{\cos\alpha}\left[\frac{\phi^3}{6} + \left(\frac{\phi^2}{4} - \frac{1}{8}\right)\sin 2\phi + \phi\,\frac{\cos 2\phi}{4}\right]$$

$$+ \frac{R_2^3}{EI_r}\,\frac{1}{\cos\alpha}\left[\frac{3}{2}\,\phi\cos^2\alpha + \frac{\cos 2\alpha}{4}\,(\sin 2\phi - 8\sin\phi) + \sin^2\alpha\,(\phi\cos 2\phi - 4\,\phi\cos\phi) + \sin^2\alpha\right.$$

$$\left.\tan^2\alpha\left\{\frac{\phi^3}{6} - \left(\frac{\phi^2}{4} - \frac{1}{8}\right)\sin 2\phi - \frac{\phi}{4}\cos 2\phi\right\}\right] + \frac{R_2^3}{GJ}$$

$$\frac{\sin^2\alpha}{\cos\alpha}\left[\frac{\phi^3}{6} - \sin 2\phi\left(\frac{\phi^2}{4} - \frac{5}{8}\right) + \frac{3}{4}\,\phi\,(2 - \cos 2\phi)\right.$$

$$\left. + 2\phi\cos\phi - 4\sin\phi\right]$$

$$D_{22} = \frac{R_2^3}{EI_r}\,\frac{\tan^2\alpha}{\cos\alpha}\left[\frac{\phi^3}{6} - \left(\frac{\phi^4}{4} - \frac{1}{8}\right)\sin 2\phi - \frac{\phi}{4}\cos 2\phi\right] +$$

$$\frac{R_2^3}{EI_s}\,\frac{1}{\cos\alpha}\left[\frac{\phi}{2}\left(\cos^2\alpha - \sin^2\alpha\cos 2\phi\right) - \frac{\cos 2\alpha}{4}\right.$$

$$\sin 2\phi + \sin^2\alpha\tan^2\alpha\left\{\frac{\phi^3}{6} + \left(\frac{\phi^2}{4} - \frac{1}{8}\right)\sin 2\phi + \frac{\phi}{4}\right.$$

$$\left.\left.\cos 2\phi\right\}\right] + \frac{R_2^3}{GJ}\,\frac{\sin^2\alpha}{\cos\alpha}\left[\frac{\phi^3}{6} + \left(\frac{\phi^2}{4} - \frac{5}{8}\right)\sin 2\phi\right.$$

$$\left. + \frac{\phi}{4}\,(3\cos 2\phi + 2)\right]$$

$$D_{33} = \frac{R_2^3}{EI_r}\,\frac{1}{\cos\alpha}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right] + \left[\frac{R_2^3}{EI_s}\,\frac{\sin^2\alpha}{\cos\alpha} + \frac{R_2^3}{GJ}\cos\alpha\right]$$

$$\left[\frac{3\phi}{2} + \frac{\sin 2\phi}{4} - 2\sin\phi\right]$$

$$D_{44} = \frac{R_2}{EI_r}\frac{1}{\cos\alpha}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right] + \left[\frac{R_2}{EI_s}\frac{\sin^2\alpha}{\cos\alpha} + \frac{R_2}{GJ}\cos\alpha\right]\left[\frac{\phi}{2} + \frac{\sin 2\phi}{4}\right]$$

$$D_{55} = \frac{R_2}{EI_r}\frac{1}{\cos\alpha}\left[\frac{\phi}{2} + \frac{\sin 2\phi}{4}\right] + \left[\frac{R_2}{EI_s}\frac{\sin^2\alpha}{\cos\alpha} + \frac{R_2}{GJ}\cos\alpha\right]\cdot\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]$$

$$D_{66} = R_2\phi\left[\frac{\cos\alpha}{EI_s} + \frac{\sin^2\alpha}{GJ\cos\alpha}\right]$$

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By Maxwell 's law of reciprocal deflections

$$D_{12} = D_{21} = \frac{R_2^3}{EI_r}\frac{\tan^2\alpha}{\cos\alpha}\left[\frac{\phi}{4}\sin 2\phi + \frac{\cos 2\phi}{8}(1 - 2\phi^2) - \frac{1}{8}\right] + \frac{R_2^3}{EI_s}\frac{1}{\cos\alpha}\left[\sin^2\alpha\left(\phi\frac{\sin 2\phi}{2} - \phi\sin\phi\right) - \frac{\cos 2\alpha}{4}(\cos 2\phi - 4\cos\phi + 3) - \sin^2\alpha\tan^2\alpha\left\{\phi\frac{\sin 2\phi}{4} + \frac{\cos 2\phi}{8}(1 - 2\phi^2) - \frac{1}{8}\right\}\right] + \frac{R_2^3}{GJ}\frac{\sin^2\alpha}{\cos\alpha}\left[\frac{\cos 2\phi}{8}(2\phi^2 - 5) + \frac{\phi}{4}(4\sin\phi - 3\sin 2\phi) + 2\cos\phi - \frac{11}{8}\right]$$

$$D_{13} = D_{31} = \frac{R_2^3}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\frac{\sin 2\phi}{8} - \frac{\phi\cos 2\phi}{4}\right] + \frac{R_2^3}{EI_s}\sin\alpha\left[\left(\frac{3\phi}{2} + \frac{\sin 2\phi}{4} - 2\sin\phi\right) - \tan^2\alpha\left(\frac{\sin 2\phi}{8} - \frac{\phi\cos 2\phi}{4} - \sin\phi + \phi\cos\phi\right)\right] + \frac{R_2^3}{GJ}\sin\alpha\left[3\sin\phi - \frac{3\sin 2\phi}{8} - \phi\cos\phi + \frac{\phi}{4}(\cos 2\phi - 6\phi)\right]$$

$$D_{14} = D_{41} = \frac{R_2^2}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\frac{\sin 2\phi}{8} - \frac{\phi\cos 2\phi}{4}\right] + \frac{R_2^2}{EI_s}\sin\alpha\left[\left(\frac{\phi}{2} + \frac{\sin 2\phi}{4} - \sin\phi\right) - \tan^2\alpha\left(\frac{\sin 2\phi}{8} - \frac{\phi\cos 2\phi}{4}\right)\right] + \frac{R_2^2}{GJ}\sin\alpha\left[\frac{\phi}{4}(\cos 2\phi - 2) - \frac{3}{8}\sin 2\phi + \sin\phi\right]$$

$$D_{15} = D_{51} = -\frac{R_2^2}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\frac{\phi^2}{4} + \frac{\phi\sin 2\phi}{4} + \frac{\cos 2\phi}{8} - \frac{1}{8}\right] + \frac{R_2^2}{EI_s}\sin\alpha$$

$$\left[\cos\phi - \frac{\cos 2\phi}{4} - \frac{3}{4} - \tan^2\alpha\left(\frac{\phi^2}{4} - \frac{\cos 2\phi}{8} - \frac{\phi\sin 2\phi}{4} + \frac{1}{8}\right)\right] +$$

$$\frac{R_2^2}{GJ}\sin\alpha\left[\frac{\phi\sin 2\phi}{4} + \frac{3\cos 2\phi}{8} - \cos\phi - \frac{\phi^2}{4} + \frac{5}{8}\right]$$

$$D_{16} = D_{61} = \frac{R_2^2}{EI_s}\cos\alpha\left[\sin\phi - \phi - \tan^2\alpha\,(\sin\phi - \phi\cos\phi)\right] +$$

$$\frac{R_2^2}{GJ}\frac{\sin^2\alpha}{\cos\alpha}\left[2\sin\phi - \phi\cos\phi - \phi\right]$$

$$D_{23} = D_{32} = \frac{R_2^3}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\frac{\phi^2}{4} - \frac{\cos 2\phi - 1}{8} - \frac{\phi\sin 2\phi}{4}\right] + \frac{R_2^3}{EI_s}\sin\alpha$$

$$\left[\tan^2\alpha\left(\frac{\phi^2}{4} + \frac{\cos 2\phi}{8} + \frac{\phi\sin 2\phi}{4} - \cos\phi - \phi\sin\phi + \frac{7}{8}\right) + \left(\cos\phi\right.\right.$$

$$\left.\left.-\frac{\cos 2\phi}{4} - \frac{3}{4}\right)\right] + \frac{R_2^3}{GJ}\sin\alpha\left[\frac{\phi^2}{4} + \frac{3\cos 2\phi}{8} + \frac{\phi\sin 2\phi}{4} - \phi\sin\phi -\right.$$

$$\left. 2\cos\phi + \frac{13}{8}\right]$$

$$D_{24} = D_{42} = \frac{R_2^2}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\frac{\phi^4}{4} - \frac{\cos 2\phi}{8} - \frac{\phi\sin 2\phi}{4} + \frac{1}{8}\right] + \frac{R_2^2}{EI_s}\sin\alpha$$

$$\left[\tan^2\alpha\left(\frac{\phi^2}{4} + \frac{\cos 2\phi}{8} + \frac{\phi\sin 2\phi}{4} - \frac{1}{8}\right) - \frac{1}{4}(\cos 2\phi - 1)\right] + \frac{R_2^2}{GJ}$$

$$\sin\alpha\left[\frac{\phi^2}{4} + \frac{3\cos 2\phi}{4} + \frac{\phi\sin 2\phi}{4} - \frac{3}{8}\right]$$

$$D_{25} = D_{52} = \frac{R_2^2}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\frac{\phi\cos 2\phi}{4} - \frac{\sin 2\phi}{8}\right] + \frac{R_2^2}{EI_s}\sin\alpha\left[\tan^2\alpha\right.$$

$$\left.\left(\frac{\sin 2\phi}{8} - \frac{\phi\cos 2\phi}{4}\right) + \left(\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right)\right] + \frac{R_2^2}{GJ}\sin\alpha\left[\frac{3\sin 2\phi}{8} -\right.$$

$$\left.\frac{\phi\cos 2\phi}{4} - \frac{\phi}{2}\right]$$

$$D_{26} = D_{62} = \frac{R_2^2}{EI_s}\frac{1}{\cos\alpha}\left[\phi\sin\phi\sin^2\alpha - \cos 2\alpha\,(\cos\phi - 1)\right] - \frac{R_2^2}{GJ}$$

$$\frac{\sin^2\alpha}{\cos\alpha}\left[2\cos\phi + \phi\sin\phi - 2\right]$$

$$D_{34} = D_{43} = \frac{R_2^2}{EI_r}\,\frac{1}{\cos\alpha}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right] + \frac{R_2^2}{EI_s}\,\frac{\sin^2\alpha}{\cos\alpha}\left[\frac{\phi}{2} + \frac{\sin 2\phi}{4} - \sin\phi\right] + \frac{R_2^2}{GJ}\cos\alpha\left[\frac{\phi}{2} + \frac{\sin 2\phi}{4} - \sin\phi\right]$$

$$D_{35} = D_{53} = \frac{R_2^2}{EI_r}\,\frac{1}{\cos\alpha}\left[\frac{\cos 2\phi - 1}{4}\right] + \frac{R_2^2}{EI_s}\,\frac{\sin^2\alpha}{\cos\alpha}\left[\cos\phi - \frac{\cos 2\phi}{4} - \frac{3}{4}\right] + \frac{R_2^2}{GJ}\cos\alpha\left[\cos\phi - \frac{\cos 2\phi}{4} - \frac{3}{4}\right]$$

$$D_{36} = D_{63} = R_2^2 \sin\alpha\,(\sin\phi - \phi)\left(\frac{1}{EI_s} - \frac{1}{GJ}\right)$$

$$D_{45} = D_{54} = \frac{R_2}{\cos\alpha}\,\frac{(\cos 2\phi - 1)}{4}\left[\frac{1}{EI_r} - \frac{\sin^2\alpha}{EI_s} - \frac{\cos^2\alpha}{GJ}\right]$$

$$D_{46} = D_{64} = R_2 \sin\alpha \sin\phi\left(\frac{1}{EI_s} - \frac{1}{GJ}\right)$$

$$D_{56} = D_{65} = R_2 \sin\alpha\,(1 - \cos\phi)\left(\frac{1}{EI_s} - \frac{1}{GJ}\right)$$

The flexibility coefficient of unit uniform load on whole staircase are expressed as follow :

$$D_{1w} = \frac{R_1 R_2^3}{EI_r}\,\frac{\tan\alpha}{\cos\alpha}\left[\frac{\phi^2}{4} + \frac{\phi\sin 2\phi}{4} - \phi\sin\phi + \frac{\cos 2\phi}{8} - \cos\phi + \frac{7}{8}\right]$$
$$+ \frac{R_2^4}{EI_s}\sin\alpha\left[\cos\phi - 1 + \phi\sin\phi - \frac{\phi^2}{2} - \tan^2\alpha\left\{2\phi\sin\phi + \cos\phi\,(2 - \phi^2) - 2\right\} + \frac{R_1}{R_2}\left(1 - \cos\phi - \frac{\sin^2\phi}{2}\right) + \frac{R_1}{R_2}\tan^2\alpha\left(\frac{\phi^2}{4} - \frac{\cos 2\phi}{8} - \frac{\phi\sin 2\phi}{4} + \frac{1}{8}\right)\right]$$
$$+ \frac{R_2^4}{GJ}\sin\alpha\left[\frac{\phi^2}{2}(1 + 2\cos\phi) - 3(\phi\sin\phi + \cos\phi - 1) + \frac{R_1}{R_2}\left(\frac{\phi^2}{4} - \frac{3\cos 2\phi}{8} - \frac{\phi\sin 2\phi}{4} + \cos\phi - \frac{5}{8}\right)\right]$$

$$D_{2w} = \frac{R_1 R_2^3}{EI_r} \frac{\tan\alpha}{\cos\alpha} \left[\frac{\sin 2\phi}{8} - \frac{\phi\cos 2\phi}{4} - \sin\phi + \phi\cos\phi \right] + \frac{R_2^4}{EI_s} \tan\alpha$$

$$\left[\tan\alpha \sin\alpha \; (\phi^2 \sin\phi + 2\phi\cos\phi - 2\sin\phi) + \cos\alpha \, (\sin\phi - \phi \right.$$

$$\cos\phi) - \frac{R_1}{R_2} \left\{ \frac{\tan\alpha \sin\alpha}{8} (\sin 2\phi - 2\phi\cos 2\phi) + \frac{\cos\alpha}{4} (2\phi \right.$$

$$\left. \left. - \sin 2\phi) \right\} \right] + \frac{R_2^4}{GJ} \sin\alpha \left[3\phi\cos\phi + \sin\phi \; (\phi^2 - 3) - \frac{R_1}{R_2} \right.$$

$$\left. \left(\frac{3\sin 2\phi}{8} - \frac{\phi}{4} (\cos 2\phi + 2) \right] \right.$$

$$D_{3w} = \frac{R_1 R_2^3}{EI_r} \frac{1}{\cos\alpha} \left[\cos\phi - \frac{\cos 2\phi}{4} - \frac{3}{4} \right] + \frac{R_2^4}{EI_s} \frac{\sin^2\alpha}{\cos\alpha} \left[\phi\sin\phi + \right.$$

$$\left. \cos\phi - 1 - \frac{\phi^2}{2} + \frac{R_1}{R_2} \left(\frac{\cos 2\phi}{4} - \cos\phi + \frac{3}{4} \right) \right] + \frac{R_2^4}{GJ} \cos\alpha \left[\phi\sin\phi \right.$$

$$\left. + \cos\phi - 1 - \frac{\phi^2}{2} + \frac{R_1}{R_2} \left(\frac{\cos 2\phi}{4} - \cos\phi + \frac{3}{4} \right) \right]$$

$$D_{4w} = \frac{R_1 R_2^2}{EI_r} \frac{1}{\cos\alpha} \left[\cos\phi - \frac{\cos 2\phi}{4} - \frac{3}{4} \right] + \frac{R_2^3}{EI_s} \frac{\sin^2\alpha}{\cos\alpha} \left[\phi\sin\phi \right.$$

$$\left. + \cos\phi - 1 + \frac{R_1}{R_2} \left(\frac{\cos 2\phi - 1}{4} \right) \right] + \frac{R_2^3}{GJ} \cos\alpha \left[\phi\sin\phi + \cos\phi \right.$$

$$\left. - 1 + \frac{R_1}{R_2} \left(\frac{\cos 2\phi - 1}{4} \right) \right]$$

$$D_{5w} = \frac{R_1 R_2^2}{EI_r} \frac{1}{\cos\alpha} \left[\sin\phi - \frac{\phi}{2} - \frac{\sin 2\phi}{4} \right] + \frac{R_2^3}{EI_s} \frac{\sin^2\alpha}{\cos\alpha} \left[\sin\phi \right.$$

$$\left. - \phi\cos\phi - \frac{R_1}{R_2} \left(\frac{\phi}{2} - \frac{\sin 2\phi}{4} \right) \right] + \frac{R_2^3}{GJ} \cos\alpha \left[\sin\phi - \phi\cos\phi - \frac{R_1}{R_2} \right.$$

$$\left. \left(\frac{\phi}{2} - \frac{\sin 2\phi}{4} \right) \right]$$

$$D_{6w} = \frac{R_2^3}{EI_s} \sin\alpha \left[\frac{\phi^2}{2} + \frac{R_1}{R_2} (\cos\phi - 1) \right] + \frac{R_2^3}{GJ} \sin\alpha \left[\frac{R_1}{R_2} (1 - \right.$$

$$\left. \cos\phi) - \frac{\phi^2}{2} \right]$$

2.5 Evaluation of redundants and internal forces

The solution of eq(1) through (6) should be determined by substituting the known values of the flexibility coefficients into equation. These equations can be evaluated by any suitable method, then the six equations may be solved simultaneously for the values of the redundants X_1, X_2, X_3, X_4, X_5 and X_6. In this study, these equations are rewritten in matrix form as follow :

$$\begin{bmatrix} D_{ij} \end{bmatrix} \begin{Bmatrix} X_j \end{Bmatrix} = \begin{bmatrix} D_i - D_{iw} \end{bmatrix}$$

where

$\begin{bmatrix} D_{ij} \end{bmatrix}$ = the 6 X 6 flexibility matrix of displacements corresponding to the redundants X_i due to the redundants X_j

$\begin{Bmatrix} X_j \end{Bmatrix}$ = the 6 X 1 matrix of the redundants

$\begin{bmatrix} D_i \end{bmatrix}$ = the 6 X 1 matrix of actual displacements corresponding to the redundants

$\begin{bmatrix} D_{iw} \end{bmatrix}$ = the 6 X 1 matrix of displacements corresponding to the redundants X_i due to unit uniform load

Therefore the solution of the redundants may be written as follow:

$$\begin{Bmatrix} X_j \end{Bmatrix} = \begin{bmatrix} D_{ij} \end{bmatrix}^{-1} \begin{bmatrix} -D_{iw} \end{bmatrix} \quad \text{-----------------(7)}$$

the matrix $\begin{bmatrix} D_i \end{bmatrix}$ equal to zero since the helical stair is fixed at both ends.

The internal forces at any location θ from the lower support may be determined as follow :

$$M_r = m_{r1}X_1 + m_{r2}X_2 + m_{r3}X_3 + m_{r4}X_4 + m_{r5}X_5 + m_{r6}X_6 + m_{rw}$$

$$M_s = m_{s1}X_1 + m_{s2}X_2 + m_{s3}X_3 + m_{s4}X_4 + m_{s5}X_5 + m_{s6}X_6 + m_{sw}$$

$$M_t = m_{t1}X_1 + m_{t2}X_2 + m_{t3}X_3 + m_{t4}X_4 + m_{t5}X_5 + m_{t6}X_6 + m_{tw}$$

$$Q_r = q_{r1}X_1 + q_{r2}X_2 + q_{r3}X_3 + q_{r4}X_4 + q_{r5}X_5 + q_{r6}X_6 + q_{rw}$$

$$Q_s = q_{s1}X_1 + q_{s2}X_2 + q_{s3}X_3 + q_{s4}X_4 + q_{s5}X_5 + q_{s6}X_6 + q_{sw}$$

$$Q_t = q_{t1}X_1 + q_{t2}X_2 + q_{t3}X_3 + q_{t4}X_4 + q_{t5}X_5 + q_{t6}X_6 + q_{tw}$$

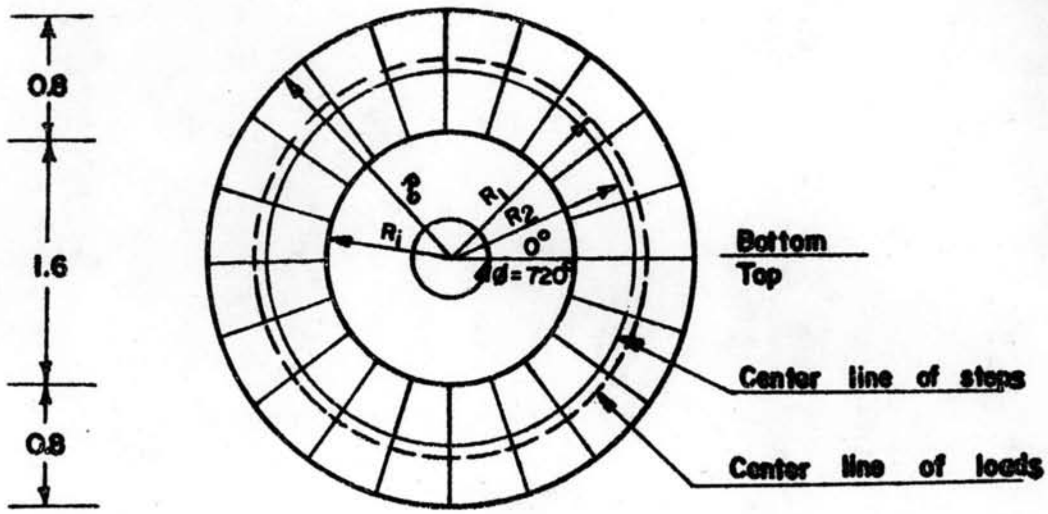
2.5.1 Data of the helical stair

The data of the helical stair under study are given as follow: (see Fig 5)

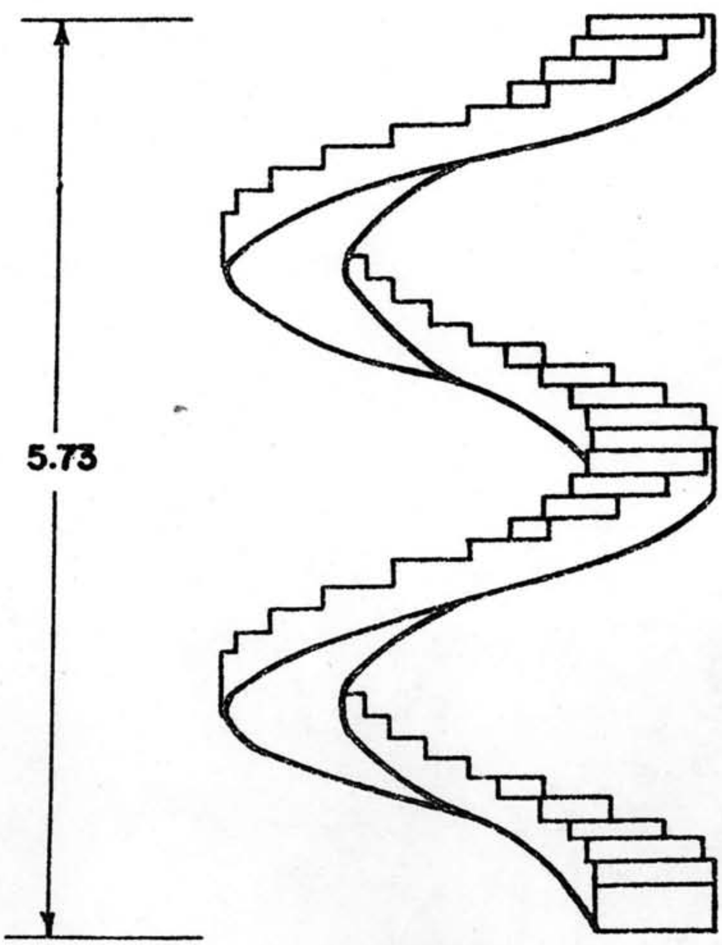
Total central angle , ϕ	=	720	degrees
Angle of slope , α	=	20.8	"
External radius , R_o	=	1.6	m.
Internal radius , R_i	=	0.8	m.
Height from top to ground floor , H_t	=	5.73	m.
Risers, 40 No.	=	0.143	m.
Width of staircase , b	=	0.8	m.
Depth of section at both supports	=	0.3	m.
Depth of section at mid point of stair	=	0.2	m.
Average depth of staircase for analysis , h	=	0.25	m.
Assume G/E	=	3/7	
Uniformly distributed live load	=	300	kg/m^2
Dead weight of concrete	=	2400	kg/m^3

Therefore

Live load , L.L.	=	239	kg/m
Dead load , D.L.	=	658	kg/m



Plan View



Elevation

Fig 5 : Dimension of Prototype Helical stair

Total load per unit projected
lenght of center line of step , W = $1.4 \times 658 + 1.7 \times 239$ kg/m.
= 1326 kg/m.

2.5.2 Solution

The redundants at the lower support and the internal forces at any section are calculated by computer. The computer pro-gramme is written in fortran IV language and shown in Appendix A. It is applicable to the helical stair under uniform load with central angle 720 degrees only. The input data W, H_t, R_o, R_i, h are required for the programme. Tabulated values of the internal forces are shown in Table 4.

2.6 Vertical deflection

The vertical deflections of the helical stair are determined by unit load method . In the analysis, a unit vertical load is applied at any point P, an angle β from the lower support. Then the deflection at point P due to unit value of the redundants and unit uniform load are separately calculated. By using the principal of superposition, total deflection are obtained as follow :

$$\Delta = D_{p1}X_1 + D_{p2}X_2 + D_{p3}X_3 + D_{p4}X_4 + D_{p5}X_5 + D_{p6}X_6 + D_{pw}W \qquad (7)$$

X_i = redundants at the lower support

D_{pi}, D_{pw} = Vertical deflection at any location β from lower support due to unit value of redundants and unit uniform load respectively

From equation (7), the vertical deflection D_{pi}, D_{pw} are determined and given below :

$$D_{pi} = \int_0^{\phi} \frac{m_{rp} m_{ri}}{EI_r} ds + \int_0^{\phi} \frac{m_{sp} m_{si}}{EI_s} ds + \int_0^{\phi} \frac{m_{tp}\, m_{ti}}{GJ} ds$$

$$D_{pw} = \int_0^{\phi} \frac{m_{rp} m_{rw}}{EI_r} ds + \int_0^{\phi} \frac{m_{sp} m_{sw}}{EI_s} ds + \int_0^{\phi} \frac{m_{tp} m_{tw}}{GJ} ds$$

where m_{rp}, m_{ri}, m_{rw} = moment about r-axis at any section due to unit vertical load at p , unit value of redundant X_i and unit uniform load respectively.

m_{sp}, m_{si}, m_{sw} = moment about s-axis at any section due to unit vertical load at p , unit value of redundant X_i and unit uniform load respectively.

m_{tp}, m_{ti}, m_{tw} = Torsional moment about t-axis at any section due to unit vertical load, unit value of redundant X_i and unit uniform load respectively

Table III: Internal force due to unit vertical load at location β from lower support

Internal forces	Expression of internal forces due to unit vertical load	
	$0 \leq \theta \leq \beta$	$\beta \leq \theta \leq \phi$
m_r	–	$R_2 \sin(\theta - \beta)$
m_s	–	$R_2 \sin\alpha\left[1 - \cos(\theta - \beta)\right]$
m_t	–	$R_2 \cos\alpha\left[1 - \cos(\theta - \beta)\right]$

The following equations below are the vertical deflections at any location β from lower support due to unit value of redundant and unit uniform load.

$$D_{pw} = \frac{R_1 R_2^3}{EI_r}\frac{1}{\cos\alpha}\left[\frac{\cos\beta}{4}\cos 2\theta + \frac{\sin\beta}{4}(2\theta + \sin 2\theta) - \cos(\theta - \beta)\right]_\beta^\phi + \left[\frac{R_2^4\sin^2\alpha}{EI_s\cos\alpha} + \frac{R_2^4}{GJ}\cos\alpha\right]\left[\frac{\theta^2}{2} + \frac{R_1}{R_2}\left\{\cos\theta - \frac{\cos\beta\cos 2\theta}{4} + \sin\beta\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right\} - \cos\beta(\cos\theta + \theta\sin\theta) - \sin\beta(\sin\theta - \theta\cos\theta)\right]_\beta^\phi$$

$$D_{p1} = \frac{-R_2^3}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\frac{\sin(2\theta - \beta)}{8} - \frac{\theta}{4}\cos(2\theta - \beta) - \frac{\theta^2\sin\beta}{4}\right]_\beta^\phi + \frac{R_2^3}{EI_s}\sin\alpha\left[\sin\theta - \frac{\theta}{2}(2 + \cos\beta) - \frac{\sin(2\theta - \beta)}{4} + \sin(\theta - \beta) - \tan^2\alpha(\sin\theta - \theta\cos\theta) + \frac{\tan^2\alpha\cos\beta}{2}\left(\frac{\sin 2\theta}{4} - \frac{\theta\cos 2\theta}{2}\right) + \tan^2\alpha\sin\beta\left(\frac{\theta^2}{4} - \frac{\theta\sin 2\theta}{4} - \frac{\cos 2\theta}{8}\right)\right]_\beta^\phi + \frac{R_2^3}{GJ}\sin\alpha\left[\theta - 2\sin\theta - \sin(\theta - \beta) + \frac{\sin(2\theta - \beta)}{8} - \frac{\theta\cos(2\theta - \beta)}{4} + \frac{\sin(2\theta - \beta)}{4} + \frac{\theta^2\sin\beta}{4} + \theta\cos\theta + \frac{\theta}{2}\cos\beta\right]_\beta^\phi$$

$$D_{p2} = \frac{-R_2^3}{EI_r}\frac{\tan\alpha}{\cos\alpha}\left[\cos\beta\left(\frac{\theta^2}{4} - \frac{\theta\sin 2\theta}{4} - \frac{\cos 2\theta}{8}\right) - \frac{\sin\beta}{2}\left(\frac{\sin 2\theta}{4} - \frac{\theta\cos 2\theta}{2}\right)\right]_\beta + \frac{R_2^3}{EI_s}\sin\alpha\left[\tan^2\alpha\left\{\cos\theta + \theta\sin\theta - \frac{\cos\beta}{8}(2\theta^2 + 2\theta\sin 2\theta + \cos 2\theta) - \frac{\sin\beta}{8}(\sin 2\theta - 2\theta\right.\right.$$

$$\left.\cos 2\theta\ )\right\} - \cos\theta - \frac{\theta\sin\beta}{2} + \left.\frac{\cos(2\theta - \beta)}{4}\right]_{\beta}^{\phi} + \frac{R_2^3}{GJ}\sin\alpha$$

$$\left[2\cos\theta + \frac{\theta}{2}(\ 2\sin\theta + \sin\beta) - \frac{\theta^2}{4}\cos\beta - \frac{\theta}{4}\sin(2\theta - \beta)\right.$$

$$\left. - \frac{\cos(2\theta - \beta)}{8} - \frac{\cos(2\theta - \beta)}{4}\right]_{\beta}^{\phi}$$

$$D_{p3} = \frac{R_2^3}{EIr}\frac{1}{\cos\alpha}\left[\frac{\sin(2\theta - \beta)}{4} - \frac{\theta\cos\beta}{2}\right]_{\beta}^{\phi} + \left[\frac{R_2^3}{EIs}\frac{\sin^2\alpha}{\cos\alpha} + \right.$$

$$\left.\frac{R_2^3}{GJ}\cos\alpha\right]\left[\sin(\theta - \beta) - \frac{\sin(2\theta - \beta)}{4} + \sin\theta - \frac{\theta}{2}(\ 2 +\right.$$

$$\left.\cos\beta\ )\right]_{\beta}^{\phi}$$

$$D_{p4} = \frac{R_2^2}{EI_r}\frac{1}{\cos\alpha}\left[\frac{\sin(2\theta - \beta)}{4} - \frac{\theta\cos\beta}{2}\right]_{\beta}^{\phi} + \left[\frac{R_2^2}{EI_s}\frac{\sin^2\alpha}{\cos\alpha}\right.$$

$$\left. + \frac{R_2^2}{GJ}\cos\alpha\right]\left[\sin\theta - \frac{\theta\cos\beta}{2} - \frac{\sin(2\theta - \beta)}{4}\right]_{\beta}^{\phi}$$

$$D_{p5} = \frac{R_2^2}{EI_r}\frac{1}{\cos\alpha}\left[-\frac{\cos 2\theta\cos\beta}{4} - \sin\beta\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\right]_{\beta}^{\phi} +$$

$$\left[\frac{R_2^2}{EI_s}\frac{\sin^2\alpha}{\cos\alpha} + \frac{R_2^2}{GJ}\cos\alpha\right]\left[\frac{\cos(2\theta - \beta)}{4} - \frac{\theta\sin\beta}{2} - \cos\theta\right]_{\beta}^{\phi}$$

$$D_{p6} = \left[\frac{R_2^2}{EI_s}\sin\alpha - \frac{R_2^2}{GJ}\sin\alpha\right]\left[\theta - \sin\theta\cos\beta + \cos\theta\sin\beta\right]_{\beta}^{\phi}$$

In this study , computer is used to calculate the vertical deflections and shown in Appendix B. The value of horizontal angle ϕ equal 4π radians is first substituted into the equation in order to shortening the programme.

2.7 Principle of symmetry

In the previous analysis, the helical stair was analysed as a three dimensional structure indeterminate to the sixth degree. This method of analysis can be applied to any type of external load. In the case of external load is uniform load on the whole staircase, the principle of symmetry are used. By select -ing the redundants at mid span, all but two of the redundants equal to, zero. The helical stair is then reduced to a structure indeterminate to the second degree. This greatly simplified the analysis. Morgan, Holmes, Scordelis and Bergman take advantage of symmetry to analyse the helical stair.

2.7.1 Morgan's method

Fig. 6 is the horizontal projection of a helical stair with uniform loading, fixed at both ends. The stairs are cut at mid span as shown in Fig 7. A double arrowhead represents a moment vector, the direction of the vector being established by the left hand rule. Each half then becomes a statically determinate structure. The general equations for bending moment, twisting moment, axial force and shear force at any section are :

$$M_r = M_v\cos\theta_1 + HR_2\theta_1\tan\alpha\sin\theta_1 - WR_1^2(1-\cos\theta_1) \tag{8}$$

$$M_s = M_v\sin\theta_1\sin\alpha - HR_2\tan\alpha\cos\theta_1\sin\alpha - HR_2\sin\theta_1\cos\alpha + (WR_1^2\sin\theta_1 - WR_1R_2\theta_1)\sin\alpha \tag{9}$$

$$M_t = (M_v\sin\theta_1 - HR_2\theta_1\cos\theta_1\tan\alpha + WR_1^2\sin\theta_1 - WR_1R_2\theta_1)\cos\alpha + HR_2\sin\theta_1\sin\alpha \tag{10}$$

$$Q_r = -H\sin\theta_1\cos\alpha - WR_1\theta_1\sin\alpha \tag{11}$$

$$Q_s = WR_1\theta_1\cos\alpha - H\sin\theta_1\sin\alpha \tag{12}$$

$$Q_t = H\cos\theta_1 \tag{13}$$

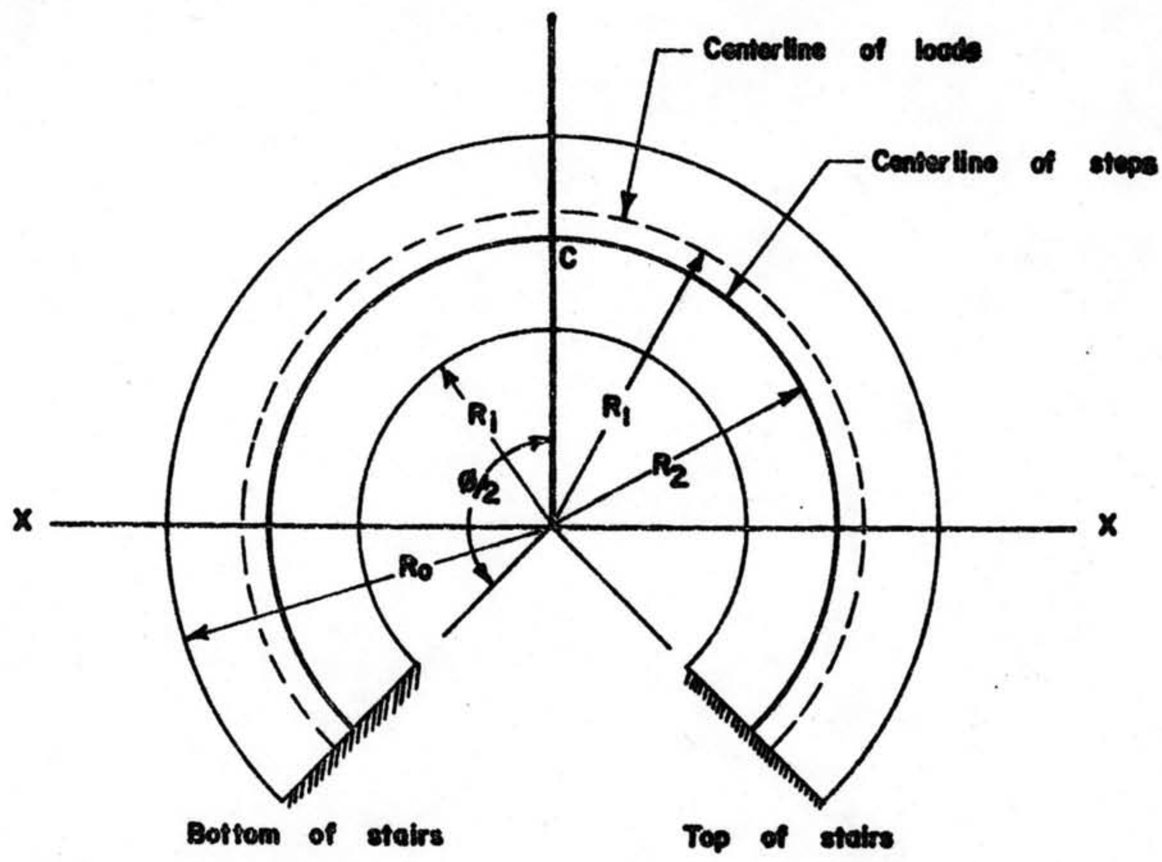


Fig 6 : Horizontal projection of helical stair

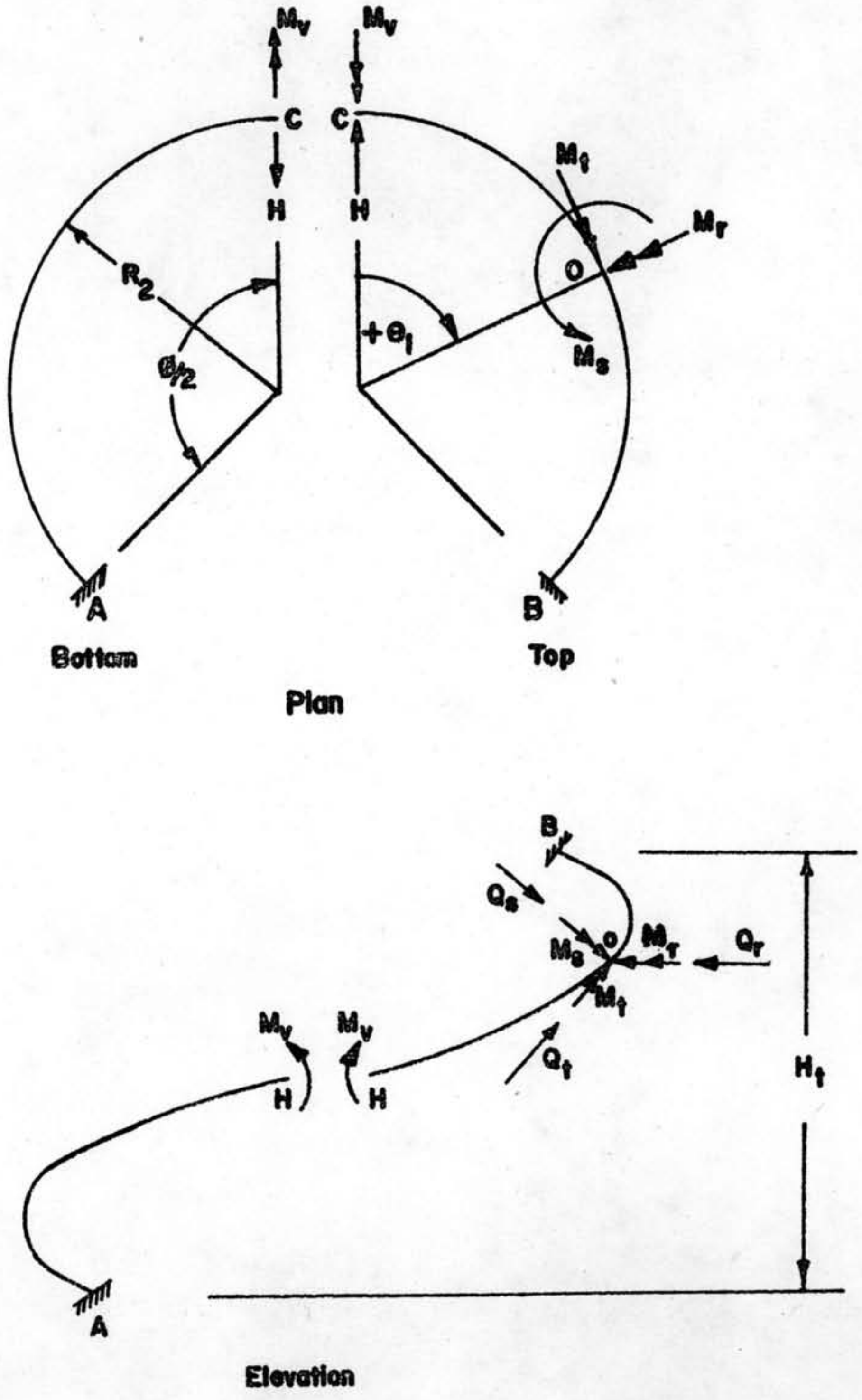


Fig 7 : Positive direction of redundants and stress resultants for upper half of the stairs considering only the effect of bending moment and twisting moment, the strain energy stored for one-half of the stairs is

$$U = \int_0^{\phi/2} \left(\frac{M_r^2}{2EI_r} + \frac{M_s^2}{2EI_s} + \frac{M_t^2}{2GJ} \right) ds$$

where

$$ds = \frac{R_2 d\theta_1}{\cos\alpha}$$

W = uniform loading per horizontal projected length of center line of load

Taking the derivatives with respect to M_v and H, and setting them equal to zero, i.e.

$$\frac{\partial U}{\partial M_v} = \int_0^{\phi/2} \left[\frac{M_r}{EI_r} \frac{\partial M_r}{\partial M_v} + \frac{M_s}{EI_s} \frac{\partial M_s}{\partial M_v} + \frac{M_t}{GJ} \frac{\partial M_t}{\partial M_v} \right] \frac{R_2 d\theta_1}{\cos\alpha} = 0 \qquad (14)$$

$$\frac{\partial U}{\partial H} = \int_0^{\phi/2} \left[\frac{Mr}{EI_r} \frac{\partial Mr}{\partial H} + \frac{Ms}{EI_s} \frac{\partial Ms}{\partial H} + \frac{Mt}{GJ} \frac{\partial Mt}{\partial H} \right] \frac{R_2 d\theta_1}{\cos\alpha} = 0 \qquad (15)$$

From equation (14) and (15), the following simplified equations are obtained

$$M_v \left[\frac{GJ}{EI_r}(m + 0.5\sin 2\theta_1) + sm \right] + H \left[- \frac{GJkR_2\tan\alpha}{EI_r} + skR_2\tan\alpha + R_2\sin\alpha \cos\alpha\, m\left(1 - \frac{GJ}{EI_s}\right) \right] + WR_1 \left[\frac{GJ}{EI_s} R_1(n + 0.5\sin 2\theta_1 - \sin\theta_1) + R_1 ms + nR_2 s \right] = 0 \qquad (16)$$

$$M_v \left[- \frac{GJ}{EI_r} k + sk + \left(s - \frac{GJ}{EI_s}\right)m \right] + H \left[\frac{GJ}{EI_r} \frac{R_2\tan\alpha}{2} \left(\frac{\theta_1^3}{3} - \frac{\theta_1^2}{2} \sin 2\theta_1 - 2k \right) + \frac{sR_2\tan\alpha}{2} \left(\frac{\theta_1^3}{3} + \frac{\theta_1^2}{2}\sin 2\theta_1 + 2k \right) + \left(s - \frac{GJ}{EI_s}\right) 2kR_2\tan\alpha + mR_2\cos^2\alpha \left(\tan\alpha + \frac{GJ}{EI_s}\cot\alpha\right) \right] + WR_1 \left[\frac{GJ}{EI_r} R_1(n - k) + skR_1 + sR_2 (\theta_1^2\sin\theta_1 + 2n) + \left(s - \frac{GJ}{EI_s}\right) mR_1 + \left(s - \frac{GJ}{EI_s}\right) nR_2 \right] = 0 \qquad (17)$$

where $k = \frac{\theta_1 \cos 2\theta_1}{4} - \frac{\sin 2\theta_1}{8}$

$m = \frac{\theta_1}{2} - \frac{\sin 2\theta_1}{4}$

$n = \theta_1 \cos\theta_1 - \sin\theta_1$

$S = \cos^2\alpha + \frac{GJ}{EI_s} \sin^2\alpha$

and $\theta_1 = \phi/2$

Equation (16) and (17) may be rewritten as follow :

$$b_1 M_v + c_1 H + d_1 = 0 \quad (18)$$

$$b_2 M_v + c_2 H + d_2 = 0 \quad (19)$$

In which b_1, c_1, d_1, b_2, c_2, and d_2 represent the corresponding coefficients of similtaneous equations (15) and (16)

Solving equation (18) and (19) simultaneously for M_v and H, these two redundants can be obtained, and the stress resultants at any section can then be determined by substituting the value of M_v and H in equation (8) to (13).

2.7.2 Computer programme

The solution of the internal forces analysed by Morgan's method are calculated by computer. The programme is written in general form and it is applicable directly to any fixed-ended helical stair under uniform loading. In Appendix C to F. are the computer programmes which are written by using Morgan, Bergman, Holmes and Scordelis method respectively. The results are shown in Table 5 to 8 .

Table 4: Values of moments and forces

Angle degrees	Mr (Kg - m)	Ms (Kg - m)	Mt (Kg - m)	Qr (Kg)	Qs (Kg)	Qt (Kg)
0	3378.45	-3257.71	-8573.53	-986.29	9345.78	-3551.23
40	1436.80	-3496.36	-7059.81	-755.54	8082.16	-3749.28
80	58.60	-3779.28	-6664.73	-171.27	6923.92	-3670.03
120	-350.40	-3703.48	-6861.03	493.14	5927.10	-3165.93
160	129.28	-3099.92	-7018.62	926.81	5072.25	-2288.24
200	1095.72	-2104.57	-6678.41	926.81	4273.47	-1262.98
240	2096.71	-1059.68	-5674.57	493.15	3418.62	-385.28
280	2841.34	-307.75	-4091.51	-171.26	2421.81	118.82
320	3253.10	3.92	-2131.69	-755.54	1263.57	198.08
360	3378.44	0.00	0.09	-986.29	0.29	-0.05
400	3253.13	-3.74	2131.71	-755.55	-1263.67	-198.02
440	2841.39	307.93	4091.57	-171.27	-2421.91	-118.78
480	2096.76	1059.87	5674.69	493.14	-3418.72	385.31
520	1095.76	2104.75	6678.51	926.81	-4273.57	1263.01
560	129.29	3100.09	7018.74	926.81	-5072.36	2288.26
600	-350.42	3703.67	6861.16	493.16	-5927.19	3165.96
640	58.54	3779.47	6664.81	-171.25	-6923.99	3670.06
680	1436.75	3496.55	7059.85	-755.54	-8082.26	3749.32
720	3379.41	3257.88	8573.54	-986.29	-9345.87	3551.28

Remark: Angle measured from the lower support

Table 5 : Values of moments and forces by Morgan method

Angle degrees	M_r (kg-m)	M_s (kg-m)	M_t (kg-m)	Q_r kg	Q_s kg	Q_t kg
0	3379.30	0.00	0.00	-987.99	0.00	0.00
20	3348.72	-24.91	1076.32	-928.41	-639.37	-118.60
40	3254.19	-5.20	2132.37	-756.85	-1264.27	-199.09
60	3088.44	96.79	3146.20	-493.99	-1861.97	-207.98
80	2842.84	305.85	4093.12	-171.56	-2423.03	-120.40
100	2511.94	629.84	4946.05	171.56	-2942.40	76.89
120	2098.28	1058.26	5677.28	493.99	-3420.11	383.89
140	1616.68	1563.44	6261.85	756.85	-3861.17	787.37
160	1096.61	2104.47	6681.97	928.41	-4275.02	1262.45
180	582.12	2633.19	6931.93	987.99	-4674.40	1775.64
200	128.79	3101.30	7022.35	928.41	-5073.79	2288.82
220	-202.23	3467.85	6983.13	756.85	-5487.64	2763.90
240	-352.32	3705.69	6863.93	493.99	-5928.70	3167.38
260	-274.79	3806.17	6731.86	171.56	-6406.40	3474.38
280	56.12	3781.19	6666.09	-171.56	-6925.78	3671.67
300	637.83	3662.31	6749.71	-493.99	-7486.83	3759.26
320	1435.29	3496.97	7059.77	-756.85	-8084.54	3750.57
340	2380.90	3342.19	7656.49	-928.41	-8709.44	3669.87
360	3379.30	3256.68	8573.27	-987.99	-9348.81	3551.27

Remark : Angle measured from mid point of center line of step

Table 6 : Values of Moments and Forces by Bergman method

Angle degrees	M_r (kg-m.)	M_t (kg-m.)	Q_s (kgs)
0	3522.62	0.00	0.00
20	3425.33	1218.26	-555.43
40	3145.21	2369.97	-1110.87
60	2716.03	3396.61	-1666.30
80	2189.57	4254.75	-2221.73
100	1629.32	4921.27	-2777.17
120	1102.85	5396.18	-3332.60
140	673.67	5702.58	-3888.03
160	393.55	5883.90	-4443.47
180	296.26	5998.68	-4998.90
200	393.55	6113.47	-5554.33
220	673.67	6294.79	-6109.77
240	1102.85	6601.19	-6665.20
260	1629.31	7076.09	-7220.63
280	2189.56	7742.61	-7776.07
300	2716.03	8600.74	-8331.50
320	3145.20	9627.38	-8886.93
340	3425.33	10779.09	-9442.37
360	3522.62	11997.35	-9997.80

Remark : Angle measured from mid point of center line of step

Table 7 : Values of Moments and Forces by Scordelis method

Angle degrees	M_r (kg-m.)	M_s (kg-m.)	M_t (kg-m.)	Q_r (kgs)	Q_s (kgs)	Q_t (kgs)
0	3414.58	0.00	0.00	-987.88	0.00	0.00
20	3381.85	-20.48	1087.56	-928.31	-639.23	-118.55
40	3281.13	3.13	2153.48	-756.76	-1263.98	-199.01
60	3105.89	108.03	3174.59	-493.94	-1861.52	-207.87
80	2848.64	318.66	4125.32	-171.55	-2422.42	-120.27
100	2505.32	642.69	4978.09	171.55	-2941.63	77.03
120	2079.98	1069.61	5705.19	493.94	-3419.16	384.01
140	1588.82	1571.94	6282.13	756.76	-3860.04	787.45
160	1062.47	2109.11	6692.03	928.31	-4273.72	1262.49
180	545.74	2633.41	6930.36	987.88	-4672.92	1775.62
200	94.46	3097.08	7009.13	928.31	-5072.12	2288.76
220	-230.45	3459.68	6959.61	756.76	-5485.79	2763.70
240	-371.12	3694.55	6832.67	493.94	-5926.67	3167.24
260	-281.97	3793.38	6696.34	171.55	-6404.20	3474.22
280	61.35	3768.27	6630.26	-171.54	-6923.41	3671.52
300	654.78	3650.81	6717.55	-493.94	-7484.31	3759.12
320	1461.85	3488.25	7034.78	-756.76	-8081.85	3750.26
340	2413.83	3337.29	7641.27	-928.31	-8706.60	3669.81
360	3414.57	3256.18	8569.28	-987.88	-9345.83	3551.25

Remark : Angle measur ed from mid point of center line of step

Table 8 : Values of Moments of Forces by Holmes method

Angle degrees	M_r (kg-m.)	M_s (kg-m.)	M_t (kg-m.)	Q_r (kgs)	Q_s (kgs)	Q_t (kgs)
0	3347.29	0.00	0.00	-980.92	0.00	0.00
20	3314.00	-17.02	1088.62	-921.77	-638.50	-116.28
40	3211.71	9.54	2155.24	-751.43	-1262.62	-194.74
60	3034.21	116.47	3176.43	-490.46	-1859.72	-202.10
80	2774.50	327.88	4126.43	-170.34	-2420.44	-113.69
100	2429.04	651.35	4977.72	170.33	-2939.76	83.65
120	2002.42	1076.43	5702.77	490.46	-3417.70	389.90
140	1511.23	1575.90	6277.43	751.43	-3859.25	791.93
160	986.31	2109.61	6685.29	921.77	-4273.78	1265.06
180	472.42	2630.34	6922.26	980.92	-4673.93	1776.01
200	25.11	3090.86	7000.82	921.77	-5074.09	2286.96
220	-295.23	3451.23	6952.47	751.43	-5488.62	2760.09
240	-431.42	3685.12	6828.22	490.46	-5930.17	3162.12
260	-338.61	3784.42	6695.91	170.34	-6408.11	3468.38
280	6.84	3761.21	6634.82	-170.33	-6927.43	3665.71
300	600.37	3646.85	6727.48	-490.46	-7488.14	3754.13
320	1405.24	3488.19	7049.73	-751.43	-8085.25	3746.76
340	2352.80	3341.41	7660.17	-921.77	-8709.37	3668.31
360	3347.28	3264.19	8590.35	-980.92	-9347.87	3552.03

Remark : Angle measured from mid point of center line of step

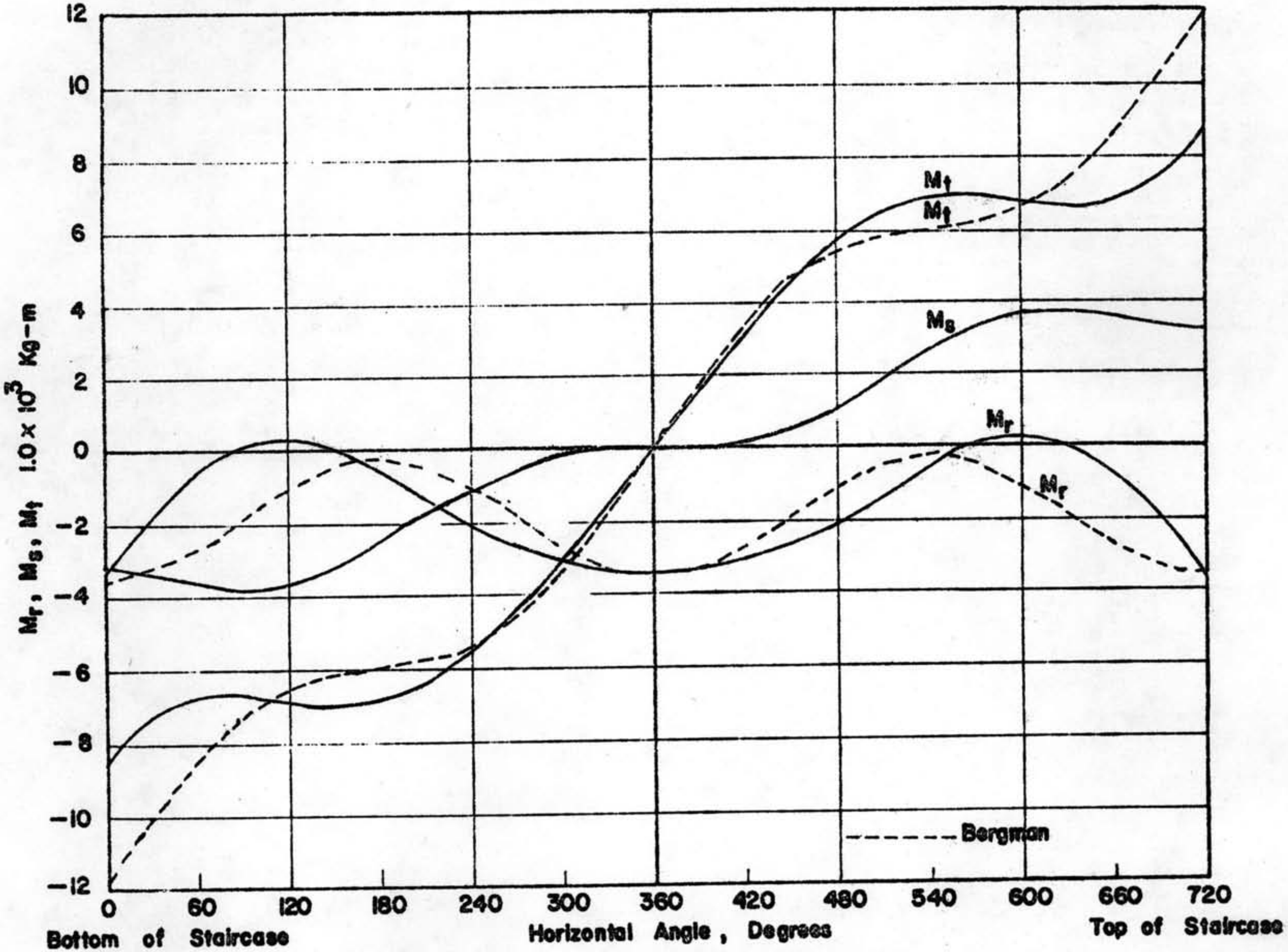


Fig 8 : Curve for Moment and Torsion

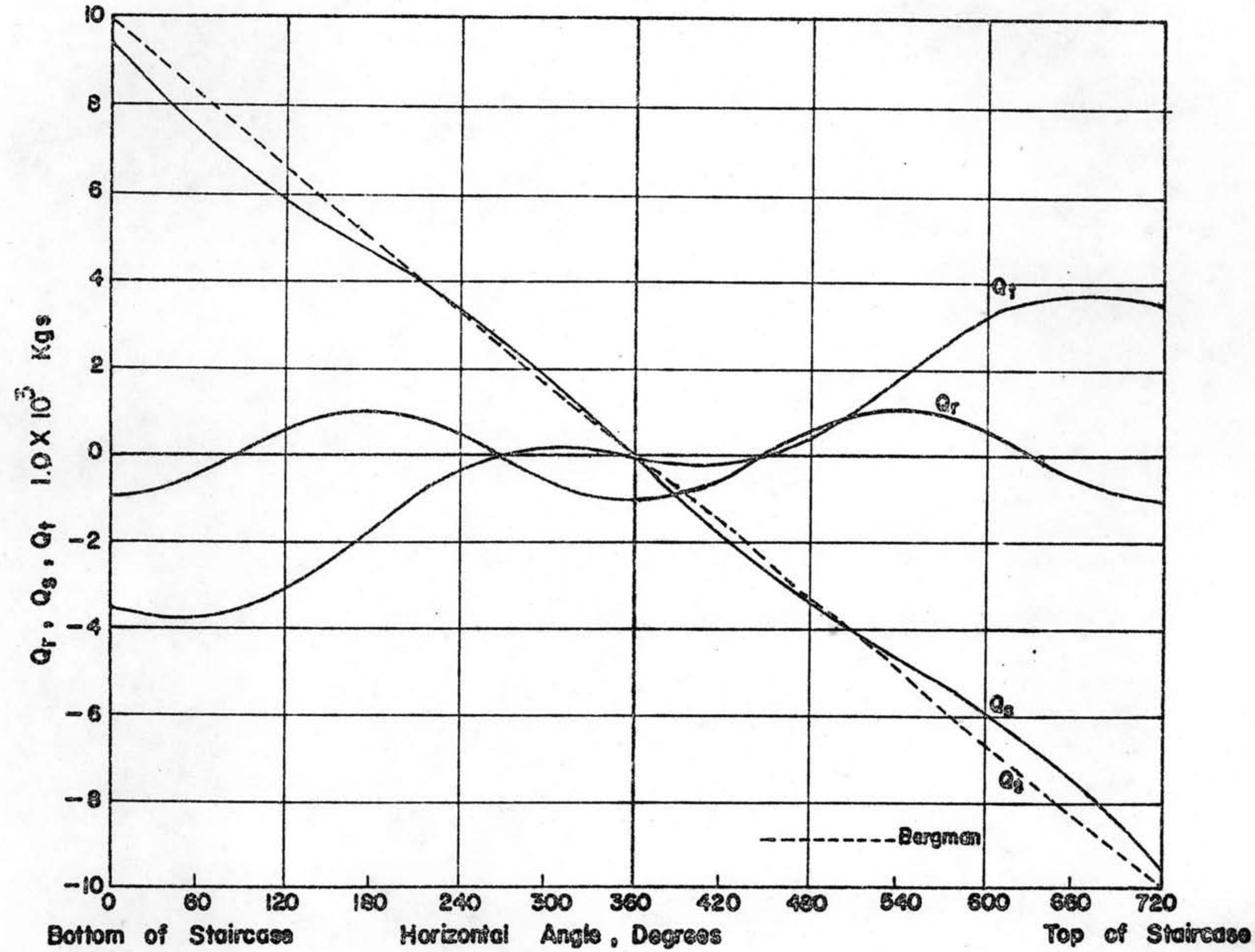


Fig 9 : Curve for Shears and Thrust