CHAPTER I.

INTRODUCTION

1.1 Basic Concepts of Buckling

Thin plate elements used in structural design are often subjected to normal and shearing forces acting in the plane of the plate. If these in-plane forces are sufficiently small, the equilibrium is stable and the resulting deformations are characterized by the absence of lateral displacements. As the magnitude of these in-plane forces increases, at a certain load intensity, a marked change in the character of the deformation pattern takes place. That is, simultaneously with the in-plane deformations, lateral displacements are introduced. In this condition, the originally stable equilibrium becomes unstable and the plate is said to have buckled. The load producing this condition is called the critical load. The importance of the critical load is the initiation of a deflection pattern, which, if the load is further increased, rapidly leads to very large lateral deflections and eventually to complete failure of the plate.

It is important to note that in the classical buckling theory the path leading from a stable to an unstable equilibrium always passes through a neutral state of equilibrium. In the elastic stability problems of plates, a bifurcation of the deformations is assumed as the neutral equilibrium. That is, at the critical load, of the possible two paths of deformations associated with the stable equilibrium and the unstable equilibrium conditions, the plate always takes the buckled form, as shown in Fig. 1.1. In addition to the existence of this bifurcation of equilibrium paths, the elastic stability analysis of plates assumes the validity of Hooke's law.

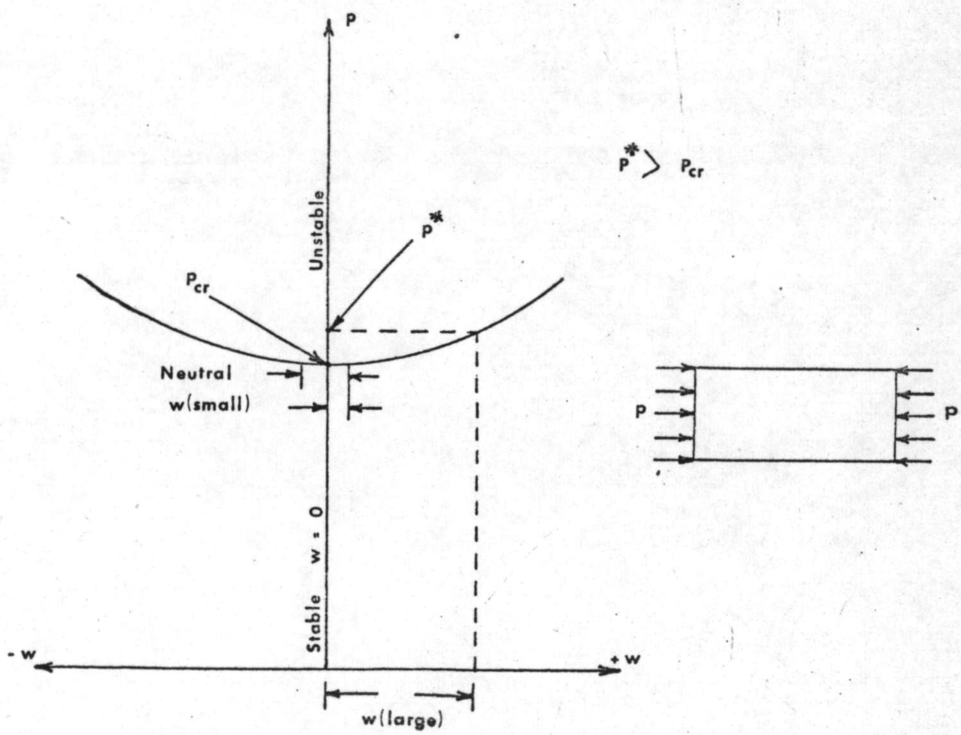

Fig. 1.1 Bifurcation diagram.

Besides this classical buckling theory, the behavior of flat plates after buckling is of considerable practical interest. The postbuckling analysis of plates is usually difficult, since it is basically a nonlinear problem. Slightly curved plates, subjected to simultaneous action of in-plane compressive forces and lateral loads, exhibit a third kind of instability behavior called snap-through buckling, which is characterized by reversal deflections, produced by the nonlinear relationship between the buckling load and the deflections. During continuous loading the plate may begin to deflect in one direction, but at a certain load, it buckles in the reverse direction, assuming again a stable shape.

1.2 Review of Literature

The previous works of buckling of thin circular plates can be roughly classified as isotropic and anisotropic plates, with and without a central hole, and symmetric and asymmetric modes. Among the researchs of thin circular plates, Bryan's[1] was the first one which was studied in 1891. He obtained the minimum buckling load for an isotropic circular plate of a radially symmetric buckling mode without a central hole. Dean[2] and Willers[16] considered the same case of a plate as Bryan, but they subjected the plate to different loading conditions. The plate of Dean's study was subjected to shearing forces distributed along the edges, while the other one was subjected to bending moment caused by initial stresses. The buckling of a circular annular isotropic plate clamped at the outer edge with the inner edge free and subjected to uniform radial compression at the outer edge was first studied by Meissner[6]. The buckling mode was still assumed radially symmetric. After Meissner's, there were also a few studies of the cases with radially symmetric buckling modes. Those were the ones studied by Olsson[8] and Schubert[10]. In 1959, Yamaki[19] showed that a radially symmetric buckling mode did not correspond to the lowest buckling load. The same problem as Meissner's was repeated in 1971 by Majumdar[5], but he allowed the various numbers of waves around the circumference to occur.

The buckling of thin isotropic annular plates was studied by Wiwat[17] as well. Only the case of applying uniform radial compression force along the outer edge was studied. He employed Galerkin's method to find the solutions of the two combinations of boundary conditions; one was the outer and the inner edges fixed, the other was the outer edge fixed and the inner edge simply supported. The other distinction of the study from the forementioned studies was that the axisymmetric buckling mode assumption was relaxed.

The stability of polar orthotropic circular plates without holes was studied by several researchers. Woinowsky-Krieger[18] considered the case of plates subjected to uniform in-plane radial pressure and obtained solutions in the form of Bessel functions for axisymmetric buckling modes. Two years later, Mossakowski[7] repeated the same problem and offered a general solution in hypergeometric series for both symmetric and asymmetric modes. Instead of the hypergeometric series, Pandalai and Patel[9] used the technique of power series expansion to obtain some approximate values for critical buckling loads in the axisymmetric case. Although many techniques were applied to reach the solution of a polar orthotropic circular plate, they were found difficult to employ when the plate had a hole. Lately, the analysis of buckling of orthotropic annular plates, due to in-plane compressive loads, was investigated by Uthgenannt and Brand[13] in 1970. He determined the critical buckling loads for several boundary conditions and two types of loadings, with equal in-plane compressive loads at both inner and outer edges and only in-plane compressive load at the outer edge, by employing finite-difference equations and the Vianello-Stodola iterative procedure. Figures showing relations between the dimensionless critical buckling load parameter and the ratio of radii were given for various moduli of elasticity in the circumferential and radial directions. The assumption of axisymmetry was still kept in the analysis. Vijayakumar and Joga Rao[14] employed the Rayleigh-Ritz method with simple polynomials to analyze axisymmetric buckling of orthotropic annular plates with the ratio of the inner and outer radii of 0.5. Numerical estimates to the critical buckling loads were obtained for all combinations of free, simply supported, and clamped edge conditions for two loading cases; the outer edge alone was subjected to uniform in-plane pressure and both edges were subjected to equal in-plane pressures.

1.3 Statement of the Problem

On the research of Vijayakumar and Joga Rao, the orthotropic plate was assumed to buckle axisymmetrically. As known from Yamaki's, a radially symmetric buckling mode did not always agree with the actual lowest buckling load for the case of an isotropic plate. Thus, the asymmetric buckling mode may be expected for the case of an orthotropic plate as well. This study considers the buckling of orthotropic annular plates with various edge conditions, and several values of the ratios of the inner and outer radii, under the internal and external in-plane compression. The constraint of axisymmetric buckling is relaxed. Various combinations of boundary conditions considered are as follows.

	outer edge	inner edge
case 1 :	clamped	clamped
case 2 :	clamped	simply supported
case 3 :	simply supported	clamped
case 4 :	simply supported	simply supported

The ratios of the inner to outer in-plane compression are 0, 0.5, and 1.0. The rigidity ratio is also varied, since the materials of different ratios of moduli of elasticity in the circumferential and the radial directions are often found in practice.

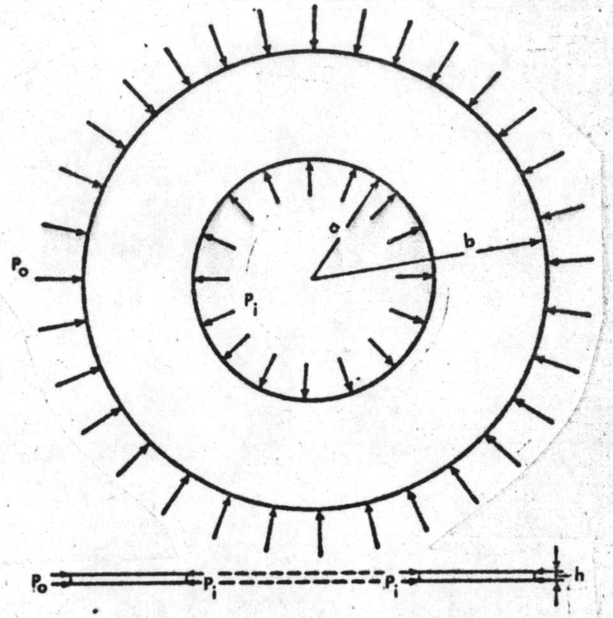

Fig. 1.2 An annular plate with radial compression.

The buckling equation of a polar orthotropic annular plate is derived from the variational method. At this point, Galerkin's method is used to solve the governing differential equation. Then, the dimensionless parameters of the critical buckling loads will be plotted against different ratios of the inner to outer radii.

Assumptions for the present problem are as follows

1). To avoid being a ring, the difference of the hole radius to the plate radius, b-a, is much greater than its thickness, h.

2). The plate is made of homogeneous orthotropic material.

3). The buckling is elastic.

4). Hooke's law is applicable.