CHAPTER III
PROCEDURE

3.1 Grass-root Design for Heat Exchanger Network

The grass-root design of heat exchanger network is designing the new effective heat exchanger network by using the stream data such as inlet and outlet temperature of each stream, stream flowrate and heat transfer coefficient of each stream. The optimal network from the grass-root model is the cheapest-cost network under the constraint functions.

The methodology used for heat exchanger network design consists of

3.1.1 Study and Test The MILP Model

Study the model to understand how it works and analyze the result of the optimal heat exchanger network. Generally, the MILP model is composed of two parts which are

3.1.1.1 Input or Set Parameters

Parameters such as heat transfer zone, number of temperature intervals, heat content in each interval, upper and lower temperature in each interval, flow rate, heat capacity and heat transfer coefficient for hot and cold process streams, logarithmic mean temperature difference, utility cost, price of heat exchanger, any assumptions for example non-isothermal mixing and constraints indicating the allowed or forbidden matching between each pair of hot and cold streams.

3.1.1.2 Set of Equations

This part is used to generate the optimum heat exchanger network that operated at minimum total cost while the transshipment model concept is applied. There are many equations and constraints presented into this zone and all explanations are mentioned in the previous chapter.

Bounds on cumulative heat transfer for cold process streams

$$q_{ijn}^{L} Y_{ijn}^{z,C} \leq \hat{q}_{ijn}^{z,C} \leq \Delta H_{jn}^{z,C} Y_{ijn}^{z,C} \qquad z \in Z; n \in M^{z}; i \in H^{z}; j \in C_{n}^{z}; j \notin CU^{z}; i \in P_{jn}^{C} \tag{2.15}$$

Bounds on cumulative heat transfer for heating utilities

$$q_{ijm}^{L} Y_{ijm}^{z,H} \leq \hat{q}_{ijm}^{z,H} \leq F_{i}^{U} \left(T_{m}^{U} - T_{m}^{L}\right) \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; i \in HU^{z}; j \in C^{z}; j \in P_{im}^{H} \tag{2.16}$$

Bounds on cumulative heat transfer for cooling utilities

$$q_{ijn}^{L} Y_{ijn}^{z,C} \leq \hat{q}_{ijn}^{z,C} \leq F_{j}^{U} \left(T_{n}^{U} - T_{n}^{L}\right) \qquad z \in Z; n \in M^{z}; i \in H^{z}; j \in C_{n}^{z}; j \in CU^{z}; i \in P_{jn}^{C} \tag{2.17}$$

Heat exchanger beginning for hot streams – $(i,j) \notin B$

$$K_{ijm}^{z,H} \geq Y_{ijm}^{z,H} \qquad z \in Z; m \in M^{z}; m = m_{i}^{0}; i \in H^{z}; j \in C^{z}; j \in P_{im}^{H}; (i,j) \notin B \tag{2.18}$$

$$\left.\begin{array}{ll} K_{ijm}^{z,H} \leq 2 - Y_{ijm}^{z,H} - Y_{ijm-1}^{z,H} & (2.19) \\ K_{ijm}^{z,H} \leq Y_{ijm}^{z,H} & (2.20) \\ K_{ijm}^{z,H} \geq Y_{ijm}^{z,H} - Y_{ijm-1}^{z,H} & (2.21) \\ K_{ijm}^{z,H} \geq 0 & (2.22) \end{array}\right\} \quad z \in Z; m \in M^{z}; i \in H_{m}^{z} \cap H_{m-1}^{z}; j \in C^{z}; j \in P_{im}^{H} \cap P_{im-1}^{H}; (i,j) \notin B$$

Heat exchanger ending for hot streams – $(i,j) \notin B$

$$\hat{K}_{ijm}^{z,H} \geq Y_{ijm}^{z,H} \qquad z \in Z; m \in M^{z}; m = m_{i}^{f}; i \in H^{z}; j \in C^{z}; j \in P_{im}^{H}; (i,j) \notin B \tag{2.23}$$

$$\left.\begin{array}{ll} \hat{K}_{ijm}^{z,H} \leq 2 - Y_{ijm}^{z,H} - Y_{ijm+1}^{z,H} & (2.24) \\ \hat{K}_{ijm}^{z,H} \leq Y_{ijm}^{z,H} & (2.25) \\ \hat{K}_{ijm}^{z,H} \geq Y_{ijm}^{z,H} - Y_{ijm+1}^{z,H} & (2.26) \\ \hat{K}_{ijm}^{z,H} \geq 0 & (2.27) \end{array}\right\} \quad z \in Z; m \in M^{z}; i \in H_{m}^{z} \cap H_{m+1}^{z}; j \in C^{z}; j \in P_{im}^{H} \cap P_{im+1}^{H}; (i,j) \notin B$$

Heat exchanger existence on hot streams - $(i,j) \in B$

$$Y_{ijm}^{z,H} = \sum_{\substack{l \in M_{i}^{z} \\ l \leq m \\ j \in P_{il}^{H}}} K_{ijl}^{z,H} - \sum_{\substack{l \in M_{i}^{z} \\ l \leq m-1 \\ j \in P_{il}^{H}}} \hat{K}_{ijl}^{z,H} \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; j \in C^{z}; j \in P_{im}^{H}; (i,j) \in B \tag{2.28}$$

The example shown in Figure 2.8 for a match $(i,j) \notin B$, only one heat exchanger is allowed, will explain how the previous sets of constraints work. The hot side of heat exchanger spans from interval 3 to 8 of stream i, heat transferred to cold stream j is not shown. Since, only one heat exchanger is permitted for this match, variables $Y_{ijm}^{z,H}$ are defined as binary while $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are continuous. The values for all variables are given in Table 2.1. These numbers correspond to the set of constraints in (2.18)-(2.22) and (2.23)-(2.27).

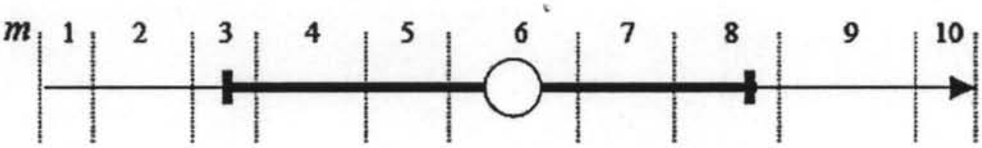


Figure 2.8 Heat exchanger definition when $(i,j) \notin B$.

Table 2.2 Values of $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ variables when $(i,j) \notin B$

m	$Y_{ijm}^{z,H}$	$K_{ijm}^{z,H}$	$\hat{K}_{ijm}^{z,H}$
1	0	0	0
2	0	0	0
3	1	1	0
4	1	0	0
5	1	0	0
6	1	0	0
7	1	0	0
8	1	0	1
9	0	0	0
10	0	0	0

Following Figure 2.8, whenever $Y_{ijm}^{z,H} = 0$ then it follows that $K_{ijm}^{z,H} = 0$ and $\hat{K}_{ijm}^{z,H} = 0$, explain in constraint (2.17) and (2.22). At any interval where $Y_{ijm-1}^{z,H} =$ 1, constraint (2.21) becomes trivial and thus $K_{ijm}^{z,H}$ is getting to be zero because when $Y_{ijm}^{z,H}$ =1, constraint (2.19) gives $K_{ijm}^{z,H}$ to zero.

The possibility of allowing two heat exchangers between the same pair of streams is considered. In Figure 2.9, there are two heat exchangers between the shown hot stream and a certain cold stream, $(i,j) \in B$. Both exchangers are placed in series for the hot stream without any other unit in between. Then, the constraint (2.28) is used for defining heat exchangers existence. Additionally, variables $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are declared as binary while $Y_{ijm}^{z,H}$ are stated as continuous which the values of these variables are shown in Table 2.3.

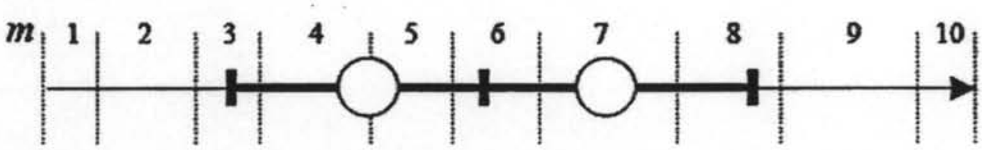


Figure 2.9 Heat exchanger definition when $(i,j) \in B$.

Table 2.3 Values of $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ variables when $(i,j) \in B$

m	Y_{ijm}	K_{ijm}	$\hat{K}_{ijm}$
1	0	0	0
2	0	0	0
3	1	1	0
4	1	0	0
5	1	0	0
6	2	1	1
7	1	0	0
8	1	0	1
9	0	0	0
10	0	0	0

Whenever a heat exchanger begins or ends, the binary variables $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are set to one. Then constraint (2.28) leads the values of $Y_{ijm}^{z,H}$ equal to one for all intervals m between the beginning and end of a heat exchanger. Note that, when a heat exchanger between the same pair of stream ends and another one begins in the same interval (interval 6 for this example) then $Y_{ijm}^{z,H}$ is equal to two. Since $Y_{ijm}^{z,H}$ =2 is not feasible if the Y are declared as binary variables and constraints (2.21) and (2.22) are used, this is why a different set of equations and variable declarations is required when $(i,j)\in B$, at a cost of increasing the number of binary variables.

A similar set of equations is used to define the location of a heat exchanger for cold streams. These expressions are presented next without further explanation.

Heat exchanger beginning for cold streams - $(i,j)\notin B$

$$K_{ijn}^{z,C} \geq Y_{ijn}^{z,C} \qquad z\in Z; n\in M^z; n=n_j^0; i\in H^z; j\in C_n^z; i\in P_{jn}^C; (i,j)\notin B \tag{2.29}$$

$$\left.\begin{array}{ll} K_{ijn}^{z,C} \leq 2-Y_{ijn}^{z,C}-Y_{ijn-1}^{z,C} & (2.30) \\ K_{ijn}^{z,C} \leq Y_{ijn}^{z,C} & (2.31) \\ K_{ijn}^{z,C} \geq Y_{ijn}^{z,C}-Y_{ijn-1}^{z,C} & (2.32) \\ K_{ijn}^{z,C} \geq 0 & (2.33) \end{array}\right\} z\in Z; n\in M^z; i\in H^z; j\in C_n^z \cap C_{n-1}^z; i\in P_{jn}^C \cap P_{jn-1}^C; (i,j)\notin B$$

Heat exchanger ending for cold streams - $(i,j)\notin B$

$$\hat{K}_{ijn}^{z,C} \geq Y_{ijn}^{z,C} \qquad z\in Z; n\in M^z; n=n_j^0; i\in H^z; j\in C_n^z; i\in P_{jn}^C; (i,j)\notin B \tag{2.34}$$

$$\left.\begin{array}{ll} \hat{K}_{ijn}^{z,C} \leq 2-Y_{ijn}^{z,C}-Y_{ijn-1}^{z,C} & (2.35) \\ \hat{K}_{ijn}^{z,C} \leq Y_{ijn}^{z,C} & (2.36) \\ \hat{K}_{ijn}^{z,C} \geq Y_{ijn}^{z,C}-Y_{ijn-1}^{z,C} & (2.37) \\ \hat{K}_{ijn}^{z,C} \geq 0 & (2.38) \end{array}\right\} z\in Z; n\in M^z; i\in H^z; j\in C_n^z \cap C_{n-1}^z; i\in P_{jn}^C \cap P_{jn-1}^C; (i,j)\notin B$$

Heat exchanger existence on cold streams - $(i,j)\in B$

$$Y_{ijn}^{z,C} = \sum_{\substack{l\in N_j^z \\ l\leq n \\ i\in P_{jl}^C}} K_{ijl}^{z,C} - \sum_{\substack{l\in N_j^z \\ l\leq n-1 \\ i\in P_{jl}^C}} \hat{K}_{ijl}^{z,C} \qquad z\in Z; n\in M^z; i\in H^z; j\in C_n^z; i\in P_{jn}^C; (i,j)\in B \tag{2.39}$$

Lastly, by counting the number of beginnings or endings of heat exchanger, the number of heat exchanger units between a given pair of streams, E_{ij}^z, can be figured out. The beginnings number is calculated by equation (2.37) to (2.38) and equation (2.39) to (2.40) is used to generate the endings number. For the last equation, (2.42), the number of shell, U_{ij}^{z}, need to be greater or equal to the number of heat exchanger units, E_{ij}^{z}. Because a single heat exchanger does not mean only one shell, the shell number should be need to satisfy the required area for each match.

Number of heat exchangers between hot stream *i* and cold stream *j* – (*i,j*) $\notin B$

$$E_{ij}^{z} = \sum_{m \in M_i^z;\, j \in P_{im}^H} K_{ijm}^{z,H} \tag{2.40}$$

$$E_{ij}^{z} = \sum_{n \in N_j^z;\, i \in P_{jn}^C} K_{ijn}^{z,C} \tag{2.41}$$

$$E_{ij}^{z} = \sum_{m \in M_i^z;\, j \in P_{im}^H} \hat{K}_{ijm}^{z,H} \tag{2.42}$$

$$E_{ij}^{z} = \sum_{n \in N_j^z;\, i \in P_{jn}^C} \hat{K}_{ijn}^{z,C} \tag{2.43}$$

$$z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P \quad \text{(for 2.40–2.43)}$$

$$E_{ij}^{z} \le 1 \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B \tag{2.44}$$

$$E_{ij}^{z} \le E_{ij}^{z,\max} \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \in B \tag{2.45}$$

However, each shell number will be counted as a separate heat exchanger whenever the condition of more than one exchanger is presented. The constraints for this situation are shown below.

Number of heat exchangers between hot stream *i* and cold stream *j* - (*i,j*) $\in B$

$$U_{ij}^{z} = \sum_{m \in M_i^z;\, j \in P_{im}^H} K_{ijm}^{z,H} \tag{2.46}$$

$$U_{ij}^{z} = \sum_{m \in M_i^z;\, j \in P_{im}^H} \hat{K}_{ijm}^{z,H} \tag{2.47}$$

$$U_{ij}^{z} = \sum_{n \in N_j^z;\, i \in P_{jn}^C} K_{ijn}^{z,C} \tag{2.48}$$

$$U_{ij}^{z} = \sum_{n \in N_j^z;\, i \in P_{jn}^C} \hat{K}_{ijn}^{z,C} \tag{2.49}$$

2.5.4 Heat Transfer Consistency

To explain the heat load of each exchanger unit for multiple heat exchange, heat transfer consistency constraints are necessary to be addressed. When heat exchanges from hot stream to cold stream with two exchangers exist in series, for example in Figure 2.10, the cumulative heat of hot stream in interval 6, $\hat{q}_{ij6}^{z,H}$ is transfer to the cold stream in interval 5 and the heat left of hot stream, $\tilde{q}_{ij6}^{z,H}$, is sent into interval 8 of cold stream. The amount of heat that is transferred to the next heat exchanger in series, $\tilde{q}_{ijm}^{z,H}$, is used to calculate the heat load and area calculations in each heat exchanger. Table 2.4 expressed the values of the variables involved in heat load calculation which are the heat exchanger existence, beginning and ending of each heat exchanger unit and the value of $\tilde{q}$. Another variable need to initiate is called $X_{im,jn}^{z}$ which used to find out the ending interval for each heat exchanger connected in sequence for match (*i,j*). So, the value of $X_{im,jn}^{z}$ will be zero whenever m and n are cold-end intervals and be higher than zero in all other situations.

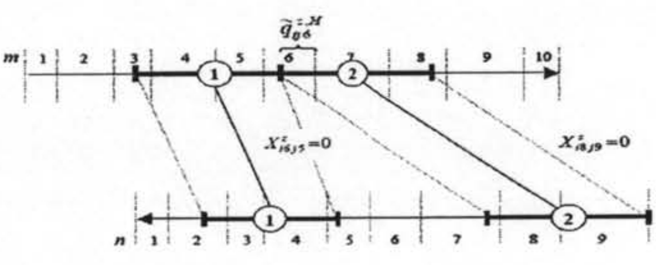


Figure 2.10 Heat transfer consistency example when $(i,j) \in B$.

Table 2.4 Values of variables $K_{ijm}^{z,H}$, $\hat{K}_{ijm}^{z,H}$, $Y_{ijm}^{z,H}$ and $\tilde{q}_{ijm}^{z,H}$ when $(i,j) \in B$

Load Unit 1 $= \hat{q}_{ij3}^{z,H} + \hat{q}_{ij4}^{z,H} + \hat{q}_{ij5}^{z,H} + \hat{q}_{ij6}^{z,H} - \tilde{q}_{ij6}^{z,H} = \hat{q}_{ij2}^{z,C} + \hat{q}_{ij3}^{z,C} + \hat{q}_{ij4}^{z,C} + \hat{q}_{ij5}^{z,C}$

M	Y_m	K_m	$\hat{K}_m$	$\tilde{q}_m$
1	0	0	0	0
2	0	0	0	0
3	1	1	0	0
4	1	0	0	0
5	1	0	0	0
6	2	1	1	≥0
7	1	0	0	0
8	1	0	1	0
9	0	0	0	0
10	0	0	0	0

Load Units 1+2 $= \hat{q}_{ij3}^{z,H} + \hat{q}_{ij4}^{z,H} + \hat{q}_{ij5}^{z,H} + \hat{q}_{ij6}^{z,H} + \hat{q}_{ij7}^{z,H} + \hat{q}_{ij8}^{z,H} = \hat{q}_{ij2}^{z,C} + \hat{q}_{ij3}^{z,C} + \hat{q}_{ij4}^{z,C} + \hat{q}_{ij5}^{z,C} + \hat{q}_{ij7}^{z,C} + \hat{q}_{ij8}^{z,C} + \hat{q}_{ij9}^{z,C}$

n	Y_n	K_n	$\hat{K}_n$	$\tilde{q}_m$
1	0	0	0	0
2	1	1	0	0
3	1	0	0	0
4	1	0	0	0
5	1	0	1	0
6	0	0	0	0
7	1	1	0	0
8	1	0	0	0
9	1	0	1	0

The heat transfer consistency constraints for multiple heat exchangers are expressed here. Heat transfer consistency for multiple heat exchangers between the same pair of streams.

$$\sum_{\substack{l\in M_i^z\\ l\le m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \le \sum_{\substack{l\in N_j^z\\ l\le n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} + 4X_{im,jn}^{z}\, Max\{ \sum_{\substack{l\in M_i^z\\ l\le m\\ j\in P_{il}^H}} \Delta H_{il}^{z,H} ; \sum_{\substack{l\in M_i^z\\ l\le n\\ i\in P_{jl}^C}} \Delta H_{jl}^{z,C} \} \tag{2.50}$$

$$\sum_{\substack{l\in M_i^z\\ l\le m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \ge \sum_{\substack{l\in N_j^z\\ l\le n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} - 4X_{im,jn}^{z}\, Max\{ \sum_{\substack{l\in M_i^z\\ l\le m\\ j\in P_{il}^H}} \Delta H_{il}^{z,H} ; \sum_{\substack{l\in M_i^z\\ l\le n\\ i\in P_{jl}^C}} \Delta H_{jl}^{z,C} \} \tag{2.51}$$

$$X_{im,jn}^{z} = 2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C} + \frac{1}{4}\sum_{\substack{l\in N_j^z\\ l\le n}} \hat{K}_{ijl}^{z,C} - \frac{1}{4}\sum_{\substack{l\in M_i^z\\ l\le m}} \hat{K}_{ijl}^{z,H} \tag{2.52}$$

$$z\in Z\, ;\, m,n\in M^z ;\; T_n^L \le T_m^U ;\, (i,j)\in B ;\; i\in H_m^z ;\, j\in C_n^z ;\; i\in P_{jn}^C ;\, j\in P_{im}^H$$

$$\sum_{\substack{l\in M_i^z\\ l\le m\\ j\in P_{il}^H}} \hat{K}_{ijl}^{z,H} - \sum_{\substack{l\in N_j^z\\ l\le n\\ i\in P_{jl}^C}} \hat{K}_{ijl}^{z,C} \ge 0 \qquad z\in Z; m,n\in M^z; T_n^L < T_m^U ; T_n^L \ge T_m^L \; (i,j)\in B;\; i\in H_m^z ; j\in C_n^z ; i\in P_{jn}^C ; j\in P_{im}^H \tag{2.53}$$

$$\sum_{\substack{l\in M_i^z\\ l\le m\\ j\in P_{il}^H}} \left(K_{ijl}^{z,H} - \hat{K}_{ijl}^{z,H}\right) \le 1 \tag{2.54}$$

$$\sum_{\substack{l\in N_j^z\\ l\le n\\ i\in P_{jl}^C}} \left(K_{ijl}^{z,C} - \hat{K}_{ijl}^{z,C}\right) \le 1 \tag{2.55}$$

$$z\in Z ; m\in M^z ; (i,j)\in B;\; i\in H_m^z ; j\in P_{im}^H$$

$$\tilde{q}_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} \tag{2.56}$$

$$\tilde{q}_{ijm}^{z,H} \le K_{ijm}^{z,H} \Delta H_{im}^{z,H} \tag{2.57}$$

$$\tilde{q}_{ijm}^{z,H} \le \hat{K}_{ijm}^{z,H} \Delta H_{im}^{z,H} \tag{2.58}$$

$$\tilde{q}_{ijm}^{z,H} \ge 0 \tag{2.59}$$

$$z\in Z ; m\in M^z ; (i,j)\in B;\; i\in H_m^z ; j\in P_{im}^H$$

$$\tilde{q}_{ijn}^{z,C} \le \hat{q}_{ijn}^{z,C} \tag{2.60}$$

$$\tilde{q}_{ijn}^{z,C} \le K_{ijn}^{z,C} \Delta H_{in}^{z,C} \tag{2.61}$$

$$\tilde{q}_{ijn}^{z,C} \le \hat{K}_{ijn}^{z,C} \Delta H_{in}^{z,C} \tag{2.62}$$

$$\tilde{q}_{ijn}^{z,C} \ge 0 \tag{2.63}$$

$$z\in Z ; n\in M^z ; (i,j)\in B;\; j\in C_n^z ; i\in P_{jn}^C$$

Main constraints for the heat transfer consistency are the equation (2.50) to (2.52). All these constraints show that whatever calculated from hot or cold stream, the heat load of heat exchanger also be the same. In addition, in case where there is the cold-end interval, $X^{z}_{im,jn}$=0, the equation (2.50) and (2.51) become an equality as

$$\sum_{\substack{l\in M_i^z \\ l\leq m}} \hat{q}^{z,H}_{ijl} - \tilde{q}^{z,H}_{ijn} = \sum_{\substack{l\in N_j^z \\ l\leq n}} \hat{q}^{z,C}_{ijl} - \tilde{q}^{z,C}_{ijm}$$

For example in Figure 2.10, at interval 6 of hot stream and interval 5 for cold stream, the constraint (2.50) and (2.51) will be summary to

$$\hat{q}^{z,H}_{ij3} + \hat{q}^{z,H}_{ij4} + \hat{q}^{z,H}_{ij5} + \hat{q}^{z,H}_{ij6} - \tilde{q}^{z,H}_{ij6} = \hat{q}^{z,C}_{ij2} + \hat{q}^{z,C}_{ij3} + \hat{q}^{z,C}_{ij4} + \hat{q}^{z,C}_{ij5} - \tilde{q}^{z,C}_{ij5}$$

And the heat exchanger does not start at interval 5 of cold stream, so the value of $\tilde{q}^{z,C}_{ij5}$ is zero. This lead the equation become

$$\hat{q}^{z,H}_{ij3} + \hat{q}^{z,H}_{ij4} + \hat{q}^{z,H}_{ij5} + \hat{q}^{z,H}_{ij6} - \tilde{q}^{z,H}_{ij6} = \hat{q}^{z,C}_{ij2} + \hat{q}^{z,C}_{ij3} + \hat{q}^{z,C}_{ij4} + \hat{q}^{z,C}_{ij5}$$

The next constraint, (2.53), is produced to make sure that there is feasible temperature difference between hot and cold stream at the cold-end, that is the hot stream temperature is forced to be higher than the cold stream temperature at the cold-end of the heat exchanger. Figure 2.10 will show more clear in description. Following constraints, (2.54)-(2.55), are used to describe that a new exchanger can only start, in the same interval with the first one sequentially, when the previous exchanger has ended. Last sets of constraint, (2.59) to (2.63), are used to specify the value of variable $\tilde{q}$. This variable is created to be zero for all intervals except the connection interval between two exchangers which continuous constructed in series, first heat exchanger ends and the second exchanger starts in the same interval.

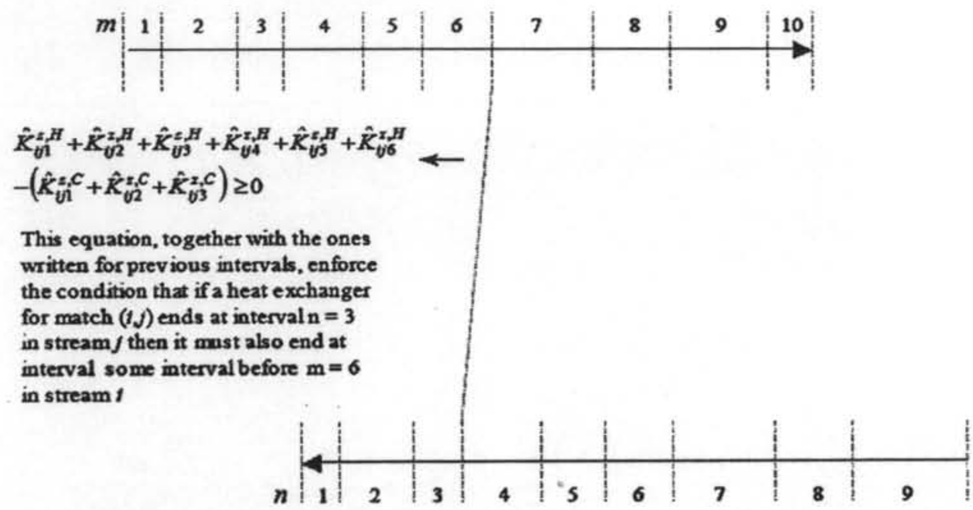


Figure 2.11 Integer cut for heat exchanger end when $(i,j) \in B$.

2.5.5 Flow Rate Consistency Within Heat Exchangers

The assumption that constant flow rate passed through heat exchanger is applied to the MILP model. The next equation group expresses the consistency of flow rate within a heat exchanger. In Figure 2.11 depicts an example of heat exchanger which exchange heat during the interval 3 to interval 8 of hot stream i with the cold stream j. Next, new word need to be introduced, they are called "extreme intervals" which are the intervals 3 and 8 for this example while "exchanger-internal intervals" are referred to the retired intervals which are the interval 4 to 7.

Let explain more details for this example where allow only one exchanger for match, $(i,j) \in B$. For the exchanger-internal intervals, interval 4 to 7, the flow rate can be consistently established as the ratio of the cumulative heat transfer, the heat capacity and the interval temperature difference. In contrast, this equation can not be used for the extreme intervals because the real temperature difference between upper and lower bound of interval are not the same as normal range, it is smaller. Consequently, flow rate for the interval 3 and 8 can be solved by the inequality constraints as mention in Figure2.12.

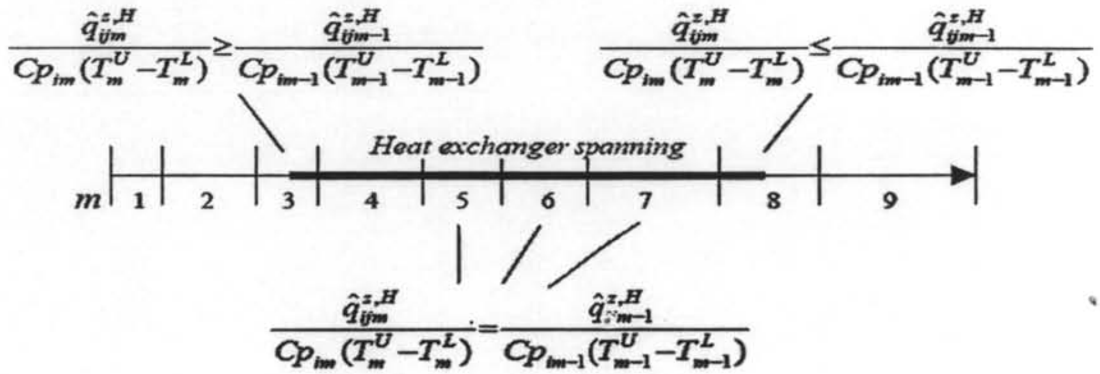


Figure 2.12 Flow rate consistency equations.

The equations used for classify which interval is exchanger-internals or extreme intervals are introduced couple with the variable α. Actually, it is defined as continuous but the following constraints enforces it to be one when the interval is exchanger-internal and zero for all others.

Definition of exchanger-internal intervals for hot streams

$$\alpha^{z,H}_{ijm} \leq 1 - K^{z,H}_{ijm} - K^{z,H}_{ijm-1} \tag{2.64}$$

$$\alpha^{z,H}_{ijm} \leq 1 - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,H}_{ijm-1} \qquad z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H; j \in C^z \tag{2.65}$$

$$\alpha^{z,H}_{ijm} \geq Y^{z,H}_{ijm} - K^{z,H}_{ijm} - K^{z,H}_{ijm-1} - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,H}_{ijm-1} \tag{2.66}$$

$$\alpha^{z,H}_{ijm} \geq 0 \tag{2.67}$$

At exchanger-internal interval, there is no exchanger begins or ends, so $K^{z,H}_{ijm}$, $K^{z,H}_{ijm-1}$, $\hat{K}^{z,H}_{ijm}$, $\hat{K}^{z,H}_{ijm-1}$ are all zero and $Y^{z,H}_{ijm}=1$. The constraint (2.66) gives the value of $\alpha^{z,H}_{ijm}$ to be one. On the other hand, for the extreme intervals, at least one of $K^{z,H}_{ijm}$, $K^{z,H}_{ijm-1}$, $\hat{K}^{z,H}_{ijm}$, $\hat{K}^{z,H}_{ijm-1}$ will be equal to one or $Y^{z,H}_{ijm}=0$. So, $\alpha^{z,H}_{ijm}$ will become to zero.

However, there is another condition, which effect to these constraint equations. When splitting stream flow rate is allowed, the flow rate consistency equation will be

Flow rate consistency for hot streams in exchanger-internal intervals-$i \in S^H$, $(i,j) \notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \le \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + \left(1-\alpha_{ijm}^{z,H}\right)\cdot F_i \\
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \ge \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1-\alpha_{ijm}^{z,H}\right)\cdot F_i
\end{aligned}\right\} \begin{aligned} &z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z \\ &i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H \end{aligned} \qquad \begin{aligned}&(2.68)\\&(2.69)\end{aligned}$$

Flow rate consistency for hot streams in extreme intervals - $i \in S^H$, $(i,j) \notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \ge \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1+\hat{K}_{ijm-1}^{z,H} + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H}\right)\cdot F_i \\
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \le \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + \left(1+K_{ijm-1}^{z,H} + K_{ijm}^{z,H} - \hat{K}_{ijm}^{z,H}\right)\cdot F_i
\end{aligned}\right\} \begin{aligned} &z \in Z; m \in M^z \\ &i \in H_m^z \cap H_{m-1}^z \\ &j \in P_{im}^H \cap P_{im-1}^H \\ &i \in S^H; j \in C^z; (i,j) \notin B \end{aligned} \qquad \begin{aligned}&(2.70)\\&(2.71)\end{aligned}$$

For the exchanger-internal interval, α=1, that is the last term in the right hand side of both constraints, (2.68) and (2.69), are canceled out and the constraints perform as equality. In contrast, constraint (2.70) and (2.71) are defined for the extreme intervals. Constraint (2.70) is referred to the beginning of heat exchanger and the end of exchanger is expressed in constraint (2.71). Consider (2.70), at the end of exchanger, the last term in the right hand side is deleted. The last term in (2.71) can also be erased whenever there is a starting of exchanger.

However, the effect of stream splitting also needs to be concerned. The possibility of appearing two different heat exchangers in the same interval is used to construct the constraints for stream splitting.

Flow rate consistency for hot streams in extreme intervals - $i \in S^{H}$, $(i,j) \in B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1 + \hat{K}_{ijm-1}^{z,H} + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H}\right) \cdot F_i && (2.72) \\
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\tilde{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(2 + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H} - Y_{ijm-1}^{z,H}\right) \cdot F_i && (2.73) \\
&\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + \left(2 + K_{ijm-1}^{z,H} - \hat{K}_{ijm}^{z,H} - Y_{ijm}^{z,H}\right) \cdot F_i && (2.74)
\end{aligned}\right\}
\begin{array}{l}
z \in Z;\, m \in M^z \\
i \in H_m^z \cap H_{m-1}^z \\
j \in P_{im}^H \cap P_{im-1}^H \\
i \in S^H;\, j \in C^z;\, (i,j) \in B
\end{array}$$

When a heat exchanger starts at interval m-1, the constraint (2.72) is applied while the constraint (2.73) is used to identify when another heat exchanger between the same pair of hot and cold stream that ends at the interval m-1. Constraint (2.74) expresses at the end of a heat exchanger which the possibility of having two heat exchangers that start at the same interval is concerned. All constraints, (2.70) to (2.74), can be simplified for the case that stream split is not allowed because the flow rate for exchanger-internal intervals is equal to the actual flow rate.

Flow rate consistency for hot streams - $i \notin S^{H}$

$$\hat{q}_{ijm}^{z,H} \geq \left(Y_{ijm-1}^{z,H} + Y_{ijm}^{z,H} - Y_{ijm+1}^{z,H} - 2\right) \cdot \Delta H_{im}^{z,H} \qquad \begin{array}{l} z \in Z;\, m \in M^z;\, i \in H_{m-1}^z \cap H_m^z \cap H_{m+1}^z;\, i \notin S^H \\ (i,j) \notin B;\, j \in C^z;\, j \in P_{im-1}^H \cap P_{im}^H \cap P_{im+1}^H \end{array} \qquad (2.75)$$

$$\left.\begin{aligned}
&\hat{q}_{ijm}^{z,H} \geq \left(Y_{ijm}^{z,H} - K_{ijm}^{z,H} - \hat{K}_{ijm}^{z,H}\right) \cdot \Delta H_{im}^{z,H} && (2.76) \\
&\hat{q}_{ijm}^{z,H} \geq \left(Y_{ijm}^{z,H} + K_{ijm}^{z,H} + \hat{K}_{ijm}^{z,H} - 2\right) \cdot \Delta H_{im}^{z,H} && (2.77)
\end{aligned}\right\}
\begin{array}{l} z \in Z;\, m \in M^z;\, i \in H_{m-1}^z \cap H_m^z \cap H_{m+1}^z;\, i \notin S^H \\ j \in C^z;\, j \in P_{im-1}^H \cap P_{im}^H \cap P_{im+1}^H \end{array}$$

In case that only one exchanger is permitted, expressed in constraint (2.75), the heat flow is equivalent to the amount of enthalpy change for any internal interval. However, for the multiple exchangers, the variables Y is probably higher than one. Therefore, two following constraint, (2.76) and (2.77), are set to satisfy the concept of equivalent between heat flow and enthalpy change.

Consequently, flow rate consistency constraints for cold streams are shown below.

Definition of exchanger-internal intervals for cold streams $\quad j\in S^{C}$

$$\left.\begin{aligned}
&\alpha_{ijn}^{z,C}\le 1-K_{ijn}^{z,C}-K_{ijn-1}^{z,C} && (2.78)\\
&\alpha_{ijn}^{z,C}\le 1-\hat{K}_{ijn}^{z,C}-\hat{K}_{ijn-1}^{z,C} && (2.79)\\
&\alpha_{ijn}^{z,C}\ge Y_{ijn}^{z,C}-K_{ijn}^{z,C}-K_{ijn-1}^{z,C}-\hat{K}_{ijn}^{z,C}-\hat{K}_{ijn-1}^{z,C} && (2.80)\\
&\alpha_{ijn}^{z,C}\ge 0 && (2.81)
\end{aligned}\right\}\; z\in Z;n\in M^{z};j\in C_{n}^{z}\cap C_{n-1}^{z};j\in S^{C};i\in H^{z};i\in P_{jn}^{C}\cap P_{jn-1}^{C}$$

Flow rate consistency for cold streams in exchanger-internal intervals-$j\in S^{C}$,$(i,j)\notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}\le\frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^{U}-T_{n-1}^{L})}+\left(1-\alpha_{ijn}^{z,C}\right)\cdot F_{j} && (2.82)\\
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}\ge\frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^{U}-T_{n-1}^{L})}-\left(1-\alpha_{ijn}^{z,C}\right)\cdot F_{j} && (2.83)
\end{aligned}\right\}\; \begin{gathered} z\in Z;n\in M^{z};\ j\in S^{C}\\ j\in C_{n}^{z}\cap C_{n-1}^{z};i\in H^{z};\ i\in P_{jn}^{C}\cap P_{jn-1}^{C}\end{gathered}$$

Flow rate consistency for cold streams in extreme intervals - $j\in S^{C}$,$(i,j)\notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}\ge\frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}-\left(1+\hat{K}_{ijn-1}^{z,C}+\hat{K}_{ijn}^{z,C}-K_{ijn-1}^{z,C}\right)\cdot F_{i} && (2.84)\\
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}\le\frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{in-1}(T_{n-1}^{U}-T_{n-1}^{L})}+\left(1+K_{ijn-1}^{z,C}+K_{ijn}^{z,C}-\hat{K}_{ijn}^{z,C}\right)\cdot F_{i} && (2.85)
\end{aligned}\right\}\; \begin{gathered} z\in Z;n\in M^{z};(i,j)\notin B\\ j\in S^{C};j\in C_{n}^{z}\cap C_{n-1}^{z}\\ i\in H^{z};\ i\in P_{jn}^{C}\cap P_{jn-1}^{C}\end{gathered}$$

Flow rate consistency for cold streams in extreme intervals - $j\in S^{C}$,$(i,j)\in B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}\ge\frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^{U}-T_{n-1}^{L})}-\left(1+\hat{K}_{ijn-1}^{z,C}+\hat{K}_{ijn}^{z,C}-K_{ijn-1}^{z,C}\right)\cdot F_{j} && (2.86)\\
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}\ge\frac{\tilde{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^{U}-T_{n-1}^{L})}-\left(2+\hat{K}_{ijn}^{z,C}-K_{ijn-1}^{z,C}-Y_{ijn-1}^{z,C}\right)\cdot F_{j} && (2.87)
\end{aligned}\right\}\; \begin{gathered} z\in Z;n\in M^{z};(i,j)\in B\\ j\in S^{C};j\in C_{n}^{z}\cap C_{n-1}^{z}\\ i\in H^{z};\ i\in P_{jn}^{C}\cap P_{jn-1}^{C}\end{gathered}$$

$$\frac{\hat{q}_{ijn}^{z,C}-\tilde{q}_{ijn}^{z,C}}{Cp_{jn}(T_{n}^{U}-T_{n}^{L})}\le\frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^{U}-T_{n-1}^{L})}+\left(2+K_{ijn-1}^{z,C}-\hat{K}_{ijn}^{z,C}-Y_{ijn}^{z,C}\right)\cdot F_{j}\qquad \begin{gathered} z\in Z;n\in M^{z};(i,j)\in B\\ j\in S^{C};j\in C_{n}^{z}\cap C_{n-1}^{z}\\ i\in H^{z};\ i\in P_{jn}^{C}\cap P_{jn-1}^{C}\end{gathered}\qquad (2.88)$$

Flow rate consistency for cold streams - $j \notin S^C$

$$\hat{q}_{ijn}^{z,C} \geq \left(Y_{ijn-1}^{z,C} - Y_{ijn}^{z,C} - Y_{ijn+1}^{z,C} - 2\right) \cdot \Delta H_{jn}^{z,C} \qquad \begin{matrix} z \in Z; n \in M^z;\ j \in C_{n-1}^z \cap C_n^z \cap C_{n+1}^z; i \notin S^C \\ (i,j) \notin B;\ i \in H^z; i \in P_{jn-1}^C \cap P_{jn}^C \cap P_{jn+1}^C \end{matrix} \tag{2.89}$$

$$\left.\begin{aligned} \hat{q}_{ijn}^{z,C} &\geq \left(Y_{ijn}^{z,C} - K_{ijn}^{z,C} - \hat{K}_{ijn}^{z,C}\right) \cdot \Delta H_{jn}^{z,C} && (2.90) \\ \hat{q}_{ijn}^{z,C} &\geq \left(Y_{ijn-1}^{z,C} + K_{ijn}^{z,C} + \hat{K}_{ijn+1}^{z,C} - 2\right) \cdot \Delta H_{jn}^{z,C} && (2.91) \end{aligned}\right\} \begin{matrix} z \in Z; n \in M^z;\ j \in C_{n-1}^z \cap C_n^z \cap C_{n+1}^z; i \notin S^C \\ (i,j) \in B;\ i \in H^z; i \in P_{jn-1}^C \cap P_{jn}^C \cap P_{jn+1}^C \end{matrix}$$

2.5.6 Temperature Difference Enforcing

This part is necessary to generate in order to assure the heat transfer feasible. Firstly, Figure 2.13, constraint (2.92) and (2.93) introduce the temperature difference of extreme interval for the condition that there are no splits are allowed. Additionally, constraint (2.94) to (2.99) further explain in case where stream splits are allowed.

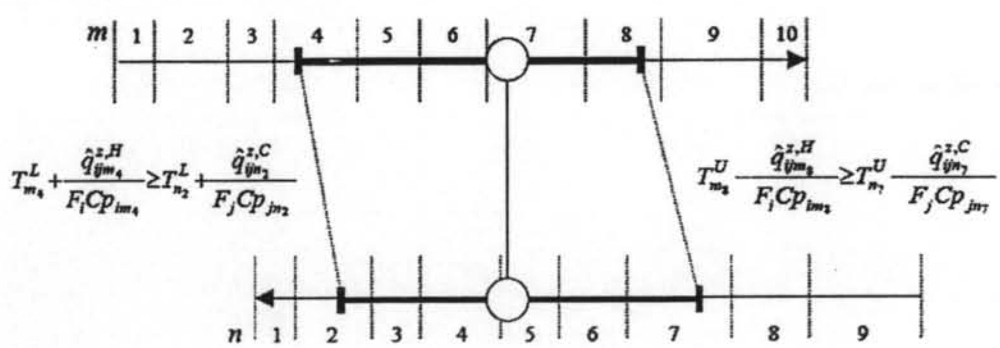


Figure 2.13 Temperature difference assurance when splits are not allowed.

Temperature feasibility constraints - $\qquad i \notin S^H, j \notin S^C$

$$\left.\begin{aligned} T_m^L + \frac{\hat{q}_{ijm}^{z,H}}{F_i Cp_{im}} &\geq T_n^L + \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right) \cdot T_n^U && (2.92) \\ T_m^U - \frac{\hat{q}_{ijm}^{z,H}}{F_i Cp_{im}} &\geq T_n^U - \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - \left(2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}\right) \cdot T_n^U && (2.93) \end{aligned}\right\} \begin{matrix} z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L \\ i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H \end{matrix}$$

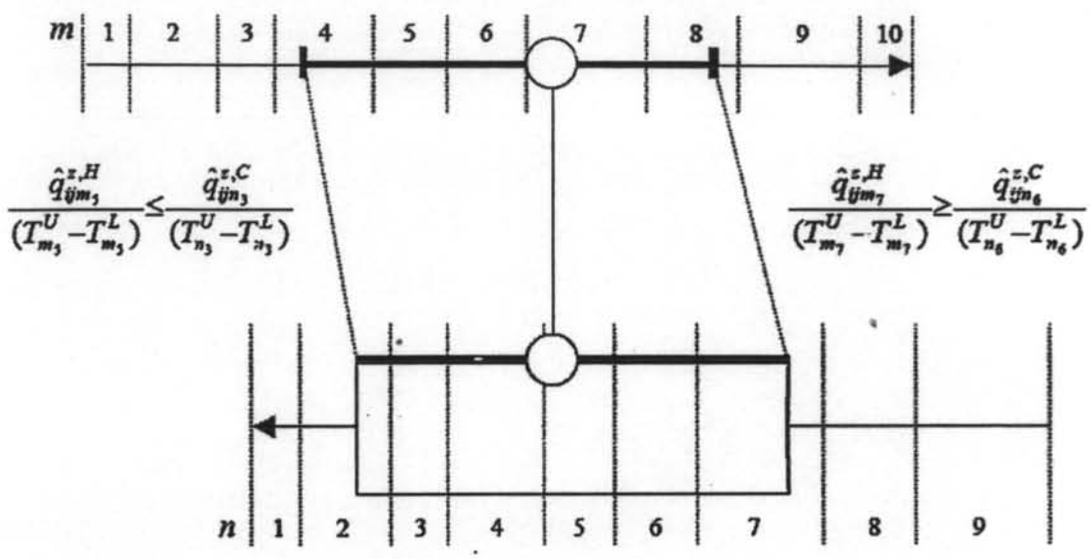


Figure 2.14 Temperature difference assurance when splits are allowed.

Temperature feasibility constraints - $\quad i \in S^H, j \in S^C, (i,j) \notin B$

$$\left.\begin{aligned}
&\hat{K}^{z,C}_{ijn} \le 2 - K^{z,H}_{ijm} - K^{z,C}_{ijn} && (2.94)\\
&\frac{\hat{q}^{z,C}_{ijn}}{T^U_m - T^L_n} \le \frac{\hat{q}^{z,C}_{ijn+1}}{T^U_{n+1} - T^L_{n+1}} \frac{Cp_{jn}}{Cp_{jn+1}} + \left(2 - K^{z,H}_{ijm} - K^{z,C}_{ijn}\right) \cdot \frac{\Delta H^{z,C}_{jn}}{T^U_m - T^L_n} && (2.95)\\
&\frac{\hat{q}^{z,H}_{ijm}}{Min\{T^U_m ; T^U_n\} - T^L_m} \ge \frac{\hat{q}^{z,H}_{ijm+1}}{T^U_{m+1} - T^L_{m+1}} \frac{Cp_{im}}{Cp_{im+1}} - \left(2 - K^{z,H}_{ijm} - K^{z,C}_{ijn}\right) \frac{\Delta H^{z,H}_{im+1}}{T^U_{m+1} - T^L_{m+1}} && (2.96)
\end{aligned}\right\}
\begin{array}{c}
z \in Z ; m,n \in M^z \; ; i \in S^H \\
j \in S^C ; T^L_n < T^U_m ; T^U_n > T^L_m \\
i \in H^z_m \cap H^z_{m+1} ; j \in C^z_n \cap C^z_{n+1} ; \\
i \in P^C_{jn} \cap P^C_{jn+1} ; j \in P^H_{im} \cap P^H_{im+1}
\end{array}$$

$$\left.\begin{aligned}
&K^{z,H}_{ijm} \le 2 - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,C}_{ijn} && (2.97)\\
&\frac{\hat{q}^{z,H}_{ijm}}{T^U_m - T^L_n} \le \frac{\hat{q}^{z,H}_{ijm-1}}{T^U_{m-1} - T^L_{m-1}} \frac{Cp_{im}}{Cp_{im-1}} + \left(2 - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,C}_{ijn}\right) \frac{\Delta H^{z,H}_{im}}{T^U_m - T^L_n} && (2.98)\\
&\frac{\hat{q}^{z,C}_{ijn}}{T^U_n - Max\{T^L_m ; T^L_n\}} \ge \frac{\hat{q}^{z,C}_{ijn-1}}{T^U_{n-1} - T^L_{n-1}} \frac{Cp_{jn}}{Cp_{jn-1}} + \left(2 - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,C}_{ijn}\right) \frac{\Delta H^{z,C}_{jn-1}}{T^U_{n-1} - T^L_{n-1}} && (2.99)
\end{aligned}\right\}
\begin{array}{c}
z \in Z ; m,n \in M^z \; ; i \in S^H \\
j \in S^C ; T^L_n < T^U_m ; T^U_n > T^L_m \\
i \in H^z_m \cap H^z_{m+1} ; j \in C^z_n \cap C^z_{n+1} ; \\
i \in P^C_{jn} \cap P^C_{jn+1} ; j \in P^H_{im} \cap P^H_{im+1}
\end{array}$$

All these next constraints are performed only for overlapping pairs of intervals where $T_n^L < T_m^U$ and $T_n^U > T_m^L$ which m and n are the overlapping intervals of hot and cold stream at the hot end of heat exchanger. Constraint (2.94) is generated to guarantee that the cold end of the cold stream of heat exchanger will not be located at the same interval with the hot end. Feasible heat transfer forces the constraint (2.95) in valid. That is the hot end temperature for the cold stream is less than the hot stream. Moreover, constraint (2.96) stated that the hot end temperature of the hot stream equal to *Min* $\{T_m^U ; T_n^U\}$ as illustrated in Figure 2.15. Finally, the constraints for the case of multiple heat exchangers are presented next.

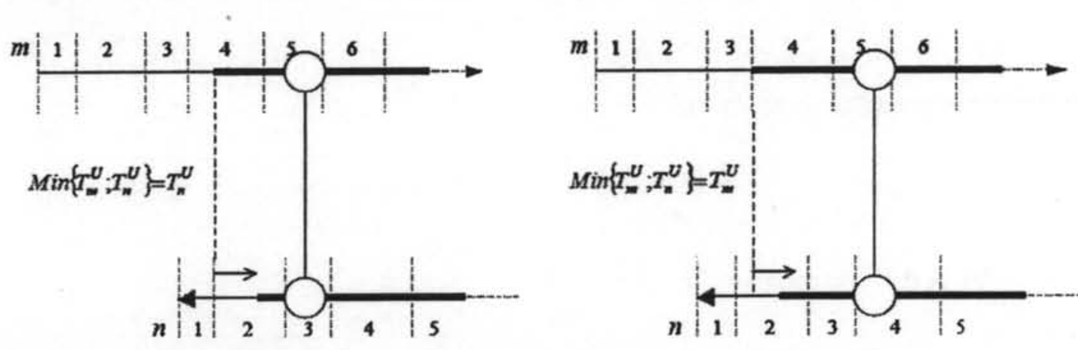


Figure 2.15 Temperature difference assurance at the hot end of an exchanger - $i \in S^H, j \in S^C, (i,j) \notin B$.

Temperature feasibility constraints $i \in S^H, j \in S^C, (i,j) \in B$

$$\hat{K}_{ijn}^{z,C} \le 1 + Y_{ijn}^{z,C} - K_{ijm}^{z,H} - K_{ijn}^{z,C} \tag{2.100}$$

$$\frac{\hat{q}_{ijn}^{z,C}}{T_m^U - T_n^L} \le \frac{\hat{q}_{ijn+1}^{z,C}}{T_{n+1}^U - T_{n+1}^L}\frac{Cp_{jn}}{Cp_{jn+1}} + \left(1 + Y_{ijn}^{z,C} - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{jn}^{z,C}}{T_m^U - T_n^L} \tag{2.101}$$

$$\frac{\tilde{q}_{ijn}^{z,C}}{T_m^U - T_n^L} \le \frac{\hat{q}_{ijn+1}^{z,C}}{T_{n+1}^U - T_{n+1}^L}\frac{Cp_{jn}}{Cp_{jn+1}} + \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{jn}^{z,C}}{T_m^U - T_n^L} \tag{2.102}$$

(2.100)–(2.102): $z \in Z; m,n \in M^z; i \in S^H;$ $j \in S^C; T_n^L < T_m^U; T_n^U > T_m^L$ $i \in H_m^z \cap H_{m+1}^z; j \in C_n^z \cap C_{n+1}^z$ $i \in P_{jn}^C \cap P_{jn+1}^C; j \in P_{im}^H \cap P_{im+1}^H$

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U ; T_n^U\} - T_m^L} \ge \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L}\frac{Cp_{im}}{Cp_{im+1}} - \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{im+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L} \tag{2.103}$$

$$K_{ijm}^{z,H} \le 1 + Y_{ijm}^{z,H} - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C} \tag{2.104}$$

$$\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{T_m^U - T_n^L} \le \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U - T_{m-1}^L} \frac{Cp_{im}}{Cp_{im-1}} + \left(2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}\right) \cdot \frac{\Delta H_{im}^{z,H}}{T_m^U - T_n^L} \tag{2.105}$$

$$\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{T_n^U - Max\{T_m^L, T_n^L\}} \le \frac{\hat{q}_{ijn-1}^{z,C}}{T_{n-1}^U - T_{n-1}^L} \frac{Cp_{jn}}{Cp_{jn-1}} - \left(2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}\right) \cdot \frac{\Delta H_{jn-1}^{z,C}}{T_{n-1}^U - T_{n-1}^L} \tag{2.106}$$

$z \in Z; m,n \in M^z; i \in S^H;$
$j \in S^C; T_n^L < T_m^U; T_n^U > T_m^L$
$i \in H_m^z \cap H_{m-1}^z; j \in C_n^z \cap C_{n-1}^z$
$i \in P_{jn}^C \cap P_{jn-1}^C; j \in P_{im}^H \cap P_{im-1}^H$

2.5.7 Heat Exchanger Area Calculation

The area of heat exchanger can be determined by considering the heat transfer of any stream match.

Heat transfer area for one heat exchanger is permitted

$$A_{ij}^z = \sum_{m \in M_i^z} \sum_{\substack{n \in N_j^z; T_n^L < T_m^U \\ j \in P_{im}^H; i \in P_{jn}^C}} \left[\frac{q_{im,jn}^z \left(h_{im} + h_{jn}\right)}{\Delta T_{mn}^{ML} h_{im} h_{jn}} \right] \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P \tag{2.107}$$

For multiple heat exchangers between streams i and j are allowed, each exchanger area can be formulated by this following constraints.

Heat transfer area for multiple heat exchangers

$$\hat{A}_{ij}^{z,k} \le \sum_{\substack{l \in M_i^z \\ l \le m}} \sum_{\substack{n \in N_j^z \\ T_n^L < T_m^U \\ j \in P_{im}^H \\ i \in P_{jn}^C}} \left[\frac{\left(q_{il,jn}^z - \breve{q}_{il,jn}^z\right) \cdot \left(h_{il} + h_{jn}\right)}{\Delta T_{ln}^{ML} \cdot h_{il} \cdot h_{jn}} \right] - \sum_{h=1}^{k-1} A_{ij}^{z,h} + A_{ijmax}^z \left(2 - \hat{K}_{ijm}^{z,H} - G_{ijm}^{z,k}\right) \tag{2.108}$$

$$\hat{A}_{ij}^{z,k} \ge \sum_{\substack{l \in M_i^z \\ l < m}} \sum_{\substack{n \in N_j^z \\ T_n^L < T_m^U \\ j \in P_{im}^H \\ i \in P_{jn}^C}} \left[\frac{\left(q_{il,jn}^z - \breve{q}_{il,jn}^z\right) \cdot \left(h_{il} + h_{jn}\right)}{\Delta T_{ln}^{ML} \cdot h_{il} \cdot h_{jn}} \right] - \sum_{h=1}^{k-1} A_{ij}^{z,h} - A_{ijmax}^z \left(2 - \hat{K}_{ijm}^{z,H} - G_{ijm}^{z,k}\right) \tag{2.109}$$

$$\hat{A}_{ij}^{z,k} \ge A_{ij}^z - \sum_{h=1}^{k-1} \hat{A}_{ij}^{z,h} \tag{2.110}$$

$$\sum_{h=1}^{k_{max}} h \cdot G_{ijm}^{z,h} = \sum_{\substack{l \in M_i^z; l \le m \\ j \in P_{im}^H}} K_{ijl}^{z,H} + 1 - Y_{ijm}^{z,H} \tag{2.111}$$

$z \in Z; m \in M^z$
$i \in H_m^z; j \in C^z$
$j \in P_{im}^H; (i,j) \in B$
$k = 1, \ldots, k_{max} - 1$

$$\left.\begin{array}{l}\sum_{\substack{n\in N_j^z;T_n^L<T_m^U \\ j\in P_{im}^H;i\in P_{jn}^C}} \breve{q}_{im,jn}^{z} = \widetilde{q}_{ijm}^{z,H} \qquad (2.112) \\ \\ \breve{q}_{im,jn}^{z} \le q_{im,jn}^{z} \qquad (2.113)\end{array}\right\} \begin{array}{l} z\in Z; m\in M^z \\ i\in H_m^z; j\in C^z \\ j\in P_{im}^H; (i,j)\in B \\ k=1,\ldots,k_{\max}-1\end{array}$$

The maximum number of heat exchangers allowed per match, k_{max}, is required for area calculation. The heat exchanger area of the k-th heat exchanger is calculated by subtracting the area of the former exchangers, k-1, from the total accumulated area until the end of the k-th exchanger. The binary variables, $\hat{X}_{ijm}^{z,h}$, are used to specify which exchanger is present at a certain temperature interval. Obviously, all constraints (2.108) to (2.113) are constructed for hot stream intervals only because hot and cold stream intervals can generate the same heat exchanger area.

2.5.8 Number of Shells

The variable U_{ij}^{z} is used to define as the number of shells.

Maximum Shell Area

$$A_{ij}^{z} \le A_{ij\,max}^{z} U_{ij}^{z} \qquad z\in Z;\ i\in H^z; j\in C^z; (i,j)\in P; (i,j)\notin B \qquad (2.114)$$

$$\hat{A}_{ij}^{z,k} \le A_{ij\,max}^{z} \hat{U}_{ij}^{z,k} \qquad z\in Z;\ i\in H^z; j\in C^z; (i,j)\in P; (i,j)\in B \qquad (2.115)$$

2.5.9 Objective Function

The objective function of the MILP model is to minimize the annualized total cost, this is composed of the operating and capital cost. The simply assumption of linear relation is used to approximate the total cost. The equation applied to calculate the objective value is indicated below. The first term represents the cost of hot utility, the second referred to cooling utility cost, followed by the fixed cost for heat exchanger and end up with the area cost.

$$Min\ \ Cost = \sum_{z}\sum_{i\in HU^z}\sum_{\substack{j\in C^z\\(i,j)\in P}} c_i^H F_i^H \Delta T_i + \sum_{z}\sum_{j\in CU^z}\sum_{\substack{i\in H^z\\(i,j)\in P}} c_j^C F_j^C \Delta T_j + \left[\sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P}}\left(c_{ij}^F U_{ij}^z + c_{ij}^A A_{ij}^z\right)\right]_{(i,j)\notin B} + \left[\sum_{k}\left\{\sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P}}\left(c_{ij}^F \hat{U}_{ij}^{z,k} + c_{ij}^A \hat{A}_{ij}^{z,k}\right)\right\}\right]_{(i,j)\in B} \quad (2.116)$$

2.6 Model for Retrofit Heat Exchanger Network

Not only designing an optimal heat exchanger network, but the problem of heat exchanger network analysis is also play attention in the retrofit part. The MILP model is extended by adding some constraints for being the retrofit configuration. An Existing heat exchanger network is necessarily identified into the model, the location of the presented exchanger units are needed to introduce. A certain reconstruction and financial investment of adding new exchangers or area expanding in an existing process can considerably reduce the total cost of the existing plant. These options are targeted to decrease the total cost by enhancing the heat integration among process streams.

2.6.1 Area Additions for Existing and New Heat Exchanger Units

The number of heat exchanger unit in each match is considering for the additional area. Firstly, for the case where only one heat exchanger unit is allowed per matching, $(i,j)\notin$ B, both possibility of adding the exchanger area in the same shell and a new one are proposed. However, when $(i,j)\in$ B, there are more than one exchanger exists in the same pair of hot and cold stream matching, the area expansion possibility can be generated by adding area to the existing exchangers and also set up the new units. The following set of constraints is used to identify when a heat exchanger unit is equipped with the existing network.

Area addition to the existing heat exchangers – $(i,j) \notin B$

$$\left.\begin{aligned} & A_{ij}^{z} \le A_{ij}^{z^0} + \Delta A_{ij}^{z^0} + A_{ij}^{z^N} && (2.117) \\ & \Delta A_{ij}^{z^0} \le \Delta A_{ij\max}^{z^0} && (2.118) \\ & A_{ij}^{z^N} \le A_{ij\max}^{z^N} \cdot \left(U_{ij}^{z} - U_{ij}^{z^0}\right) && (2.119) \\ & U_{ij}^{z} \le U_{ij\max}^{z} && (2.120) \end{aligned}\right\} \quad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} \ge 1$$

The area of exchanger per match (i,j) which presented only one exchanger should not over a summation of the existing area ($A_{ij}^{z^0}$), the area added to the existing shells ($\Delta A_{ij}^{z^0}$) and the area placed into the new shells ($A_{ij}^{z^N}$). The extended area into the existing shells and number of new shell need to be assigned as maximum. Additionally, a new shell is counted whenever the area is increased that shown in constraint (2.119). However, another set of equations is presented for the case in which there is no exchanger unit settled between a pair of hot and cold process streams.

Area required for new matches – $(i,j) \notin B$

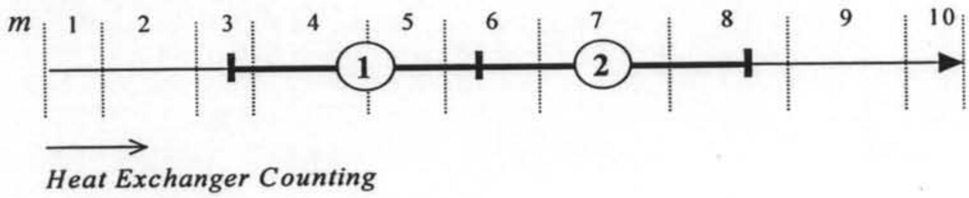
$$A_{ij}^{z^N} \le A_{ij\max}^{z^N} \cdot U_{ij}^{z} \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} = 0 \qquad (2.121)$$

$$U_{ij}^{z} \le U_{ij\max}^{z} \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} = 0 \qquad (2.122)$$

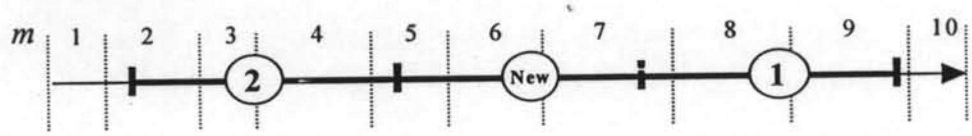
On the other hand, when there is more than one exchanger unit presented in the same pair of streams, $(i,j) \in B$, the position and order of each unit is necessary to record. A variable δ_{hk} is used to identify the exchanger location, an example is shown in Figure 2.16. For example, variable $\delta_{13} = 1$ indicate that the exchanger presented in the first location in the original network and it has been equipped in the third position in the retrofitted design network. Definition of variable δ_{hk} is defined below

$$\delta_{hk} = \begin{cases} 1 & \text{If the } h\text{-th original heat exchanger is placed in the } k\text{-th position in the retrofitted network} \\ 0 & \text{Otherwise} \end{cases}$$

Before Retrofit



After Retrofit



δ_{hk}		k: 1	2	3
h	1	0	0	1
	2	1	0	0

Figure 2.16 Area computation when $(i,j) \in B$.

The area of the k-th existing exchanger between streams i and j after retrofit should smaller or equal to the combination of original area of h-th exchanger ($\sum_{h=1}^{k_e} A_{ij}^{z,h^0}\delta_{ij}^{z,hk}$), the area added to the existing shells ($\Delta A_{ij}^{z,k^0}$) and the area for new shells (A_{ij}^{z,k^N}). Whenever an existing h-th exchanger unit is analyzed to relocate into k-th position, $\sum_{h=1}^{k_e}\delta_{ij}^{z,hk} = 1$, there is no new heat exchanger unit for the retrofit network, therefore the retrofit exchanger area will be the original area combine with the addition area. On the contrary, original area term in constraint (2.124) for retrofit match will be canceled where as the new heat exchanger unit is placed, $\sum_{h=1}^{k_e}\delta_{ij}^{z,hk} = 0$.

Area addition to existing and new heat exchangers when $(i,j)\in B$

$$\left.\begin{aligned}
&A_{ij}^{z,k} \le \sum_{h=1}^{k_e} A_{ij}^{z,h^0}\delta_{ij}^{z,hk} + \Delta A_{ij}^{z,k^0} + A_{ij}^{z,k^N} && (2.123)\\
&\Delta A_{ij}^{z,k^0} \le \sum_{h=1}^{k_e}\left(\Delta A_{ij\max}^{z,h}\delta_{ij}^{z,hk}\right) && (2.124)\\
&A_{ij}^{z,k^N} \le A_{ij\max}^{z^N}\cdot\left(1-\sum_{h=1}^{k_e}\delta_{ij}^{z,hk}\right) && (2.125)\\
&\sum_{h=1}^{k_e}\delta_{ij}^{z,hk} \le 1 && (2.126)
\end{aligned}\right\} \quad z\in Z;\ i\in H^z;\ j\in C^z;(i,j)\in P;(i,j)\in B;1\le k\le k_{\max}$$

$$\sum_{k=1}^{k_{\max}}\delta_{ij}^{z,hk} \le 1 \qquad z\in Z;\ i\in H^z;\ j\in C^z;(i,j)\in P;(i,j)\in B;1\le h\le k_e \tag{2.127}$$

$$\sum_{k=1}^{k_{\max}}\sum_{h=1}^{k_e}\delta_{ij}^{z,hk} = k_e \qquad z\in Z;\ i\in H^z;\ j\in C^z;(i,j)\in P;(i,j)\in B \tag{2.128}$$

In addition, the number of new heat exchanger unit placed into the existing network would be specified as the following constraint.

$$\sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P}}\left(U_{ij}^{z}-U_{ij}^{z^0}\right)\le U_{\max}^{N} \tag{2.129}$$

2.6.2 Objective Function

In retrofit situation, the exchanger investment cost-functions are different from the grassroot design. The objective function for the retrofit heat exchanger network structure also subjects to minimize the total annualized cost but the retrofit programming model has complicated functions for the area cost. Not only count for the number of exchanger unit, but there are also the existing units which need to optimize for area addition or new able place an exchanger. Therefore, the exchanger area for the retrofit target is consisted of area addition to the initial structure and the new exchanger area. All other terms, the hot and cold utility cost, seem to be the same as the grassroot design model. However, fixed charge for the exchanger unit is need to count as the increasing number of unit which correspond to minus the number of exchanger unit, U_{ij}^{z}, with the initial unit, $U_{ij}^{z^0}$.

$$Min\ \ Cost=\sum_{z}\sum_{i\in HU^z}\sum_{\substack{j\in C^z\\(i,j)\in P}} c_i^H F_i^H \Delta T_i+\sum_{z}\sum_{j\in CU^z}\sum_{\substack{i\in H^z\\(i,j)\in P}} c_j^C F_j^C \Delta T_j+\sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P}} c_{ij}^F\left(U_{ij}^z-U_{ij}^{z^0}\right)$$
$$+\sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P\\(i,j)\notin B}}\left(c_{ij}^{A^0}\Delta A_{ij}^{z^0}+c_{ij}^{A^N}A_{ij}^{z^N}\right)+\sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P\\(i,j)\in B}}\sum_{k=1}^{k_{\max}}\left(c_{ij}^{A^0}\Delta A_{ij}^{z,k^0}+c_{ij}^{A^N}A_{ij}^{z,k^N}\right) \qquad (2.130)$$

2.7 Model for Heat Exchanger Network Included Pump Around

In a crude fractionation unit the pump-around is used to provide high level temperature sources that can help in increasing the energy efficiency of crude units. The MILP grass root model is extended by adding some constraints for including the pump-around into the design the heat exchanger network. The candidate values of each pump-around are necessarily identified into the model. In this model has to define the new set, variables and equations. The new set PA^Z is introduced as pump around streams in zone z, which are the function of i. The new parameter $FPR_{i,r}^H$ is the candidate values for pump-around flowrate which are the function of i and r, Q_{PA} is total of pump-around duty.

2.7.1 Heat Balance Equations

These groups of equation are almost same as the equations in the group 2.5.2 but some part are different.

Heat balance for hot process streams – $i \notin NI^H$ *:*

$$\Delta H_{im}^{z,H}=\sum_{\substack{n\in M^z\\T_n^L<T_m^U}}\ \sum_{\substack{j\in C_n^z\\j\in P_{im}^H\\i\in P_{jn}^C}} q_{im,jn}^z \qquad z\in Z;m\in M^z;i\in H_m^z;i\notin HU^z;i\notin NI^H;i\notin PA^z \qquad (2.131)$$

$$FP_i^H Cp_{i,m}^H(T_m^U-T_m^L)=\sum_{\substack{n\in M^z\\T_n^L<T_m^U}}\ \sum_{\substack{j\in C_n^z\\j\in P_{im}^H\\i\in P_{jn}^C}} q_{im,jn}^z \quad z\in Z;m\in M^z;i\in H_m^z;i\notin HU^z;i\notin NI^H;i\in PA^z \qquad (2.132)$$

Equation (2.131) and (2.132) are same as the equation (2.6).Which equation (2.132) is used to calculate heat balance of pump-around but equation (2.131) is not.

Heat balance for hot streams (non-isothermal mixing allowed):

$$\Delta H_{im}^{z,H} = \sum_{\substack{n\in M^z \\ T_n^L<T_m^U}} \sum_{\substack{j\in C_n^z \\ j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z + \sum_{\substack{n\in M^z \\ n>m}} \sum_{i\in H_n^z} \overline{q}_{imn}^{z,H} - \sum_{\substack{n\in M^z \\ n<m}} \sum_{i\in H_n^z} \overline{q}_{imn}^{z,H}$$

$$z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; i\in NI^H; i\notin PA^z \qquad (2.133)$$

$$FP_i^H Cp_{i,m}^H (T_m^U - T_m^L) = \sum_{\substack{n\in M^z \\ T_n^L<T_m^U}} \sum_{\substack{j\in C_n^z \\ j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z + \sum_{\substack{n\in M^z \\ n>m}} \sum_{i\in H_n^z} \overline{q}_{imn}^{z,H} - \sum_{\substack{n\in M^z \\ n<m}} \sum_{i\in H_n^z} \overline{q}_{imn}^{z,H}$$

$$z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; i\in NI^H; i\in PA^z \qquad (2.134)$$

Equation (2.133) and (2.134) are same as the equation (2.10).Which equation (2.134) is used to calculate heat balance of pump-around but equation (2.133) is not.

2.7.2 Heat Exchanger Definition and Count

This equation are almost same as the equations in the group 2.5.3 but some part are different.

$$q_{ijm}^L Y_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} \le \Delta H_{im}^{z,H} Y_{ijm}^{z,H}$$

$$z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; j\in C^z; j\in P_{im}^H; i\notin PA^z \qquad (2.135)$$

$$q_{ijm}^L Y_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} \le Cp_{i,m}^H (T_m^U - T_m^L) YFP_{ijm}^{z,H}$$

$$z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; j\in C^z; j\in P_{im}^H; i\in PA^z \qquad (2.136)$$

$$YFP_{ijm}^{z,H} = \sum_{r} FPR_{i,r}^{H} YW_{ijm}^{z,H} \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; i \notin HU^{z}; j \in C^{z}; j \in P_{im}^{H}; i \in PA^{z} \qquad (2.137)$$

$$YW_{ijm}^{z,H} - Y_{ijm}^{z,H} \le 0 \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; i \notin HU^{z}; j \in C^{z}; j \in P_{im}^{H}; i \in PA^{z} \qquad (2.138)$$

$$YW_{ijm}^{z,H} \le W_{i,r} \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; i \notin HU^{z}; j \in C^{z}; j \in P_{im}^{H}; i \in PA^{z} \qquad (2.139)$$

$$YW_{ijm}^{z,H} \ge Y_{ijm}^{z,H} + W_{i,r} - 1 \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; i \notin HU^{z}; j \in C^{z}; j \in P_{im}^{H}; i \in PA^{z} \qquad (2.140)$$

Equations (2.135) to (2.140) are same as the equation (2.14). Which equation (2.136) to (2.140) are used for heat exchanger defining heat balance of pump-around but equation (2.135) is not.

2.7.3 Heat Transfer Consistency

These groups of equation are almost same as the equations in the group 2.5.4 but some part are different.

Heat transfer consistency for multiple heat exchangers between the same pair of streams:

$$\sum_{\substack{l \in M_i^z \\ l \le m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \le \sum_{\substack{l \in N_j^z \\ l \le n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} + 4X_{im,jn}^{z} \, Max\{ \sum_{\substack{l \in M_i^z \\ l \le m \\ j \in P_{il}^H}} \Delta H_{il}^{z,H}; \sum_{\substack{l \in M_i^z \\ l \le n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C} \}$$

$$z \in Z; m,n \in M^{z}; T_{n}^{L} \le T_{m}^{U}; (i,j) \in B; i \in H_{m}^{z}; j \in C_{n}^{z}; i \in P_{jn}^{C}; j \in P_{im}^{H}; i \notin PA^{z} \qquad (2.141)$$

$$\sum_{\substack{l \in M_i^z \\ l \le m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \le \sum_{\substack{l \in N_j^z \\ l \le n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} + 4XM_{im,jn}^{z}$$

$$z \in Z; m,n \in M^{z}; T_{n}^{L} \le T_{m}^{U}; (i,j) \in B; i \in H_{m}^{z}; j \in C_{n}^{z}; i \in P_{jn}^{C}; j \in P_{im}^{H}; i \in PA^{z} \qquad (2.142)$$

$$XM_{im,jn}^{z} - T_{im,jn}^{z} \Omega_{im,jn}^{z} \le \sum XW_{im,jn,r}^{z} FPR_{i,r}^{H} \sum_{\substack{l \in M_i^z \\ l \le m \\ j \in P_{il}^H}} Cp_{i,l}^{H} (T_{l}^{U} - T_{l}^{L})$$

$$z \in Z; m,n \in M^{z}; T_{n}^{L} \le T_{m}^{U}; (i,j) \in B; i \in H_{m}^{z}; j \in C_{n}^{z}; i \in P_{jn}^{C}; j \in P_{im}^{H}; i \in PA^{z} \qquad (2.143)$$

$$XM_{im,jn}^{z} - T_{im,jn}^{z} \Omega_{im,jn}^{z} \ge \sum XW_{im,jn,r}^{z} FPR_{i,r}^{H} \sum_{\substack{l \in M_i^z \\ l \le m \\ j \in P_{il}^H}} Cp_{i,l}^{H} (T_{l}^{U} - T_{l}^{L})$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{2.144}$$

$$XM_{im,jn}^z - (1 - T_{im,jn}^z)\Omega_{im,jn}^z \leq X_{im,jn}^z \sum_{\substack{l \in M_i^z \\ l \leq n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C}$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{2.145}$$

$$XM_{im,jn}^z \geq X_{im,jn}^z \sum_{\substack{l \in M_o^z \\ l \leq n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C}$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{2.146}$$

$$XW_{im,jn,r}^{z,H} - \Gamma_{im,jn}^{z,H} W_{i,r} \leq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{2.147}$$

$$(X_{im,jn}^{z,H} - XW_{im,jn,r}^{z,H}) - (1 - W_{i,r})\Gamma_{im,jn}^{z,H} \leq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{2.148}$$

$$(X_{im,jn}^{z,H} - XW_{im,jn,r}^{z,H}) \geq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{2.149}$$

Equations (2.141) to (2.149) are same as the equation (2.50).Which equation (2.142) to (2.149) are used to calculate heat transfer of pump-around but equation (2.141) is not.

$$\sum_{\substack{l \in M_i^z \\ l \leq m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \geq \sum_{\substack{l \in N_j^z \\ l \leq n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} - 4X_{im,jn}^z \, Max\{ \sum_{\substack{l \in M_i^z \\ l \leq m \\ j \in P_{il}^H}} \Delta H_{il}^{z,H}; \sum_{\substack{l \in M_i^z \\ l \leq n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C} \}$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \notin PA^z \tag{2.150}$$

$$\sum_{\substack{l \in M_i^z \\ l \leq m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \geq \sum_{\substack{l \in N_j^z \\ l \leq n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} - 4XM_{im,jn}^{z}$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \qquad (2.151)$$

Equation (2.150) and (2.151) are same as the equation (2.51).Which equation (2.151) is used to calculate heat transfer of pump-around but equation (2.150) is not.

$$\tilde{q}_{ijm}^{z,H} \leq K_{ijm}^{z,H} \Delta H_{im}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.152)$$

$$\tilde{q}_{ijm}^{z,H} \leq KFP_{ijm}^{z,H} Cp_{i,m}^H (T_m^U - T_m^L) \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.153)$$

$$KFP_{ijm}^{z,H} = \sum_r FPR_{i,r}^H \, KW_{ijm,r}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.154)$$

$$KW_{ijm,r}^{z,H} - K_{ijm}^{z,H} \leq 0 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.155)$$

$$KW_{ijm,r}^{z,H} \leq W_{i,r} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.156)$$

$$KW_{ijm,r}^{z,H} \geq K_{ijm}^{z,H} + W_{i,r} - 1 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.157)$$

Equations (2.152) to (2.157) are same as the equation (2.57).Which equation (2.153) to (2.157) are used to calculate heat transfer of pump-around but equation (2.152) is not.

$$\tilde{q}_{ijm}^{z,H} \leq \hat{K}_{ijm}^{z,H} \Delta H_{im}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \notin PA^z \qquad (2.158)$$

$$\tilde{q}_{ijm}^{z,H} \leq \hat{K}FP_{ijm}^{z,H} Cp_{i,m}^H (T_m^U - T_m^L) \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.159)$$

$$\hat{K}FP_{ijm}^{z,H} = \sum_r FPR_{i,r}^H \, \hat{K}W_{ijm,r}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (2.160)$$

$$\hat{K}W^{z,H}_{ijm,r} - \hat{K}^{z,H}_{ijm} \leq 0 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H^z_m; j \in P^H_{im}; i \in PA^z \qquad (2.161)$$

$$\hat{K}W^{z,H}_{ijm,r} \leq W_{i,r} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H^z_m; j \in P^H_{im}; i \in PA^z \qquad (2.162)$$

$$\hat{K}W^{z,H}_{ijm,r} \geq \hat{K}^{z,H}_{ijm} + W_{i,r} - 1 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H^z_m; j \in P^H_{im}; i \in PA^z \qquad (2.163)$$

Equations (2.158) to (2.163) are same as the equation (2.58).Which equation (2.159) to (2.163) are used to calculate heat transfer of pump-around but equation (2.158) is not.

2.7.4 Flow Rate Consistency Within Heat Exchangers

These groups of equation are almost same as the equations in the group 2.5.5 but some part are different.

$$\frac{\hat{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \leq \frac{\hat{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} + \left(1 - \alpha^{z,H}_{ijm}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; i \in S^H; j \in C^z; j \in P^H_{im} \cap P^H_{im-1}; i \in PA^z \qquad (2.164)$$

$$\frac{\hat{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \leq \frac{\hat{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} + (FP^H_i - FP\alpha^{z,H}_{ijm})$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; i \in S^H; j \in C^z; j \in P^H_{im} \cap P^H_{im-1}; i \in PA^z \qquad (2.165)$$

$$FP\alpha^{z,H}_{ijm} = \sum FPR^H_{i,r} \, W\alpha^{z,H}_{ijm,r}$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; i \in S^H; j \in C^z; j \in P^H_{im} \cap P^H_{im-1}; i \in PA^z \qquad (2.166)$$

$$W\alpha^{z,H}_{ijm,r} - \alpha^{z,H}_{ijm} \leq 0$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; i \in S^H; j \in C^z; j \in P^H_{im} \cap P^H_{im-1}; i \in PA^z \qquad (2.167)$$

$$W\alpha^{z,H}_{ijm,r} \leq W_{i,r}$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; i \in S^H; j \in C^z; j \in P^H_{im} \cap P^H_{im-1}; i \in PA^z \qquad (2.168)$$

$$W\alpha^{z,H}_{ijm,r} \geq \alpha^{z,H}_{ijm} + W_{i,r} - 1$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; i \in S^H; j \in C^z; j \in P^H_{im} \cap P^H_{im-1}; i \in PA^z \qquad (2.169)$$

Equations (2.164) to (2.169) are same as the equation (2.68). Which equation (2.165) to (2.169) are used to calculate flowrate within heat exchanger of pump-around but equation (2.164) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1 - \alpha_{ijm}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \notin PA^z \qquad (2.170)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - (FP_i^H - FP\alpha_{ijm}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (2.171)$$

Equation (2.170) and (2.171) are same as the equation (2.69). Which equation (2.171) is used to calculate flowrate within heat exchanger of pump-around but equation (2.170) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1 + \hat{K}_{ijm-1}^{z,H} + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \notin PA^z \qquad (2.172)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - (FP_i^H + \hat{K}FP_{ijm-1}^{z,H} + \hat{K}FP_{ijm}^{z,H} - KFP_{ijm-1}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \in PA^z \qquad (2.173)$$

Equation (2.172) and (2.173) are same as the equation (2.70). Which equation (2.173) is used to calculate flowrate within heat exchanger of pump-around but equation (2.171) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + \left(1 + K_{ijm-1}^{z,H} + K_{ijm}^{z,H} - \hat{K}_{ijm}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \notin PA^z \qquad (2.174)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \le \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + (FP_i^H + KFP_{ijm-1}^{z,H} + KFP_{ijm}^{z,H} - \hat{K}FP_{ijm}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \in PA^z \qquad (2.175)$$

Equation (2.174) and (2.175) are same as the equation (2.71). Which equation (2.175) is used to calculate flowrate within heat exchanger of pump-around but equation (2.174) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \ge \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1 + \hat{K}_{ijm-1}^{z,H} + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \in B; i \notin PA^z \qquad (2.176)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \ge \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - (FP_i^H + \hat{K}FP_{ijm-1}^{z,H} + \hat{K}FP_{ijm}^{z,H} - KFP_{ijm-1}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \in B; i \in PA^z \qquad (2.177)$$

Equation (2.176) and (2.177) are same as the equation (2.72). Which equation (2.177) is used to calculate flowrate within heat exchanger of pump-around but equation (2.176) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \ge \frac{\tilde{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(2 + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H} - Y_{ijm-1}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \in B; i \notin PA^z \qquad (2.178)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \ge \frac{\tilde{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - (2FP_i^H + \hat{K}FP_{ijm}^{z,H} - KFP_{ijm-1}^{z,H} - YFP_{ijm-1}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \in B; i \in PA^z \qquad (2.179)$$

Equation (2.178) and (2.179) are same as the equation (2.73).Which equation (2.179) is used to calculate flowrate within heat exchanger of pump-around but equation (2.178) is not.

$$\frac{\hat{q}^{z,H}_{ijm} - \tilde{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \leq \frac{\hat{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} + \left(2 + K^{z,H}_{ijm-1} - \hat{K}^{z,H}_{ijm} - Y^{z,H}_{ijm}\right) \cdot F_i$$

$$z \in Z\,;\, m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H\,;\, j \in C^z; (i,j) \in B; i \notin PA^z \qquad (2.180)$$

$$\frac{\hat{q}^{z,H}_{ijm} - \tilde{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \leq \frac{\hat{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} + (2FP^H_i + KFP^{z,H}_{ijm-1} - \hat{K}FP^{z,H}_{ijm} - YFP^{z,H}_{ijm})$$

$$z \in Z\,;\, m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H\,;\, j \in C^z; (i,j) \in B; i \in PA^z \qquad (2.181)$$

Equation (2.180) and (2.181) are same as the equation (2.74).Which equation (2.181) is used to calculate flowrate within heat exchanger of pump-around but equation (2.180) is not

$$\hat{q}^{z,H}_{ijm} \geq \left(Y^{z,H}_{ijm} - K^{z,H}_{ijm} - \hat{K}^{z,H}_{ijm}\right) \cdot \Delta H^{z,H}_{im} \qquad \begin{array}{c} z \in Z; m \in M^z;\ i \in H^z_{m-1} \cap H^z_m \cap H^z_{m+1}; i \notin S^H \\ j \in C^z;\ j \in P^H_{im-1} \cap P^H_{im} \cap P^H_{im+1}; i \notin PA^z \end{array} \qquad (2.182)$$

$$\hat{q}^{z,H}_{ijm} \geq \left(YFP^{z,H}_{ijm} - KFP^{z,H}_{ijm} - \hat{K}FP^{z,H}_{ijm}\right) \cdot Cp^H_{i,m}(T^U_m - T^L_m) \qquad \begin{array}{c} z \in Z; m \in M^z;\ i \in H^z_{m-1} \cap H^z_m \cap H^z_{m+1}; i \notin S^H \\ j \in C^z;\ j \in P^H_{im-1} \cap P^H_{im} \cap P^H_{im+1}; i \in PA^z \end{array} \qquad (2.183)$$

Equation (2.182) and (2.183) are same as the equation (2.75).Which equation (2.183) is used to calculate flowrate within heat exchanger of pump-around but equation (2.182) is not

2.7.5 Temperature Difference Enforcing

These groups of equation are almost same as the equations in the group 2.5.6 but some part are different.

$$T_m^L + \frac{\hat{q}_{ijm}^{z,H}}{F_i Cp_{im}} \geq T_n^L + \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right) \cdot T_n^U$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \notin PA^z \quad (2.184)$$

$$T_m^L + \frac{FP\hat{q}_{ijm}^{z,H}}{Cp_{im}} \geq T_n^L + \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - (2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}) \cdot T_n^U$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (2.185)$$

$$FP\hat{q}_{ijm}^{z,H} = \sum_r \frac{W\hat{q}_{ijm,r}^{z,H}}{FPR_{i,r}^H}$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (2.186)$$

$$W\hat{q}_{ijm,r}^{z,H} - \Gamma_{im,jn}^{z,H} W_{i,r} \leq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (2.187)$$

$$(\hat{q}_{ijm,r}^{z,H} - W\hat{q}_{ijm,r}^{z,H}) - (1 - W_{i,r}) \Gamma_{im,jn}^{z,H} \leq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (2.188)$$

$$(\hat{q}_{ijm,r}^{z,H} - W\hat{q}_{ijm,r}^{z,H}) \geq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (2.189)$$

Equations (2.184) to (2.189) are same as the equation (2.88).Which equation (2.185) to (2.189) are used to control temperature for pump-around heat exchanger but equation (2.184) is not.

$$T_m^U - \frac{\hat{q}_{ijm}^{z,H}}{F_i\, Cp_{im}} \geq T_n^U - \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - (2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}).T_n^U$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \notin PA^z \quad (2.190)$$

$$T_m^U - \frac{FP\hat{q}_{ijm}^{z,H}}{Cp_{im}} \geq T_n^U - \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - (2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}).T_n^U$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; T_n^U \geq T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (2.191)$$

Equation (2.190) and (2.191) are same as the equation (2.89).Which equation (2.191) used to control temperature for pump-around heat exchanger but equation (2.190) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U; T_n^U\} - T_m^L} \geq \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L} \frac{Cp_{im}}{Cp_{im+1}} - \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right) \frac{\Delta H_{im+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L}$$

$$z \in Z; m,n \in M^z \;; i \in S^H; j \in S^C; T_n^L < T_m^U; T_n^U > T_m^L; i \in H_m^z \cap H_{m+1}^z; j \in C_n^z \cap C_{n+1}^z; i \in P_{jn}^C \cap P_{jn+1}^C; j \in P_{im}^H \cap P_{im+1}^H; i \notin PA^z \quad (2.192)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U; T_n^U\} - T_m^L} \geq \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L} \frac{Cp_{im}}{Cp_{im+1}} - \left(2FP_i^H - KFP_{ijm}^{z,H} - KFP_{ijn}^{z,C}\right) \frac{Cp_{i,m+1}^H (T_{m+1}^U - T_{m+1}^L)}{T_{m+1}^U - T_{m+1}^L}$$

$$z \in Z; m,n \in M^z \;; i \in S^H; j \in S^C; T_n^L < T_m^U; T_n^U > T_m^L; i \in H_m^z \cap H_{m+1}^z; j \in C_n^z \cap C_{n+1}^z; i \in P_{jn}^C \cap P_{jn+1}^C; j \in P_{im}^H \cap P_{im+1}^H; i \in PA^z \quad (2.193)$$

Equation (2.192) and (2.193) are same as the equation (2.92).Which equation (2.193) used to control temperature for pump-around heat exchanger but equation (2.192) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{T_m^U - T_n^L} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U - T_{m-1}^L} \frac{Cp_{im}}{Cp_{im-1}} + \left(2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}\right) \frac{\Delta H_{im}^{z,H}}{T_m^U - T_n^L}$$

$$z \in Z; m,n \in M^z \;; i \in S^H; j \in S^C; T_n^L < T_m^U; T_n^U > T_m^L; i \in H_m^z \cap H_{m-1}^z; j \in C_n^z \cap C_{n-1}^z; i \in P_{jn}^C \cap P_{jn-1}^C; j \in P_{im}^H \cap P_{im-1}^H; i \notin PA^z \quad (2.194)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{T_m^U - T_n^L} \le \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U - T_{m-1}^L} \frac{Cp_{im}}{Cp_{im-1}} + \left(2FP_i^H - \hat{K}FP_{ijm}^{z,H} - \hat{K}FP_{ijn}^{z,C} \right) \frac{Cp_{i,m}^H (T_m^U - T_m^L)}{T_m^U - T_n^L}$$

$$z \in Z;\ m,n \in M^z;\ i \in S^H;\ j \in S^C;\ T_n^L < T_m^U;\ T_n^U > T_m^L;\ i \in H_m^z \cap H_{m-1}^z;\ j \in C_n^z \cap C_{n-1}^z;\ i \in P_{jn}^C \cap P_{jn-1}^C;\ j \in P_{im}^H \cap P_{im-1}^H;\ i \in PA^z \quad (2.195)$$

Equation (2.194) and (2.195) are same as the equation (2.94). Which equation (2.195) used to control temperature for pump-around heat exchanger but equation (2.194) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U; T_n^U\} - T_m^L} \ge \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L} \frac{Cp_{im}}{Cp_{im+1}} - \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C} \right) \cdot \frac{\Delta H_{im+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L}$$

$$\begin{aligned} &z \in Z;\ m,n \in M^z;\ i \in S^H;\ j \in S^C;\ T_n^L < T_m^U;\ T_n^U > T_m^L;\ i \in H_m^z \cap H_{m+1}^z; \\ &j \in C_n^z \cap C_{n+1}^z;\ i \in P_{jn}^C \cap P_{jn+1}^C;\ j \in P_{im}^H \cap P_{im+1}^H;\ i \notin PA^z \end{aligned} \quad (2.196)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U; T_n^U\} - T_m^L} \ge \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L} \frac{Cp_{im}}{Cp_{im+1}} - \left(2FP_i^H - KFP_{ijm}^{z,H} - KFP_{ijn}^{z,C} \right) \cdot \frac{Cp_{i,m+1}^H (T_{m+1}^U - T_{m+1}^L)}{T_{m+1}^U - T_{m+1}^L}$$

$$\begin{aligned} &z \in Z;\ m,n \in M^z;\ i \in S^H;\ j \in S^C;\ T_n^L < T_m^U;\ T_n^U > T_m^L;\ i \in H_m^z \cap H_{m+1}^z; \\ &j \in C_n^z \cap C_{n+1}^z;\ i \in P_{jn}^C \cap P_{jn+1}^C;\ j \in P_{im}^H \cap P_{im+1}^H;\ i \in PA^z \end{aligned} \quad (2.197)$$

Equation (2.196) and (2.197) are same as the equation (2.99). Which equation (2.197) used to control temperature for pump-around heat exchanger but equation (2.196) is not.

$$\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{T_m^U - T_n^L} \le \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U - T_{m-1}^L} \frac{Cp_{im}}{Cp_{im-1}} + \left(2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C} \right) \cdot \frac{\Delta H_{im}^{z,H}}{T_m^U - T_n^L}$$

$$\begin{aligned} &z \in Z;\ m,n \in M^z;\ i \in S^H;\ j \in S^C;\ T_n^L < T_m^U;\ T_n^U > T_m^L;\ i \in H_m^z \cap H_{m-1}^z; \\ &j \in C_n^z \cap C_{n-1}^z;\ i \in P_{jn}^C \cap P_{jn-1}^C;\ j \in P_{im}^H \cap P_{im-1}^H;\ i \notin PA^z \end{aligned} \quad (2.198)$$

$$\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{T_m^U - T_n^L} \le \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U - T_{m-1}^L} \frac{Cp_{im}}{Cp_{im-1}} + \left(2FP_i^H - \hat{K}FP_{ijm}^{z,H} - \hat{K}FP_{ijn}^{z,C}\right) \cdot \frac{Cp_{i,m}^H (T_m^U - T_m^L)}{T_m^U - T_n^L}$$

$$z \in Z; m,n \in M^z; i \in S^H; j \in S^C; T_n^L < T_m^U; T_n^U > T_m^L; i \in H_m^z \cap H_{m-1}^z; \quad (2.199)$$
$$j \in C_n^z \cap C_{n-1}^z; i \in P_{jn}^C \cap P_{jn-1}^C; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z$$

Equation (2.198) and (2.199) are same as the equation (2.101).Which equation (2.199) used to control temperature for pump-around heat exchanger but equation (2.198) is not.

3.1.2 Apply The Grass-root Model

The studied variable in this work is the temperature intervals. The target is the total cost for the heat exchange process. The simulations are proposed to test with four examples and the optimal number of temperature intervals is found to get the stable cost heat exchanger network.

3.1.3 Analyze The Objective Value Stability

These included other costs which are utility, fixed and area cost.

3.2 Retrofit Design for Heat Exchanger Network

The retrofit design of heat exchanger network is improving the existing heat exchanger network by adding area or adding the new heat exchanger to the existing heat exchanger. The network from the retrofit model is cheaper-cost than one of the existing network.

For retrofit model, additional equations are required to specify the existing heat exchanger or indicate the existing matches between hot and cold streams. The constraint equations 2.117 to 2.129 are mentioned in Chapter II. The new parameters of existing network are added into the model.

3.3 Design of Heat Exchanger Network for Crude Fractionation Unit

A heat exchanger network in this part is designed by using the grass-root constraint functions and constraint functions of pump-around. Not only is this model used to design the heat exchanger network but it is used to select the best values of flowrate of each pump-around also.

The methodology used for designing a heat exchanger network of crude fractionation unit is consisted of

3.3.1 Relationship between Duty of Pump-Around and Side Stripper

The simulation software is used in this part to simulate a crude fractionation unit consisting of crude fractionation column, pump-around and side stripper. The product specification (,gap) is controlled by amount of steam in each side stripper. In this part, the duty of each pump-around is varied to find the steam used in each side stripper. By keeping total duty of pump-around constant.

3.3.2 New Parameters and Equations

From the grass-root design of crude fractionation unit, the new parameters such as the candidate values of pump-around flow rate and the constraint functions of pump-around shown in the previous chapter are added into the grass root model.

3.4 Retrofit of Heat Exchanger Network for Crude Fractionation Unit

The retrofit model for crude fractionation unit is used to improve the existing heat exchanger network by adding area to the existing heat exchanger or adding the new heat exchanger to reduce utility consumption. It can be done by add constraint equations of retrofit from section 3.2 into the grass root design model of section 3.3