CHAPTER II
BACKGROUND AND LITERATURE SURVEY

2.1 The Atmospheric Distillation Unit

The atmospheric distillation unit consists of a desalter, an atmospheric tower, three side strippers and a debutanizer. Crude oil is preheated by exchanging heat with pump-around reflux streams and then sent to a desalter, where salts, solids and water are removed. The desalted crude oil is further preheated by exchanging heat with products and pump-around reflux stream, and finally heated by a crude furnace to a temperature which provides the required degree of vaporization. The heated crude oil is then introduced to the flash zone of the atmospheric tower. The liquid portion of the flashed crude oil flows down to a bottom stripping section of the atmospheric tower, where distillate fractions dissolved in the liquid are vaporized with steam stripping. As shown in figure 2.1

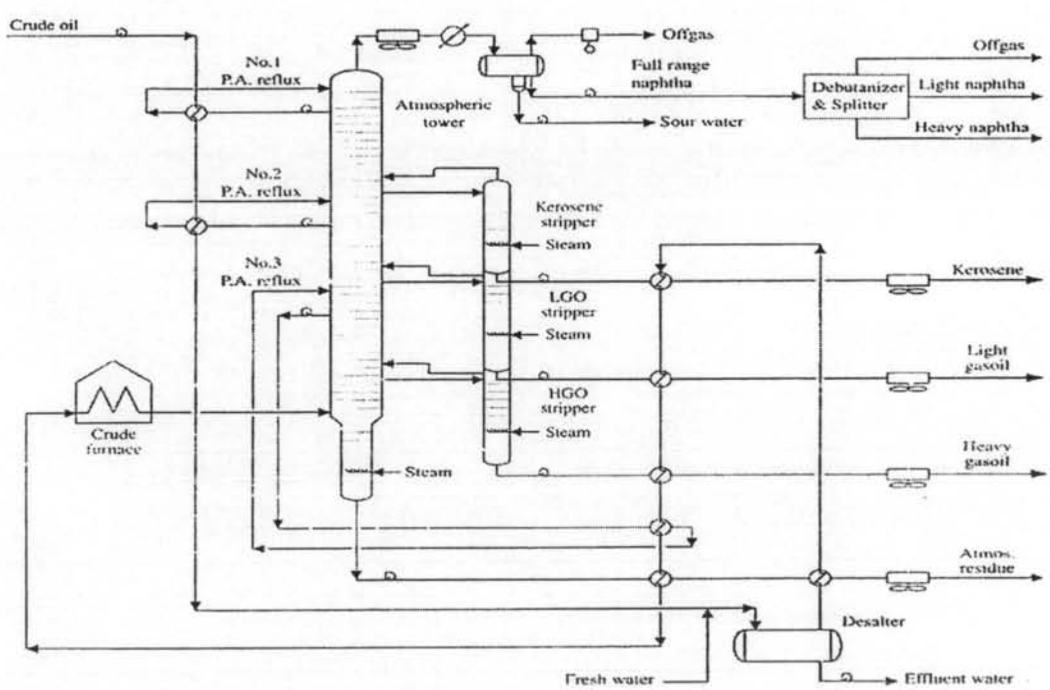

Figure 2.1 Process Flow Scheme of an atmospheric distillation unit (Lorenz *et al.*, 1997).

The mixed vapor stream contacts down-flowing internal reflux liquid on the trays, where condensation and fractionation of distillate products take place. The internal reflux liquid is created by condensation of the ascending oil vapor that has contacted cooled pump-around liquid. Use of the several pump-around reflux systems prepares reflux streams of different temperature levels, and enables effective utilization of the reflux heat load for heating the crude oil feed, which improve the amount of energy consumption in distillation unit. The condensed liquid is withdrawn as side-stream products such as kerosene, light gas oil and heavy gas oil. These streams are sent to side strippers, where the lighter gas and oil fractions are removed by steam stripping for adjustment of the flash point. The bottoms of the side strippers are withdrawn as distillate products such as kerosene, light gas oil and heavy gas oil. The overhead vapor of the atmospheric tower is condensed by an overhead condenser(s). The condensed liquid, called full boiling range naphtha, is sent to a debutanizer to remove the butane and lighter in it. The debutanizer offgas and gases not condensed in condenser(s) of the atmospheric tower are sent to a gas concentration unit to recover propane and butane (LPG). The debutanized full range naphtha is separated into light naphtha and heavy naphtha by a splitter.

2.1.1 Crude Oil Characterization

Crude oil is a mixture of hundreds of hydrocarbons of all type (Aromatic, Aliphatic, Cycloalyphatic, etc) with water, salts, sulfur and nitrogen containing compounds and some metal complexes. Thus, instead of straight composition, alternative measures are used to characterize it.

2.1.1.1 Density

The main classification of crude is by referring to their density. Light crudes have a specific gravity (d) smaller than 0.825, medium crudes between 0.825 and 0.875, heavy crudes between 0.875 and 1.000 and extra heavy crudes larger than 1.000. The petroleum industry uses another scale (API)

$$API = \frac{141.5}{d} - 131.5 \qquad (2.1)$$

Thus, light oils have a large API gravity and heavy ones a low one. Typical low values are close to 15 (some Venezuelan oils) to about 40 (some middle east oils).

2.1.1.2 True Boiling Point (TBP)

The true boiling point curve is basically a plot of boiling point of each component of the mixture as a function of the cumulative volumetric fraction distilled. In practice, the ASTM D2892 test method (ASTM is an acronym for American Society for testing and Materials) is used. It consists of using a 15 to 18 tray distillation column operating at a 5:1 reflux ratio. An example of such curve is shown in Figure 2.2

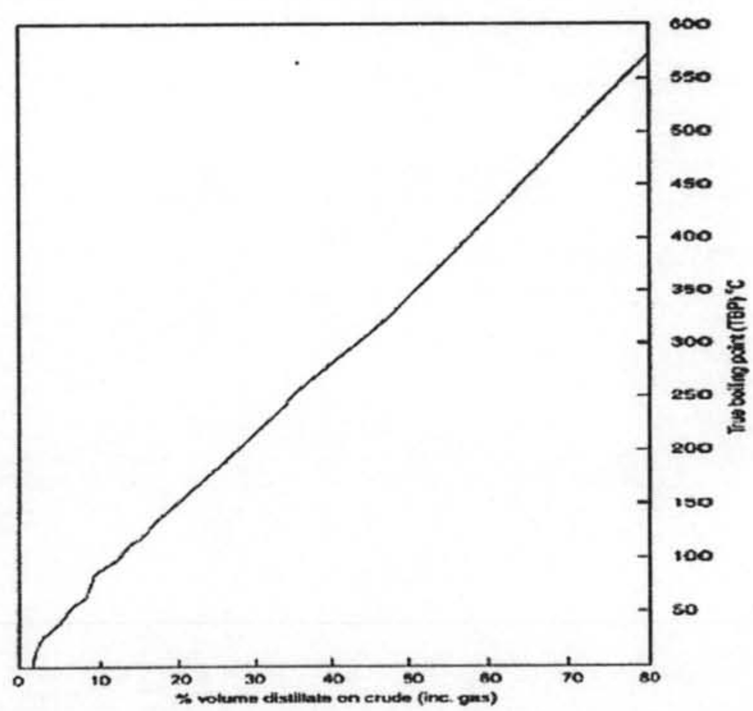

Figure 2.2 TBP curve for Kuwait Crude (Jones D., 1995).

2.1.2 Product Properties

The TBP curve, together with the density curve can be used to characterize the products that will result from fractionation. This is important, especially if one wants to generate some initial value to aid simulations to converge.

To do this, one needs to define cut points. These are the temperatures that represent the limits of the fraction of the TBP curve that will be product. This is illustrated in Figure 2.3

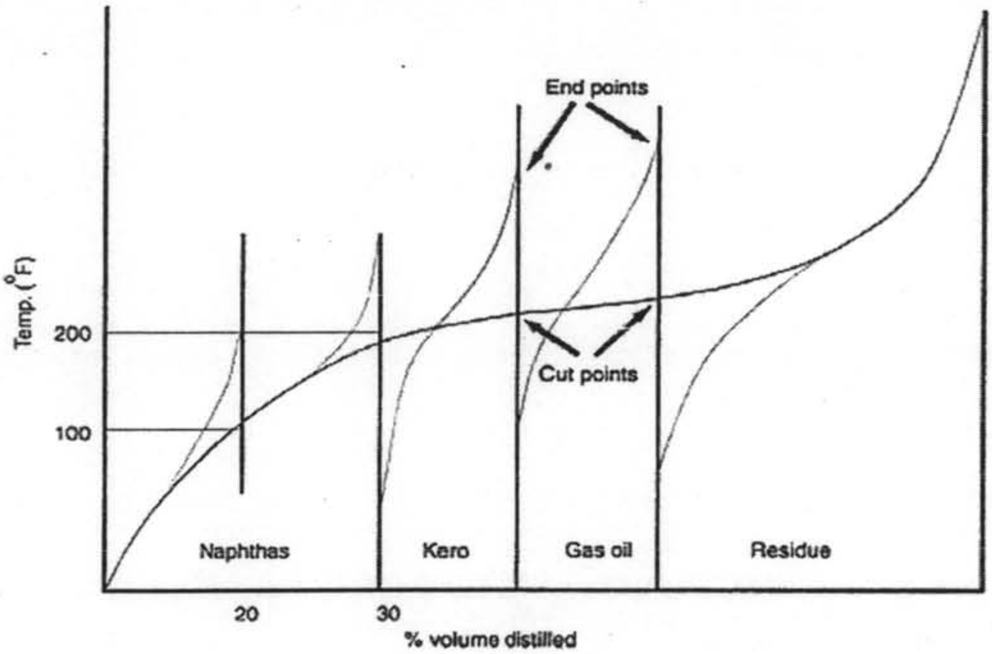

Figure 2.3 Cut points and end points (Bagajewicz, 2004).

Once the cut points are established, when the fractions are analyzed independently, their end points will be higher than the upper cut point temperature. This is due to the absence of the heavy components. The same can be said for the initial points, which will be lower than that of the mixture. This is also illustrated in Figure 2.3

The concept of Gap, which is used to quantify the overlap between adjacent fractions Consider two adjacent fractions. The (5-95) Gap is the difference between the 5% ASTM temperature of a heavy fraction (H) minus the 95% ASTM temperature of the lighter fraction (L), that is:

$$Gap_{H,L} = T_5^H - T_{95}^L \tag{2.2}$$

Table 2.1 The Gaps recommended by Watkins

Products	Specification
Kerosene - Naphtha	(5-95) Gap ≥ 16.7 °C
Diesel - Kerosene	(5-95) Gap ≥ 0 °C
Gas Oil - Diesel	(5-95) Gap -5.6 °C ≥ -11 °C

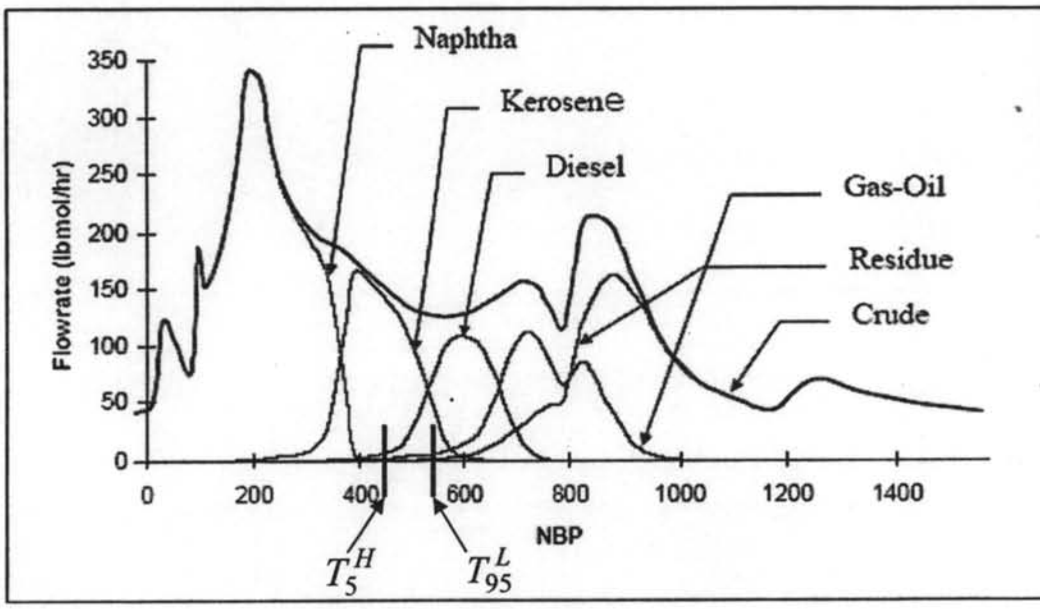

Figure 2.4 Pseudo-Component Flow rate Distribution (Bagajewicz, 2004).

The figure 2.4 shows the flow rate of pseudo-component and show the sample of gap between diesel and kerosene.

Another property needed to specify a particular product is its flash point, which is defined as the temperature at which the vapour above the oil will momentarily flash or explode. Usually, this is determined in the laboratory. It is also related to the ASTM 5% point by means of the following correlation:

$$Flash\ Pt.\,(°F) = 0.77\,(ASTM\ 5\%\ °F - 150\ °F) \qquad (2.3)$$

2.2 State of the Arts for Heat Exchanger Network Synthesis

Heat exchanger network (HEN) synthesis is one of the most extensively studied problems in industrial process synthesis. This is attributed to the importance of determining the energy costs for a process and improving the energy recovery in industrial sites. The first systematic method to which was energy recovery was the thermodynamic approach, the concept of pinch introduced during the 1970s.

The first approaches in the 1960s and early 1970s treated the HEN synthesis problem without applying decomposition into sub-tasks. The limitations of optimization techniques were the bottleneck of the mathematical approaches at that time. For the synthesis problem of the HEN, the thermodynamic approach of pinch analysis was introduced by the work of Hohmann (1971) and Linnhoff and Flower (1978). As a result of the pinch concept, the single task approaches were shifted to procedures introducing techniques for decomposing the problem into three subtasks; minimum utility cost, minimum number of units and minimum investment cost network configurations. The main advantage of decomposing the HEN synthesis problem is that sub-problems can be treated in a much easier fashion than the original single-task problem. The sub-problems are the following

2.2.1 Minimum Utility Cost Target

The maximum energy recovery can be achieved in a feasible HEN for a fixed heat recovery approach temperature (HRAT), allowing for the elimination of several non-energy efficient HEN structures. Minimum utility cost was first introduced by Hohmann (1971) and.Linnhoff and Flower (1978) and later as an LP transportation model by Cerda et al. (1983), being an improvement of the LP transshipment model of Papoulias and Grossmann (1983).

2.2.2 Minimum Number of Units Target

The match combination can be determined with the minimum number of units and their load distribution for a fixed utility cost. The MILP transportation model of Cerda and Westerberg (1983) and the MILP transshipment model of Papoulias and Grossmann (1983) are the most common, while the vertical heat transfer formulation of Gundersen and Grossmann (1990) and Gundersen, Duvold and Hashemi-Ahmady (1996) are also used.

2.2.3 Minimum Investment Cost Network Configurations

It is based on the heat load and match information of previous targets. Using the superstructure-based formulation, developed by Floudas et al. (1986), the NLP problem is formulated and optimized for the minimum total cost of the network. The objective function in this model is the investment cost of the heat exchangers that are postulated in a superstructure.

However, limitation of decomposition-based methods is that costs due to energy, units and area cannot be optimized simultaneously, and as a result the trade-offs are not taken into account appropriately. Thus, simultaneous heat exchanger network synthesis methods are taken place. The simultaneous approaches purpose to find the optimal network with or without some decomposed problem. The simultaneous optimization generally results in MINLP formulations, which assumptions exist to simplify these complex models.

Floudas and Ciric (1989) proposed a match-network hyperstructure model to simultaneously optimize all of the capital costs related to the heat exchanger network. This MINLP formulation is based on the combination of the transshipment model of Papoulias and Grossmann (1983) for match selection, and the minimum investment cost network configuration model of Floudas and Grossmann (1986) for determining the heat exchanger areas, temperatures and the flow rate in the network. The proposed simultaneous synthesis may still lead to suboptimal networks, since the value for HRAT must be specified before the design stage.

In 1990, Yee and Grossmann formulated another simultaneous synthesis where within each stage exchanges of heat can occur between each hot and cold stream. This model can simultaneously target for area and energy cost while properly accounting for the differences in heat transfer coefficients between the streams. The match-network hyperstructure model was then further modified by Ciric and Floudas (1991) to treat HRAT as an explicit optimization variable. This MINLP formulation included any decomposition into design targets and simultaneously optimizes trade-offs between energy, units and area. Ciric and Floudas (1991) also demonstrated the benefit of a simultaneous approach versus sequential methods.

In 1986, Floudas and Grossmann introduced a multiperiod MILP model for the minimum utilities cost and minimum number of match of target problems, based on Papoulias and Grossmann's (1983) transshipment model. In this model the changes in the pinch point and utility required at each time period are taken into account. Extensions were presented first by Floudas and Grossmann (1987), and NLP formulation based on a superstructure presentation of possible network topologies to derive automatically network configurations that feature minimum investment cost, minimum number of units, and minimum utility cost for each time period.

Ji and Bagajewicz (2001) introduced the rigorous procedure for the design of conventional atmospheric crude fractionation units. Part I aims to find the best scheme of a multipurpose crude distillation unit which can process the various crude. Heat demand-supply diagrams are used as a guide for optimal scheme instead of grand composite curves. Thus, the total energy consumption from stream, heater and cooler is clearly shown and this leads the process to be easily optimal. In part II, 2001, Soto and Bagajewicz attempted to design a multipurpose heat exchanger network that can handle in variety of crude. In order to overcome the smaller gap between hot and cold composite curves, models that fixed the heat recovery by using the minimum heat recovery approximation temperature (HRAT) and the exchanger minimum approach temperature (EMAT) was performed. In 2003, Part III, Soto and Bagajewicz established a model to determine a heat exchanger network with only two branches above and below desalter. The total annualized costs, operating cost and depreciation of capital, of solution limited to one or two branches are compared with the results of four branches. In this part, the present model is based on a transshipment model and the vertical heat exchange constraints combined with HRAT/EMAT. In addition, investment cost is not directly controlled by this model, but further indirectly controlled by limiting of the minimum unit numbers. The smaller number of units leads to minimal capital cost and energy consumption simultaneously.

New rigorous one-step MILP formulation for heat exchanger network synthesis was developed by Barbaro and Bagajewicz (2002). This methodology does neither rely on traditional supertargeting network design by the pinch technology, nor is a nonlinear model, but further use only one-step to optimize the solution. Cost-optimal networks, cost-effective solutions, can be obtained at once by using this model.

2.3 Concepts for Using Mathematical Programming in Process Integration

Mathematical programming is a class of methods for solving constrained optimization problems. Since both continuous and binary variables can be used in the corresponding mathematical programming models, these methods are perfectly suited for typical design tasks encountered in process synthesis and process integration.

Generally, a mathematical programming model consists of an objective function (typically some economic criteria) and a set of equality constraints as well as inequality constraints. The general form is indicated below

$$\text{Min } f(x,y)$$

Subject to

$$g(x,y) \leq 0$$

$$h(x,y) = 0$$

where

$$x \in R^n$$

$$y \in [0,1]^m$$

It should be noticed that the variables x and y in general are vectors of variables, and that the constraints g and h similarly are vectors of functions. The objective function (f) is assumed to be a scalar.

The mathematical modeling of the systems lead to different types of formulations, such as Linear Programming (LP), Mixed Integer Linear Programming

(MILP), Non-Linear Programming (NLP) and Mixed Integer Non-Linear Programming (MINLP) models.

If there are no binary variables, and all functions f, g and h are linear, we have the simplest class of problems, the Linear Programming (LP) models. Using the simplex algorithm, for example, LP models with hundreds of thousands variables and constraints can be solved in reasonable times with today's computer resources. If there are no binary variables, and at least one of the functions f, g and h are non-linear, we have a Non-Linear Programming (NLP) problem. These are generally much harder to solve, especially if the non-linearities are non-convex, because a local optimum may be found.

If there are binary variables in the model, and all functions f, g and h are linear, we have a Mixed Integer Linear Programming (MILP) problem. These can be solved to global optimality provided the number of binary variables does not cause a combinatorial explosion. Finally, if there are binary variables in the model, and at least one of the functions f, g and h are non-linear, we have the hardest class of problems, Mixed Integer Non-Linear Programming (MINLP) models. Unfortunately, most real design problems are of the MINLP type with significant problems related to computer time and local optima.

2.4 Model for Grass-Root Synthesis

This MILP model is based on the transportation transshipment scheme and it has the following features

- Counts heat exchangers units and shells
- Approximates the area required for each exchanger unit or shell
- Controls the total number of units
- Implicitly determines flow rates in splits
- Handles non-isothermal mixing
- Identifies bypasses in split situations when convenient
- Controls the temperature approximation (HRAT/EMAT of ΔT_{min}) when desired

- Can address block-design through the use of zones
- Allows multiple matches between two streams

2.5 Mathematical Model

2.5.1 Set Definitions

A set of several heat transfer zones is defined, namely $Z - \{z \mid z$ is a heat transfer zone$\}$

Use of zones can be used to separate the design in different sub-networks that are not interrelated, simplifying the network and the problem complexity. Next, the following sets are used to identify hot streams, cold streams, hot utilities and cold utilities.

H^z = { $i \mid i$ is a hot stream present in zone z }

C^z = { $j \mid j$ is a cold stream present in zone z }

HU^z = { $i \mid i$ is a heating utility present in zone z } $(HU^z \subset H^z)$

CU^z = { $j \mid j$ is a heating utility present in zone z } $(CU^z \subset C^z)$

Moreover, several temperature intervals are considered in each zone, in order to perform the heat balances and the area calculations. The different sets related to the temperature intervals are defined as

M^z = { $m \mid m$ is a temperature interval in zone z }

M_i^z = { $m \mid m$ is a temperature interval belonging to zone z, in which hot stream i is presented }

N_j^z := { $n \mid n$ is a temperature interval belonging to zone z, in which cold stream j is presented }

H_m^z = { $i \mid i$ is a hot stream present in temperature interval m in zone z }

C_n^z = { $j \mid j$ is a cold stream present in temperature interval n in zone z }

m_i^0 = { m | m is the starting temperature interval for hot stream i }

n_j^0 = { n | n is the starting temperature interval for cold stream j }

m_i^f = { m | m is the final temperature interval for hot stream i }

n_i^f = { n | n is the final temperature interval for cold stream j }

The MILP model uses the temperature intervals to perform energy balances and mass flow balances. At each temperature interval, the variables $\hat{q}_{ijm}^{z,H}$ account for the overall heat exchanged in interval m of hot stream i and all the intervals of cold stream j, in zone z. Familiar with $\hat{q}_{ijm}^{z,H}$, the variables $\hat{q}_{ijn}^{z,C}$ are used to compute the overall heat received by cold stream j at interval n from all intervals of hot stream i. The variables $q_{im,jn}^{z,H}$ are used to formulate the heat transportation from interval to interval between both streams.

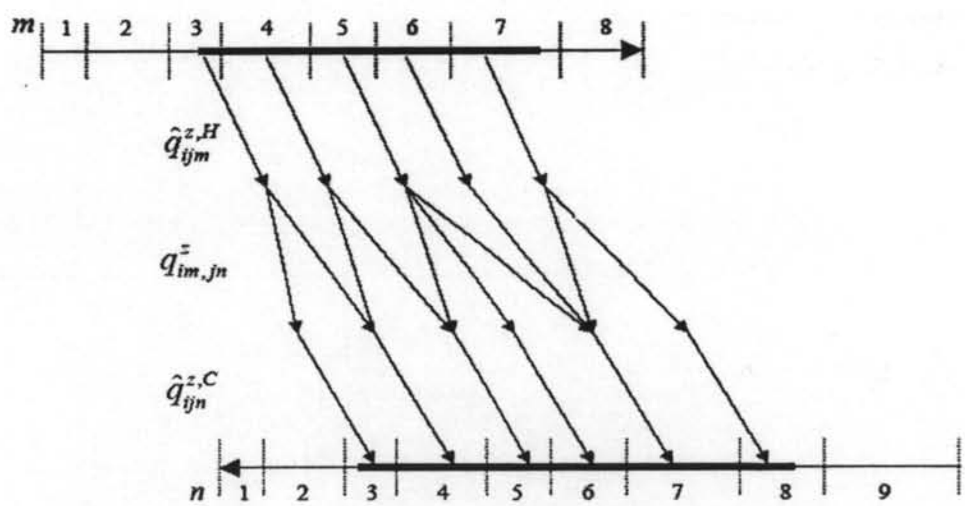

Figure 2.5 Basic scheme of the transportation/transshipment model.

A number of sets are introduced to define all possible sources and destinations for heat transfer in this transportation scheme.

P = { (i,j) | heat exchange match between hot stream i and cold stream j is permitted }

P^H_{im} = { j | heat transfer from hot stream i at interval m to cold stream j is permitted }

P^C_{jm} = { i | heat transfer from hot stream i to cold stream j at interval n is permitted }

Set P defines as allowed matching between hot and cold streams. In order not to against the thermodynamically possible, permitted and forbidden heat exchange matches can be set up by the designer. Sets P^H_{im} and P^C_{jm} define as feasible heat transfer flows at each temperature interval.

Finally, the following sets allow the designer to manage additional features of the formulation.

NI^H = { i | non-isothermal mixing is permitted for hot stream i }

NI^C = { j | non-isothermal mixing is permitted for cold stream j }

S^H = { i | splits are allowed for hot stream i }

S^C = { j | splits are allowed for cold stream j }

B = { (i,j) | more than one heat exchanger unit is permitted between hot stream i and cold stream j }

The sets NI^H and NI^C are used to specify whether non-isothermal mixing of stream splits is permitted, while sets S^H and S^C establish the possibility of stream splits. Finally, set B is used to allow more than one heat exchanger match between two streams, as shown in Figure 2.6 for match (i_1,j_1). Thus, this model is able to distinguish situations where more than one heat exchanger unit is required to perform a heat exchange match. Next, the different equations of the model for grass-root design of heat exchanger networks are introduced.

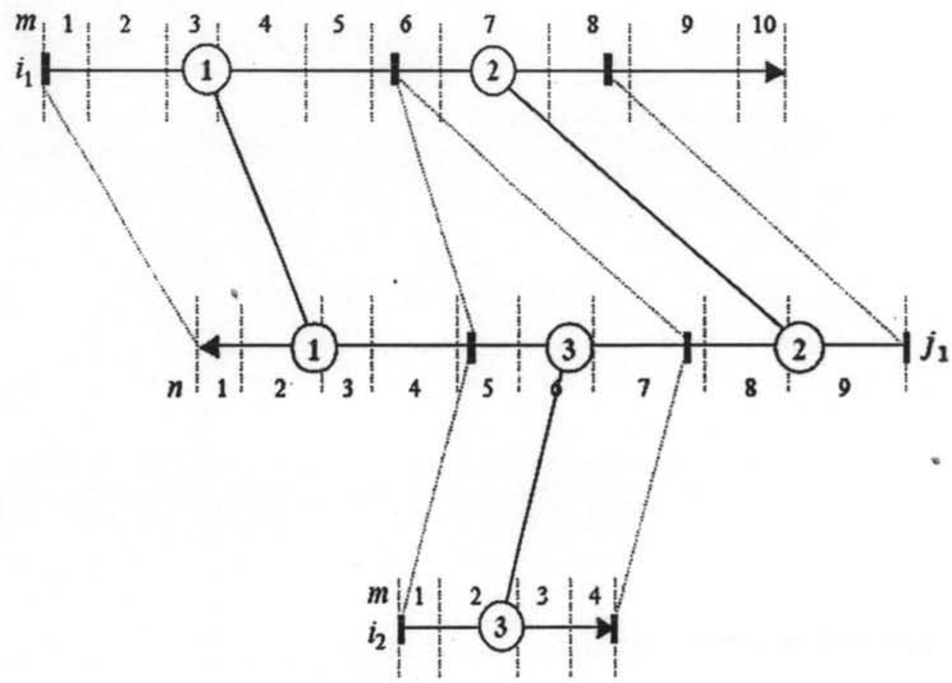

Figure 2.6 A case where more than one heat exchanger unit is required for a match (i,j).

2.5.2 Heat Balance Equations

The total heat available on each hot streams or the total heat demand of cold streams is equal to the heat transferred to the specific intervals. For heating and cooling utilities, these balances are described by the following equations.

Heat balance for heating utilities

$$F_i^H\left(T_m^u - T_m^L\right) = \sum_{\substack{n\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j\in C_n^z \\ j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z \qquad z\in Z; m\in M^z; i\in H_m^z; i\in HU^z \qquad (2.4)$$

Heat balance for cooling utilities

$$F_i^C\left(T_n^u - T_n^L\right) = \sum_{\substack{m\in M^Z \\ T_n^L < T_m^U}} \sum_{\substack{i\in H_m^z \\ i\in P_{jn}^C \\ j\in P_{im}^H}} q_{im,jn}^z \qquad z\in Z; n\in M^z; j\in C_n^z; j\in CU^z \qquad (2.5)$$

The heat balances for process streams where only isothermal mixing of splits is considered are stated below.

Heat balance for hot process streams $- i \notin NI^H$

$$\Delta H_{im}^{z,H} = \sum_{\substack{n \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j \in C_n^z \\ j \in P_{im}^H \\ i \in P_{jn}^C}} q_{im,jn}^z \qquad z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; i \notin NI^H \quad (2.6)$$

Heat balance for cold process streams $- j \notin NI^C$

$$\Delta H_{jn}^{z,C} = \sum_{\substack{m \in M^Z \\ T_n^L < T_m^U}} \sum_{\substack{i \in H_m^z \\ i \in P_{jn}^C \\ j \in P_{im}^H}} q_{im,jn}^z \qquad z \in Z; n \in M^z; j \in C_n^z; j \notin CU^z; i \notin NI^C \quad (2.7)$$

The hot and cold cumulative heat transfer is defined in the next sets of equations. This cumulative transfer is introduced for presentation convenience because it is related to the equations that define the existence of heat exchangers in the different temperature intervals.

Cumulative heat transfer from hot stream i at interval m to cold stream j

$$\hat{q}_{ijm}^{z,H} = \sum_{\substack{n \in M^z ; T_n^L < T_m^U \\ j \in C_n^z ; i \in P_{jn}^C}} q_{im,jn}^z \qquad z \in Z ; m \in M^z ; i \in H_m^z ; j \in C^z ; j \in P_{im}^H \quad (2.8)$$

Cumulative heat transfer to cold stream j at interval n from hot stream i

$$\hat{q}_{ijn}^{z,C} = \sum_{\substack{m \in M^z ; T_n^L < T_m^U \\ i \in H_m^z ; j \in P_{im}^H}} q_{im,jn}^z \qquad z \in Z; n \in M^z; i \in H^z; j \in C_n^z; i \in P_{jn}^C \quad (2.9)$$

2.5.2.1 Heat Balance Equations for Streams Allowed to Have Non-Isothermal Splii Mixing

A new variable ($\overline{q}$) is introduced to account for heat flows between intervals of the same stream that correspond to such mixing. Heat is artificially transferred from one interval to another within the same stream to account for non-isothermal mixing conditions. Figure 2.7 illustrates how this non-isothermal mixing of stream splits is taken into account.

Following the Figure 2.7, cold stream j has been split to exchange heat between stream i_1 and i_2 and non-isothermal mixing between these splits is allowed. This figure shows the upper portion, the split in the cold stream spans temperature intervals 3 and 8, while the lower portion spans from interval 5 to interval 8. However, the whole stream spans from interval 4 to interval 8 after mixing and the non-split part spans the rest of the intervals. In order to complete the non-isothermal mixing which allow one branch to reach a larger temperature as shown in the Figure 2.7, interval 3 get more heat than its demand $\left(\Delta H_{j3}^{z,C}\right)$ and transfer this surplus heat to interval 4 and 5. Interval 4 and 5 receive less heat than their demand from the hot streams, with the difference being transferred from interval 3 by the heat $\bar{q}$. The heat balance equations for non-isothermal mixing of split are shown as

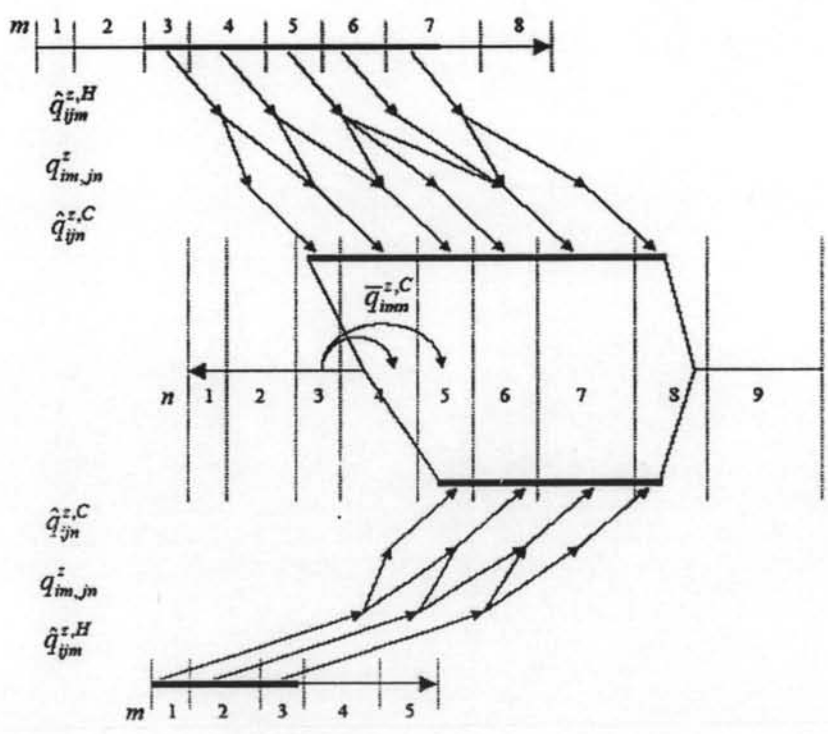

Figure 2.7 Non-isothermal split mixing.

Heat balance for hot streams (non-isothermal mixing allowed)

$$\Delta H_{im}^{z,H} = \sum_{\substack{n \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j \in C_n^z \\ j \in P_{im}^H \\ i \in P_{jn}^C}} q_{im,jn}^z + \sum_{\substack{n \in M^z \\ n > m}} \sum_{i \in H_n^z} \overline{q}_{inm}^{z,H} - \sum_{\substack{n \in M^z \\ n < m}} \sum_{i \in H_n^z} \overline{q}_{imn}^{z,H} \qquad z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; i \in NI^H$$

(2.10)

Heat balance for cold streams (non-isothermal mixing allowed)

$$\Delta H_{jn}^{z,C} = \sum_{\substack{m \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{i \in H_m^z \\ i \in P_n^C \\ j \in P_m^H}} q_{im,jn}^z + \sum_{\substack{m \in M^z \\ m < n}} \sum_{j \in C_m^z} \overline{q}_{jmn}^{z,C} - \sum_{\substack{m \in M^z \\ m > n}} \sum_{j \in C_m^z} \overline{q}_{inm}^{z,C} \qquad z \in Z; n \in M^z; j \in C_n^z; j \notin CU^z; j \in NI^C$$

(2.11)

In addition, the condition that heat cannot be transferred within a stream if there is no heat transfer with other stream need to be established in the model. Consequently, these equations force $\overline{q}$ to be zero whenever there is no heat transferred with other streams.

Heat balance for hot streams – i $\in NI^H$

$$\sum_{\substack{n \in M^z \\ n < m}} \sum_{i \in H_n^z} \overline{q}_{inm}^{z,H} \leq \sum_{\substack{n \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j \in C_n^z; j \in P_{im}^H \\ i \in P_{jn}^C}} q_{im,jn}^z \qquad z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; i \in NI^H \qquad (2.12)$$

Heat balance for cold streams – i $\in NI^C$

$$\sum_{\substack{m \in M^z \\ m > n}} \sum_{j \in C_m^z} \overline{q}_{inm}^{z,C} \leq \sum_{\substack{m \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{i \in H_m^z; i \in P_n^C \\ j \in P_m^H}} q_{im,jn}^z \qquad z \in Z; n \in M^z; j \in C_n^z; j \notin CU^z; j \in NI^C \qquad (2.13)$$

2.5.3 Heat Exchanger Definition and Count

The model is defined as a consecutive series of heat exchange shells between a hot and a cold stream. For each temperature interval, heat transfer is accounted using the cumulative heat $(\hat{q})$, while the existence of a heat exchanger for a given interval is defined by a new variable (Y), which determines whether heat exchange takes place or not at that interval. In addition, two new variables (K and $\hat{K}$), which are closely related to the Y variables, are introduced in order to indicate whether a heat exchanger begins or ends at a specific interval. The use of these new variables to count units has been previously proposed by Bagajewicz and Rodera (1998) and later used by Bagajewicz and Soto (2001, 2003) and Ji and Bagajewicz (2002).

Even placing the multiple shells, this seems to be as a single heat exchanger. Nevertheless, there are cases where non-consecutive series of shells could be allowed. For those cases, different heat exchangers have to be defined for each series. In order to consider the possibility of multiple heat exchangers between the same pair of streams, the additional equations are required.

For the case where only one exchanger is allowed per match between streams i and j, $(i,j)\notin B$, then binary variable $Y_{ijm}^{z,H}$, and two continuous variables $K_{ijm}^{z,H}$, $\hat{K}_{ijm}^{z,H}$ are used. The binary variable $Y_{ijm}^{z,H}$, indicates that there is a match between stream i at interval m receiving heat from some intervals of stream j. In turn, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ indicate the beginning and end of a string of intervals for which the binary variable is active. Conversely, when $(i,j)\in B$, $Y_{ijm}^{z,H}$ is declared as continuous and $K_{ijm}^{z,H}$, $\hat{K}_{ijm}^{z,H}$ are set up as binary. The Y variables are probably greater or equal than one if a heat exchanger exists for the correspondent streams and interval. However, all variables $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are getting to be zero when no heat exchanger exists matching streams i and j.

The following group of constraints is used to determine the existence of a heat exchanger for a given pair of streams and temperature intervals. When only one heat exchanger is allowed per match, constraint (2.18)–(2.22) and (2.23)–(2.27) are valid. The equation (2.28) applies further in cases where more than one exchanger is permitted. However, equations (2.18) and (2.23) only apply to the first and last interval of a hot stream, respectively, while the sets of equations (2.19)–(2.22) and (2.24)–(2.27) are used for all intervals.

Bounds on cumulative heat transfer for hot process streams

$$q_{ijm}^{L} Y_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} \le \Delta H_{im}^{z,H} Y_{ijm}^{z,H} \qquad z\in Z;\, m\in M^{z};\, i\in H_{m}^{z};\, i\notin HU^{z};\, j\in C^{z};\, j\in P_{im}^{H} \qquad (2.14)$$