

**A MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL FOR
DESIGNING/RETROFITTING HEAT EXCHANGER NETWORK OF
CRUDE FRACTIONATION UNIT**

Kitisak Junlobol

A Thesis Submitted in Partial Fulfilment of the Requirements
for the Degree of Master of Science
The Petroleum and Petrochemical College, Chulalongkorn University
in Academic Partnership with
The University of Michigan, The University of Oklahoma,
Case Western Reserve University and Institut Français du Pétrole
2006

ISBN 974-9937-44-9

Thesis Title: A Mixed Integer Linear Programming (MILP) Model for Designing/Retrofitting Heat Exchanger Network of Crude Fractionation unit
By: Kitisak Junlobol
Program: Petroleum Technology
Thesis Advisors: Asst. Prof. Kitipat Siemanond
Prof. Miguel Bagajewicz

Accepted by the Petroleum and Petrochemical College, Chulalongkorn University, in partial fulfilment of the requirements for the Degree of Master of Science.

Nantaya Yanumet
..... College Director
(Assoc. Prof. Nantaya Yanumet)

Thesis Committee:

Kitipat Siemanond
.....
(Asst. Prof. Kitipat Siemanond)

Miguel Bagajewicz
.....
(Prof. Miguel Bagajewicz)

Thirasak Rirksomboon
.....
(Assoc. Prof. Thirasak Rirksomboon)

Pramoch Rangsunvigit
.....
(Assoc. Prof. Pramoch Rangsunvigit)

ABSTRACT

4773006063: Petroleum Technology Program
Kitisak Junlobol: A Mixed Integer Linear Programming (MILP)
Model for Designing/Retrofitting Heat Exchanger Network of Crude
Fractionation Unit.
Thesis Advisors: Asst. Prof. Kitipat Siemanond, and
Prof. Miguel Bagajewicz 225 pp. ISBN 974-9937-44-9

Keywords: Heat Exchanger Network/ Mixed Integer Linear Programming/
Crude Fractionation

Today, an increasing trend of crude oil price is one of many causes of high production cost because oil is a major energy source used in industry. An effective design of heat exchanger network (HEN) is one of the solutions that can reduce production cost because the heat exchanger network can recover the energy. A crude fractionation unit consumes high energy about 30-40% of a refinery. In this thesis, a strategy to design efficient HENs of the refinery is proposed as the Mixed Integer Linear Programming (MILP) formulation based on the special transshipment structure concept. This methodology can generate networks where utility cost, heat exchanger areas and selection of matches are optimized simultaneously. And it can design the appropriate flow rate of pump-around. In addition, the simplicity in model assumption, non-isothermal mixing, comes with handling constraints such as stream splitting and allowed/forbidden matches which bring the model structure more convenient to use. This research gives the efficient HENs to a fractionation unit of light, intermediate and heavy crudes by saving the energy cost, capital cost and area cost about 123.91% from the existing network.

บทคัดย่อ

กิตติศักดิ์ จุลโกลบ: แบบจำลอง MILP เพื่อการออกแบบเครือข่ายแลกเปลี่ยนความร้อนของหอกลั่นน้ำมันดิบ (A Mixed Integer Linear Programming (MILP) Model for Designing/Retrofitting a Heat Exchanger Network for Crude Fractionation Column)
 อ. ที่ปรึกษา: ผศ. ดร. กิติพัฒน์ สีมานนท์ และ ศ. ดร. มิเกล บากาเฮวิช 225 หน้า
 ISBN 974-9937-44-9

ในปัจจุบันนี้แนวโน้มการเพิ่มสูงขึ้นของราคาน้ำมัน เป็นหนึ่งในหลายสาเหตุที่ทำให้ค่าใช้จ่ายในกระบวนการผลิตเพิ่มสูงตาม เพราะน้ำมันเป็นแหล่งพลังงานหลักที่ใช้ในอุตสาหกรรม การออกแบบระบบแลกเปลี่ยนความร้อนที่มีประสิทธิภาพ ถือเป็นอีกหนึ่งทางเลือกที่สามารถช่วยลดค่าใช้จ่ายในกระบวนการผลิต เพราะการออกแบบเครือข่ายแลกเปลี่ยนความร้อนสามารถนำพลังงานกลับมาใช้ใหม่ได้ หอกลั่นน้ำมันดิบใช้ปริมาณพลังงานสูง ถึงประมาณ 30-40 เปอร์เซ็นต์ ของปริมาณความร้อนในโรงกลั่นน้ำมัน วิทยานิพนธ์นี้ นำเสนอวิธีการออกแบบเครือข่ายแลกเปลี่ยนความร้อนของหอกลั่นน้ำมันดิบ โดยอาศัยแบบจำลองทางคณิตศาสตร์ มิกส์อินทีเจอร์ ลีเนียโปรแกรมมิง โดยอ้างถึงหลักการของ ทรานซิพเมนต์โมเดล โดยวิธีการที่ใช้จะสามารถคำนวณค่าใช้จ่ายทั้งหมดในเครือข่ายการแลกเปลี่ยนความร้อน ซึ่งประกอบด้วย ค่าใช้จ่ายของ ๑) สารหล่อเย็น พื้นที่ และจำนวนของเครื่องแลกเปลี่ยนความร้อน ในเวลาเดียวกัน และยังสามารถออกแบบอัตราการไหลของป้อนอะรวานที่เหมาะสม นอกจากนี้ การสมมุติค่าที่ทำให้โมเดลง่ายในการคำนวณแบบจำลองทางคณิตศาสตร์นี้ยังสามารถประยุกต์ใช้กับกระบวนการที่มีข้อจำกัดต่างๆ เช่น ข้อจำกัดในการจับคู่ของสายสตรีมร้อนและสายสตรีมเย็น การแบ่งสายสตรีมเพื่อเพิ่มความสามารถในการแลกเปลี่ยนความร้อนเป็นต้น ในวิทยานิพนธ์ฉบับนี้ได้ แสดงระบบแลกเปลี่ยนความร้อนที่มีประสิทธิภาพของโรงกลั่นน้ำมัน สำหรับน้ำมันดิบแบบเบา กลางและหนัก โดยได้ประหยัดค่าใช้จ่ายด้านพลังงาน ค่าติดตั้ง และค่าขนาดพื้นที่ของเครื่องแลกเปลี่ยนความร้อน ได้ถึง 123.91 เปอร์เซ็นต์

ACKNOWLEDGEMENTS

This work has been a very good experience. It would not have been successful without the assistance of the following individuals and organization.

This thesis work is partially funded by Postgraduate Education and Research Programs in Petroleum and Petrochemical Technology (PPT Consortium).

I gratefully acknowledge Prof. Miguel Bagajewicz, and Asst. Prof. Kitipat Siemanond, my advisors, for several enlightens suggestions, discussions, and encouragement throughout the course of my work. This thesis would never have been completed without their consistent help.

I would also thank Assoc. Prof. Pramoch Rangsunvigit, and Assoc. Prof. Thirasak Rirksomboon, my thesis committee, for their well-intentioned suggestions and comments is greatly acknowledged.

I would like to take this opportunity to thank all The Petroleum and Petrochemical College's staffs who supported me throughout my thesis works and finally forward my special thanks to my friends and all senior students for their friendly help, cheerfulness and suggestions.

TABLE OF CONTENTS

	PAGE
Title Page	i
Abstract (in English)	iii
Abstract (in Thai)	iv
Acknowledgements	v
Table of Contents	vi
List of Tables	x
List of Figures	xii
Abbreviations	xv
 CHAPTER	
I INTRODUCTION	1
 II BACKGROUND AND LITERATURE SURVEY	
2.1 Atmospheric Distillation Unit	3
2.1.1 Crude Oil Characterization	4
2.1.2 Product Properties	5
2.2 State of the Arts for Heat Exchanger Network Synthesis	8
2.2.1 Minimum Utility Cost Target	8
2.2.2 Minimum Number of Units Target	8
2.2.3 Minimum Investment Cost Network Configurations	9
2.3 Basic Concepts for Using Mathematical Programming in Process Integration	11
2.4 Model for Grass-Root Synthesis	12
2.5 Mathematical Model	13
2.5.1 Set Definitions	13
2.5.2 Heat Balance Equations	16

CHAPTER	PAGE
2.5.3 Heat Exchanger Definition and Count	19
2.5.4 Heat Transfer Consistency	26
2.5.5 Flow Rate Consistency Within Heat Exchangers	29
2.5.6 Temperature Difference Enforcing	34
2.5.7 Heat Exchanger Area Calculation	37
2.5.8 Number of Shells	38
2.5.9 Objective Function	38
2.6 Model for Retrofit Heat Exchanger Network	39
2.6.1 Area Additions for Existing and New Heat Exchanger Unit	39
2.6.2 Objective Function	42
2.7 Model for Heat Exchanger Network Included Pump Around	43
2.7.1 Heat Balance Equations	43
2.7.2 Heat Exchanger Definition and Count	44
2.7.3 Heat Transfer Consistency	45
2.7.4 Flow Rate Consistency Within Heat Exchangers	48
2.7.5 Temperature Difference Enforcing	51
 III PROCEDURE	 56
3.1 Grass-root Design for Heat Exchanger Network	56
3.1.1 Study and Test The MILP Model	56
3.1.2 Apply The Automatic MILP Model	57
3.1.3 Analyze The Objective Value Stability	57
3.2 Retrofit Design for Heat Exchanger Network	57
3.3 Design of Heat Exchanger Network for Crude Fractionation Unit	57

CHAPTER	PAGE
3.3.1 Relationship between Duty of Pump Around and Side Stripper	57
3.3.2 New Parameters and Equations	58
3.4 Retrofit of Heat Exchanger Network for Crude Fractionation Unit	58
IV RESULTS AND DISCUSSION	59
4.1 Grass-roots Design for HEN	59
4.1.1 Case study 4.1 (Problem 4.1, Vipaturat's work)	59
4.1.2 Case study 4.2 (Problem 4.2, Vipaturat's work)	63
4.1.3 Case study 4.3 (Problem 4.4, Vipaturat's work)	68
4.2 Retrofit for Heat Exchanger Networks	73
4.2.1 Case study 4.4 (Problem 4.5, Vipaturat's work)	73
4.2.2 Case study 4.5 (Problem 4.6, Vipaturat's work)	75
4.2.3 Case study 4.6 (Problem 4.7, Vipaturat's work)	79
4.3 Design of Heat Exchanger Network for Crude Fractionation Unit	82
4.3.1 Relationship between Duty of Pump Around and Steam of Side Stripper	82
4.3.2 Preparation the Streams Data for Finding a Heat Exchanger Network	89
4.3.3 Heat Exchanger Network for Crude Fractionation Unit	93

CHAPTER	PAGE
4.4 Retrofit of Heat Exchanger Network for Crude Fractionation Unit	103
4.4.1 Retrofit of Heat Exchanger Network for Light Crude	103
V CONCLUSIONS AND RECOMMENDATIONS	106
REFERENCES	108
APPENDICES	
Appendix A Programming Model for Grass-Roots Design	110
Appendix B Programming Model for Heat Exchanger Network Retrofit	132
Appendix C Programming Model for Design Heat Exchanger Network of Crude Fractionation Unit	157
Appendix D Programming Model for Retrofit Heat Exchanger Network of Crude Fractionation Unit	190
CURRICULUM VITAE	225

LIST OF TABLES

TABLE	PAGE
2.1 The Gap recommended by Watkins	7
2.2 Values of $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ variables when $(i,j) \notin B$	22
2.3 Values of $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ variables when $(i,j) \in B$	23
2.4 Values of variables $K_{ijm}^{z,H}$, $\hat{K}_{ijm}^{z,H}$, $Y_{ijm}^{z,H}$ and $\tilde{q}_{ijm}^{z,H}$ when $(i,j) \in B$	26
4.1 Properties of stream for Problem 4.1	59
4.2 Cost data for Problem 4.1	59
4.3 Result of increasing number of temperature intervals for Problem 4.1	60
4.4 Properties of stream for Problem 4.2	63
4.5 Cost data for Problem 4.2	64
4.6 Result of increasing number of temperature intervals for Problem 4.2	68
4.7 Properties of stream for Problem 4.3	69
4.8 Cost data for Problem 4.3	69
4.9 Result of increasing number of temperature intervals for Problem 4.3	72
4.10 Properties of stream for Problem 4.4	73
4.11 Cost data for Problem 4.4	73
4.12 Resulting of retrofit heat exchanger for Problem 4.4	75
4.13 Annual cost comparison between original and retrofit network for Problem 4.4	75
4.14 Properties of stream for Problem 4.5	76
4.15 Cost data for Problem 4.5	76
4.16 Resulting of retrofit heat exchanger for Problem 4.5	78
4.17 Annual cost comparison between original and retrofit network for Problem 4.5	79

TABLE	PAGE
4.18 Properties of stream for Problem 4.6	79
4.19 Cost data for Problem 4.6	80
4.20 Resulting of retrofit heat exchanger for Problem 4.6	81
4.21 Annual cost comparison between original and retrofit network for Problem 4.6	81
4.22 Feedstock used for design	82
4.23 TBP data	83
4.24 Light-end composition of crude	83
4.25 Product specifications and withdrawal tray	86
4.26 Result for light crude	87
4.27 Relationship between duty of pump-around and steam of side stripper for light crude	88
4.28 Result for heavy crude	88
4.29 Relationship between duty of pump-around and steam of side stripper for heavy crude	89
4.30 Result for intermediate crude	89
4.31 Relationship between duty of pump-around and steam of side stripper for intermediate crude	89
4.32 Stream data of each crude type	90
4.33 Function of heat capacity for light crude	91
4.34 Function of heat capacity for intermediate crude	92
4.35 Function of heat capacity for heavy crude	93
4.36 Utility and heat exchanger cost for crude fractionation unit	93
4.37 Cost of steam stripper	93
4.38 The result of light crude oil	96
4.39 The result of intermediate crude oil	99
4.40 The result of heavy crude oil	102
4.41 Resulting of retrofit heat exchanger for light crude	104
4.42 Annual cost comparison between original and retrofit network for light crude	104

LIST OF FIGURES

FIGURE	PAGE
2.1 Process flow Scheme of an atmospheric distillation unit	3
2.2 TBP curve for Kuwait Crude	5
2.3 Cut points and end points	6
2.4 Pseudo-Component flow rate Distribution	7
2.5 Basic scheme of the transportation/transshipment model	14
2.6 A case where more than one heat exchanger unit is required for a match (i,j)	16
2.7 Non-isothermal split mixing	18
2.8 Heat exchanger definition when $(i,j) \notin B$	22
2.9 Heat exchanger definition when $(i,j) \in B$	23
2.10 Heat transfer consistency example when $(i,j) \in B$	26
2.11 Integer cut for heat exchanger end when $(i,j) \in B$	29
2.12 Flow rate consistency equations	30
2.13 Temperature difference assurance when splits are not allowed	34
2.14 Temperature difference assurance when splits are allowed	35
2.15 Temperature difference assurance at the hot end of an exchanger - $i \in S^H, j \in S^C, (i,j) \notin B$	36
2.16 Area computation when $(i,j) \in B$	41
4.1 Heat exchanger network for case study 4.1 at 26 temperature intervals	60
4.2 Trend of varying the intervals for case study 4.1	61
4.3 Heat exchanger network for case study 4.1	62-63
4.4 Heat exchanger network for case study 4.2	64-67
4.5 Trend of varying the intervals for case study 4.2	68
4.6 Heat exchanger network for case study 4.3	70-71
4.7 Trend of varying the intervals for case study 4.3	72

FIGURE	PAGE
4.8 The existing heat exchanger network for case study 4.4	74
4.9 The retrofit heat exchanger network for case study 4.4	74
4.10 The existing heat exchanger network for case study 4.5	77
4.11 The retrofit heat exchanger network for case study 4.5	78
4.12 The existing heat exchanger network for case study 4.6	80
4.13 The retrofit heat exchanger network for case study 4.6	80
4.14 Ji's model of crude fractionation column	84
4.15 Modified Ji's model of crude fractionation column	85
4.16 Setting calculator to calculate the gap specification	86
4.17 Setting controller to control steam of side stripper	87
4.18 Changing of heat capacity with temperature for light crude	90
4.19 Changing of heat capacity with temperature for intermediate crude	91
4.20 Changing of heat capacity with temperature for heavy crude	92
4.21 Heat exchanger network for light crude type 1	94
4.22 Heat exchanger network for light crude type 2	95
4.23 Heat exchanger network for light crude type 3	95
4.24 Heat exchanger network for light crude type 4	96
4.25 Heat exchanger network for intermediate crude type 1	97
4.26 Heat exchanger network for intermediate crude type 2	98

4.27	Heat exchanger network for intermediate crude type 3	98
4.28	Heat exchanger network for intermediate crude type 4	99
4.29	Heat exchanger network for heavy crude type 1	100
4.30	Heat exchanger network for heavy crude type 2	101
4.31	Heat exchanger network for heavy crude type 3	101
4.32	Heat exchanger network for heavy crude type 4	102
4.33	The existing heat exchanger network for light crude	103
4.34	The retrofit heat exchanger network for light crude	105

ABBREVIATIONS

Sets

- B = $\{ (i,j) \mid \text{more than one heat exchanger unit is permitted between hot stream } i \text{ and cold stream } j \}$
- C^z = $\{ j \mid j \text{ is a cold stream present in zone } z \}$
- C_n^z = $\{ j \mid j \text{ is a cold stream present in temperature interval } n \text{ in zone } z \}$
- CU^z = $\{ j \mid j \text{ is a heating utility present in zone } z \}$ ($CU^z \subset C^z$)
- H^z = $\{ i \mid i \text{ is a hot stream present in zone } z \}$
- H_m^z = $\{ i \mid i \text{ is a hot stream present in temperature interval } m \text{ in zone } z \}$
- HU^z = $\{ i \mid i \text{ is a heating utility present in zone } z \}$ ($HU^z \subset H^z$)
- M^z = $\{ m \mid m \text{ is a temperature interval in zone } z \}$
- M_i^z = $\{ m \mid m \text{ is a temperature interval belonging to zone } z, \text{ in which hot stream } i \text{ is presented} \}$
- m_i^0 = $\{ m \mid m \text{ is the starting temperature interval for hot stream } i \}$
- m_i^f = $\{ m \mid m \text{ is the final temperature interval for hot stream } i \}$
- N_j^z = $\{ n \mid n \text{ is a temperature interval belonging to zone } z, \text{ in which cold stream } j \text{ is presented} \}$
- NI^H = $\{ i \mid \text{non-isothermal mixing is permitted for hot stream } i \}$
- NI^C = $\{ j \mid \text{non-isothermal mixing is permitted for cold stream } j \}$
- n_j^0 = $\{ n \mid n \text{ is the starting temperature interval for cold stream } j \}$
- n_j^f = $\{ n \mid n \text{ is the final temperature interval for cold stream } j \}$
- P = $\{ (i,j) \mid \text{heat exchange match between hot stream } i \text{ and cold stream } j \text{ is permitted} \}$
- P_{im}^H = $\{ j \mid \text{heat transfer from hot stream } i \text{ at interval } m \text{ to cold stream } j \text{ is permitted} \}$

P_{jm}^C	= { i heat transfer from hot stream i to cold stream j at interval n is permitted }
PA^z	= { i I is a hot stream present in zone z and is a pump around stream }
S^H	= { i splits are allowed for hot stream i }
S^C	= { j splits are allowed for cold stream j }
Z	= { z z is a heat transfer zone }

Parameters

$A_{ij}^{a^0}$	Area of an existing exchanger between streams i and j in zone z prior to retrofit
A_{ij}^{z,k^0}	Area of the k -th existing exchanger between streams i and j in zone z prior to retrofit
$A_{ij \max}^z$	Maximum shell area for an exchanger matching hot stream i and cold stream j in zone z
$A_{ij \max}^{z^N}$	Maximum area for a new heat exchanger matching hot stream i and cold stream j in zone z
$\Delta A_{ij \max}^{z^0}$	Maximum area addition for an existing heat exchanger matching hot stream i and cold stream j in zone z
$\Delta A_{ij \max}^{z,k^0}$	Maximum area addition for the k -th existing heat exchanger matching hot stream i and cold stream j in zone z
\hat{C}_{im}	Heat capacity of hot stream i at temperature interval m
\hat{C}_{jn}	Heat capacity of cold stream j at temperature interval n
c_i^H	Cost of heating utility i
c_j^C	Cost of cooling utility j
c_{ij}^F	Fixed charge cost for a heat exchanger matching hot stream i and cold stream j

$c_{ij}^{A^N}$	Variable cost for a new heat exchanger matching hot stream i and cold stream j
$c_{ij}^{A^0}$	Area addition cost for an existing heat exchanger matching hot stream i and cold stream j
$CHEAD_{im,jn}$	Temperature difference between interval m of hot stream i and interval n of cold stream j at cold end
F_i	Flow rate of hot process stream i
F_j	Flow rate of cold process stream j
F_i^U	Upper bound for the flow rate of heating utility i
F_j^U	Upper bound for the flow rate of cooling utility j
$FPR_{i,r}^H$	Candidate values for pump around flowrate i
h_{im}	Film heat transfer coefficient for hot stream i in interval m
h_{jn}	Film heat transfer coefficient for cold stream j in interval n
$\Delta H_{im}^{z,H}$	Enthalpy change for hot stream i at interval m of zone z
$\Delta H_{jn}^{z,C}$	Enthalpy change for cold stream j at interval n of zone z
$HHEAD_{im,jn}$	Temperature difference between interval m of hot stream i and interval n of cold stream j at hot end
k_{\max}	Maximum number of heat exchangers allowed between hot stream i and cold stream j in zone z when $(i,j) \in B$
k_e	Number of existing heat exchangers between hot stream i and cold stream j in zone z when $(i,j) \in B$
k_e'	Number of existing heat exchangers in the original network
q_{ijm}^L	Lower bound for heat transfer from hot stream i at interval m to cold stream j

q_{ijn}^L	Lower bound for heat transfer from hot stream i to cold stream j at interval n
Q_{PA}	Total PA load
T_m^U	Upper temperature of interval m
T_m^L	Lower temperature of interval m
T_n^U	Upper temperature of interval n
T_n^L	Lower temperature of interval n
ΔT_i	Temperature range of stream i
ΔT_j	Temperature range of stream j
ΔT_{mn}^{ML}	Mean logarithmic temperature difference between intervals m and n
U_{ij}^z	Number of existing heat exchangers between hot stream i and cold stream j in zone z
U_{ij}^{zN}	Maximum number of new heat exchangers allowed for the retrofit design
$\Gamma_{im,jn}^{z,H}$	Upper bound

Variables

A_{ij}^z	Total required area for a match between hot stream i and cold stream j in zone z
A_{ij}^{z,k^0}	Area of the k -th existing heat exchanger between hot stream i and cold stream j in zone z after retrofit
A_{ij}^{zN}	Area of a new heat exchanger between hot stream i and cold stream j in zone z
$\Delta A_{ij}^{z^0}$	Area addition for an existing heat exchanger between hot stream i and cold stream j in zone z

$\Delta A_{ij}^{z,k^0}$	Area addition for the k -th existing heat exchanger between hot stream i and cold stream j in zone z
FP_i^H	Flowrate of pumparound stream
$K_{ijm}^{z,H}$	Determines the beginning of a heat exchanger at interval m of zone z for hot stream i with cold stream j . Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
$K_{ijn}^{z,C}$	Determines the beginning of a heat exchanger at interval n of zone z for cold stream j with hot stream i . Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
$\hat{K}_{ijm}^{z,H}$	Determines the end of a heat exchanger at interval m of zone z for hot stream i with cold stream j . Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
$\hat{K}_{ijn}^{z,C}$	Determines the end of a heat exchanger at interval n of zone z for cold stream j with hot stream i . Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
k_e	Number of existing heat exchangers between hot stream i and cold stream j in zone z when $(i,j) \in B$
$q_{im,jn}^z$	Heat transfer from hot stream i at interval m to cold stream j at interval n in zone z
$\bar{q}_{imn}^{z,H}$	Non-isothermal mixing heat transfer for hot stream i between intervals m and n in zone z
$\bar{q}_{jmn}^{z,C}$	Non-isothermal mixing heat transfer for cold stream j between intervals m and n in zone z
$\hat{q}_{ijm}^{z,H}$	Heat transfer from hot stream i at interval m to cold stream j in zone z
$\hat{q}_{ijn}^{z,C}$	Heat transfer to cold stream j at interval n from hot stream i in zone z
$\tilde{q}_{ijm}^{z,H}$	Auxiliary continuous variable utilized to compute the hot side heat load of each heat exchanger when several exchangers exist between hot stream i and cold stream j in zone z

$\tilde{q}_{ijn}^{z,C}$	Auxiliary continuous variable utilized to compute the cold side heat load of each heat exchanger when several exchangers exist between hot stream i and cold stream j in zone z
$\tilde{q}_{im,jn}^{z,H}$	Auxiliary continuous variable utilized to compute the area of individual heat exchangers between hot stream i with cold stream j in zone z when $(i,j) \in B$
QP_i^z	Pumparound load
U_{ij}^z	Number of heat exchangers between hot stream i and cold stream j in zone z
$X_{im,jn}^z$	Auxiliary continuous variable equals to zero when an exchanger ends at interval m for hot stream i and at interval n for cold stream j . A value of one corresponds to all other cases
$\hat{X}_{ijm}^{z,k}$	Auxiliary binary variable that determines whether the k -th between hot stream i with cold stream j in zone z exists at interval m of when $(i,j) \in B$
$Y_{ijm}^{z,H}$	Determines whether heat is being transferred from hot stream i at interval m to cold stream j . Defined as binary when $(i,j) \notin B$ and as continuous when $(i,j) \in B$
$Y_{ijn}^{z,C}$	Determines whether heat is being transferred from hot stream i to cold stream j at interval n . Defined as binary when $(i,j) \notin B$ and as continuous when $(i,j) \in B$
$\delta_{ij}^{z,k}$	Auxiliary binary variable used for heat exchanger relocation. Auxiliary binary variable that determines whether the k -th original heat exchanger of zone z has is serving the match between hot stream i and cold stream j , when $(i,j) \notin B$
$\delta_{ij}^{z,hk}$	Auxiliary binary variable used for heat exchanger relocation. This variable determines whether the k -th original heat exchanger of zone z has is serving the h -th exchanger streams i and j , when $(i,j) \in B$

$\alpha_{ijm}^{z,H}$ Auxiliary continuous variable equal to one when heat transfer from interval m of hot stream i to cold stream j occurs in zone z and it does not correspond to the beginning nor the ending of a heat exchanger. A value of zero corresponds to all other cases

$\alpha_{ijn}^{z,C}$ Auxiliary continuous variable equal to one when heat transfer from hot stream i to interval n of cold stream j occurs in zone z and it does not correspond to the beginning nor the ending of a heat exchanger. A value of zero corresponds to all other cases