# CHAPTER II

# LITERATURE REVIEW

A warrant pricing model was originally developed based on the option pricing model of Black and Scholes. In 1973, Black and Scholes proposed a model to price an option. They claimed that in many cases their model "can be used as an approximation to give an estimate of the warrant value." However, the unrealistic assumptions such as no dividend payment, no early exercise, constant volatility, and interest rate cause biases in pricing a warrant. The option pricing literature (See, for example, Galai and Schneller (1978), Lauterbach and Schultz (1990), and Schulz and Trautmann (1994)) has long recognized that warrants should be valued differently from standard options. With warrants, the underlying asset is not the stock price but the total equity price; i.e. the stock price plus the warrant price. Consequently, several alternative models are developed to improve warrant pricing e.g. dividend-adjusted Black-Scholes model, square-root CEV model of Cox and Ross (1976), and jump-diffusion model of Merton (1976).

Relative to the standard Black-Scholes call option model, warrant valuation requires two adjustments to the standard option model, the share dilution and the use of total equity value and its volatility as model inputs. When warrants and/or ESOs exist, neither the volatility of the traded stock return nor the implied volatility of traded equity options should be used directly as the inputs of Black-Scholes option model.

Schulz and Trautmann (1994) compare the outcomes of competitive warrant valuation model with the outcomes of the original option-like Black-Scholes model. They find that to obtain warrant values with acceptable accuracy, adjustments to the Black-Scholes formula are not needed except perhaps for deep-out-of-the-money warrants. For in-the-money and near-maturity warrants, omitting the dilution effect in the option-like formula is overcompensated by the lower model input parameters, which are, the stock price and stock volatility.

Kremer and Roenfeldt (1993) investigate the warrant pricing abilities of dilution-adjusted version of the Black-Scholes and Jump-Diffusion option pricing model. They conclude that the jump-diffusion model is often the least biased model but is the most efficient only for one subset of warrants. Large reductions in bias accompanied by relatively minor losses in efficiency indicate that the jump-diffusion model probably should be considered when valuing out-of-the-money, noncallable nonsenior security warrants with maturities in excess of one year, or warrants with underlying stocks exhibiting a historically jump impact.

Shastri and Sirodom (1995) examine the pricing structure of warrants that were traded on the Stock Exchange of Thailand. A comparison of model prices with the market prices of the warrants shows that the Black-Scholes model misprices warrants on average by 4 percent, while the corresponding figure for the Cox Square Root (CSR) model is 3.84 percent.

Hauser and Lauterbach (1997) examine five warrant pricing models, including the Black-Scholes call option model, the dilution-adjusted Black-Scholes model, the Longstaff extendible-warrant model, the square-root CEV model, and the free-theta CEV Model. They find that the free-theta CEV model generates the lowest average absolute pricing error in most of the warrants examined. The dilution-adjusted Black-Scholes model, however, remains a reasonable economical alternative in many cases.

Huang and Chen (2002) investigate the stochastic volatility option pricing model of Hull and White (1987). They examine the out-of-sample performance of the pricing model applied to the value of covered warrants traded on the Taiwan Stock Exchange (TSE). Warrants values from Hull and White's stochastic volatility model and those from Black-Scholes model are compared with observed option premiums. Empirical results indicate that the Hull-White model with implied volatility outperforms others in predicting the warrant prices, indicating that the pricing model incorporated with stochastic volatility feature can improve the pricing of warrants.

Recently, the multiple series of warrants pricing has gained much interest from many researchers since firms usually issue warrants/ESOs at different maturities and strike prices. Pricing each warrant series independently may lead to overprice sub-sequent warrant series. This perhaps is one of the reasons that causes mispricing in warrants. Consequently, some working papers have addressed the simultaneity issue associated with valuing multiple series warrants/ESOs issued by the same firm.

Darsinos and Satchell (2002) extend the theoretical framework of Galai and Schneller (1978) for valuing multiple series warrants and ESOs and derive distribution free formulae for firms with warrants and ESOs of several maturities and strikes. They only test their model by valuing multiple series of warrants issued from Cisco Systems company at July 28, 2001. Their model is used as a benchmark and is compared to the single issuance approach, and option-like approach[1]. They illustrate that, when applied in multiple settings, the single issuance approach can result in significant overpricing. Option-like valuation, on the contrary, is not that good an approximate in multiple settings and will typically result in underpricing.

Lim and Terry (2003) state that a cross-dilution effect[2] and a subtle slippage effect[3] cause the existing models such as Galai-Schneller model to be inappropriate for pricing multiple series of warrants. They propose a multiple series warrants pricing model. The sensitivities of the pricing errors to dilution ratio, degree of moneyness, and time-to-maturity are examined. From their analysis, the highest pricing errors for the Black-Scholes model and Galai-Schneller model occur when the potential diltuion of the warrants is high, when the warrants are out-of-the-money, or when the warrants are further from expiry. The practical example is illustrated to show the presence of the slippage and the cross dilution effect. Using warrants data from the Strike Engineering company in Singapore, they find that the cross-dilution effect is stronger than the subtle slippage effect.

Whereas Darsinos and Satchell only consider a cross-dilution effect, Lim and Terry, in turn, consider both cross-dilution effect and subtle slippage effect. In other words, for the Darsinos-Satchell model, the earlier warrant series is valued the same as in Galai-Schneller approach, omitting the effect from the later warrant series. All in all, it seems like Lim-Terry approach is more appropriate than Darsinos-Satchell approach. However, as was investigated in Lim and Terry paper, the cross-dilution effect is stronger than the subtle slippage effect. The subtle slippage effect might have low value and thus can be ignored. Consequently, the prices of earlier warrant series from both models might not differ much. In addition, Darsinos and Satchell model is much easier to extend to value more than two warrant series since there is a closed-

[1]The single issuance approach is refered as the textbook treatment for the valuation of warrants which is accounting for dilution and values a warrant as a call option on the value of the firm. Unlike the single issuance approach, the option-like approach values warrants as identical to value standard call options,

[2]It is the effect that potential dilution of the earlier warrant series affects the fair price of the later warrant series.

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form formula for each warrant series. In case of the Lim-Terry model, only formulas for two warrant series are derived and it is harder for practitioners to obtain formula for more than two warrants series.

Dennis and Rendleman (2006) develop a binomial model in which up to 30 employee stock options issued by the same firm can be valued simultaneously. They also develop a version of the model to assess the valuation impact associated with employee options that are expected to be issued in the future. The two-period example is illustrated to show how the value of each ESOs series can be obtained. In the example, they assume that the firm issues only two employee stock options. For more realistic scenarios, each framework is generated and results from the multiple employee option pricing model are compared to those from the standard binomial option valuation. As a result, the standard binomial values are all slightly less than those computed via the multiple employee option pricing model. For firms with a large fraction of employee options outstanding, not accounting for the simultaneity can result in under-valuation on the order of ten percent.

Unlike the other two papers, Dennis and Rendleman model does not have a closed-form formula to value multiple warrant series. Instead, they use the typical approach, the binomial model. Their model takes into account both of the cross-dilution effect and the subtle slippage effect. The binomial model is more flexible to price than a closed-form formula. For example, risk-free rate of interest and firm volatility can be adjusted during each period. Thus, the model can capture risk-free rate of interest and firm volatility better than the other two models. Furthermore, as all the models mentioned above are suitable for just warrant pricing or European option pricing, the binomial model, in an effective way, is also suitable for American option pricing. This is because with the binomial model it is possible to check at every point in an option's life (i.e. at every step of the binomial tree) for the possibility of early exercise. The binomial model likewise can price warrants in case that there is an anticipated issuance of new warrant series. Nevertheless, it is more time-consuming to calculate the price of warrant than other models. In practice, because of current computing limitation (using 2-Ghz Pentium 4 computer), this model can practically value up to 30 multiple warrant series.

From all literatures above, it is obvious that there is no empirical evidence on multiple warrants pricing models. All the models proposed in the literatures are simply tested in the given scenerio or just one data set of multiple warrants from a firm. Given advantages and disadvantages of each model, it is still ambiguous that which multiple warrants models is the fittest model and should be used in pricing multiple warrants. Consequently, this paper tries to perform an empirical study in Thailand about multiple warrants pricing in order to determine the model that is suitable for Thai market.