# Chapter 1

# Introduction

The physics of nonlinear composite materials has attracted much interest for both strongly nonlinear [1,2] and weakly nonlinear composites [3,4,6-15] because of their applications in engineering and physics [3-5]. The optical composite materials, one type of nonlinear composite materials, play the important roles in developing telecommunications [3,4], high-power lasers [3,4] and photonic devices [5]. Therefore, it is useful to study the electric field response of nonlinear composite materials.

The perturbation expansion method was firstly developed for solving nonlinear harmonic oscillator problems [16]. This method have been extended to derive the field equations and the boundary conditions for solving the electrostatic boundary value problems of weakly nonlinear composite [6-8]. These expressions have included only the second-order perturbation expansion of the electric potential and the third-order nonlinearity. The general formulae for effective nonlinear coefficients of weakly nonlinear composites have been derived by using the perturbation expansion method based on electric potentials up to the second order [6-8], including only the third-order nonlinearity [6,7] and then extended to those including the fifth-order nonlinear coefficient [8]. These formulae can be used to estimate the bulk response of nonlinear composites in an external electric field.

It is well known that the lower-order nonlinear coefficients can lead to higher-order effective nonlinearities in the nonlinear composite system. The higher-order effective nonlinear coefficients of weakly nonlinear composite have received much attention for theoretical predictions [9,11,12]. In addition, the experiments [3,4,13,14] have supported that it is necessary to include these higher-order coefficients for accurate predictions of electric field response of composites. For theoretical predictions, the effective coefficients up to the ninth order have been derived by Liu and Li [9]. In their work, the expressions of the effective nonlinear coefficients are obtained, following the method of Zeng *et al.* [10], by using the decoupling approximation. These expressions have been applied to weakly nonlinear dielectric composites in dilute limit and arbitrary inclusion packing fractions based on linear effective coefficient using Maxwell-Garnett approximation. The spectral representation theory for higher-order nonlinear responses formulated in random composites based on decoupling approximation was reported by Gao [11] and Gu *et al.* [12]. In their work, the effective nonlinear coefficients of arbitrary order for varying inclusion volume fraction have been investigated by using Bruggeman effective medium approximation and Maxwell-Garnett approximation.

In this research, we consider the electric field response at static limit (low frequency of the applied field), in which case the linear and nonlinear coefficients are real and independent of the field frequency. In order to obtain more accurate results of the electric potentials of weakly nonlinear composites, the derivation of the field equations and the boundary conditions based on the third-order perturbation expansion of the electric potentials including the fifth-order nonlinearity is presented here. In order to show the reliability of these expressions, we have applied the results to solve for the electric potentials up to the third order of weakly nonlinear dielectric composite consisting of dilute nonlinear spherical inclusions randomly dispersed in a linear host medium subjected to a uniform external electric field [15]. The perturbation expansion method is used to derive for the general formulae for effective nonlinear coefficients up to the ninth order, including the fifth-order nonlinearity which the extension of the work of Gu and Yu [6] considering only the third-order nonlinearity is given here. Our formulae are applied to the dielectric composites consisting of dilute weakly nonlinear cylindrical inclusions randomly dispersed in a linear host medium. In order to confirm our formulae, the results are reduced to compare with those obtained by using the method of Gu and Yu [6]. These formulae are also applied to a more complicated weakly nonlinear composite consisting of dilute linear cylindrical inclusions randomly dispersed in a nonlinear host medium.

In Chapter 2, the details of the derivation for the field equations and the boundary conditions by using the perturbation expansion method, including the third-order electric potential and fifth-order nonlinear coefficient, are presented. In Chapter 3, the field equations and the boundary conditions, as derived in chapter 2, are applied to solve the electric potentials up to the third-order of weakly nonlinear composite. The general formulae for effective nonlinear coefficients up to the ninth-order by using the perturbation expansion method are derived in Chapter 4. In Chapter 5, the general formulae for effective coefficients of this work are applied to weakly nonlinear composites. The last chapter is conclusions of this research.