CHAPTER II

THEORETICAL CONSIDERATIONS

PART 1 Dynamometer Requirements

Two requirements that are always in opposition in dynamometer design are sensitivity and rigidity. The sensitivity of a good research dynamometer should be such that determinations are accurate to within ± 1%. That is, if a dynamometer is designed for a mean force of 100 pounds, one pound increments should be easily readable.

Some deformation is associated with the operation of every dynamometer. However, a dynamometer should be rigid enough so that the cutting operation is not influenced by the accompanying deflections. Frequently, the dominating stiffness criterion is the natural frequency of the dynamometer. All machine tools operate with some vibrations, and in certain cutting operations these vibrations may have large amplitudes.

In order that the recorded force (or the actual cut) shall not be influenced by any vibrating motion of the dynamometer, it's natural frequency must be large (at least 4 times as large) compared to the frequency of the exciting vibration. For purposes of analysis any dynamometer can be reduced to a mass supported by a spring. The natural frequency (ν_n) of such a system is equal to

$$\nu_n = \frac{1}{2\pi}\left(\frac{K}{M}\right)^{1/2} \quad \text{c.p.s}$$

Where K is the spring constant in lb/in

M is the mass in $\frac{\text{lb(sec)}^2}{\text{in}}$

In terms of the supported weight of the dynamometer (w)

$$\nu_n = \frac{1}{2\pi}\left(\frac{386K}{w}\right)^{1/2}$$

In general, a dynamometer must measure at least two force components in order to determine a two dimensional resultant cutting force. In a three dimensional cutting operation, three force components are necessary, while in drilling or tapping, only a torque and a thrust are required. It is usually most convenient to measure force relative to a set of rectangular coordinates (x,y,z) and it is advisable that there be no cross-sensitivity between these components. That is an applied force in the x direction should give no reading in y or z directions. If mutual interference of the force measuring elements exists, determination of the force components requires the solution of simultaneous equations.

This prevents the immediate interpretation of the data. When certain of the electric transducers are suitably located and connected, unwanted strain component can often be cancelled electrically. How this may be done will be subsequently illustrated.

It is convenient to use a system having a linear calibration. In such a case, the force is determined with the precision with which a strain increment can be measured relative to an arbitrary datum. If the system is not linear, it is then necessary that the zero load point be accurately known as well as the strain increment. This introduces an additional quantity that must be carefully measured.

A dynamometer should be stable with respect to time, temperature, and humidity once a calibration is made & it should only have to be checked occasionally. Many existing dynamometers use devices for the separation of force components which involve friction, (i.e. rollers, balls and sliding surfaces) As friction condition are usually variable due to dirt, temperature, etc. such instruments are of limited usefulness.[2]

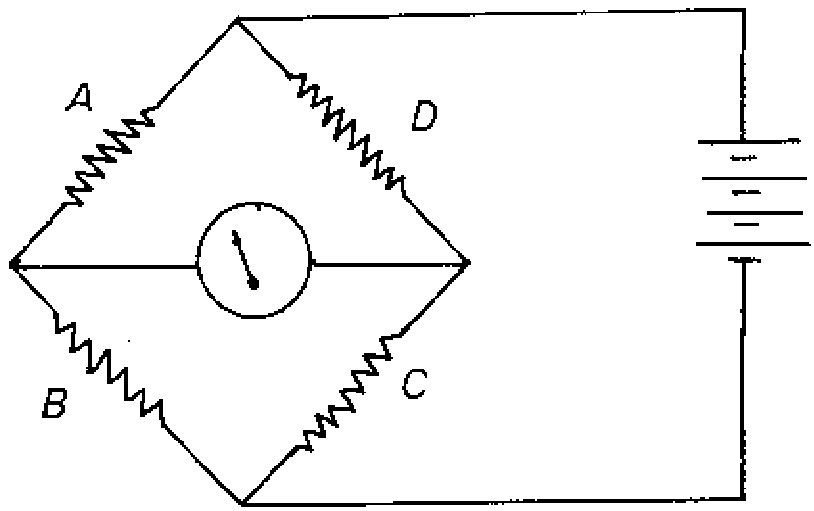

FIG. 2.1 Wheatstone bridge

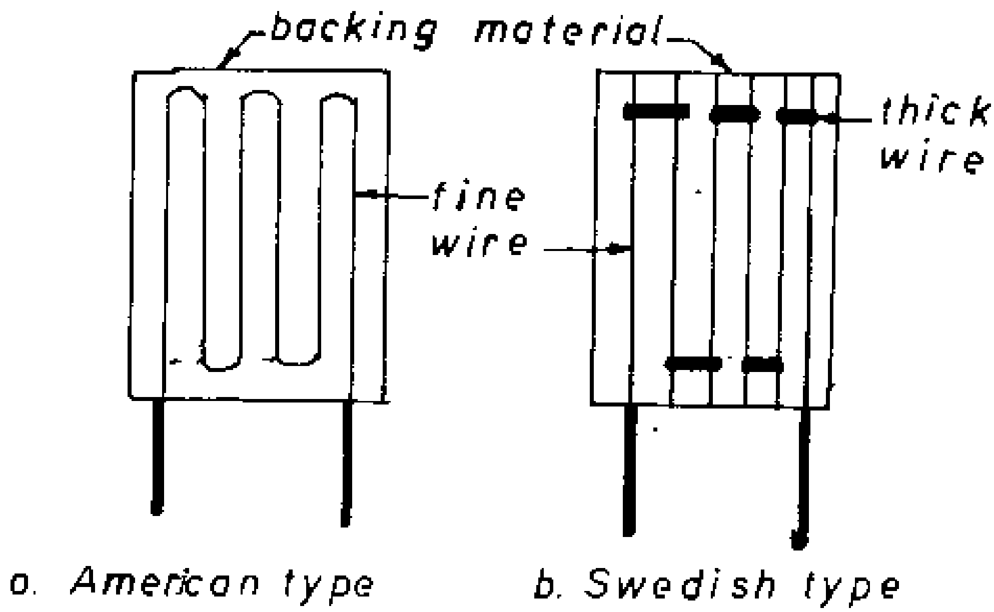

FIG. 2.2 STRAIN GAUGE

Force measurement a variety of devices have been used to measure forces in dynamometers but the two component lathe dynamometers as designed here is a wire resistance strain gauge type.

Principle of operation of strain gauge A strain gauge works on the principle discovered by Lord Kelvin that when a conductor is stretched, its resistance increases. If, there fore, a conductor (the strain gauge) is bonded intimately to the surface of a specimen, then any subsequent strain in the specimen is experienced by the gauge, resulting in a change of resistance.

As the strains normally experienced in structures are small (mirco inches / inch) The change in resistance of a strain gauge is also small (micro ohms / ohm) and therefore a Wheatstone Bridge net-work is usually used to measure these very small values Fig. 2.1.

The four arms of the bridge may be made up in various ways according to the type of measurement required and the instrumentation used. For measurement of strain at a point, a single strain gauge is used as one arc A., with three fixed resistances making up the bridge B., C. and D. Most static strain recording instruments have two résistors built in C. and D. and the other two arms are made up of the active gauge cemented to the specimen and a dummy gauge (compensating gauge) cemented to a piece of similar material which experiences the same temperature conditions as the test specimen but is unstrained. This is called a half bridge. It gives temperature compensation and eliminates apparent strain due to thermal expansion or contraction of specimen. Instruments incorporating two fixed resistors are usually of the null balance type, in which out of balance signals are corrected by means of a slide wire which is calibrated directly in percentage change. This system has the advantage of being independent of in-put voltage, but it is only suitable for static or slowly changing strains. For measuring loads or displacements in a calibrated system, the signal can be increased by making more than one arm of the bridge *active*. In this case strain gauges are used for all four arms of the bridge, known as *full bridge*, and the signal is fed directly to a galvanometer, a galvanometer recorder, or a sensitive micro-ammeter. The optimum for measuring loads or deflections is the cantilever beam where gauges A. and C. are mounted on the upper surface and C. and D. on the lower surface make all four arms active. The change of resistance of gauges A. and C. is equal and opposite to that of gauge B. and D. so that the signal obtained is four times that from a single active gauge and is usually sufficiently large to drive a low resistance 0-50 micro-ammeter direct with an energising voltage of between 6 and 12volts DC. Alternatively, the signal can be put directly into a galvanometer recorder to give a permanent trace at frequencies up to 3,000 cycles/sec.

The disadvantage of the above systems, which are called *deflection* methods, as opposed to the *null balance* method, is that the signal obtained is directly proportional to the applied voltage, making some kind of voltage stabilized supply necessary for accurate work.[4]

Bonded strain gauge construction Bonded strain gauges consist of fine metal wire of 0.0004 to 0.001 in. thickness, depending on the type of gauge, forming a grid pattern which is mounted on a backing material as shown in fig. 2.2

The gauges are manufactured with differing metal and backing materials as necessary.

Gauge factor The gauge factor is the relationship between the change in resistance of the gauge and the strain, and is defined as difficulty in obtaining good heat transfer between the two without transmitting strain to the dummy block. Often it is preferable to maintain the dummy block at a known constant temperature and calibrate the active gauge against temperature.

If the two soldered joints on a gauge are at the same temperature then the thermal e.m.f's will be equal and opposite and will cancel out.

When working at elevated temperatures a check for stray e.m.f's should always be made by disconnecting the power supply to the bridge and varying the temperature of the specimen. If there are zero-shifts with temperature and these can not be eliminated, a graph of zero-shift against temperature must be made and used to correct readings made under load.

Copper has a high temperature coefficient so that, although the lead resistances are usually low, the errors introduced by leads in the bridge circuit being either in an elevated temperature area or subjected to varying temperatures can be appreciable.

It is always preferable to complete a bridge at the point of measurement in order to keep the lengths of wire in the bridge as short as possible. The errors introduced by long leads ex-ternal to the bridge circuit are generally negligible a part from voltage drop.

Leads in high temperature zones should always be kept as short as possible. They should be matched for length and resistance, and bound together so that they all pass through the same environment.[4]

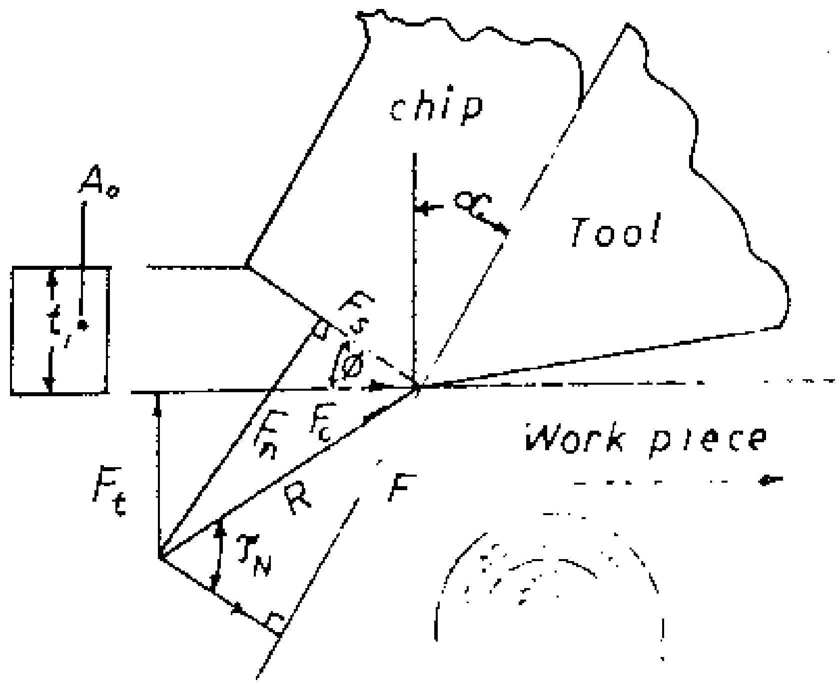

FIG. 2.3 Force diagram for orthogonal cutting

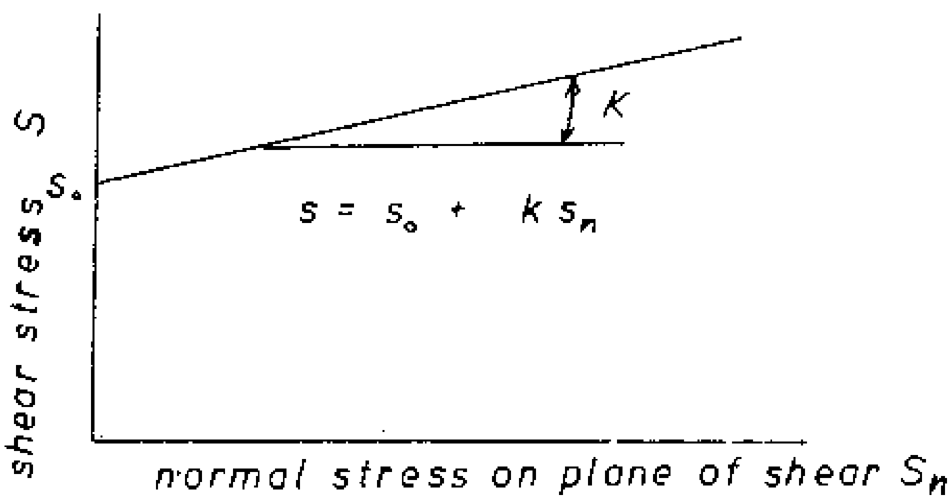

FIG. 2.4 Dependance of s on s_n assumed in Merchant s second theory

PART 2 Mechanics of cutting

Theory of Ernst and Merchant

Although an attempt had been made to solve this problem by Piispanen in 1937 the first complete analysis resulting in a so-called shear angle solution was presented by Ernst and Merchant.

(Fig. 2.3 , p. 11)

R	=	resultant tool force
F_c	=	cutting force
F_t	=	thrust force
F_s	=	shear force on shear plane
F_n	=	normal force on shear plane
F	=	friction force on rake face
ϕ	=	shear angle
α	=	tool rake angle
τ	=	mean friction angle
A_o	=	cross-section area of uncut chip
t_1	=	undeformed chip thickness
t_2	=	chip thickness

In this analysis the chip is assumed to be a rigid body held in equilibrium by the action of the forces transmitted across the chip-tool interface and across the shear plane.

For convenience, in Fig. 2.3 the resultant tool force R. is shown acting at the tool cutting edge and is resolved into components N. and F. in directions normal to and along the tool face respectively and into component F_n and F_s normal to and along the shear plane respectively. The cutting (F_c) and thrust (F_t) components of the resultant tool force are also shown.

It is assumed that the whole of the resultant cutting force is transmitted across the chip-tool interface and that no force acts on the tool edge or flank (i.e. the ploughing force P = 0)

The basis Ernst and Merchant's theory was the suggestion that the shear angle ϕ would take up such a value as to make the work done in cutting a minimum. Since, for given cutting conditions, the work done in cutting is proportional to F_c it was necessary to develop an expression for F_c in terms of ϕ and then to obtain the value of ϕ for which F_c is a minimum.

From Fig. 2.3

$$F_s = R \cos (\phi + \tau - \alpha) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

and

$$F_s = S A_s$$

$$= \frac{S A_o}{\sin \phi} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

where S = shear strength of material on the shear plane

A_s = area of shear plane

Stress $S = S_o$ This assumption agreed with the work of Bridgman where, in experiments on poly crys-talline metals, the shear strength was shown to be dependent on the normal stress on the plane of shear

Now from Fig. 2.3

$$F_n = R \sin(\phi + \tau - \alpha) \qquad \ldots\ldots\ldots\ldots\ldots(8)$$

and

$$F_n = S_n A_o$$

$$= \frac{S_n A_o}{\sin\phi} \qquad \ldots\ldots\ldots\ldots\ldots(9)$$

From (8) and (9)

$$S_n = \frac{\sin\phi}{A_o} R \sin(\phi + \tau - \alpha) \qquad \ldots\ldots\ldots\ldots\ldots(10)$$

Combining equation (3) and (10)

$$S = S_n \cot(\phi + \tau - \alpha) \qquad \ldots\ldots\ldots\ldots\ldots(11)$$

and from equations (7) and (11)

$$S = \frac{S_o}{1 - K \tan(\phi + \tau - \alpha)} \qquad \ldots\ldots\ldots\ldots\ldots(12)$$

This equation shows how the value of S may be affected by changes in ϕ and is now inserted in equation (5) to give a new equation for F_c in terms of ϕ ;

$$F_c = \frac{S_o A_o \cos(\tau - \alpha)}{\sin\phi \cos(\phi + \tau - \alpha) \, [1 - K \tan(\phi + \tau - \alpha)]} \qquad \ldots\ldots(13)$$

It is now assumed that K and S_o are constants for the particular work material; and that A_o and are constants for the cutting operation. Thus eq. (13) may be differentiated to give the new value of

A_o = cross-sectional area of undeformed chip

τ = mean angle of friction between chip and tool ($=\tan^{-1} F/N$)

α = tool rake angle

From (1) and (2)

$$R = \frac{S\ A_o}{\sin \phi}\ \frac{1}{\cos(\phi+\tau-\alpha)} \qquad \text{..........(3)}$$

Now by geometry

$$F_c = R \cos(\tau - \alpha) \qquad \text{..........(4)}$$

Hence from (5) and (4)

$$F_c = \frac{S\ A_o}{\sin \phi}\ \frac{\cos(\tau-\alpha)}{\cos(\phi+\tau-\alpha)} \qquad \text{..........(5)}$$

Equation (5) may now be differentiated with respect to ϕ and equated to zero to find the value of ϕ for which F_c is a minimum. The required value is given by

$$2\phi + \tau - \alpha = 90^\circ \qquad \text{..........(6)}$$

Merchant found that this theory gave good agreement with experimental results obtained when cutting synthetic plastics but gave poor agreement for steel machined with a sintered carbide tool.

It should be noted that, in differentiating equation (5) with respect to ϕ, it was assumed that A_o, and would be in-dependent of ϕ. On reconsidering these assumptions, Merchant decided to include in a new theory the relationship

$$S = S_o + K\ S_n \qquad \text{..........(7)}$$

which indicates that the shear strength of the material increases linearly with increase in normal stress on the shear plane (Fig.2.3) at zero normal.

The resulting expression is

$$2\phi + \tau - \alpha = C \qquad \dots\dots\dots\dots\dots\dots (14)$$

where $C = \cot^{-1} K$ and is a constant for the work material.[1]