APPENDIX

Deformation of elasto-plastic hub

$$\epsilon_r = \frac{dU}{dr} \quad , \quad \epsilon_t = \frac{U}{r}$$

The compressibility condition assumed to be valid both in elastic and plastic, i.e. $\epsilon_r + \epsilon_t = 0$

$$\frac{dU_r}{d_r} + \frac{U_r}{r} = 0$$

Integrating gives $U_r = \frac{C}{r}$

at hub bore, $U_{ri} = \frac{C}{r_i}$

$$U_r = \frac{U_{ri}\, r_i}{r}$$

from item (8) Table I $\quad \delta = U_{ri} = \frac{P}{2E} \cdot \left[\frac{dD^2 + d^3}{D^2 - d^2} + \mu d \right]$

Substituting $\quad P = P_i, \quad r_i = \frac{d}{2} = \rho$

$$U_\rho = \rho \frac{P_i}{E} \left[\frac{D^2 + (2\rho)^2}{D^2 - (2\rho)^2} + \mu \right]$$

from $\quad S_y = \frac{2D^2}{D^2 - (2\rho)^2} \cdot P_x$

at $\rho = \frac{d}{2}, \quad P_x = P_i$

$$P_i = \left[1 - \frac{(\rho)^2}{(r_o)^2} \right] \cdot \frac{S_y}{2}$$

hence $\quad U_\rho = \frac{S_y}{2} \cdot \left[1 - \left(\frac{\rho}{r_o}\right)^2 \right] \cdot \frac{1}{2E} \cdot \left[\frac{2\rho \left[1 + \left(\frac{\rho}{r_o}\right)^2 \right]}{1 - \left(\frac{\rho}{r_o}\right)^2} + 2\mu\rho \right]$

$$= \rho \frac{S_y}{2E} \left[(1 + \mu) + \left(\frac{\rho}{r_o}\right)^2 \cdot (1 - \mu) \right]$$

To obtain the displacement of the inner surface U_{ri} that can be used as a characteristic parameter of the process of deformation,

Substitute $r = \rho$ in eq. $U_r = \frac{U_{ri} r_i}{r}$

$$U_{ri} = \frac{U_\rho \cdot \rho}{r_i}$$

hence, substituting U gives

$$U_{ri} = \frac{S_y \rho^2}{2 E r_i} \left[(1 + \mu) + \left(\frac{\rho}{r_o}\right)^2 (1 - \mu) \right]$$

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