DESIGN CALCULATION

Design of a 20-Ampere, 220-Volt, 50-Cycle, Single-Phase, Core-Type Variable Inductor, with Minimum Losses.

Design of Core and Winding

The core of the inductor is made of annealed steel laminations, silicon steel having about 4.0 per cent silicon is used, for this material provides a good compromise in cost, workability, low hysteresis and eddy-current losses, and high permeability at relatively high flux density. For 50-cycle inductor a 4.0 per cent silicon sheet steel 0.014 in. thick is used. The flux density in the core must be below the saturation point and is generally

$$B = 55{,}000 \text{ to } 75{,}000 \text{ lines per sq in.}$$

For low voltage, the rectangular-shaped core section is generally used and is not expensive to build. Circular coils are preferred for large capacity inductor because of their superior mechanical characteristics.

The flux density B in the core can be chosen equal to 60,000 lines per sq in. or 9,300 gausses. From the iron loss curves in the Appendix for 0.014-in.(0.35 mm), 4.0-4.5 per cent silicon steel, the loss per pound for this density

$$w_c = 0.60 \times 0.8 \times 1.12 = 0.536 \text{ watt/lb.}$$

$$= 1.18 \text{ watts per kilogram,}$$

if the additional losses due to bending and shearing strains, imperfect insulation between sheet, etc., are taken equal to 12.0 per cent of the fundamental frequency loss. And the ampere-turns per inch at this flux density B_c = 4.70 or 1.85 amp-turns per cm. From Eq. 2,

$$\phi_{max} = \frac{220}{4.44 \times 50 \times N \times 10^{-8}} \text{ lines,}$$

where E = 220 volts, f = 50 cps. Then

$$\phi_{max} = \frac{0.99099 \times 10^{8}}{N} \text{ lines.} \qquad (55)$$

With no air gap in the magnetic circuit, the effective value of the exciting current in the inductor is 20 amperes when the winding turns are varied for the maximum current. From Eq. 5, 6, and 7,

$$\begin{aligned} P_c &= G_c w_c \\ &= l_c A_c d_c w_c \\ &= \frac{I_m N}{H_c} \times A_c d_c w_c \quad \text{watts.} \qquad (56) \end{aligned}$$

When A_c is in sq cm, H_c = 1.85/$\sqrt{2}$ amp rms, d_c = 0.272 lb/cu in. or 7.5 grams/cu cm, therefore

$$P_c = \frac{I_m N}{1.85/\sqrt{2}} \times A_c \times \frac{7.5}{1000} \times 1.18 \quad \text{watts.}$$

$$P_c = \frac{I_m N A_c}{148} \quad \text{watts.}$$

From Eq. 8,

$$I_c = \frac{I_m N A_c}{148 \times 220} = 0.307 \times 10^{-4} I_m N A_c \text{ amp.}$$

and from Eq. 4,

$$I^2 = I_m^2 + I_c^2$$

therefore
$$20^2 = I_n^2 + 0.307 \times 10^{-4} I_n N A_n{}^2$$

$$I_n = \frac{20}{\left[1 + (0.307 \times 10^{-4} N A_n)^2\right]^{1/2}} \quad . (57)$$

From Eq. 55 if B = 9,300 gausses and the winding turns = 350, the cross-sectional area of the core is 33.83 sq cm with the stacking factor equal to 0.9. Then

$$I_n = \frac{20}{\left[1 + (0.307 \times 10^{-4} \times 350 \times 33.83)^2\right]^{1/2}}$$

$$= \frac{20}{1.065} = 18.78 \text{ amperes.}$$

In Eq. 55, if the flux density is kept constant the product of A_n and N will be also constant, and also is I_n. The length of the flux path will be

$$\ell_n = \frac{18.78\ N}{1.89/\sqrt{2}} \text{ cm.}$$

With these conditions, the core losses at various core areas when the flux density B, 60 kilolines/sq in., is kept constant are shown in the Table I .

From TABLE I, the core loss current

$$I_c = \frac{1505}{220} = 6.85 \text{ amperes,}$$

and I_n = 18.78 amperes, therefore

$$I = \sqrt{(18.78)^2 + (6.85)^2}$$

$$= 20 \text{ amperes (checked).}$$

TABLE I

CORRELATION OF CORE LOSS AND CORE AREA AT CONSTANT B

$B = 9{,}300$ gausses

Winding Turns N	Total Flux ϕ kilolines	Core Area $A_c = \frac{\phi}{0.9B}$ sq cm	Length of Flux Path ℓ_c cm	Weight of Core G_c kilograms	Core Losses P_c watts
100	991	118.4	1435	1274	1503
150	661	78.9	2153	1274	1503
200	495	59.2	2870	1274	1503
250	396	47.4	3588	1274	1503
300	330	39.5	4305	1274	1503
350	283	33.9	5023	1274	1503
400	248	29.6	5740	1274	1503
450	220	26.3	6458	1274	1503
500	198	23.7	7175	1274	1503

Next, the inductor with the core of constant cross-sectional area, 36.00 sq cm or 5.58 sq in., is considered. With 20 ampere exciting current and 200 turn winding, from Eq. 55,

$$B = \frac{0.99099 \times 10^8}{200 \times 36 \times 0.9}$$

$$= 15.3 \text{ kilogausses,}$$

$$= 98.7 \text{ kilolines/sq in.,}$$

then H_c is equal to 100 amperes/in. or 39 amp-turns/cm (peak value).

and the loss per kilogram at this density

$$= v_c \times 2.2 = 1.75 \times 0.8 \times 1.12 \times 2.2$$

$$= 3.45 \text{ watts/kg.}$$

From Eq. 36,

$$P_c = \frac{I_n \times 200}{39/\sqrt{2}} \times \frac{36 \times 7.5}{1000} \times 3.45$$

$$= 6.75\, I_n.$$

and

$$I_c = \frac{6.75\, I_n}{220} = 0.0307\, I_n.$$

And from Eq. 4,

$$20^2 = I_n^2 + (0.0307\, I_n)^2$$

$$I_n = \frac{20}{(1 \approx 0.0009)^{1/2}} = 19.99 \text{ amperes.}$$

The length of the flux path

$$l_c = \frac{19.99 \times 200}{39/\sqrt{2}} = 144.95 \text{ cm.}$$

$$= 57.07 \text{ inches.}$$

The weight of the core

$$G_c = \frac{36.0 \times 144.95 \times 7.5}{1000}$$

$$= 39.14 \text{ kilograms or}$$

$$= 86.10 \text{ pounds.}$$

The total core loss $P_c = G_c\, v_c$

$$= 39.14 \times 3.45 = 135 \text{ watts,}$$

$$I_c = 135/220 = 0.614 \text{ amperes,}$$

and

$$I = \sqrt{(19.99)^2 + (0.614)^2}$$

$$= 20 \text{ amperes, checked.}$$

With these conditions, the exciting current and core loss at the various values of flux density are shown in the Table II

TABLE II

CORRELATION OF FLUX DENSITY, CORE LOSS AND EXCITING CURRENT

$A_c = 36$ sq cm, $l_c = 57.07$ cm

Winding Turns N	Total Flux ϕ kilolines	Flux Density $B = \frac{\phi}{0.9A_c}$ kilolines/sq in	Magnetizing Current I_m amperes rms	Core Loss P_c watts	Exciting Current I amperes rms
100	991	197.3	saturated	-	-
150	661	131.6	"	-	-
200	495	98.7	19.99	135	20.00
250	396	78.9	1.78	60	1.80
300	330	65.8	0.74	54	0.78
350	283	56.4	0.46	44	0.50
400	248	49.5	0.33	33	0.37
450	220	43.9	0.23	26	0.26
500	198	39.5	0.18	23	0.21

It can be seen from TABLE I that if the flux density is kept constant and the winding turns are increased, the cross-sectional area of the core will decrease but the length of the flux path will increase, the core loss P_c is constant and very large. it is equal to 1903 watts. And in TABLE II, if the cross-sectional area is kept constant, 36.00 sq cm, the exciting current is 20 amp.

when the winding turns are 200 and it increases very rapidly when the winding turns are decreased from 250 turns because of saturation in the core. The iron core loss is still large, it is 155 watts.

With an air gap inserted in the magnetic circuit the core dimensions can be decreased and the core loss will be much decreased, the exciting current can be varied in a very wide range.

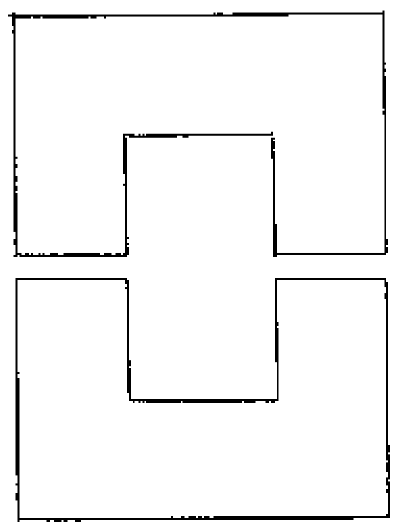


Fig. 3. Magnetic structure of the core

The magnetic structure shown in Fig. 3 is used. The coils will be placed on the legs containing the air gaps. The inductance is affected by changes in the gap lengths.

Let the cross-sectional dimensions of the core be 6.0 x 6.0 cm. The coil of 350 turns will be used. From Eq. 2, the total flux

$$\phi_{max} = \frac{220}{4.44 \times 50 \times 350 \times 10^{-8}}$$

$$= 283,140 \quad \text{lines.}$$

Conductor Size.- To make the copper loss small the current density in the coil must be low, 2.5 amp.per sq cm is used. For the maximum current of 20 amperes, the cross-sectional area of the conductor is 20/2.5 or 8 sq mm. The conductor 4.0 x 2.0 mm bare, 4.4 x 2.4 mm insulated, 8 sq mm area is selected. The conductor is paper insulated.

Winding.— The total number of the winding is 350 turns. Two coils, each of 175 turns, are used for each leg. Each coil has 7 layers, each layer has 25 turns.

The height of the coil = (25 x 4.4) + 4.4

= 114.4 mm or 11.44 cm,

and the thickness of the coil = 2.4 x 7

= 16.8 mm or 1.68 cm.

The insulation between the coil and core consists of a bakelite sheet of 3 mm thick, and the clearance between the coil and the core is 0.5 mm. The total depth of the winding per core leg is then

= 16.8 + 3.0 + 0.5 = 20.3.

The clearance of 11 mm at each end of the winding for insulating and supporting collars is satisfactory. The window height is then

= 114.4 + (2 x 11) = 136.4 = 136 mm.

The space of 35 mm between the coils is satisfactory. The window width is then = 35 + (20.3 + 20.3) = 75.6 = 76 mm.

Then the dimensions of the window are

the height = 136 mm, the width = 76 mm.

For the cross-sectional area of the core 6.0 x 6.0 cm, the mean length of the flux path

$$\ell_m = (136 + 60)2 + (76 + 60)2 = 664 \text{ mm} = 66.4 \text{ cm}.$$

When no air gap is present, and the window dimensions are kept unchanged, the values of the core loss at various cross-sectional areas of the core are shown in the Table III.

TABLE III

CORRELATION OF CORE LOSS AND CORE AREA

N = 350

Core Area A_c sq cm	Core Area A_c sq in.	Flux Density $B = \frac{\phi}{0.9A_c}$ kilolines/sq in.	Length of Flux Path l_c cm	Weight of Core G_c Kg.	Core Loss P_c watts
16.00	2.48	127.0	58.4	7.01	saturated
20.25	3.14	100.1	60.4	9.17	37.60
25.00	3.88	81.1	62.4	11.70	25.00
30.25	4.69	67.1	64.4	14.61	21.30
36.00	5.58	56.4	66.4	17.93	18.30
42.25	6.55	48.0	68.4	21.67	17.05
49.00	7.60	41.4	70.4	25.81	16.05
56.25	8.72	36.1	72.4	30.54	16.90
64.00	9.92	31.7	74.4	35.71	17.60

It can be seen that when the core area increases, the corresponding core loss decreases.

The coils are in the rectangular form. The space for the head of the bolt and the nut clamping the core is 0.7 cm on each side of the core. Then the inner dimensions of the winding, for the cross-sectional area of the core = 6.0 x 6.0 cm. is

$$= (6.0 + 0.3 + 0.3 + 0.1)(6.0 + 0.7 + 0.7 + 0.3 + 0.3)$$

$$= 6.7 \times 8.0 \text{ cm.}$$

The length of the mean-turn for the winding

$$l_t = (6.7 + 1.68 + 8.0 + 1.68) \times 2 = 36.2 \text{ cm.}$$

The winding, two 175 turns coils in series, has 350 turns. Therefore the length of the conductor

$$= 36.2 \times 350 = 12{,}670 \text{ cm}$$

$$126.70 \text{ meters.}$$

From the page 9, the resistance of the annealed copper conductor 8 sq mm at 20°C is

$$r = \frac{17.241}{8} = 2.155 \text{ ohm/ Km.}$$

and the total direct-current resistance of the winding is

$$R_d = 2.155 \times \frac{126.70}{1000} = 0.273 \text{ ohm.}$$

The resistance of the winding at 55°C, from Eq. 20,

$$R = 0.273 \left[1 + 0.00393(55 - 20)\right]$$

$$= 0.273 (1 + 0.1375) = 0.3105 \text{ ohm.}$$

At 20 amperes,

$$I^2R = 20^2 \times 0.3105 \times 1.10 = 136.6 \text{ watts.}$$

The total losses, core loss + I^2R

$$= 18.3 + 136.6 = 155 \text{ watts.}$$

The total losses with the exciting current at 20 amp. at different cross-sectional area of the core are shown in the Table IV.

TABLE IV

CORRELATION OF TOTAL LOSSES AND CORE AREA

N = 350

Core Area A_c sq cm	Core Losses P_c watts	Length of Mean Turn ℓ_m cm	R_{dc} at 55°C ohm	Copper Loss I^2R + Stray Load Loss at I=20 Amp. watts	Total Losses P watts
16.00	-	-	-	-	-
20.25	31.60	30.20	0.2991	114.0	147.3
25.00	29.80	32.200	0.2765	121.6	147.8
30.25	21.90	34.20	0.2934	129.1	150.7
36.00	18.90	36.20	0.3105	136.6	155.0
42.25	17.05	38.20	0.3278	144.2	161.1
49.00	16.85	40.20	0.3449	151.8	168.1
56.25	16.90	42.20	0.3621	159.3	176.0
64.00	17.60	44.20	0.3792	166.8	183.5

In TABLE IV at 20 amp. exciting current, the 25 sq cm cross-sectional area core has the total losses of 147.8 watts and for the 36 sq cm area core 155.0 watts. The difference of the losses is only 7.2 watts.

Effects of Air Gaps and the Inductance

Next consider the inductor with the core area of 25 sq cm and of 36 sq cm, and the gaps are present in the core.

From Eq. 10, the cross-sectional area of the gap is

$$A = (a + \delta)(b + \delta)$$

when δ does not exceed about one-fifth of a or b. For $A_g = 5.0 \times 5.0$ cm, the maximum value of

$$\delta = \frac{1}{5} \times 5.0 = 1.0 \text{ cm},$$

then the equivalent area of the gap

$$A = (5.0 + 1.0)(5.0 + 1.0)$$

$$= 36.00 \text{ sq. cm.}$$

Therefore

$$B_g = \frac{\phi}{A} = \frac{283,140}{36.00}$$

$$= 7870 \text{ gausses.}$$

From Eq. 12, $H_g = 0.796 \times 7,870 = 6,260$ amp-turns/cm

$$U_g = H_g l_g = 6,260 \times 1.0 = 6,260 \text{ amp-turns.}$$

The amp-turns required for 2 gaps (see Fig. 3) are

$$= 6,260 + 6,260 = 12,520 \text{ amp-turns.}$$

From Eq. 17

$$I_{gap} = \frac{12,520}{350/\sqrt{2}} = 25.3 \text{ amp. rms.}$$

$$I_m = \frac{U_m}{N/\sqrt{2}} = \frac{H_m l_m}{N/\sqrt{2}},$$

where $H_m = \frac{13}{2.54/\sqrt{2}}$ amp-turns/cm, l_m (from TABLE III) = 62.4 cm.

Then

$$I_m = \frac{13}{2.54/\sqrt{2}} \times \frac{62.4}{350} = 0.646 \text{ amp. rms.}$$

And from Eq. 8,

$$I_c = \frac{26.20}{220} = 0.119 \text{ amp. rms.}$$

The effective value of the exciting current, from Eq. 10,

$$I = \sqrt{(I_m + I_{gap})^2 + I_c^2}$$

$$I = \sqrt{(25.3 + 0.646)^2 + (0.119)^2}$$

$$= 25.97 \quad \text{amperes.}$$

Next $A_g = 6.0 \times 6.0$ cm is considered. The maximum value of

$$\delta = \frac{1}{5} \times 6.0 = 1.2 \quad \text{cm}$$

The equivalent area of the gap

$$A = (6.0 + 1.2)(6.0 + 1.2)$$

$$= 51.84 \quad \text{sq cm.}$$

$$B_g = \frac{283,140}{51.84} = 5,460 \quad \text{gausses.}$$

then

$$H_g = 0.796 \times 5,460 = \quad \text{amp-turns/cm.}$$

$$U_g = H_g \ell_g = 4,346 \times 1.2 = 5,215 \quad \text{amp-turns.}$$

The ampere-turns required for 2 gaps are

$$= 5,215 + 5,215 = 10,430 \quad \text{amp-turns.}$$

Then

$$I_{gap} = \frac{10,430}{350/\sqrt{2}} = 21.1 \quad \text{amp. rms.}$$

from the TABLE III,

$$I_h = \frac{4}{2.54/\sqrt{2}} \times \frac{66.4}{350} = 0.211 \quad \text{amp. rms.}$$

$$I_e = \frac{18.3}{220} = 0.083 \quad \text{amp. rms.}$$

Therefore

$$I = \sqrt{(21.1 + 0.211)^2 + 0.083)^2}$$

$$= 21.31 \quad \text{amperes rms.}$$

It can be seen from these calculations that the iron-core inductor having 350 turns is considered to be better than the other one if its core area is 36.00 sq cm and air gaps are present in in the magnetic circuit. The reasons are as follows:

1. if no air gap is present, the size of the inductor is large and the losses are too much,

2. with air gaps inserted in the magnetic circuit,, the core size decreases, the losses are small,

3. core area is 56.00 sq cm, the rules for the equation $A = (a + \delta)(b + \delta)$ is usable. If the core area A_c decreases the flux density is too large and the exciting current is large with very short gap length. If the core area increases the total loss increases and its cost also increases.

From these reasons, the iron-core inductor having 56.00 sq cm core area, 350 winding turns, is considered to be built and its total loss is considered to be minimum. The gap lengths are changed by moving up and down the upper part of the core. The dimensions of the core and winding are shown in Fig. 4.

The characteristics of the designed inductor are shown in the Table V.

The magnetic potential required by the whole iron-core circuit

$$U_c = H_c \ell_c = \frac{4}{2.54} \times 66.4 = 105 \text{ amp-turns.}$$

The core-loss current I_c =

$$I_c = \frac{18.30}{220} = 0.083 \text{ amperes.}$$

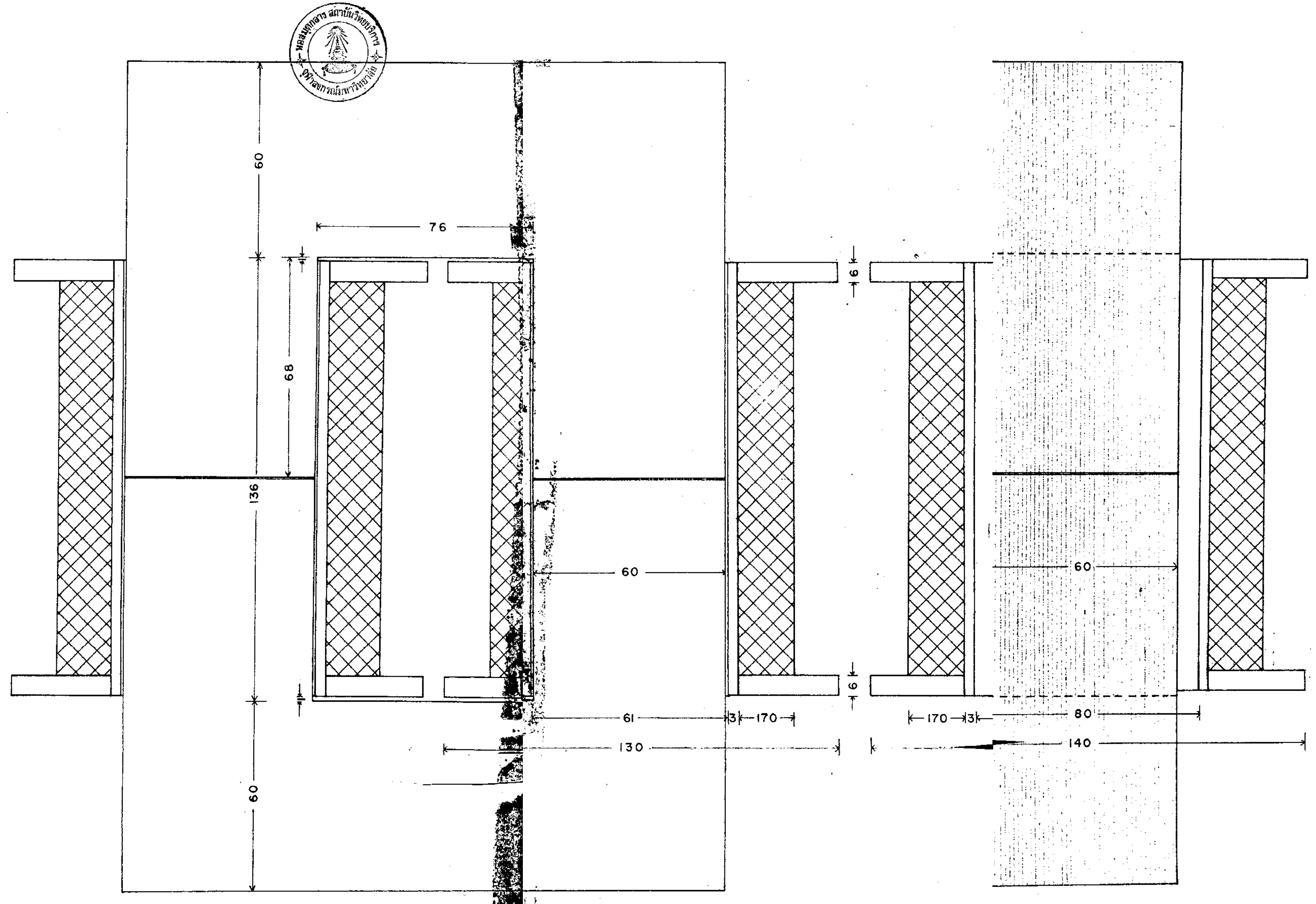


Fig. 4. mensions of core and winding.

TABLE V

CHARACTERISTICS OF THE DESIGNED VARIABLE INDUCTOR

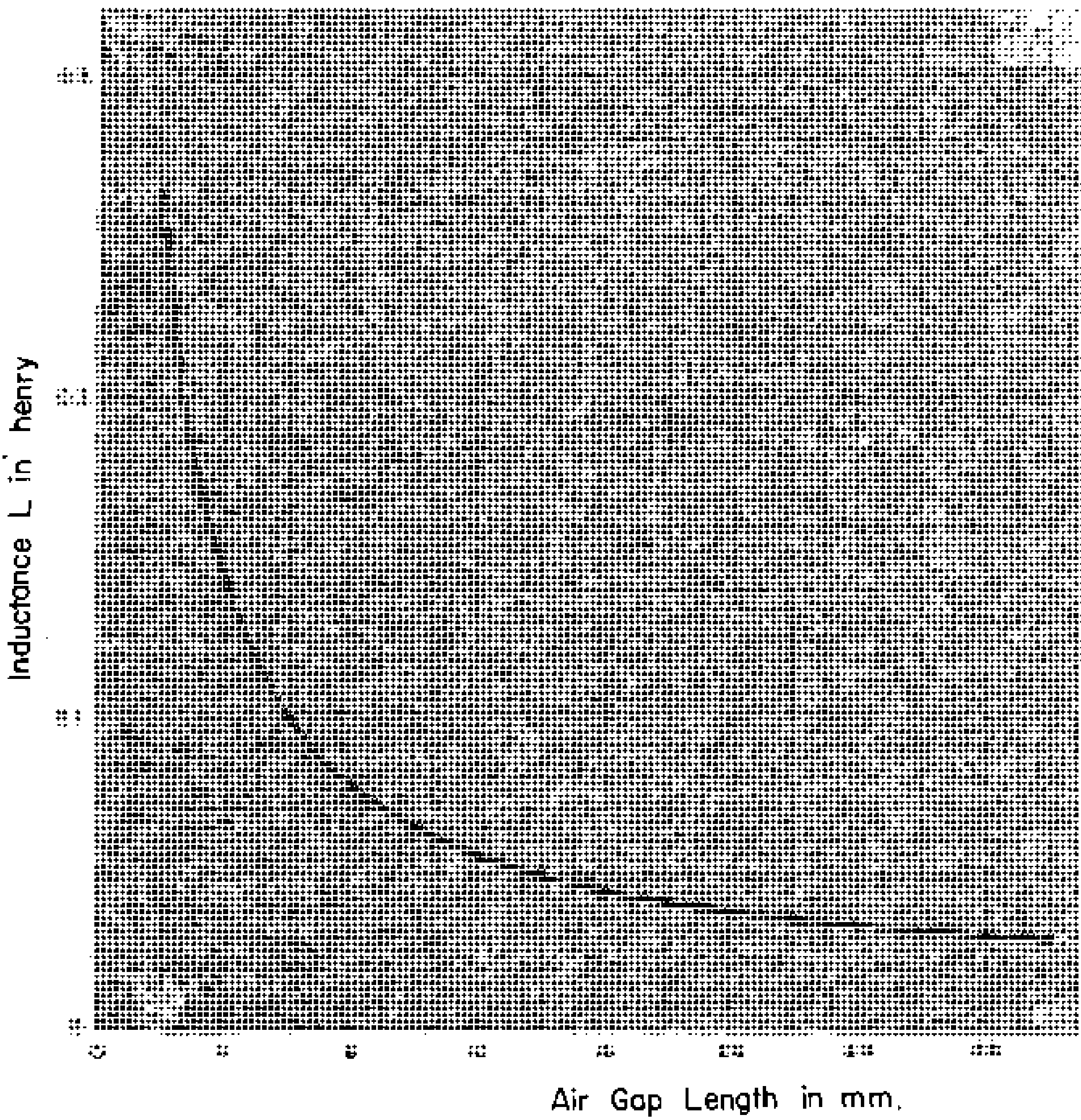
Air Gap Length δ mm	Equiv. Area of Air Gap $A=(a+\delta)(b+\delta)$ sq cm	$B_g = \frac{\phi}{A}$ gausses	$H_g = 0.796 B_g$ amp-turns/cm	$U_g = 2 H_g$ amp-turns	$F = U_c + U_g$ amp-turns
0	-	-	-	-	105
1	37.21	7,610	6,058	1,212	1,317
2	38.44	7,360	5,859	2,344	2,449
3	39.69	7,130	5,675	3,406	3,511
4	40.96	6,910	5,500	4,400	4,505
5	42.25	6,670	5,309	5,310	5,415
6	43.56	6,500	5,174	6,208	6,313
7	44.89	6,300	5,015	7,022	7,127
8	46.24	6,120	4,872	7,796	7,911
9	47.61	5,940	4,728	8,510	8,615
10	49.00	5,780	4,601	9,202	9,307
11	50.41	5,610	4,466	9,826	9,931
12	51.84	5,460	4,346	10,430	10,535
13	53.29	5,310	4,227	10,990	11,095
14	54.76	5,170	4,115	11,522	11,627
15	56.25	5,030	4,004	12,012	12,117

TABLE VI

CHARACTERISTICS OF THE DESIGNED VARIABLE INDUCTOR (CONT'D)

Air Gap Length 2δ mm	Magnetizing Current $I_m = \frac{F}{N}$ amperes	Exciting Current I amperes	Inductance $L = \frac{\phi N}{I}$ henrys	Reactance $X = \omega L$ ohms	Core Loss + I^2R-Copper Load Losses watts
0	0.300	0.228	3.074	965.73	18.30
2	3.763	2.660	0.263	82.64	20.81
4	6.997	4.950	0.142	44.46	26.76
6	10.031	7.070	0.099	30.99	35.45
8	12.871	9.100	0.077	24.18	46.60
10	15.471	10.930	0.064	20.10	59.40
12	18.037	12.750	0.055	17.24	73.80
14	20.263	14.380	0.049	15.29	88.70
16	22.603	16.000	0.044	13.75	105.40
18	24.614	17.420	0.040	12.69	121.70
20	26.591	18.800	0.037	11.62	138.90
22	28.574	20.070	0.035	10.96	154.80
24	30.100	21.310	0.033	10.33	173.20
26	31.700	22.410	0.031	9.80	189.40
28	33.200	23.450	0.030	9.36	205.90
30	34.620	24.450	0.029	8.98	222.40

Fig. 5. Correlation of Inductance and Air Gap Length.



Ratio of Reactance to Resistance and Loss Ratio

When no gaps are present, P_c = 18.30 watts, R = 0.3105 ohm.

From Eq. 33-36,

$$I_c = \frac{18.30}{220} = 0.083 \text{ amp.,}$$

$$g_o = \frac{0.083}{220} = 0.00037 \text{ mho,}$$

$$I_o = 0.212 \text{ amp.,}$$

$$b_o = -\frac{0.212}{220} = -0.00096 \text{ mho.}$$

From Eq. 39,

$$r_c = \frac{0.00037}{(0.00037)^2 + (-0.00096)^2}$$

$$= 349.55 \text{ ohms,}$$

and from Eq. 40,

$$x_o = \frac{0.00096}{0.000001059} = 906.94 \text{ ohms.}$$

Then

$$Q_o = \frac{x_o}{r_o + R} = \frac{906.94}{349.55 + 0.3105}$$

$$= 2.59 .$$

From Eq. 34, $I_o^2 R = P_c + I_c^2 R$,

then

$$I_o^2 \times 0.3105 = 18.30 + 0.083 \times 0.3105$$

$$I_o^2 = \frac{18.302}{0.3105} = 58.94 \text{ amp.}$$

$$I_o = 7.65 \text{ amperes.}$$

The value of I_o that results in the minimum loss ratio is 7.65 amp.

The lengthsof the gaps are between 3 and 4 cm.