FACTORS GOVERNING THE DESIGN

Core Area, Winding Turns and Flux Density

The effective value of E, the voltage induced by the flux, is

$$E = 4.44\, f\, N\, \Phi_{max}, \quad \text{volts,} \qquad (1)$$

where f = frequency cps,

N = number of turns in the exciting winding,

Φ_{max} = maximum value of the flux in webers,

When the resistance drop is negligible, the voltage generated by the flux very nearly equals the terminal voltage. Therefore when a sinusoidal voltage is impressed at the terminals of a winding, the maximum value of the core flux is determined by the effective value and the frequency of the applied voltage, and the number of turns in the winding. That is, from Eq. 1,

$$\Phi_{max} = \frac{E}{4.44\, f\, N}, \qquad (2)$$

when E is in volts, Φ_{max} is in webers.

If the flux density B in the core is fixed, the cross-sectional area of the core is readily determined when the total flux is known. The area of the core

$$A_m = \frac{\Phi_{max}}{B}. \qquad (3)$$

The effective value of the exciting current I is given by the magnitude of the vector sum of the core-loss component I_c and the magnetizing component I_m,

$$I = \sqrt{I_c^2 + I_m^2}. \qquad (4)$$

For a certain value of the magnetizing current component I_m, the mean length of the flux path

$$l_c = \frac{I_m N}{H_m}, \qquad (5)$$

where H_m is the ampere-turns per inch for a given flux density B. The weight of the core

$$G_c = l_c A_c d_c, \qquad (6)$$

where d_c is the density of silicon sheet steel. Therefore, the total core loss or the power absorbed

$$P_c = G_c w_c, \qquad (7)$$

where w_c is the core loss, watts per pound, at the flux density B kilolines/sq in. The core-loss current component

$$I_c = \frac{P_c}{E}. \qquad (8)$$

In actual transformer or reactor, the current increases more rapidly than the flux, the eddy-current loss varies as the square of the flux, and the hysteresis loss may often be assumed also to vary as the square of the flux.

The total losses in an iron-core reactor comprise of the effective resistance loss I^2R, and the hysteresis and eddy-current losses in the core (core loss), that is

$$P = I^2R + \text{core loss}. \qquad (9)$$

For the iron-core inductor designed for minimum total losses, the value of P is made minimum for the same volt-amperes.

In order to study the effects in design, the effects of core and of the winding must be considered carefully.

Air Gap

When the air gap is short compared with its cross-sectional dimensions and has parallel faces, magnetic circuit calculations can usually be performed with a precision approximating the limits of reliability of most magnetic data. As for the other magnetic-circuit computations, the method yields the total flux and average flux density for the air gap. The determination of actual flux distribution in the gap, the distribution of the fringing and leakage fluxes, is a field-mapping problem. The procedure here used is to neglect the effect of leakage flux. The effect of fringing is then taken into account for computation of total flux through replacing the actual parallel-face gap with its fringing by a parallel-face gap assumed to have no fringing but a reluctance equivalent to that of the actual gap. If the cross-sectional dimensions of the core are the same on both faces of the gap, the equivalent gap is assumed to have a length δ equal to the actual gap, but to have a cross-sectional area

$$A = (a + \delta)(b + \delta) \qquad (10)$$

where a and b are the cross-sectional dimensions of the actual core faces.

Experience shows that these rules ordinarily give satisfactory results if the correction applied does not exceed about one-fifth of the cross-sectional dimension to which it is applied.[1]

[1] Members of the Electrical Engineering Staff, M.I.T., *Magnetic Circuits and Transformers* (New York: John Wiley & Sons, Inc., 1958), p. 69.

Within the region of the equivalent gap the flux density is assumed to be uniform. Therefore, between the core faces,

$$H \text{ (oersteds)} = B \text{ (gausses)} \qquad (11)$$

$$H \text{ (amp-turns/cm)} = \frac{1}{0.4} B = 0.796\, B \text{ (gausses)} \qquad (12)$$

$$H \text{ (amp-turns/in.)} = \frac{1}{0.4} \times \frac{2.54}{(2.54)^2} \times 10^3\, B$$

$$= 313\, B \text{ (kilolines/sq in)} \qquad (13)$$

$$H \text{ (praoersteds)} = 10^7\, B \text{ (webers/sq m)}. \qquad (14)$$

Since the flux density is assumed to be uniform, the total flux is

$$\phi = B\, A\ . \qquad (15)$$

If the flux or flux density is known, the magnetizing force H_a in the air gap can be calculated from Eqs. 11, 12, 13, or 14. The magnetizing force H_s for the steel portion of the core can be read out from the magnetization curve in the Appendix. The magnetomotive force for the circuit is

$$F = H_a \ell_a + H_s \ell_s \qquad (16)$$

where ℓ_a and ℓ_s are the lengths of air gap and steel paths respectively.

The maximum value of the component current added to the exciting current by insertion of the gap is

$$I_{gap} = \frac{H_a \ell_a}{N}\ . \qquad (17)$$

This component current is sinusoidal, since the flux varies sinusoidally. Its effective value I_{gap} is $H_a \ell_a/\sqrt{2}N$. Since this added component current is in phase with the flux, it adds directly to the fundamental sine component I_1' of the exciting current. The other components of the exciting current are practically unchanged, since conditions in the iron portion of the magnetic circuit are essentially unaltered. The air gap therefore increases the effective value of the exciting current I to

$$\sqrt{(I_1' + I_{gap})^2 + I_0^2 + (I_3')^2 + (I_5'')^2 + \cdots\cdots} \qquad (18)$$

Although the harmonics are unchanged in ampere value, they are much smaller percentage of the increased exciting current. The waveform of the exciting current therefore is more nearly sinusoidal.

The inductance L of the inductor can be expressed as

$$L = \frac{N \phi}{I}, \qquad (19)$$

where ϕ is the rms value of the flux and I is the rms value of the exciting current.