

CHAPTER II

THE THEORY OF IDENTITY



(1) Definition of identity

" $x \equiv y$ " means "x has every property of y and y has every property of x."

(2) Rule of substitution

If $x \equiv y$, then any expression containing x is identical with the corresponding expression containing y in place of x.

(3) Th. $x \equiv x$

(4) Th. If $x \equiv y$ then $y \equiv x$

(5) Th. If $x \equiv y$ and $y \equiv z$ then $x \equiv z$

(6) Th. If $x \equiv z$ and $y \equiv z$ then $x \equiv y$

Example of proof: th. (3)

" $x \equiv x$ " means "x has every property of x and x has every property of x."

The latter statement is obviously true.

Example of proof: th. (5)

Suppose $x \equiv y$ ----- (1)

and $y \equiv z$ ----- (2)

Then, from (2), any expression containing y is identical with the corresponding expression containing z in place of y (Rule of substitution).

Therefore, from (1), " $x \equiv y$ " is identical with the " $x \equiv z$."

Therefore if $x \equiv y$ and $y \equiv z$ are true, it follows that $x \equiv z$ is true.