CHAPTER I

INTRODUCTION

The purpose of this thesis is to set up axiom systems for handling the logic used in formal mathematical proofs.

We first start with the theory of identity which consists of an informal verbal definition of identity, the rule of substitution, and "theorems" on identity. Proofs are informal verbal proofs. These "theorems" are the laws for equivalence relations (p. 31 Birkhoff and Mac Lane). The rules obtained in this section are used for making rigorous proofs in the remaining part of the thesis.

The theory of compound statements is used for handling combinations of statements, rearranging them into convenient forms, negating etc.

The truth value calculus is used for calculating the truth value of a compound statement when the truth value of each constituent statement is given. The statements fall into three classes

(1) Tautologies (always true)

(2) Contradictions (always false)

(3) Neither tautologies nor contraditions (truth value depends on the truth value of the constituent statements).

The $\vdash$ symbol is introduced as a convenient sign to indicate the truth value of a statement. Examples of its use are given in the section on tautologies and the theory of deduction.

The theory of the predicate calculus with one and two variables is given a rigorous treatment.

Each theory is constructed from the following parts.

(1) Basic undefined symbols, which are given an informal description only.

(2) Formation rules for combining the basic undefined symbols into complex expressions.

(3) Definitions for introducing new symbols.

(4) Axioms, which are unproved statements.

(5) Theorems, which are derived from the definitions and axioms by means of the rules from the theory of identity.

In each theory the consistency of the axioms has to be shown by using a model.

For practical purposes, the independence of the axioms need not be shown.

In the case of the truth value calculus the completeness of the theory for determining whether or not a given compound statement is a tautology, a contradiction, or neither is shown by giving the decision procedure for the problem.

Abbreviations

Th.	Theorem
Ax.	Axiom
Def.	Definition
Fig.	Figure