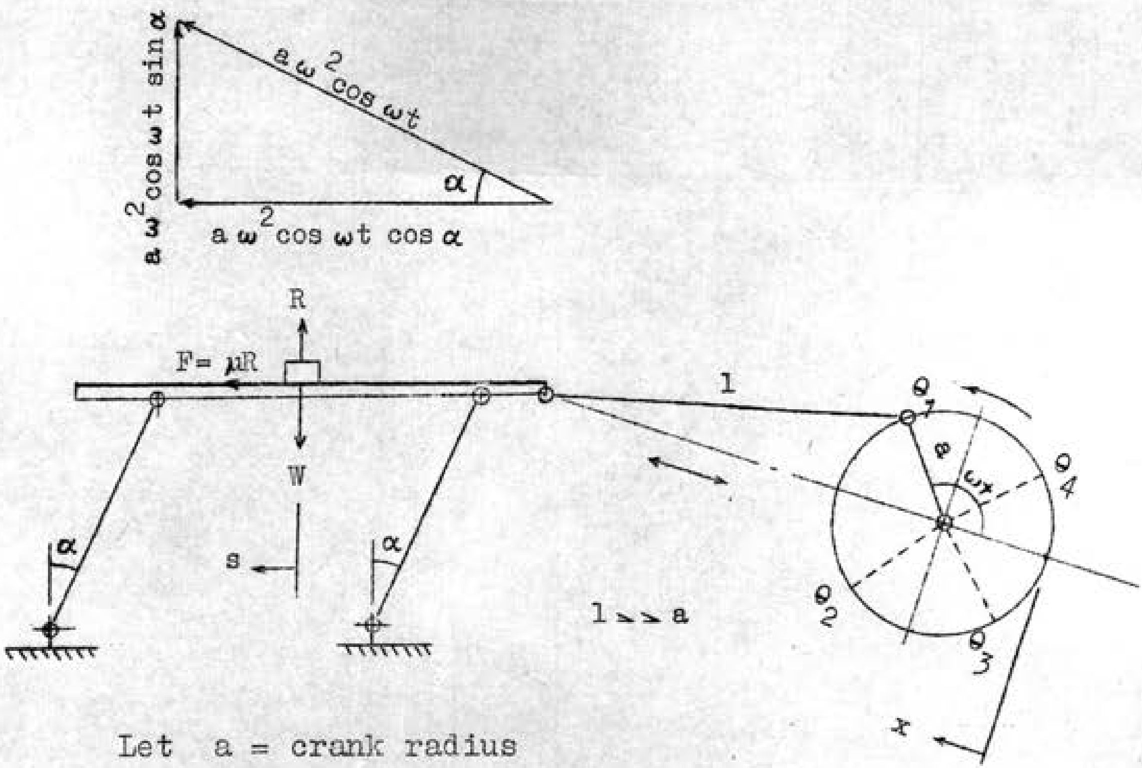
THEORETICAL ANALYSIS

Consider the motion of a small block on the simple conveyor shown in the figure.



Let a = crank radius

l = length of connecting rod

α = link inclination

W = weight of the block

R = reaction between the block and the trough

μ = coefficient of friction between the block and the trough

F = frictional force

ω = angular velocity of crank

t = time

x = displacement of the trough

and then $x = a - a\cos\omega t$

$$\frac{dx}{dt} = a\omega\sin\omega t$$

$$\frac{d^2x}{dt^2} = a\omega^2\cos\omega t$$

The acceleration vector diagram for the block will be as shown.

$$R = W + \frac{W}{g} a\omega^2\cos\omega t \sin\alpha$$

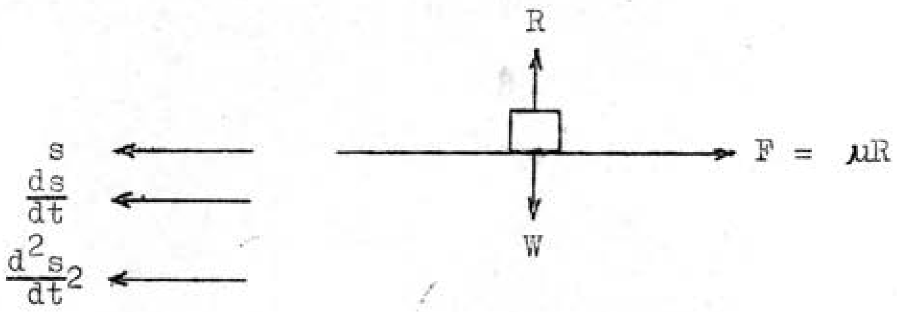
$$F = \mu R = \mu W(1 + \frac{a\omega^2}{g}\cos\omega t \sin\alpha)$$

For the reaction R to remain positive,

$$W > \frac{W}{g} a\omega^2\sin\alpha$$

i.e., $a\omega^2\sin\alpha < g$

Let slipping of the block commence at angle $\omega t = \theta_1$.



At this point,

$$\frac{W}{g} a\omega^2\cos\theta_1 \cos\alpha = -\mu W(1 + \frac{a\omega^2}{g}\cos\theta_1 \sin\alpha)$$

i.e., $$\cos\theta_1 = \frac{-\frac{\mu g}{a\omega^2}}{\cos\alpha + \mu\sin\alpha}$$

As first approximation, assume that slipping velocity is always low and so $F = \mu R$ while slipping as R varies.

Let s = displacement of the block in space measured from the point where slipping first occurs.

Therefore, $\frac{W}{g}\frac{d^2s}{dt^2} = -\mu R = -\mu W(1 + \frac{a\omega^2}{g}\cos\omega t\sin\alpha)$

$$\frac{d^2s}{dt^2} = -\mu g(1 + \frac{a\omega^2}{g}\cos\omega t\sin\alpha)$$

$$\frac{ds}{dt} = -\mu g t - \mu a\omega\sin\omega t\sin\alpha + A$$

$$s = -\frac{\mu g t^2}{2} + \mu a\cos\omega t\sin\alpha + At + B$$

When $\omega t = \theta_1$, $s = 0$ .....(1)

and $\frac{ds}{dt} = a\omega\sin\omega t\cos\alpha$ .....(2)

From (1), $0 = -\frac{\mu g}{2}(\frac{\theta_1}{\omega})^2 + \mu a\cos\theta_1\sin\alpha + A(\frac{\theta_1}{\omega}) + B$ .....(3)

From (2), $a\omega\sin\theta_1\cos\alpha = -\mu g(\frac{\theta_1}{\omega}) - \mu a\omega\sin\theta_1\sin\alpha + A$ .....(4)

$$A = a\omega\sin\theta_1(\cos\alpha + \mu\sin\alpha) + \mu g(\frac{\theta_1}{\omega})$$

Substituting A in (3), we have

$$0 = -\frac{\mu g}{2}(\frac{\theta_1}{\omega})^2 + \mu a\cos\theta_1\sin\alpha + \mu g(\frac{\theta_1}{\omega})^2 + (\frac{\theta_1}{\omega})a\omega\sin\theta_1(\cos\alpha + \mu\sin\alpha) + B$$

$$B = -\frac{\mu g}{2}(\frac{\theta_1}{\omega})^2 - \mu a\cos\theta_1\sin\alpha - a\omega\sin\theta_1(\cos\alpha + \mu\sin\alpha)(\frac{\theta_1}{\omega})$$

Therefore,

$$s = -\frac{\mu g t^2}{2} + \mu a\cos\omega t\sin\alpha + a\omega\sin\theta_1(\cos\alpha + \mu\sin\alpha)t + \mu g(\frac{\theta_1}{\omega})t - \frac{\mu g}{2}(\frac{\theta_1}{\omega})^2 - \mu a\cos\theta_1\sin\alpha - a\omega\sin\theta_1(\cos\alpha + \mu\sin\alpha)(\frac{\theta_1}{\omega}).$$

Let slipping cease at $\omega t = \theta_2$, i.e., $t = \frac{\theta_2}{\omega}$.

$$s_{\theta_1 - \theta_2} = -\frac{\mu g}{2\omega^2}(\theta_2 - \theta_1)^2 + a\left[\mu \sin\alpha\,(\cos\theta_2 - \cos\theta_1) + (\cos\alpha + \mu\sin\alpha)(\theta_2 - \theta_1)\sin\theta_1\right]$$

Now when $\omega t = \theta_2$, the accelerations of the trough and the block will be equal,

i.e., $$-\mu g\left(1 + \frac{a\omega^2}{g}\cos\theta_2 \sin\alpha\right) = a\omega^2 \cos\theta_2 \cos\alpha$$

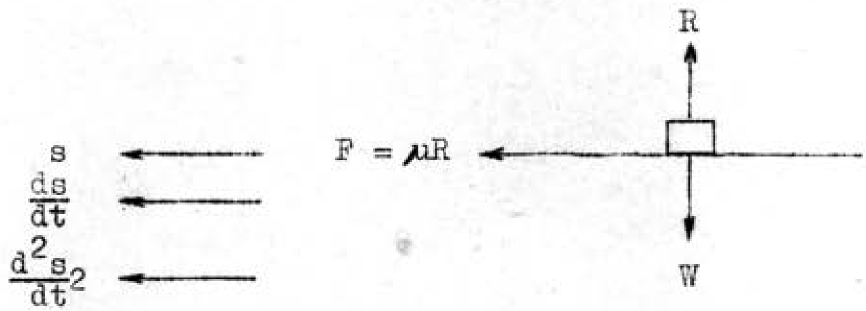
giving $$\cos\theta_2 = \frac{-\frac{\mu g}{a\omega^2}}{\cos\alpha + \mu\sin\alpha}$$

i.e., $$\cos\theta_1 = \cos\theta_2$$

hence $$s_{\theta_1 - \theta_2} = -\frac{\mu g}{2\omega^2}(\theta_2 - \theta_1)^2 + a(\cos\alpha + \mu\sin\alpha)(\theta_2 - \theta_1)\sin\theta_1$$

Displacement of the trough in space during same period equals zero, hence the above represents the relative motion of the block.

Let second slipping period commence at $\omega t = \theta_3$.



At this point,

$$\frac{W}{g} a\omega^2 \cos\theta_3 \cos\alpha = +\mu W\left(1 + \frac{a\omega^2}{g}\cos\theta_3 \sin\alpha\right)$$

i.e., $$\cos\theta_3 = \frac{+\frac{\mu g}{a\omega^2}}{\cos\alpha + \mu\sin\alpha}$$

With the same assumptions as before :-

$$\frac{W}{g}\frac{d^2s}{dt^2} = +\mu R = +\mu W(1 + \frac{a\omega^2}{g}\cos\omega t \sin\alpha)$$

$$\frac{d^2s}{dt^2} = +\mu g(1 + \frac{a\omega^2}{g}\cos\omega t \sin\alpha)$$

$$\frac{ds}{dt} = \mu g t + \mu a\omega \sin\omega t \sin\alpha + A_1$$

$$s = \frac{\mu g t^2}{2} - \mu a \cos\omega t \sin\alpha + A_1 t + B_1$$

When $\omega t = \theta_3$, $s = 0$ .....(1A)

and $\frac{ds}{dt} = a\omega \sin\omega t \cos\alpha$ .....(2A)

From (1A) $0 = \frac{\mu g}{2}(\frac{\theta}{\omega}3)^2 - \mu a \cos\theta_3 \sin\alpha + A_1(\frac{\theta}{\omega}3) + B_1$ ....(3A)

From (2A) $a\omega \sin\theta_3 \cos\alpha = \mu g(\frac{\theta}{\omega}3) + \mu a\omega \sin\theta_3 \sin\alpha + A_1$ (4A)

$$A_1 = a\omega \sin\theta_3(\cos\alpha - \mu \sin\alpha) - \mu g(\frac{\theta}{\omega}3)$$

Substituting A_1 in (3A)

$$0 = \frac{\mu g}{2}(\frac{\theta}{\omega}3)^2 - \mu a \cos\theta_3 \sin\alpha - \mu g(\frac{\theta}{\omega}3)^2$$

$$+ \theta_3 a \sin\theta_3(\cos\alpha - \mu \sin\alpha) + B_1$$

$$B_1 = \frac{\mu g}{2}(\frac{\theta}{\omega}3)^2 + \mu a \cos\theta_3 \sin\alpha$$

$$- a\theta_3 \sin\theta_3(\cos\alpha - \mu \sin\alpha)$$

Therefore, $s = \frac{\mu g t^2}{2} - \mu a \cos\omega t \sin\alpha$

$$+ a\omega \sin\theta_3(\cos\alpha - \mu \sin\alpha)t - \mu g t(\frac{\theta}{\omega}3)$$

$$+ \frac{\mu g}{2}(\frac{\theta}{\omega}3)^2 + \mu a \cos\theta_3 \sin\alpha$$

$$- a\theta_3 \sin\theta_3(\cos\alpha - \mu \sin\alpha)$$

Let slipping cease at $\omega t = \theta_4$, i.e., $t = \frac{\theta}{\omega}4$.

$$s_{\theta_3 - \theta_4} = + \frac{\mu g}{2\omega^2}(\theta_4 - \theta_3)^2 - a\left[\mu \sin\alpha(\cos\theta_4 - \cos\theta_3) - (\cos\alpha - \mu\sin\alpha)(\theta_4 - \theta_3)\sin\theta_3\right]$$

When $\omega t = \theta_4$, the accelerations of the trough and the block will be equal.

$$\mu g\left(1 + \frac{a\omega^2}{g}\cos\theta_4 \sin\alpha\right) = a\omega^2 \cos\theta_4 \cos\alpha$$

giving $\cos\theta_4 = \dfrac{+\frac{\mu g}{a\omega^2}}{\cos\alpha - \mu\sin\alpha}$

i.e., $\cos\theta_3 = \cos\theta_4$

hence $s_{\theta_3 - \theta_4} = \frac{\mu g}{2\omega^2}(\theta_4 - \theta_3)^2 + a(\cos\alpha - \mu\sin\alpha)(\theta_4 - \theta_3)\sin\theta_3$

So nett displacement of the block $S = s_{\theta_1 - \theta_2} + s_{\theta_3 - \theta_4}$

$$S = \frac{\mu g}{2\omega^2}\left[(\theta_4 - \theta_3)^2 - (\theta_2 - \theta_1)^2\right] + a(\cos\alpha + \mu\sin\alpha)(\theta_2 - \theta_1)\sin\theta_1 + a(\cos\alpha - \mu\sin\alpha)(\theta_4 - \theta_3)\sin\theta_3$$

Finally, the mean velocity of the block is

$$v = \text{Nett displacement } S \times \frac{N}{60}$$

where N = speed of crank rotation in rpm.