Chapter 3

Boonpikum and Yoksan's Model

In this model it is assumed that there is a strong interlayer interaction between the CuO_2 layer and the nearest layers. There is a coupling between the nearest neighbour layers via a process in which a pair is scattered out of a layer into a neighbour layer and a small direct hopping between adjacent sheets. A strong intralayer interaction also exists within each layer.

We can write Feynman diagrams for this model as

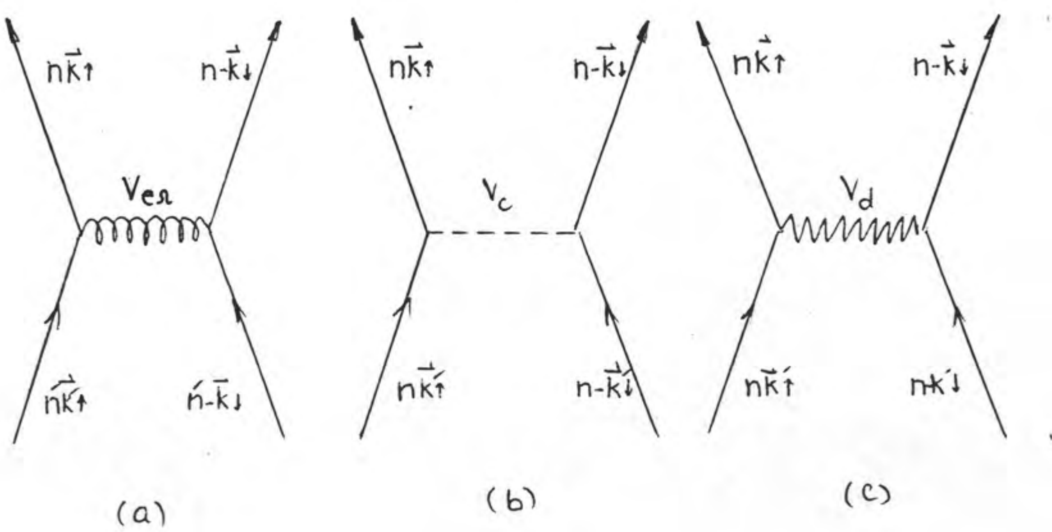

Fig. 3.1 Feynman diagrams where there is a strong interlayer scattering interaction between CuO_2 layers, V_{er}, and strong intralayer scattering interactions within the layer, V_c and V_d (53).

To implement this idea, a multilayer model of 3-sheet unit is introduced as shown in Fig.(3.1).

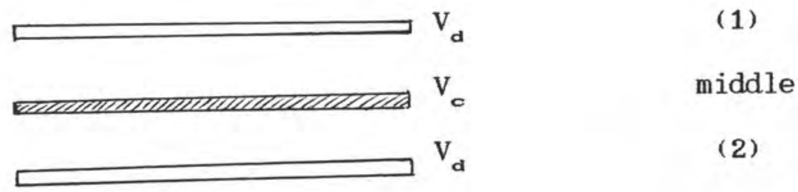

Fig. 3.2 Schematic representation of the layer structure showing a superconducting plane (solid line) and "nonsuperconducting planes" (empty lines) (53).

It should be emphasized that here a model with three layers per unit cell is investigated, the general case of n layers in a unit cell can also be treated easily.

The Hamiltonian of the model can be written as follows(53):

$$
\begin{aligned}
H_o = & \sum_{k}{}' \left(\epsilon_{ck} c^{+}_{k\uparrow} c_{k\uparrow} + \epsilon_{ck} c^{+}_{-k\downarrow} c_{-k\downarrow} \right) \\
& + \sum_{k,n=1,2}{}' \left(\epsilon_{dk} d^{+}_{nk\uparrow} d_{nk\uparrow} + \epsilon_{dk} d^{+}_{n-k\downarrow} d_{n-k\downarrow} \right) \\
& - \sum_{k,k'}{}' \left(V_c c^{+}_{k\uparrow} c^{+}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} + h.c. \right) \\
& - \sum_{k,k'} \left(V_d d^{+}_{nk\uparrow} d^{+}_{n-k\downarrow} d_{n-k'\downarrow} d_{nk'\uparrow} + h.c. \right) \\
& - \sum_{k,k',n=1,2} V_{er} \left[c^{+}_{k\uparrow} c^{+}_{-k\downarrow} d_{n-k'\downarrow} d_{nk'\uparrow} + d^{+}_{nk\uparrow} d^{+}_{n-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} \right] \\
& + \sum_{k,k',n=1,2} t \left[c^{+}_{k\uparrow} d_{nk'\uparrow} + c^{+}_{-k\downarrow} d_{n-k'\downarrow} + d^{+}_{nk\uparrow} c_{k'\uparrow} + d^{+}_{n-k\downarrow} c_{-k'\downarrow} \right]
\end{aligned}
\qquad (3.1)
$$

Here **k** represents momentum index, **n** denotes real space index for the n^{th} layer, V_c, V_d are the constants parameterizing the microscopic pair mechanism in CuO and the neighbour layers, respectively. V_{er} is the Josephson coupling between the nearest neighbour layers, and t is the small electron transfer integral between layers of which we do not specify the origin (54). σ labels the spins within the layers. The first two terms represent the kinetic energy of the non-interacting electrons with energy dispersion ϵ_{ck} and ϵ_{dk}, $c_{k\sigma}$ and $d_{nk\sigma}$ stand for carrier annihilation operators in the middle and outer layers, respectively. Except the last term in Eq.(3.1), our H_o is identical to the Hamiltonian of Ihm and Yu (46), and if we exclude the V_{er} term in Eq.(3.1) we have the Hamiltonian of Schneider and Baeriswyl (56).

To solve the problem, the Green's function method will be used.

From Eq.(3.1) we can split the interaction term $V_c c^+_{k\uparrow} c^+_{-k\downarrow} c_{-k\downarrow} c_{k\uparrow}$ into three parts (42).

$$
\begin{aligned}
V_c c^+_{k\uparrow} c^+_{-k\downarrow} c_{-k\downarrow} c_{k\uparrow} &= V_c \langle c^+_{k} c^+_{-k\downarrow} \rangle c_{-k\downarrow} c_{k\uparrow} + V_c c^+_{k\uparrow} c^+_{-k\downarrow} \langle c_{-k\downarrow} c_{k\uparrow} \rangle \\
&\quad - V_c \langle c^+_{k\uparrow} c^+_{-k\downarrow} \rangle \langle c_{-k\downarrow} c_{k\uparrow} \rangle \\
&= \Delta^+_c c_{-k\downarrow} c_{k\uparrow} + \Delta_c c^+_{k\uparrow} c^+_{-k\downarrow} - \frac{|\Delta_c|^2}{V_c}
\end{aligned}
$$

where the parameters Δ_c^{+} and Δ_c are defined as

$$\Delta_c^{+} = V_c \langle c_{k\uparrow}^{+} c_{-k\downarrow} \rangle \tag{3.2}$$

$$\Delta_c = V_c \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle \tag{3.3}$$

Similarly we have

$$V_d d_{nk\uparrow}^{+} d_{n-k\downarrow}^{+} d_{n-k'\downarrow} d_{nk\uparrow} = \Delta_n^{+} d_{n-k'\downarrow} d_{nk'\uparrow} + \Delta_n d_{nk\uparrow}^{+} d_{n-k\downarrow}^{+} - \frac{|\Delta_n|^2}{V_d}$$

where

$$\Delta_n^{+} = V_d \langle d_{nk\uparrow}^{+} d_{n-k\downarrow} \rangle \tag{3.4}$$

and

$$\Delta_n = V_d \langle d_{n-k'\downarrow} d_{nk\uparrow} \rangle \tag{3.5}$$

Here Δ_c, Δ_n are order parameters of middle and neighbour layers respectively. These parameters serve to define a T_c solution to the problem.

In the same manner, we have

$$V_{er} c_{k\uparrow}^{+} c_{-k\downarrow}^{+} d_{n-k'\downarrow} d_{nk'\uparrow} = \frac{V_{er} \Delta_c^{+} d_{n-k'\downarrow} d_{nk'\uparrow}}{V_c} + \frac{V_{er} \Delta_n c_{k\uparrow}^{+} c_{-k\downarrow}^{+}}{V_d} - \frac{V_{er} \Delta_c^{+} \Delta_n}{V_c V_d} \tag{3.6}$$

and

$$V_{er} d_{nk\uparrow} d^{+}_{n-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} = \frac{V_{er} \Delta^{+}_{n} c_{-k'\downarrow} c_{k'\uparrow}}{V_d} + V_{er} \Delta_c d^{+}_{nk\uparrow} d^{+}_{n-k\downarrow} - \frac{V_{er} \Delta^{+}_{n} \Delta_c}{V_c V_d} \quad (3.7)$$

The Hamiltonian now becomes

$$H_o = \sum_{k} (\epsilon_{ck} c^{+}_{k\uparrow} c_{k\uparrow} + \epsilon_{ck} c^{+}_{-k\downarrow} c_{-k\downarrow})$$

$$+ \sum_{k,n=1,2} (\epsilon_{dk} d^{+}_{nk} d_{nk\uparrow} + \epsilon_{dk} d^{+}_{n-k\downarrow} d_{n-k\downarrow})$$

$$- \sum_{k} (\Delta^{+}_{c} c_{-k'\downarrow} c_{k'\uparrow} + \Delta_c c^{+}_{k\uparrow} c^{+}_{-k\downarrow} - \frac{|\Delta_c|^2}{V_c}$$

$$- \sum_{k,k',n} (\Delta^{+}_{n} d_{n-k'\downarrow} d_{nk'\uparrow} + \Delta_n d^{+}_{nk\uparrow} d^{+}_{n-k\downarrow} - \frac{|\Delta_n|^2}{V_d}$$

$$- \sum_{k,k',n=1,2} (\frac{V_{er} \Delta^{+}_{c} d_{n-k'\downarrow} d_{nk'\uparrow}}{V_c} + \frac{V_{er} \Delta_n c^{+}_{k\uparrow} c^{+}_{-k\downarrow}}{V_d} - \frac{V_{er} \Delta^{+}_{c} \Delta_n}{V_c V_d})$$

$$+ \sum_{k,k',n=1,2} (\frac{V_{er} \Delta^{+}_{n} c_{-k\downarrow} c_{k'\uparrow}}{V_d} + \frac{V_{er} \Delta_c d^{+}_{nk\uparrow} d^{+}_{n-k\downarrow}}{V_c} - \frac{V_{er} \Delta^{+}_{c} \Delta_c}{V_c V_d})$$

$$- \sum_{k,k',n=1,2} t[c^{+}_{k\uparrow} d_{nk'\uparrow} + c^{+}_{-k\downarrow} d_{n-k'\downarrow} + d^{+}_{nk\uparrow} c_{k'\uparrow} + d^{+}_{n-k\downarrow} c_{-k'\downarrow}] \quad (3.8)$$

Using the Heisenberg's equation of motion for the creation and annihilation operators with the Hamiltonian given in Eq.(3.8), and by letting $\hbar=1$, we get

$$i\frac{d}{dt}c^{+}_{k\uparrow} = [c^{+}_{k\uparrow}, H_o]$$

$$i\frac{d}{dt}c^{+}_{k\uparrow} = -\epsilon_{ck}c^{+}_{k\uparrow} + \Delta_c c^{+}_{-k\downarrow} + \frac{V_{er}}{V_d}\sum \Delta_n c_{-k\downarrow} - td^{+}_{nk\uparrow} \qquad (3.9)$$

$$(i\frac{d}{dt} - \epsilon_{ck})c^{+}_{k\uparrow} + (\Delta_c + \frac{V_{er}}{V_d}\sum \Delta_n)c^{+}_{-k\downarrow} - \sum td_{nk''\uparrow} = 0 \qquad (3.10)$$

Proceeding in the same manner for each operator we obtain the following equations :

$$(i\frac{d}{dt} + \epsilon_{ck})c^{+}_{k\uparrow} - (\Delta^{+}_c + \frac{V_{er}}{V_d}\sum_n \Delta^{+}_n)c_{-k\downarrow} + \sum_{k',n} td^{+}_{nk'\uparrow} = 0 \qquad (3.11)$$

$$(i\frac{d}{dt} - \epsilon_{ck})c_{k\uparrow} + (\Delta_c + \frac{V_{er}}{V_d}\sum_n \Delta_n)c^{+}_{-k\downarrow} - \sum_{k'',n} td_{nk''\uparrow} = 0 \qquad (3.12)$$

$$(i\frac{d}{dt} + \epsilon_{ck})c^{+}_{-k\downarrow} + (\Delta^{+}_c + \frac{V_{er}}{V_d}\sum_n \Delta^{+}_n)c_{k\uparrow} + \sum_{k',n} td^{+}_{n-k'\downarrow} = 0 \qquad (3.13)$$

$$(i\frac{d}{dt} - \epsilon_{ck})c_{-k\downarrow} - (\Delta_c + \frac{V_{er}}{V_d}\sum_n \Delta_n)c^{+}_{k\uparrow} - \sum_{k'',n} td_{n-k''\downarrow} = 0 \qquad (3.14)$$

$$(i\frac{d}{dt} + \epsilon_{dk})d^{+}_{nk\uparrow} - (\Delta^{+}_n + \frac{V_{er}}{V_c}\Delta^{+}_c)d_{n-k\uparrow} + \sum_{k'} tc^{+}_{k'\uparrow} = 0 \qquad (3.15)$$

$$(i\frac{d}{dt} - \epsilon_{dk})d_{nk\uparrow} + (\Delta_n + \frac{V_{er}}{V_c}\Delta_c)d^{+}_{n-k\downarrow} - \sum_{k''} tc_{k''\uparrow} = 0 \qquad (3.16)$$

$$(i\frac{d}{dt} + \epsilon_{dk})d^{+}_{n-k\downarrow} + (\Delta^{+}_n + \frac{V_{er}}{V_c}\Delta^{+}_c)d_{nk\uparrow} + \sum_{k'} tc^{+}_{-k'\downarrow} = 0 \qquad (3.17)$$

$$(i\frac{d}{dt} - \epsilon_{dk})d_{n-k\downarrow} - (\Delta_n + \frac{V_{er}}{V_c}\Delta_c)d^{+}_{nk\uparrow} - \sum_{k''} tc_{-k''\downarrow} = 0 \qquad (3.18)$$

A 2X2 matrix Green's function in now introduced. There are four Green's functions for the model here, namely

$$G^{c}(k,k',\omega_n) = \langle\langle C_k;C^{+}_{k'}\rangle\rangle \tag{3.19}$$

$$G^{n}(k,k',\omega_n) = \langle\langle D_{nk};D^{+}_{nk'}\rangle\rangle \tag{3.20}$$

$$G^{nc}(k,k',\omega_n) = \langle\langle D_{nk};C^{+}_{k'}\rangle\rangle \tag{3.21}$$

$$G^{cn}(k,k',\omega_n) = \langle\langle C_k;D^{+}_{nk'}\rangle\rangle \tag{3.22}$$

where

$$C_k = \begin{pmatrix} c_{k\uparrow} \\ c^{+}_{-k\downarrow} \end{pmatrix}, \quad C^{+}_k = \begin{bmatrix} c^{+}_{k\uparrow} & c_{-k\downarrow} \end{bmatrix} \tag{3.23}$$

$$D_{nk} = \begin{pmatrix} d_{nk\uparrow} \\ d^{+}_{n-k\downarrow} \end{pmatrix}, \quad D^{+}_{nk} = \begin{bmatrix} d^{+}_{nk\uparrow} & d_{n-k\downarrow} \end{bmatrix} \tag{3.24}$$

Thus

$$G^{c}(k,k',\omega_n) = \begin{pmatrix} \langle\langle c_{k\uparrow};c^{+}_{k'\uparrow}\rangle\rangle & \langle\langle c_{k\uparrow};c_{-k'\downarrow}\rangle\rangle \\ \langle\langle c_{-k\downarrow};c^{+}_{k'\uparrow}\rangle\rangle & \langle\langle c^{+}_{-k\downarrow};c_{-k'\downarrow}\rangle\rangle \end{pmatrix} \tag{3.25}$$

$$G^{n}(k,k',\omega_n) = \begin{pmatrix} \langle\langle d_{nk\uparrow};d^{+}_{nk'\uparrow}\rangle\rangle & \langle\langle d_{nk\uparrow};d_{n-k'\downarrow}\rangle\rangle \\ \langle\langle d_{n-k\downarrow};d^{+}_{nk'\uparrow}\rangle\rangle & \langle\langle d^{+}_{n-k\downarrow};d_{n-k'\downarrow}\rangle\rangle \end{pmatrix} \tag{3.26}$$

$$G^{nc}(k,k',\omega_n) = \begin{pmatrix} \langle\langle d_{nk\uparrow};c^{+}_{k'\uparrow}\rangle\rangle & \langle\langle d_{nk\uparrow};c_{-k'\downarrow}\rangle\rangle \\ \langle\langle d_{n-k\downarrow};c^{+}_{k'\uparrow}\rangle\rangle & \langle\langle d^{+}_{n-k\downarrow};c_{-k'\downarrow}\rangle\rangle \end{pmatrix} \tag{3.27}$$

and

$$G^{cn}(k,k',\omega_n) = \begin{pmatrix} \langle\langle c_{k\uparrow};d^{+}_{nk'\uparrow}\rangle\rangle & \langle\langle c_{k\uparrow};d_{n-k'\downarrow}\rangle\rangle \\ \langle\langle c_{-k\downarrow};d^{+}_{nk'\uparrow}\rangle\rangle & \langle\langle c^{+}_{-k\downarrow};d_{n-k'\downarrow}\rangle\rangle \end{pmatrix} \tag{3.28}$$

The Fourier transforms of Eqs.(3.12) and (3.13) are taken to get the Green's function matrix elements.

$$(i\omega_n - \epsilon_{ck})\langle\langle c_{k\uparrow};c^{+}_{k'\uparrow}\rangle\rangle + (\Delta_c + \frac{V_{er}}{V_d}\sum_n \Delta_n)\langle\langle c^{+}_{k\uparrow};c^{+}_{k'\uparrow}\rangle\rangle - \sum_{k'',n} t\langle\langle c_{k\uparrow};c^{+}_{k'\uparrow}\rangle\rangle =$$

$$[c_{k\uparrow},c^{+}_{k'\uparrow}] = \delta_{k,k'}$$

or

$$(i\omega_n - \epsilon_{ck})G^{c}_{11}(k,k',\omega_n) + (\Delta_c + \frac{V_{er}}{V_d}\sum_n \Delta_n)G^{c}_{21}(k,k',\omega_n) - \sum_{k'',n} tG^{nc}_{11}(k,k',\omega_n)$$

$$= \delta_{k,k'} \tag{3.29}$$

and from Eq.(3.13)

$$(i\omega_n + \epsilon_{ck})G^{c}_{21}(k,k',\omega_n) + (\Delta^{+}_c + \frac{V_{er}}{V_d}\sum_n \Delta_n)G_{11}(k,k',\omega_n) - \sum_{k',n} tG^{nc}_{21}(k,k',\omega_n) = 0 \tag{3.30}$$

Eqs.(3.29) and (3.30) can therefore be written in matrix form as

$$\begin{pmatrix} i\omega_n - \epsilon_{ck} & 0 \\ 0 & i\omega_n + \epsilon_{ck} \end{pmatrix} \begin{pmatrix} G^c_{11} & G^c_{12} \\ G^c_{21} & G^c_{22} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & \Delta_c + \frac{V_{er}}{V_d}\Delta_n \\ \Delta_c + \frac{V_{er}}{V_d}\Delta_n & 0 \end{pmatrix} \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

$$- \begin{pmatrix} \sum_{k'',n} t & 0 \\ 0 & \sum_{k',n} t \end{pmatrix} \begin{pmatrix} G^{nc}_{11} & G^{nc}_{12} \\ G^{nc}_{21} & G^{nc}_{22} \end{pmatrix} = \delta_{k,k'} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

or in short hand notation

$$(i\omega_n - \epsilon_{ck}\sigma_3 + (\Delta_c + \frac{V_{er}}{V_d}\Delta_n)\sigma_1)G^c(k,k',\omega_n) - \sum t\sigma_3 G^{nc}(k,k',\omega_n) = \delta_{k,k'} \tag{3.31}$$

where we define

$$G^{co}(k,k',\omega_n) = \frac{\delta_{k,k'}}{[i\omega_n - \epsilon_{ck}\sigma_3 + (\Delta_c + \frac{V_{er}}{V_d}\sum_n \Delta_n)\sigma_1]} .$$

Therefore from Eq.(3.11)

$$(G^{co}(k,k',\omega_n))^{-1} G^c(k,k',\omega_n) - \sum_{k'',n} t\sigma_3 G^{nc}(k,k',\omega_n) = \delta_{k,k'} \tag{3.32}$$

When Eq.(3.32) is multiplied by $G^{co}(k,k',\omega_n)$, we obtain

$$G^{c}(k,k',\omega_n) = G^{co}(k,k',\omega_n) + G^{co}(k,k',\omega_n)\sum_{k'',n} t\ _3G^{nc}(k,k',\omega_n) \quad (3.33)$$

To find $G^{nc}(k,k',\omega_n)$, we start by applying the Fourier transform to Eqs. (3.16) and (3.17) and get

$$(i\omega_n - \epsilon_{ck})G^{nc}_{11}(k,k',\omega_n)+(\Delta_n + \frac{V_{er}}{V_c}\Delta_c)G^{nc}_{21}(k,k',\omega_n) - \sum_{k'',n} tG^{c}_{11}(k,k',\omega_n) = 0 \quad (3.34)$$

$$(i\omega_n + \epsilon_{ck})G^{nc}_{21}(k,k',\omega_n)+(\Delta_n + \frac{V_{er}}{V_c}\Delta_c)G^{nc}_{11}(k,k',\omega_n) + \sum_{k'',n} tG^{c}_{21}(k,k',\omega_n) = 0 \quad (3.35)$$

Eqs. (3.34) and (3.35) can be written in the matrix form as

$$\begin{bmatrix} i\omega_n - \epsilon_{dk} & 0 \\ 0 & i\omega_n + \epsilon_{dk} \end{bmatrix} \begin{bmatrix} G^{nc}_{11} & G^{nc}_{12} \\ G^{nc}_{21} & G^{nc}_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & \Delta_n + \frac{V_{er}}{V_c}\Delta_c \\ \Delta_n + \frac{V_{er}}{V_c}\Delta_c & 0 \end{bmatrix} \begin{bmatrix} G^{nc}_{11} & G^{nc}_{12} \\ G^{nc}_{21} & G^{nc}_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} \sum_{k'',n} t & 0 \\ 0 & \sum_{k'',n} t \end{bmatrix} \begin{bmatrix} G^{c}_{11} & G^{c}_{12} \\ G^{c}_{21} & G^{c}_{22} \end{bmatrix} = 0$$

or

$$[i\omega_n - \epsilon_{dk}\sigma_3 + (\Delta_n + \frac{V_{er}}{V_c}\Delta_c)\sigma_1]G^{nc}(k,k',\omega_n) = \sum_{k'',n} t\sigma_3 G^{hc}(k,k',\omega_n)$$

$$G^{nc}(k,k',\omega_n) = tG^{nco}(k,k',\omega_n)\sum_{k'',n} \sigma_3 G^{c}(k,k',\omega_n) \qquad (3.36)$$

where

$$G^{nco}(k,k',\omega_n) = \frac{\delta_{k,k'}}{[i\omega_n - \epsilon_{dk}\sigma_3 + (\Delta_n + \frac{V_{er}}{V_c}\Delta_c)\sigma_1]} \qquad (3.37)$$

By substituting $G^{nc}(k,k',\omega_n)$ into Eq.(3.33) we obtain the $G^{c}(k,k',\omega_n)$ equation

$$G^{c}(k,k',\omega_n) = G^{co}_{k,k}(k,k',\omega_n)$$

$$+ t^2 G^{co}(k,k',\omega_n)\sum_{k'',k'''} \sigma_3 G^{nco}(k'',k',\omega_n)\sigma_3 G^{c}(k''',k',\omega_n) \qquad (3.38)$$

We solve this equation by iteration, as a first approximation we write

$$G^{c}(k,k',\omega_n) \doteq G^{co}(k,k',\omega_n)$$

then Eq.(3.38) becomes

$$G^{c}(k,k',\omega_n) = G^{co}(k,k',\omega_n)$$

$$+ t^2 G^{co}(k,k',\omega_n)\sum_{k'',k''',n} \sigma_3 G^{nco}(k'',k',\omega_n)\sigma_3 G^{co}(k''',k',\omega_n) \qquad (3.39)$$

An order parameter equation (3.3) is

$$\Delta_c = V_c \langle c_{k\uparrow} c_{-k\downarrow} \rangle$$

$$= V_c T_c \sum_{k,k',\omega_n} \langle\langle c_{k\uparrow} ; c_{-k'\downarrow} \rangle\rangle$$

$$\Delta_c = V_c T_c \sum_{k,k',\omega_n} G^c_{12}(k,k',\omega_n) \qquad (3.40)$$

Obviously the element G^c_{12} is directly related to T_c solution. Substituting $G^{co}(k,k',\omega_n)$, $G^{nco}(k,k',\omega_n)$ from Eqs.(3.32) and (3.37) into Eq.(3.32), the following relation is obtained

$$G^c_{12}(k,k',\omega_n) = \frac{(\Delta_c + \sum (V_{er}/V_d)\Delta_n)}{(\omega_n^2 + \epsilon_{ck'}^2)} + t^2 \sum_{k',k''} \frac{1}{(\omega_n^2 + \epsilon_{ck''}^2)^2(\omega_n^2 + \epsilon_{ck''}^2)}$$

$$\times [\omega_n^2(\Delta_n + \frac{V_{er}}{V_c}\Delta_c) - 2\omega_n^2(\Delta_c + \sum_n \frac{V_{er}}{V_d}\Delta_n) + 2\epsilon_{ck'}\epsilon_{dk''}(\Delta_c + \sum_n \frac{V_{er}}{V_d}\Delta_n)$$

$$+ \epsilon_{ck}(\Delta_n + \frac{V_{er}}{V_c}\Delta_c)] \qquad (3.41)$$

Using $G_{12}(k,k,\omega_n)$ in Eq.(3.40) and rearranging terms, we obtain the identity

$$\Delta_c[1-V_cT_c\sum_n \frac{1}{(\omega_n^2+\epsilon_{ck}^2)} + 2V_cT_ct^2\sum_{k,k',\omega_n}\omega_n^2 H$$

$$- V_{er}T_ct^2\sum_{k,k',\omega_n}\omega_n^2 H + V_{er}T_ct^2\sum_{k,k',\omega_n}\epsilon_{ck'}H] = \sum_n \Delta_n[\frac{V_{er}V_cT_c}{V_d}\sum\frac{1}{(\omega_n^2+\epsilon_{ck}^2)}$$

$$- 2V_{er}V_cT_ct^2\sum_{k,k',\omega_n}\omega_n^2 H + V_cT_ct^2\sum_{k,k',\omega_n}\omega_n^2 H + V_cT_ct^2\sum_{k,k',n}\epsilon_{ck'}H] \qquad (3.42)$$

$$\text{where}\quad H = 1/(\omega_n^2+\epsilon_{dk'}^2)(\omega_n^2+\epsilon_{ck'}^2)^2 \qquad (3.43)$$

Consider

$$V_cT_c\sum_{k,\omega_n}\frac{1}{(\omega_n^2+\epsilon_{ck}^2)} = V_cT_cN_c\sum_{\omega_n}\int_{-\infty}^{\infty}\frac{d\epsilon}{(\omega_n^2+\epsilon^2)}$$

$$= V_cT_cN_c(2\sum_{n=0}^{\omega}\frac{\pi}{\omega_n}) = V_cN_cF$$

where we define F as

$$F = 2\pi T_c\sum_{n=0}^{\omega}\frac{1}{\omega_n} \qquad (3.44)$$

and

$$2V_cT_ct^2\sum_{k,\omega_n}\omega_n^2H = 2V_cT_ct^2\left(2\sum_{k',k'',n=0}^{\infty}\frac{\omega_n^2}{(\omega_n^2+\epsilon_{ck'}^2)^2(\omega_n^2+\epsilon_{dk''}^2)}\right)$$

$$= 2V_cT_ct^2\ 2\sum_{n=0}^{\infty}N_c\omega_n^2\int\frac{d\epsilon_{ck'}}{(\omega_n^2+\epsilon_{ck'}^2)^2}\ N_d\int\frac{d\epsilon_{dk''}}{(\omega_n^2+\epsilon_{dk''}^2)}$$

$$= 2V_cT_ct^2\ 2\sum_{n=0}^{\infty}N_cN_d\omega_n^2\ \frac{\pi}{2\omega_n^3}\ \frac{\pi}{\omega_n}$$

$$= \frac{2V_cN_cN_dt^2}{T_c}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\ ,\quad \omega_n = (2n+1)\pi T \qquad (3.45)$$

$$= \frac{2V_cN_cN_dt^2\lambda(2)}{T_c}$$

where $\lambda(2) = \sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}$

Here N_c, N_d are the densities of states at the Fermi level of the middle and outer layers, respectively.

Next we calculate

$$2V_cT_ct^2\sum_{k,\omega_n}\omega_n^2H = \frac{2V_cN_cN_dt^2\lambda(2)}{T_c} = K\ ,\ \text{say} \qquad (3.46)$$

The following relations are obtained easily

$$V_{er}T_ct^2\sum_{k,\omega_n}\omega_n^2H = \frac{V_{er}K}{2V_c} \qquad (3.47)$$

$$V_{er}T_c t^2 \sum_{k,\omega_n} \epsilon_{ck} H = \frac{V_{er}K}{2V_c} \tag{3.48}$$

$$\frac{V_{er}V_cT_c}{V_d} \sum_{k,n} \frac{1}{(\omega_n^2 + \epsilon_{ck}^2)} = \frac{V_{er}V_cN_cH}{V_d} \tag{3.49}$$

$$\frac{2V_cT_cV_{er}t^2}{V_d} \sum_{k,n} \omega_n^2 H = \frac{KV_{er}}{V_d} \tag{3.50}$$

$$V_cT_c t^2 \sum_{k,\omega_n} \omega_n^2 H = K/2 \tag{3.51}$$

Now from Eq.(3.42) we have

$$\Delta_c[1 - N_cV_cF + K(1-\frac{V_{er}}{V_d})] = (\Delta_1+\Delta_2)[\frac{V_cV_{er}N_cF}{V_d} + K(1-\frac{V_{er}}{V_d})] \tag{3.52}$$

The n^{th} layer of the system has the Green's function of G^n type. In a similar manner, we obtain

$$G^n(k,k',\omega_n) = G^{no}(k,k',\omega_n)$$

$$+ t^2G^{no}(k,k',\omega_n) \sum_{k'',k'''} \sigma_3 G^{co}(k'',k''',\omega_n)\sigma_3 G^{no}(k''',k',\omega_n) \tag{3.53}$$

Again, using the self-consistent equation (3.5), one arrives at the following equation

$$\Delta_n[1 - N_d V_d F + \acute{K}(1-\frac{V_{er}}{V_d})] = \Delta_c[\frac{V_d V_{er} N_d F}{V_c} + \acute{K}(1-\frac{V_{er}}{V_c})] \qquad (3.54)$$

where $\acute{K} = \frac{\pi^2 t^2 V_d N_c N_d}{4T_{co}}$

Eqs.(3.52) and (3.54) are now readily rewritten in the following matrix form:

$$\begin{vmatrix} 1-N_d V_d F+\acute{K}(1-\frac{V_{er}}{V_d}) & -[\frac{V_d V_{er} N_d F}{V_c}+\acute{K}(1-\frac{V_{er}}{V_c})] & 0 \\ -[\frac{V_c V_{er} N_c F}{V_d}+K(1-\frac{V_{er}}{V_d})] & 1-N_c V_c F+K(1-\frac{V_{er}}{V_c}) & -[\frac{V_c V_{er} N_c F}{V_d}+K(1-\frac{V_{er}}{V_d})] \\ 0 & -[\frac{V_d V_{er} N_d F}{V_c}+\acute{K}(1-\frac{V_{er}}{V_c})] & 1-N_d V_d F+\acute{K}(1-\frac{V_{er}}{V_d}) \end{vmatrix}$$

$$\times \begin{pmatrix} \Delta_1 \\ \Delta_c \\ \Delta_2 \end{pmatrix} = 0 \qquad (3.55)$$

The T_c formula can be obtained from this matrix equation. This objective will be done in the next chapter.