CHAPTER I

THE ELECTRIC AND MAGNETIC FIELDS DUE TO A CHARGE MOVING WITH UNIFORM VELOCITY. *

The electric field $\vec{E}$ and the magnetic field $\vec{B}$ at a point are generally defined in terms of the Lorentz force acting on a test charge placed at the point .

If $\vec{E}$ is the electric field at point A due to the charge q_2

$\vec{B}$ is the magnetic field at point A due to the charge q_2

q_2 is at the origin in the initial frame of reference Σ

as shown in figure I,

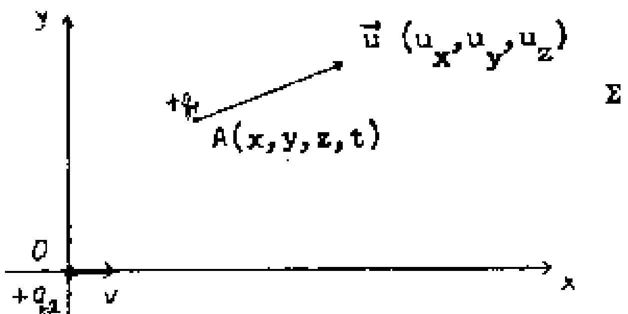

figure I

then the Lorentz force on the charge q_1 expressed in terms of the fields is given by

$$\vec{F} = q_1\vec{E} + \left(\frac{q_1}{c}\right)(\vec{u} \times \vec{B}) . \qquad \text{............ (A)}$$

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* The calculations in this chapter are by W.G.V. Rosser, Contemporary Physics , Vol.1, p.453 (1960).

Writing the Lorentz force in equation (A) in components, we have

$$F_x = q_1 E_x + \frac{q_1}{c} (u_y . B_z - u_z . B_y) , \quad \ldots\ldots\ldots \quad (1)$$

$$F_y = q_1 E_y + \frac{q_1}{c} (u_z . B_x - u_x . B_z) , \quad \ldots\ldots\ldots \quad (2)$$

$$F_z = q_1 E_z + \frac{q_1}{c} (u_x . B_y - u_y . B_x) . \quad \ldots\ldots\ldots \quad (3)$$

By using the transformations of the special theory of relativity we may calculate from Coulomb's law the force between charges moving with uniform velocity assuming the invariance of electric charge. In Σ at a given time $t = 0$ consider a charge q_2 at the origin moving with a uniform velocity v along the ox axis and a charge q_1 at a point A (x,y,z) moving with a velocity $\vec{u}$ having components u_x, u_y, u_z as shown in figure (2a) . Consider a frame Σ' moving to the right with a uniform velocity v relative to Σ along the ox axis as shown in figure (2b) . Let the origins coincide at $t = 0$. The charge q_2 remains at rest at the origin in Σ'. We want to calculate the force on q_1 due to q_2 measured in Σ at the time $t = 0$ when q_2 is at the origin.

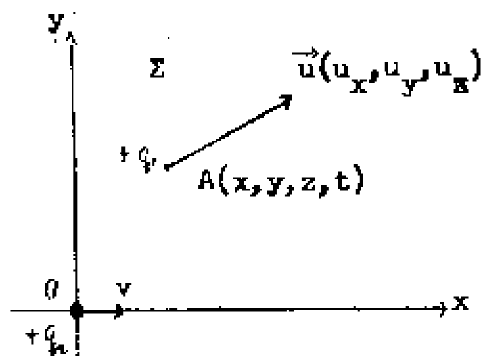

figure (2a)

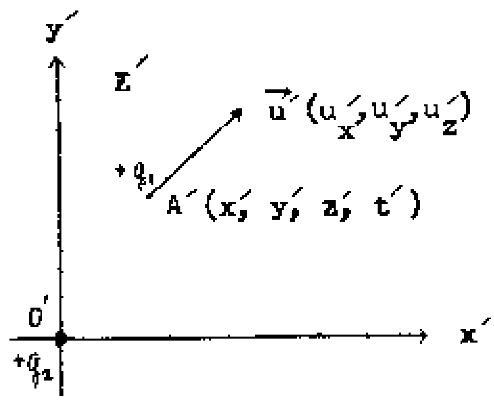

figure (2b)

The formulae for the transformation of force are *

$$F_x = F'_x + \frac{u'_y \cdot v}{(c^2 + u'_x \cdot v)} \cdot F'_y + \frac{u'_z \cdot v}{(c^2 + u'_x \cdot v)} \cdot F'_z ,$$

$$F_y = \frac{c^2 \sqrt{(1 - v^2/c^2)}}{(c^2 + u'_x \cdot v)} \cdot F'_y , \qquad \ldots\ldots(4)$$

$$F_z = \frac{c^2 \sqrt{(1 - v^2/c^2)}}{(c^2 + u'_x \cdot v)} \cdot F'_z .$$

In Σ', since the charge q_2 is always at rest at the origin, we assume that the force on q_1 due to q_2 measured at the point A in Σ' at the time t' (corresponding to $t = 0$ in Σ) is given by Coulomb's law, namely

$$F' = \frac{q_1 q_2}{O'A'^2} .$$

Writing out the components of this force and substituting for x' and $O'A'$ from the Lorentz transformation equations

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} ,$$

$$y = y' ,$$

$$z = z' ,$$

$$t = \frac{vx'/c^2 + t'}{\sqrt{1 - v^2/c^2}} ,$$

*See R.C.Tolman, Relativity, Thermodynamics and Cosmology, p 46, Oxford (1934).

we have

$$F'_x = \frac{q_1 q_2}{s^3} \cdot x \cdot (1 - \frac{v^2}{c^2}) ,$$

$$F'_y = \frac{q_1 q_2}{s^3} \cdot y \cdot (1 - \frac{v^2}{c^2})^{\frac{3}{2}} , \text{-------- (5)}$$

$$F'_z = \frac{q_1 q_2}{s^3} \cdot z \cdot (1 - \frac{v^2}{c^2})^{\frac{3}{2}} ,$$

since $O'A' = \frac{s}{\sqrt{1 - v^2/c^2}}$.

$$s = \sqrt{x^2 + (1 - \frac{v^2}{c^2})(y^2 + z^2)} ,$$

and $x' = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Substituting for F'_x, F'_y and F'_z from equations (5) and for u'_x, u'_y and u'_z we have (from the Lorentz transformation) in equations (4)

$$F_x = \frac{q_1 q_2}{s^3} \cdot (1 - \frac{v^2}{c^2}) \left\{ x + \frac{v}{c^2} (y \cdot u_y + z \cdot u_z) \right\}, \quad \text{......(6)}$$

$$F_y = \frac{q_1 q_2}{s^3} \cdot (1 - \frac{v^2}{c^2}) \left\{ 1 - (v \cdot u_x)/c^2 \right\} \cdot y , \quad \text{......(7)}$$

$$F_z = \frac{q_1 q_2}{s^3} \cdot (1 - \frac{v^2}{c^2}) \left\{ 1 - (v \cdot u_x)/c^2 \right\} \cdot z . \quad \text{..... (8)}$$

Comparing the equations (1), (2), (3) with the equations (6),(7),(8) we conclude that

$$E_x = \frac{q_2}{s^3}\left(1 - \frac{v^2}{c^2}\right) \cdot x \ ,$$

$$E_y = \frac{q_2}{s^3}\left(1 - \frac{v^2}{c^2}\right) \cdot y \ ,$$

$$E_z = \frac{q_2}{s^3}\left(1 - \frac{v^2}{c^2}\right) \cdot z \ ,$$

and

$$B_x = 0 \ ,$$

$$B_y = \frac{-q_2}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot z \ ,$$

$$B_z = \frac{q_2}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot y \ .$$

These equations are the basic equations we shall use as axioms in this thesis for deriving Maxwell's equations.