THE DERIVATION OF MAXWELL'S EQUATIONS FROM THE ELECTRIC AND MAGNETIC FIELDS OF A MOVING CHARGED PARTICLE.

by

Yindee Saklertwachara

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Dean of the Graduate School

Thesis Committee .... Chairman

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....

Thesis Supervisor ....

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ABSTRACT

The electric and magnetic fields due to a charge q at the origin moving with uniform velocity v in the x-direction are as follows

$$E_x = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) x, \quad E_y = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) y, \quad E_z = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) z,$$

and

$$B_x = 0, \quad B_y = -\frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right)\frac{v}{c}\, z, \quad B_z = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right)\frac{v}{c}\, y .$$

These expressions are obtained from Coulomb's law for a charge at rest using the Lorentz transformation and the transformation equations of force from the theory of special relativity. The calculation is given in Contemporary Physics by Rosser (1960).

These equations are the basic equations used as axioms in this thesis for deriving Maxwell's equations.

Maxwell's equations for the behaviour of the electromagnetic field in empty space are

$$\nabla \cdot \vec{E} = 0 , \quad \nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} ,$$

$$\nabla \cdot \vec{B} = 0 , \quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} ,$$

and are obtained directly by simple differentiation of the expressions for the fields.

For non-zero charge densities ρ and current densities j the first two Maxwell's equations are

$$\nabla \cdot \vec{E} = 4\pi\rho \quad ,$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi\vec{j}}{c} \, .$$

These equations are derived from the foregoing expressions for the fields, the latter expression being proved for a uniform current density $\vec{j}$ in space.

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