CHAPTER II

LITERATURE REVIEWS

2.1 Line Drawings

A *line drawing* is a 2D pictorial representation of an object. It is a straightforward method to illustrate 3D object in terms of segments and junctions. In literatures, when talking about line drawing, it means a *raw line drawing.* A raw line drawing [5] is a triple $D_{raw} = (J, S, r)$, where J is an abstract set of *junctions*, S is an abstract set of *segments*, which $S \subset J \times J$, and r is a plane location map that assign a point of the XY plane to every junction in J Raw line drawings can be divided into *natural line drawings* or *wireframe line drawings.* A natural line drawing [4] is a line drawing where only the visible edges and parts of edges are shown. Lines (segments) in the drawing represent the visible parts of edges. Lines intersected at junctions and cycles of lines subdivide the drawing into regions. Whereas a wireframe line drawing [17] is a line drawing in which lines occluded by the material of the solid object (invisible lines) are also shown in the drawing.

When considering the existing realizability methods in terms of input of line drawing, it is clearly known that input to all approaches is given in forms of *incidence structure* which can be obtained from a raw line drawing or a natural line drawing. The incidence structure is a 2D diagram made with straight line segments, called edges, and points where two or more segments meet, called vertices. Such a diagram divides the plane into several regions, called faces. Therefore, the minimum input to all classical existing work on realizability methods is defined in forms of a raw line drawing or a natural line drawing. That means, the 2D coordinates of all essential vertices must be known. From a raw line drawing or a natural line drawing, ones can obtain their own incidence structure according the proposed method.

By this limitation on the input to the existing classical methods, Ros [5] pointed out that it was difficult to directly apply the existing line drawing interpretation method to real image processing due to the strong assumption that all vertices and edges in a given line drawing must be known in advance and essential in the realization. However, our study on 3D realizability problem concerns this difficulty.

2.2 Line Drawing Interpretation

Although, in principle, any line drawing is the 2D representation of an infinite number of possible 3D objects, a counter-argument suggests that computers should be able to interpret line drawing [17]. It is common knowledge that humans have no difficulty in seeing 3D structure in 2D drawings. So, there must be some implicit assumptions and simplifications that enable us to arrive (in most cases) at unique interpretations of 3D structure [18]. Therefore, interpretation of line drawings is a complex problem with some technical assumptions.

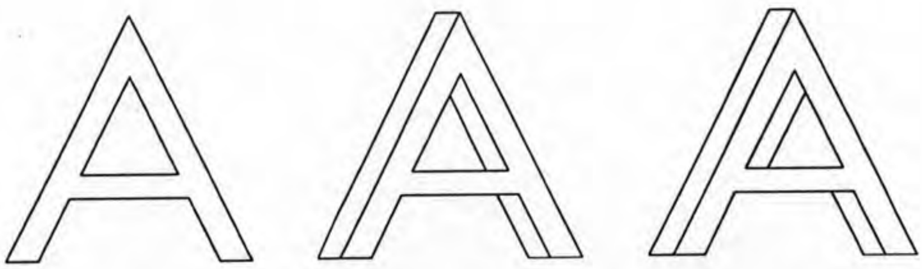
Many theories concerning line drawing interpretation problem have been extensively studied in the past decades. We can distinguish the related works on machine interpretation of line drawing into three main subproblems which are *recognition*, *reconstruction*, and *realizability* of line drawing objects. The recognition problem tries to compare the object in the line drawing with a set of known objects. Most applications on recognition method involved retrieving and identifying a 3D object in a database. Methods for recognition are as proposed by Kamgar-Parsi [19], Wong [20], and Cry [21]. The reconstruction problem attempts to recover the possible 3D shape of the line drawing object by first, faces of an object are identified and then 3D geometry reconstruction is recovered. The background and evolution of three-dimensional reconstruction of line drawing over the last thirty years of sketch-based modeling was discussed in [22] and [23]. The evolution method on faces identification was also proposed by Liu [24]. The shading information of polyhedral objects are included in recovering the 3D shape in the recent work of Shimodaira [25]. Applications on reconstruction methods are for automatical recovering 3D geometry in CAD based modeling and for simulating the real world scene. Methods on recognition and reconstruction always have been proposed under an assumption that the user has an object in mind when creating the line drawing. In other word, assume that the line drawing is realizable as a 3D object. Whereas realizability problem alone is also widely studied. Realizability is the problem of deciding whether the line drawing is realizable as a 3D object; that is, whether it is the correct projection of some 3-dimensional scene of polyhedral objects [14]. The problem is interesting because if we know the object is 3D realizable from its 2D representation, it can guarantee the possible reconstruction the underlying 3D structure of that object. The main objective on existing 3D realizable approaches are therefore proposed what conditions must be verified so that a line drawing represent a possible 3D object.

2.3 Realizability of Line Drawing

2.3.1 Realizability Problem

Because 3D realization means the physical legitimacy (correction) of the 3D object interpretation, a line drawing is said to be realizable if it is the orthographic or perspective projection of some 3D scene of polyhedral objects, and non-realizable otherwise [5]. Therefore, as mention above, realizability is the problem of deciding whether the line drawing is realizable as a 3D object; that is, whether it is the correct projection of some 3-dimensional scene of polyhedral objects. If we know the object is 3D realizable from its 2D representation, it can guarantee the possible reconstruction the underlying 3D structure of that object. As a result, the reconstruction problem and the realizability problem are the related problems.

Fig. 2.1 shows the different perceptions when a 2D representation of the line drawing can be interpreted as a 2D object, 3D realizable object, and non-3D realizable object, respectively. Fig. 2.1(a) shows the line drawing of the capital letter A that can be interpreted as a 2D object. Fig. 2.1(b) shows the line drawing of the capital letter A that can be interpreted as a 3D realizable object. Whereas Fig. 2.1(b) shows an example when the line drawing of the capital letter A is interpreted as a non-3D realizable object. The capital letter A in Fig. 2.1(c) is interpreted as a non-3D realizable object because there is an incorrect edge inside the inner triangle. Such non-3D realizable of the capital letter A in Fig. 2.1(c) is impossible to reconstruct a 3D object.



(a) (b) (c)

Figure 2.1: Three patterns of 2D representation of the line drawings of the capital letter A. (a) Line drawing of the capital letter A interpreted as a 2D object. (b) Line drawing of the capital letter A interpreted as a 3D realizable object. (c) Line drawing of the capital letter A interpreted as a non-3D realizable object.

Therefore, the main objective on classical works about the realizability of line drawing concerns what conditions must be verified so that a line drawing represent a possible 3D object (3D realizable object).

2.3.2 Classical Works on Realizability Problem

The pioneer classical method for checking the realizability of a line drawing was named as line labelling [26, 27, 28]. After that this method has been widely expanded and many other methods also have been proposed. There are mainly two communities concerning realizability problem [5]; machine vision and structural geometry. Methods in machine vision are line labelling [29, 30, 31], Generic Realizability [32], Sugihara's Algebraic Test [6, 33, 34], and Superstrictness [13], whereas most methods in machine vision are based on algebraic approach. In structural geometry community, the methods such as Maxwell's Theorem [35, 36, 37], Structural Geometry [9, 45], Algebraic Geometry [10, 39, 49] and The Cross-Section Test [7, 14, 48] were proposed. It is well known that theory in algebraic side for realizability problem was solved by the classical work by Sugihara in his continuous publications [6, 32, 33, 34]. Recently, the advance work by Ros [14] still proposed an alternative approach in geometric side complementary to Sugihara's approach. We will explain some classical methods in detail as follows.

- **Line Labelling**

  The first mathematical results about realizability of a line drawing started independently with the pioneering works by Huffman [26] and Clowes [27] known as the line labeling technique. They introduced a necessary condition for realizability of a line drawings as the 2D projection of a polyhedral scene.

  In principle, line labelling is a technique that attempts to identify each line in a line drawing as either boundary (<, >) convex (+) or concave (-). The main problem of line labelling is to determine which lines are boundary lines separating objects. Because boundary, convex, and concave lines come together at a junction in only a few ways, this restriction on junction combinations can determine the proper physical interpretation for each line in a drawing. Once the correct line interpretations are known, it is easy to use known boundary lines to divide the drawing into objects. Along the way, some impossible drawings can be detected, because there is no way to interpret all the lines consistently [40].

Hence, the primary outputs of line labelling are the junction label for each junction and the line label for each line. Some drawings have no valid labelling. In such cases, the only output is an error message to the user. And the secondary outputs of line labelling are *runner-up* labelling, which could be used as alternatives if it proves impossible to reconstruct an object on the basis of the first chosen labelling; and a merit figure for each labelling [5].

The main drawback that arises with the labelling scheme is that line drawings having consistent labelling are not guaranteed to be a projection of a polyhedron. As shows in Fig. 2.2, the line drawings in Fig. 2.2(c) and Fig. 2.2(d) are the impossible objects but they can produce the consistent labelling. In Fig. 2.2(a) and Fig. 2.2(b) show when the line labelling can decide whether a line drawing is realizable as a 3D object. The object in Fig. 2.2(a) has a consistency labelling, therefore, it is a 3D realizable object. Whereas, the object in Fig. 2.2(b) is a non-3D realizable object and has the inconsistency labelling. The inconsistency occurs at the segment marked by " ? ". A second drawback of the line labelling method is the need of pre-computed junction dictionaries. Indeed, it is impossible to enumerate all feasible label configurations for the junctions because, in principle, any number of faces can meet on each junction. However, line labelling technique was extended in several directions in [29], [30], and [31].

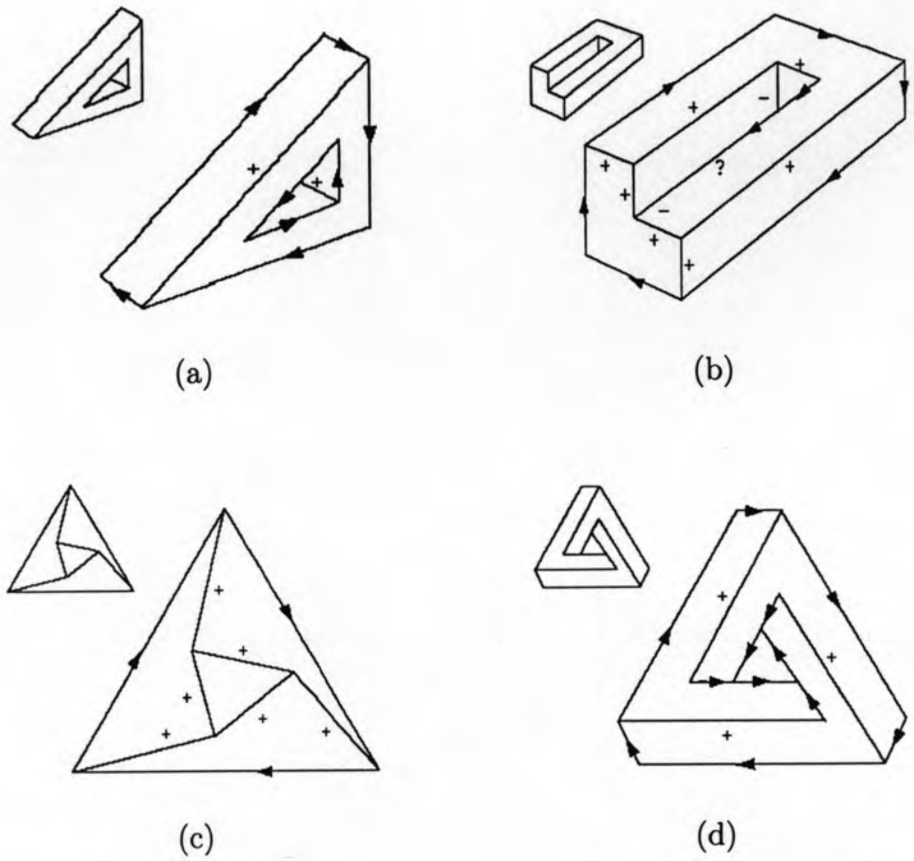


Figure 2.2: Example of line labelling method [5]. (a) A consistency labelling in a 3D realizable object. (b) An inconsistency labelling in an impossible object. The segment marked by " ? " is the impossible segment. (c) The impossible object but it has a consistency labelling. (d) Another impossible object and also has a consistency labelling.

- **Maxwell's Theorem**

  One important alternative to the labelling scheme is the use of reciprocal figures in a dual space. Reciprocal figures were already used more than a century ago by Maxwell [35] for graphical calculus in mechanics. Maxwell proved that a reciprocal figure can be constructed if and only if there exists an assignment of forces to the bars of the framework that keep it in equilibrium –i.e., a self-stress [35, 36] . The reciprocal figure is constructed as follows. First, a framework is abstractly seen as a diagram of vertices (the joints), joins in pairs by straight edges (the bars), and dividing the plane into several regions (the polygonal areas). The reciprocal figure is also a diagram of vertices and edges. It has a vertex for each region of the framework, and an edge joins two vertices if their corresponding regions are adjacent.

The idea of this reciprocal figure has been rediscovered and used repeatedly by Huffman [26], Mackworth [41] or Draper [42] as a necessary condition for labeled line drawing and classifying the inconsistency figure.

- **Sugihara's Algebraic Test**

In 1982, Sugihara finally proposed a complete set of constraints that characterizes realizable line drawings [32]. Roughly speaking, his fundamental theorem states that a labeled drawing is correct if and only if a system of linear constraints of the form $Aw = 0$ and $Bw \geq 0$ has a solution, which can be tested by linear programming techniques. Here, the vector w denotes the unknown parameters of the planar faces of the eventual realization, and A and B are matrices derived from the particular (+,− ,<,>) labelling, the positions of the junctions and the incidence relations between junctions and regions in the drawing. Although Sugihara's approach seemed to be only applicable to the special case in which line drawings solely contain trihedral junctions, Whiteley proved Sugihara's theorems in the general case [43]. A problem with Sugihara's method was that the condition $Aw = 0$ and $Bw = 0$ is too strict and slight perturbations of vertex positions can make a line drawing incorrect. In a realistic application, it is impossible to guarantee the exact position of objects in a scene and some uncertainty must be taken into consideration. In order to correct super strict incorrect pictures, Sugihara proposed a nice solution to delete from those constraints that lead to this superstrictness by using the purely combinatorial concept of position-free incidence structures [44]. In [7], however, Whiteley reported some limitations of this technique.

Ponce and Shimshoni proposed a variation of Sugihara's approach [8]. They defined a system similar to Sugihara but, unlike Sugihara, they did not eliminate constraints leading to a superstrict set of equations, but explicitly introducing uncertainty in these constraints. A necessary condition for a line drawing to be the correct projection of a polyhedron is that this system admits a solution, which again can be tested using linear programming.

- **Algebraic Geometry**

  As far as the realizability problem is concerned, the work of several combinatorial geometers is often unnoticed by the robotics community. In this sense, Crapo and Whiteley investigated the connection between the realizability of linear scenes and the rigidity of planar bar frameworks [9, 45, 46]. Interestingly, they proved that a line drawing is consistent if and only if the representing planar bar framework supports a non-null pattern of stresses on the bars. Hence, the realizability of a drawing and the rigidity of a framework was proved to be connected problems.

- **The Cross-Section Test**

  Whiteley presented another reciprocal diagram in 1991 known as the cross-section [48]. The cross-section reciprocal also offers necessary and sufficient conditions for correct drawings of sphere polyhedra like Maxwell's. The drawing is correct if and only if it is possible to draw a cross-section compatible with it. The cross-section is compatible if the line of any edge between a pair of faces contains the point of intersection of the cross-section lines of such faces. Although the cross-section reciprocal proposed by Whiteley is an elegant tool that graphically allows to test the correctness of a drawing by just verifying the concurrence conditions of lines on it, in its original form, the test can only be used on drawings of spherical polyhedra. In [14], Ros then extended the validity of the cross-section test including the case of polydisks with any number of holes. An algorithm to generate all cross-section, if there are any, was given in his work and the superstrictness problem can also be overcome with his methods in [5] and [13].