CHAPTER III

PROPOSED METHOD

3.1 Features of 3D Realizable Line Drawing Objects

Before each rule is established and proposed in an algorithm for identifying and extracting a realizable 3D object from a set of crossing lines, in this study, the features of 3D realizable line drawing objects are first explored. The significant features of 3D realizable objects and the additional features covered in this research are formally stated.

3.1.1 Significant Features

The significant feature that an object can be interpreted as a 3D realizable object from its 2D representation is the feature as shown in Fig. 3.1. Point a in Fig. 3.1 is the front most junction. Its degree is three which equal to the number of the coherence faces at its position. An object can be interpreted as an unambiguous 3D object if every faces of the front most junction of an object can be visible like an object in Fig. 3.1. Such front most junction must have degree at least three. From this feature, we can formulate the significant conditions of a 3D realizable object as follows.

(1) Given A as a set of junctions of which $degree \geq 3$, an object can be interpreted as a 3D realizable object if at least one junction in A has the property such that,

$$number\ of\ degrees = number\ of\ the\ coherence\ faces$$

(2) In any 3D realizable object, there must be at least another junction in A linked with junction in (1) with the property that,

number of the coherence faces = number of degrees - 1

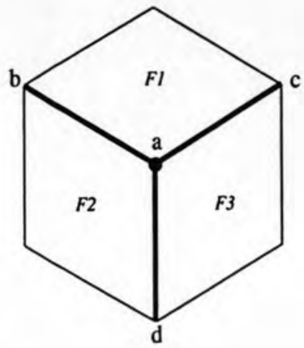
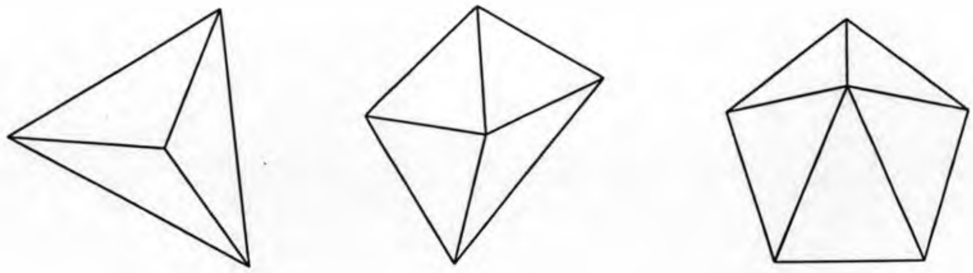


Figure 3.1: Significant feature of 3D realizable object.

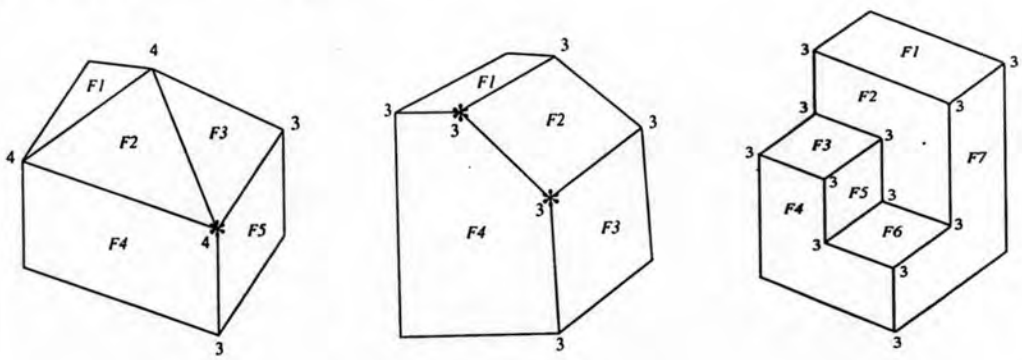
In Fig. 3.1, point a has degree equal to three and it has three coherence faces at point a which are faces $F1$, $F2$, and $F3$. This feature satisfies the first above condition. Points b, c, and d in Fig. 3.1 have the features according to the second condition. Point b links with point a and the number of coherence faces at point b equal to two. Such number of the coherence faces comes from the number of degrees at point b minus one. Point c and point d also have the same features as that of point b.

Figure 3.2 shows other simple examples of the objects that have features according to the significant feature of a 3D realizable object as mention above. Figure 3.3 shows when such significant feature exposes in the more complicated objects. Points mark by the asterisks in Fig. 3.3(a) and Fig. 3.3(b) are the points that have feature according to (1). The object in Fig. 3.3(c) also has the significant feature according to both conditions mentioned above.



(a) (b) (c)

Figure 3.2: Simple examples of the significant feature. (a) There are three coherence faces at the front most junction of this object. Such junction (the inner junction) has degree equal to three. (b) There are four coherence faces at the front most junction of this object. Such junction (the inner junction) has degree equal to four. (c) There are five coherence faces at the front most junction of this object. Such junction (the inner junction) has degree equal to five.



(a) (b) (c)

Figure 3.3: More complicated examples of the significant feature. (a) The point marked by an asterisk has the number of degrees equal to the number of the coherence faces at its position which is four. (b) The points marked by an asterisk also has the number of degrees equal to the number of the coherence faces at its position. (c) Another example of the object when it has the significant feature according to both conditions mentioned above.

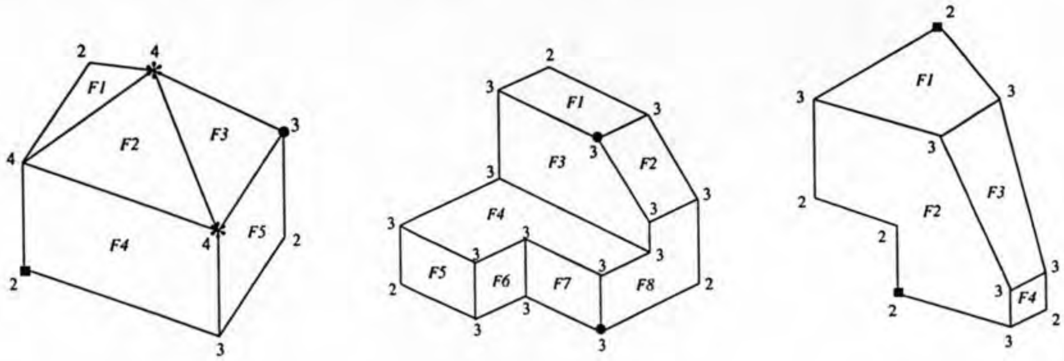
3.1.2 Features Covered in This Research

Besides the significant features mentioned in Section 3.1.1, the 3D realizable objects studied in this research also have the additional features about the degree of junction and junction pattern in each face as follows.

1. **Degree of junction.**
   (a) Degree two junction (junction that has degree equal to two) will be the coherence of only one visible face.
   (b) Degree n junction (junction that has degree equal to n) will be the coherence of at least $n-1$ visible faces, where $3 \leq n \leq 6$.

2. **Junction pattern in each face.**
   (a) Each visible face must consist of at least two junctions which have degree equal to three or more than three.
   (b) On each visible face, junctions which have degree equal to three and more than three must be connected.

The features of 3D realizable polyhedral line drawing objects about degree of junction as mentioned above are shown by the examples in Fig. 3.4. The junctions marked by the asterisks in Fig. 3.4(a) are of degree four. The upper asterisk is the junction of three faces coherence which are F_1, F_2, and F_3. The lower asterisk is the junction of four faces coherence which are F_2, F_3, F_4, and F_5. An example of degree three junction in Fig. 3.4(a) marked by the black circle. It is the coherence of two faces which are F_3, and F_5. In Fig. 3.4(b), the junctions marked by black circles are of degree three. The upper circle is the coherence of three faces whereas the lower circle is the coherence of only two faces. Examples of junctions that have degree equal to two are also shown and marked by black squares in Fig. 3.4(a) and Fig. 3.4(c). The degree two junction shown in Fig. 3.4(a) is the coherence of only face F_4. The upper degree two junction in Fig. 3.4(c) marked by black square is the coherence of only face F_1 and on the same way the lower one is the coherence of only face F_2.

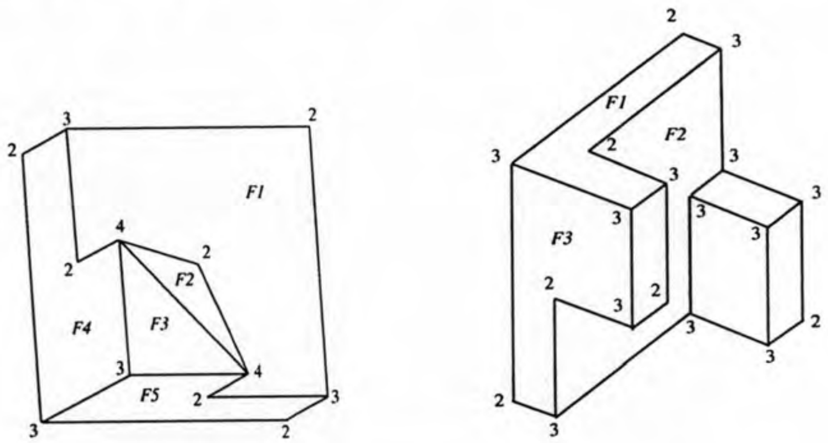


(a) (b) (c)

Figure 3.4: Features of 3D realizable objects about degree of junctions and junction patterns in each face that cover in our study. (a) Object 1. (b) Object 2. (c) Object 3.

The features about junction patterns in each face are also evident in Fig. 3.4. All faces in each object consist at least of two junctions which have degree equal to three or more than three. Such junctions in each face are also connected. In Fig. 3.4(a), junction pattern in face F_1 is 4–4–2. It consists of two junctions which have degree equal to four and such two junctions are connected (has an edge linked between such two junctions). Junction patterns in faces F_2, F_3, F_4, and F_5 are 4–4–4, 4–4–3, 4–4–3–2, and 3–4–3–2, respectively. All junctions in each face which have degree equal to three and more than three are also connected. These features expose in the same way in objects in Fig. 3.4(b), and Fig. 3.4(c).

Figure 3.5 shows the examples of objects that do not have the features constrained by the degree of junctions and junction patterns as previously mentioned. This kind of object will not be covered in our study. In face F_1 of Fig. 3.5(a), there are four junctions that have degree equal to two. Only one of such junctions is the coherence of one visible face (the right most junction) which is the feature that cover in our study. But the other three junctions are the coherence of two faces. Such three junction positions also interrupt the connection between junctions of degree three and degree four in face F_1. Faces F_2, F_4, and F_5 of this object also have the features similar to face F_1. In Fig. 3.5(b), faces F_1, F_2, and F_3 have the same features as well.



(a) (b)

Figure 3.5: Degree of junctions and junction patterns of objects that do not cover in our study. (a) Object 1. (b) Object 2.

3.2 A Proposed Rule-Based Approach

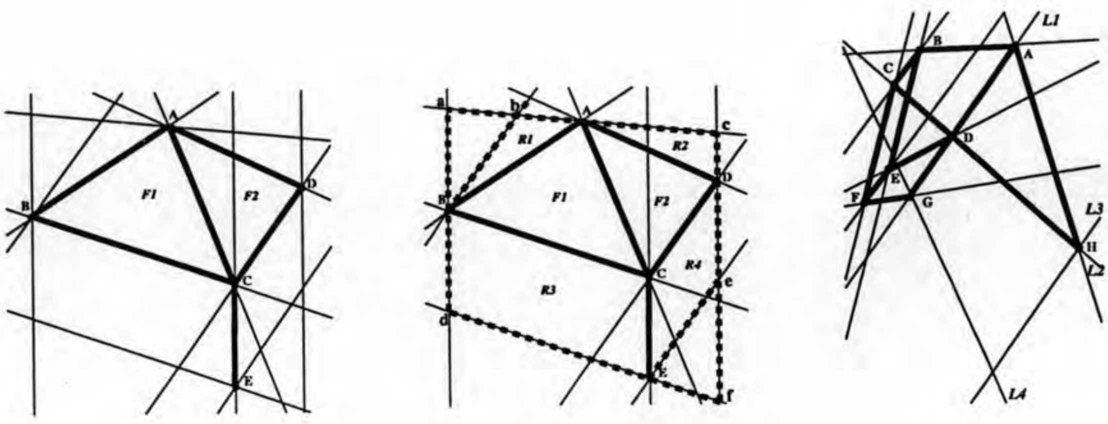
Our rule-based approach is developed under the assumption that each given line is relevant and comprised of at least one identified segment of the corresponding realizable 3D object.

3.2.1 Defined Essential Components

The definitions of all essential components used in the algorithm are defined as follows.

- *Crossing point:* a point created from at least two lines crossing.
- *Significant junction:* a point created from three lines or more than three lines crossing.
- *Significant segment:* segment that links between the significant junctions.
- *Significant face:* closed region that all junctions are significant junctions and all segments are significant segments.
- *Essential segment:* each segment in image frame along the line that have no significant segment passes through.
- *Essential junction:* each crossing point in image frame formed by the essential segments.
- *Essential face:* closed region that consists at least of two significant junctions and one essential junction.
- *Problem line:* line that has at least two identified segments and such segments are connected.
- *Unused line:* line that is not used to identify any segment of the extracted object.

Figure 3.6 shows examples of the defined components. All points created from at least two lines crossing are defined as the crossing points. Figure 3.6(a) shows when the significant junctions, A, B, C, D, and E, are located. They are the points formed by three lines or more than three lines crossing. In this example, junctions A, B, and C are formed by four lines crossing. Junctions D and E are formed by three line crossing.



(a) (b) (c)

Figure 3.6: Examples of essential components defined in the proposed algorithm. (a) The significant junctions, the significant segments (bold solid segments), and the significant faces. (b) The essential junctions, the essential segments (dashed segments), and the essential faces. (c) The problem lines (lines $L1$ and $L2$) and the unused lines (lines $L3$ and $L4$).

The bold solid segments in Fig. 3.6(a) are defined as the significant segments. They are the segments that link between the predefined significant junctions. Faces F_1 and F_2, the closed regions formed by the significant junctions and the significant segments also shown in Fig. 3.6(a), are defined as the significant faces.

The dashed segments in Fig. 3.6(b) are defined as the essential segments. They are all of the segments in image frame along the lines that have no significant segment (the bold solid segments) passes through. Points a, b, c, d, e, and f formed by the predefined essential segments are defined as the essential junctions. The closed regions R_1, R_2, R_3, and R_4 are the essential faces. All of these regions consist at least of two significant junctions and one essential junction. Region R_1 formed by junctions ABb of which junction A and B are the significant junctions and junction b is an essential junction. Region R_3 is formed by three significant junctions and one essential junctions. Lines labeled by L_1 and L_2 in Fig. 3.6(c) are defined as the problem lines. Assume that the bold solid segments in Fig. 3.6(c) are the identified segments. Segments AD and DG are identified as the connected segments on the same line which is line L_1. Therefore line L_1 is defined as the problem line. On the same way, line L_2 is the problem line because segments CD and DH are identified as the connected segments on such line. By our approach, some identified segments on the problem line are the *over-identified segments*. Lines L_3 and L_4 in Fig. 3.6(c) are the unused lines because they still are not used to identify any segments of the extracted object.

3.2.2 The Proposed Algorithm

As mentioned in Section 3.1.1, the junctions of which degree equal to three and more than three are the significant junctions in both conditions of a line drawing object that can be interpreted as a 3D realizable object. If such kind of junctions are first identified, the significant feature of a 3D realizable object can be extracted. From the structure of the crossing lines, all junctions of which degree equal to three and more than three of a line drawing object are hidden under the crossing points formed by at least three lines crossing. Therefore, by our method, the crossing points formed by at least three lines crossing are first identified as the significant junction candidates of the extracted object. Besides, the additional feature of a 3D realizable object in Section 3.1.2 about the junction pattern in each face is also mentioned that on each visible face, junctions which have degree equal to three and more than three must be connected. In the next step of our method, therefore, the segments that link between the previously identified significant junction candidates are identified as the significant segment candidates of the extracted object. The closed regions formed by the identified significant junction and the significant segments are the candidates of the visible faces of an object and we identify them as the significant face candidates.

Not all of the identified significant face candidates are the real faces of a 3D realizable object. Some identified significant junctions or significant segments can be over-identified. The verification process is, then, proposed to verify the identified significant face candidates as the real faces of the extracted object. The first set of rules that based on the observation on the characteristics of 3D realizable objects is applied to remove the over-identified significant junctions and significant segments.

Although all the significant faces are already identified and verified to be the real faces of a 3D realizable object, in most cases the object consisting of only significant faces still is not a 3D realizable object according to its properties and our assumptions. According to our assumptions, each given line is relevant and comprised of at least one segment of the corresponding realizable 3D object. If there are still some lines do representing any significant segment from the previous processes then the next process is applied to identify the remaining essential faces. All possible essential junctions and essential segments that form a realizable 3D object are, therefore, identified in this process. The verification process is, then, also proposed to remove the over-identified essential junctions and segments using the second set of rules.

After all significant and essential faces of an extracted object are already identified and verified to be the real faces of a 3D realizable object, the final rule is applied to verify the extracted object in the final process. The assumption of the extracted object is also checked in the this process. Lines that are not used to represent any segment of the extracted object are considered and used before the final 3D realizable object can be extracted.

Therefore, our proposed rule-based approach roughly described above consists mainly of five processes. Some processes are performed by applying a set of rules. The detail of processes and rules are as follows.

Algorithm: *Extracting a 3D realizable object from a set of crossing lines.*

Input: *A given set of crossing lines and set of crossing points.*

Output: *An extracted 3D realizable object.*

Process 1: Identifying the significant junctions and significant segments as the significant face candidates of a 3D realizable object.

Process 2: Verifying the significant faces candidates whether they are the real significant faces of a 3D realizable object.
(Rules $S1$, $S2$, and $S3$ are orderly applied)

Process 3: Identifying the essential junctions and essential segments as the essential faces candidates of a 3D realizable object.

Process 4: Verifying the essential face candidates whether they are the real essential faces of a 3D realizable object.
(Rules $E1$, $E2$, and $E3$ are orderly applied)

Process 5: Verifying the extracted 3D realizable object.
(Rules $SE1$ is applied and the unused lines are also considered)

- **Rules for verifying the significant faces in Process 2.**

Rule S1: *If the ends of significant segments from at least two problem lines are adjacent, such adjacent point and its links formed by the related significant segments can be removed.*

Rule S2: *If any significant segment on the problem line crosses the other significant segments, such significant segment on the problem line can be removed.*

Rule S3: *If any significant segment on the problem line forms a triangle in some* *significant faces, such significant segment can be removed.*

- **Rules for verifying the essential faces in Process 4.**

  Rule E1: *If any essential segment on the problem line crosses the significant segments, such essential segment on the problem line can be removed.*

  Rule E2: *Closed region that has no significant junction or closed region with only one significant junction cannot be the real face of a 3D realizable object. If such region is formed, it can be removed.*

  Rule E3: *If the ends of essential segments from at least two problem lines are adjacent, such adjacent point and its links formed by the related essential segments can be removed.*

- **Rules for verifying the extracted object in Process 5.**

  Rule SE1: *If each of the identified segment on a problem line is not parallel to any other identified segments in the same region, such identified segment can be removed.*

Figure 3.7 shows the flowchart of the proposed algorithm. Inputs to the algorithm is a set of crossing lines and a set of crossing points. The significant junctions and the significant segments are identified as the significant faces candidates of a 3D realizable object in Process 1. If there is no problem line from the identified significant face candidates and there is no unused line (all input lines are used to identify some segments of the extracted object) then the final 3D realizable object can be extracted by our algorithm.

If there are some problem lines from the identified significant face candidates in Process 1 then Process 2 is applied for verifying the significant face candidates as the real faces of a 3D realizable object. The over-identified significant junctions and significant segments from Process 1 are verified and can be removed after rules $S1$, $S2$, and $S3$ are orderly applied in Process 2.

After all rules in Process 2 are already applied and there are some unused lines, Process 3 is applied for identifying the essential faces of a 3D realizable object. The essential junctions and the essential segments are identified to be the essential faces candidates of a 3D realizable object in this process. If there is no problem line from the identified essential faces candidates and there is no unused line then the final 3D realizable object can be extracted.

If there are some problem lines from the identified essential face candidates in Process 3 then Process 4 is applied for verifying the essential faces candidates to be the real faces of a 3D realizable object. The over-identified essential junctions and essential segments from Process 3 are verified and can be removed after rules $E1$, $E2$, and $E3$ are orderly applied in Process 4.

After all rules in Process 4 are already applied and there are some unused lines, the final process, Process 5, is applied for verifying the extracted object. In this process, if there are some problem lines then rule $SE1$ is applied to remove the remaining over fit identified junctions and segments. If there are some unused lines, all essential junctions on such lines are considered again whether they can be the real junctions of the extracted object.

3.2.3 The Detailed Steps by Examples

In order to explain the detailed steps of the proposed method to cover with most possible cases of the existing objects, we use three realizable objects shown in Fig. 3.8 as the examples. The set of lines extracted from each realizable object shown in Fig. 3.9 is used as the input to the algorithm.

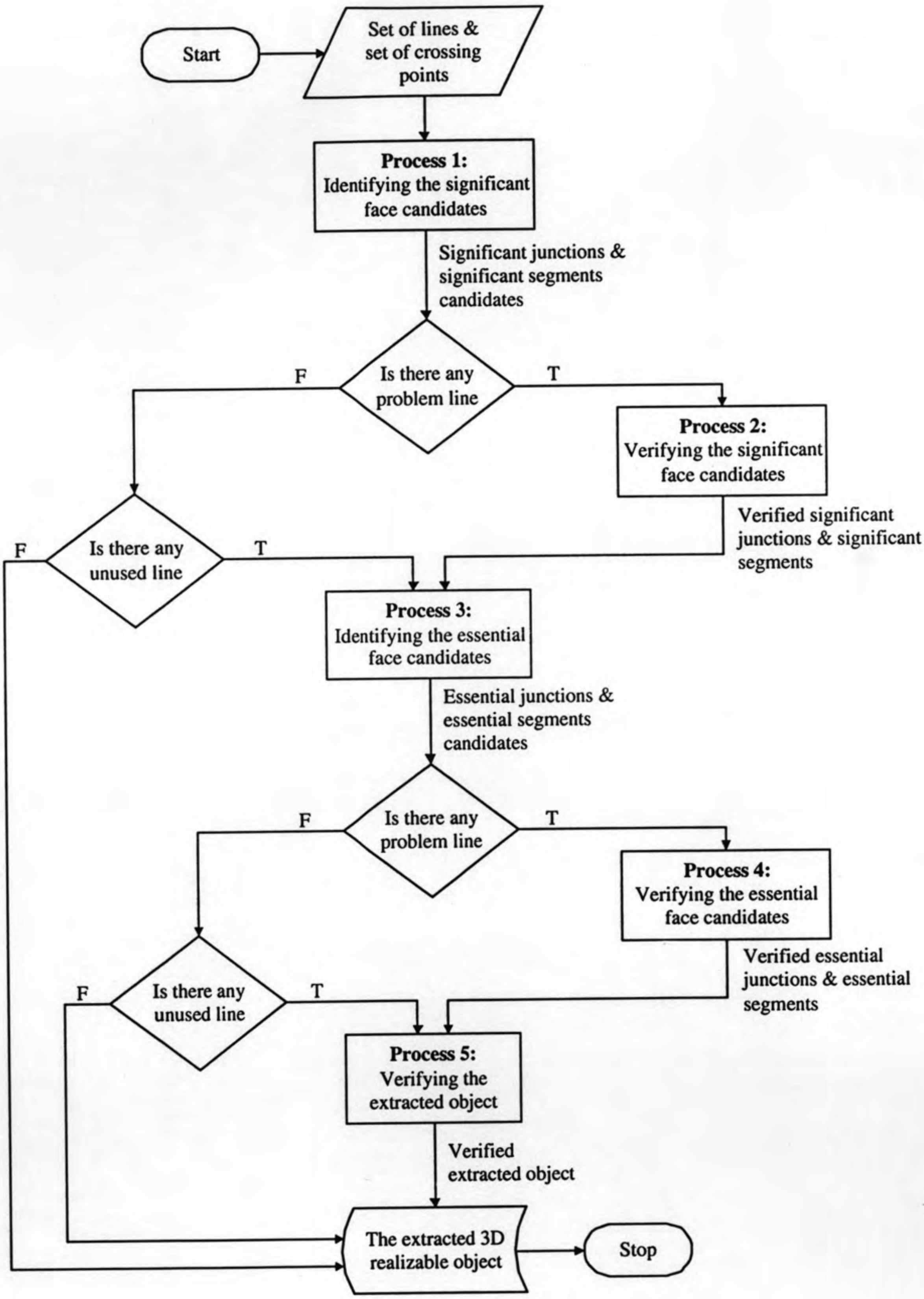
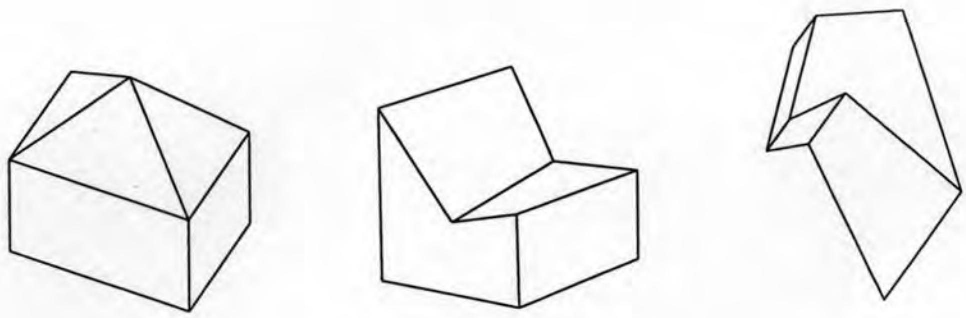


Figure 3.7: Flowchart of the proposed algorithm.



(a) (b) (c)

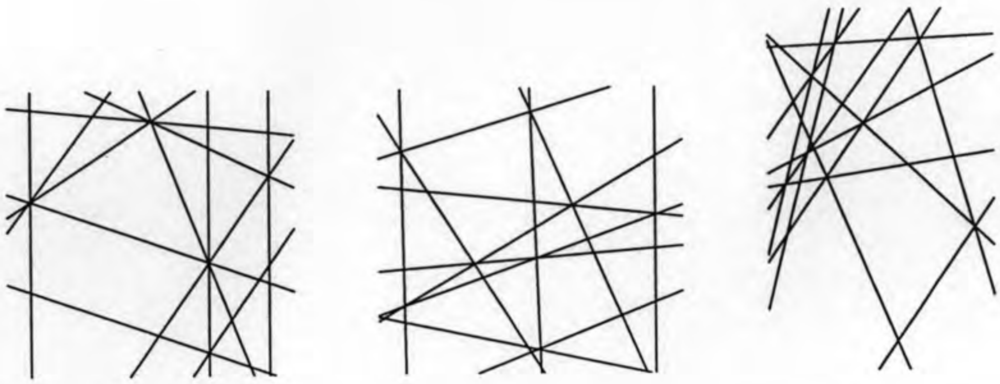
Figure 3.8: Realizable objects used in explaining the detail steps of the proposed method. (a) Object in Example 1. (b) Object in Example 2. (c) Object in Example 3.

Process 1: Identifying the significant face candidates.

In this process, the significant junctions and the significant segments are identified to be the significant face candidates of a 3D realizable object.

Because the junctions of which degree equal to three and more than three are the significant junctions in an object that can be interpreted as a 3D realizable object as mentioned in Section 3.1.1, this kind of junctions are first tried to identify by our approach. Besides, from the additional features of a 3D realizable object also mentioned in Section 3.1.2, the more the number of degrees they have at each junction point, the more the number of coherence faces can be visible in a 3D realizable object. From a given set of crossing lines with unidentified real junctions and real faces of a 3D realizable object, the most possible coherence faces should be identified if all the junctions with high degree are located.

The crossing points formed by three lines crossing and more than three lines crossing are first located. Such located points are identified as the candidates of the significant junctions of an extracted object. The segments that link between the identified significant junctions are the candidates of the significant segments of an object. The closed



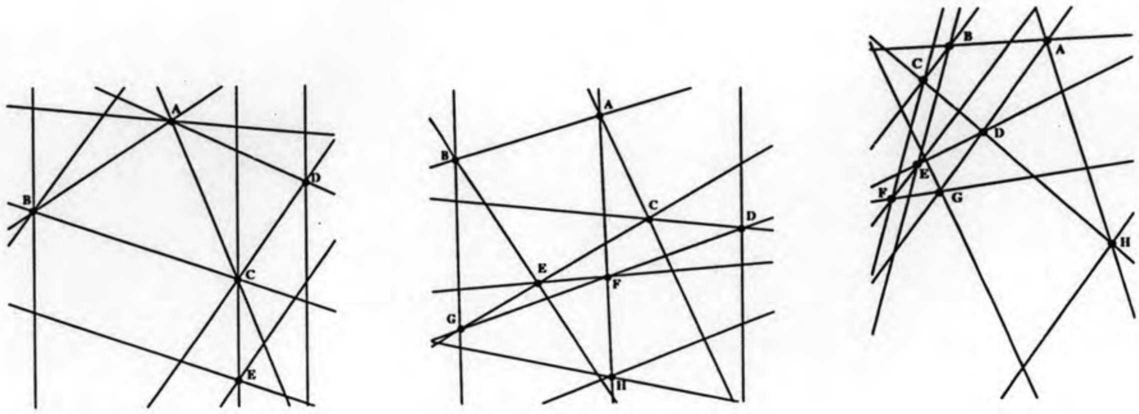
(a) (b) (c)

Figure 3.9: Set of lines extracted from each realizable object that will be used as an input to the algorithm. (a) Set of lines extracted from realizable object in Fig. 3.8(a). (b) Set of lines extracted from realizable object in Fig. 3.8(b). (c) Set of lines extracted from realizable object in Fig. 3.8(c).

regions formed by these significant junctions and significant segments are considered as the significant face candidates of the extracted object.

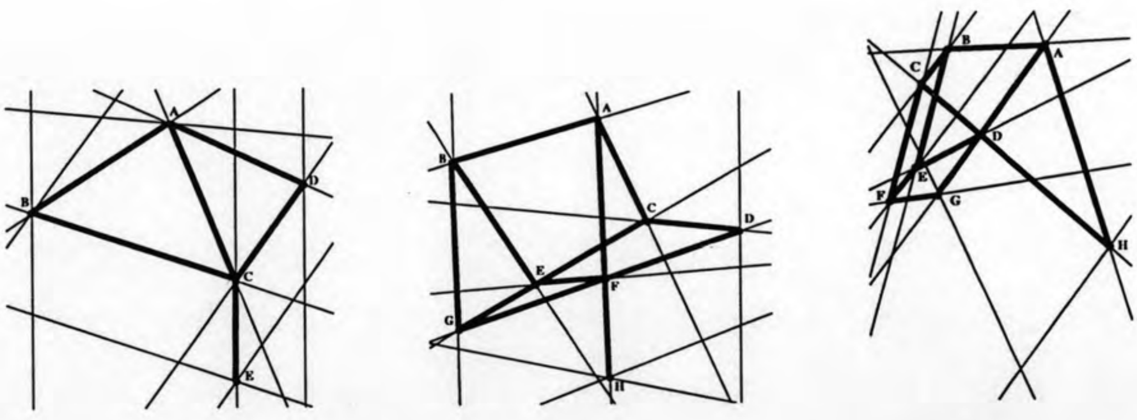
Figure 3.10 and Fig. 3.11 show Process 1 of an algorithm when the significant junctions and the significant segments are identified to be the significant faces of a 3D realizable object. All junctions formed by three lines or more than three lines crossing are marked at this step. The significant junctions of Example 1, Fig. 3.10(a), are labeled by A, B, C, D, and E. The significant junctions of Example 2, Fig. 3.10(b), are labeled by A, B, C, D, E, F, G, and H. The significant junctions of Example 3 are also labeled by A, B, C, D, E, F, G, and H in Fig. 3.10(c). The identified significant segments of each example marked by the bold solid segments are shown in Fig. 3.11. They are the segments that link between the significant junctions marked in Fig. 3.10.

Regions formed by the significant segments in Fig. 3.11 are the candidates of the real significant faces of the extracted 3D realizable object. The verification process in Process 2 is then applied to verify such significant face candidates as the real faces.



(a) (b) (c)

Figure 3.10: Process 1 of an algorithm when the significant junctions are identified. (a) The identified significant junctions of Example 1. (b) The identified significant junctions of Example 2. (c) The identified significant junctions of Example 3.



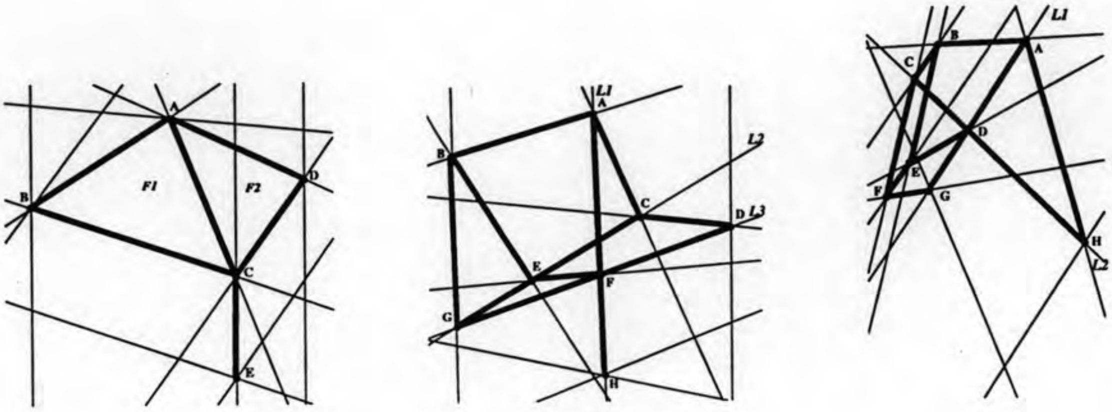
(a) (b) (c)

Figure 3.11: Process 1 of an algorithm when the significant segments are identified. (a) The identified significant segments of Example 1. (b) The identified significant segments of Example 2. (c) The identified significant segments of Example 3.

Process 2: Verifying the significant face candidates.

In this process, the significant face candidates identified in Process 1 are verified to be the real faces of a 3D realizable object. Not all of the possible significant faces identified in Process 1 are the real faces of a 3D realizable object. Some significant junctions or significant segments are over-identified. By our approach, if there exist some problem lines from the identified significant faces candidates, some significant junctions and significant segments are over-identified. The first set of rules, rules $S1$, $S2$, and $S3$ generated from the study about the characteristics of 3D realizable objects are, therefore, applied in this process to verify such problem lines. The over-identified significant junctions and significant segments can be removed after each rule is orderly applied.

The verification process for verifying the significant faces starts by checking whether the problem lines are emergent. As previously defined, the problem line is the line that has at least two identified segments and such segments are connected. If there is no problem line, by our method, we assume that all identified significant segments are correct. The regions formed by the identified segments can suddenly be the real faces of the extracted 3D realizable object. For Example 1 in Fig. 3.11(a), no problem line emerges from the identified significant segments, regions ABC and ACD formed by the solid segments are then marked as faces F_1 and F_2, respectively in Fig. 3.12(a). For Example 2 in Fig. 3.11(b), there are three problem lines after the significant segments are identified which are lines L_1, L_2, and L_3 as shown in Fig. 3.12(b). The identified segments AF and FH are connected along line L_1, therefore line L_1 is defined as the problem line. The identified segments CE and EG are connected along line L_2. In the same way, the identified segments DF and FG are connected along line L_3. Lines L_2 and L_3 are, therefore, also defined as the problem lines. Example 3 in Fig. 3.11(c), there are two problem lines labeled by L_1 and L_2.



(a) (b) (c)

Figure 3.12: Check if the problem lines are emergent. (a) There has no problem line in Example 1. The significant faces are, therefore, suddenly marked. (b) Problem lines, L_1, L_2, and L_3, of Example 2. (c) Problem lines, L_1 and L_2, of Example 3.

In our approach, if there are some problem lines after the significant segments are identified, we assume that some segments along such problem lines are over-identified. The following generated rules in our method are orderly applied to remove such over-identified significant segments.

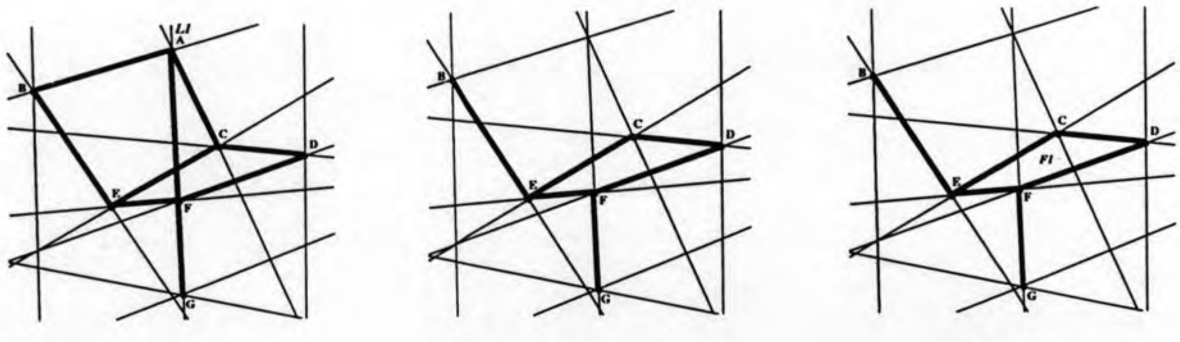
- **Rules for verifying the significant faces.**

  Rule S1: *If the ends of significant segments from at least two problem lines are adjacent, such adjacent point and its links formed by the related significant segments can be removed.*

  Rule S2: *If any significant segment on the problem line crosses the other significant segments, such significant segment on the problem line can be removed.*

  Rule S3: *If any significant segment on the problem line forms a triangle in some significant faces, such significant segment can be removed.*

Figure 3.13 shows when the rules are applied to Example 2. Figure 3.13(a) is the result after rule $S1$ is applied to Fig. 3.12(b). In Fig. 3.12(b), the ends of the significant segments from problem lines L_2 and L_3 are adjacent to point G. Therefore, point G and its related links can be removed by rule $S1$. The result after such point G and its links, segments GB, GE, and GF are removed as shown in Fig. 3.13(a). Figure 3.13(b) shows the result after rule $S2$ is applied to Fig. 3.13(a). In Fig. 3.13(a), there still is a problem line which is line L_1. The significant segment AF along line L_1 crosses the other significant segment, segment CE; therefore, segment AF can be removed by rule $S2$. The results after it is removed is shown in Fig. 3.13(b). After segment AF is removed, the crossing point A does not have the property of the significant junction because it is formed from just two crossing lines. Point A and its related segments are then also removed at this step. The result after point A and its links, segment AB and AC, are removed is shown in Fig. 3.13(b).

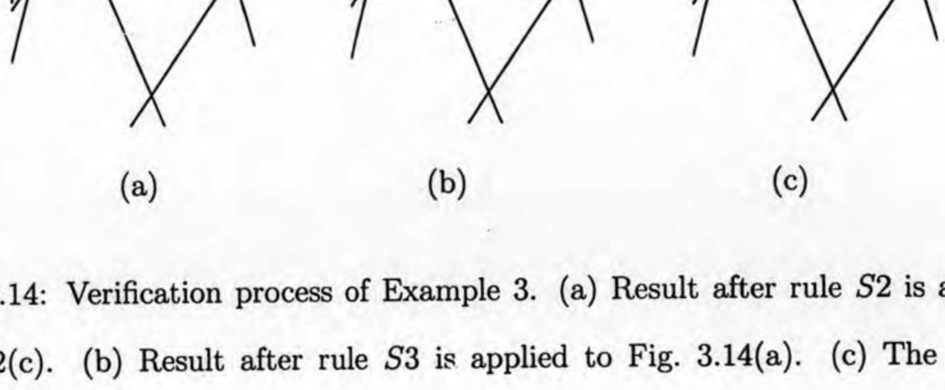


(a) (b) (c)

Figure 3.13: Verification process of Example 2. (a) Result after rule $S1$ is applied to Fig. 3.12(b). (b) Result after rule $S2$ is applied to Fig. 3.13(a). (c) The significant faces.

After removing segment AF and the other related segments in Fig. 3.13(b), there is no problem line exists. That means all over-identified significant segments are deleted. All remaining regions formed by the identified significant junctions and significant segments are then labeled as the real significant faces of the extracted 3D realizable object. Region $CEFD$ in Fig. 3.13(b) is then labeled as face F_1 in Fig. 3.13(c).

Figure 3.14 shows when the rules are applied to Example 3. Figure 3.14(a) is the result after rule $S2$ is applied to Fig. 3.12(c). The significant segment CD along line L_2 in Fig. 3.12(c) crosses the other significant segment, segment BE. By rule $S2$, segment CD can be removed. The removal results are shown in Fig. 3.14(a). After segment CD is removed, the crossing point C, now, does not have the property of the significant junction. Thus, point C and its related segments are then also removed. The result after point C and its links, segment BC and FC, are removed is also shown in Fig. 3.14(a).



(a) (b) (c)

Figure 3.14: Verification process of Example 3. (a) Result after rule $S2$ is applied to Fig. 3.12(c). (b) Result after rule $S3$ is applied to Fig. 3.14(a). (c) The identified significant faces.

At this step, there still is a problem line in Fig. 3.14(a), line L_1. The significant segment AD along line L_1 forms a triangle ADH in region $ABEDH$, rule $S3$ is, then, used to remove such segment. The result after segment AD and its related links including point A are removed is shown in Fig. 3.14(b). When all over-identified segments are erased (no problem line exists) in Fig. 3.14(b), region $DEFG$ formed by the verified significant junctions and significant segments is labeled as the real significant face, F_1, in Fig. 3.14(c).

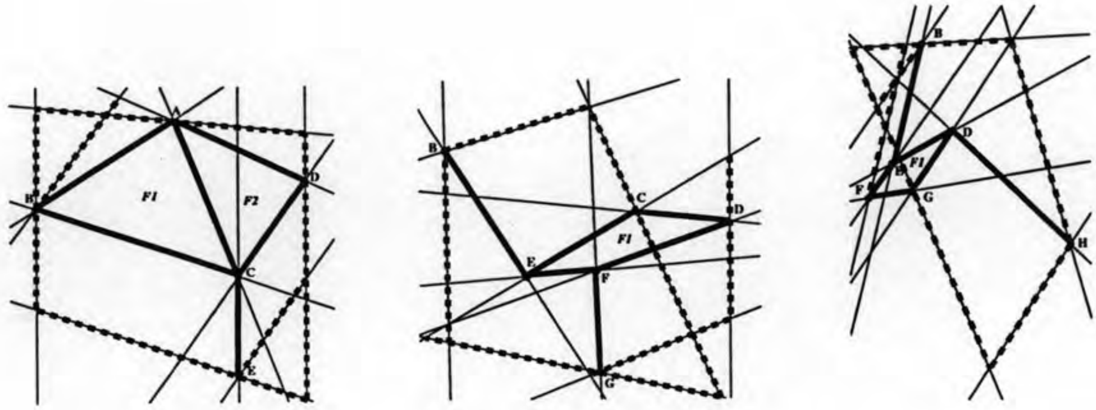
Process 3: Identifying the essential face candidates

In this process, the essential junctions and essential segments are identified to to be the essential faces candidates of a 3D realizable object.

Although all the significant faces are already identified and verified to be the real face of a 3D realizable object in Process 1 and Process 2, respectively, in most cases the object consisting of only significant faces still is not a 3D realizable object according to its properties and our assumptions. According to our assumptions, each given line is relevant and comprised of at least one segment of the corresponding realizable 3D object. If there are still some lines do not represent any significant segment from the previous two processes, Process 3 is applied to identify the remaining faces of the object.

At this process, if there are some unused lines (lines that still do not represent any segment of an extracted object), the essential junctions and essential segments are identified to be the essential faces candidates of a 3D realizable object. The dashed segments in every example in Fig. 3.15 represent all candidates of the essential segments. They are all of the remaining segments in image frame along the line that have no significant segment (solid segments) passes through.

New regions formed after identifying the essential segments are the candidates of the essential faces of the extracted 3D realizable object. The crossing points formed by



(a) (b) (c)

Figure 3.15: Identifying essential junctions and essential segments. (a) Example 1. (b) Example 2. (c) Example 3.

the adjacency of the identified essential segments in the essential faces candidates are also identified as the essential junctions. The verification process in Process 4 is, then, applied to verify which essential faces candidates can be the real essential faces.

Process 4: Verifying the essential face candidates.

In this process, the essential faces candidates identified in Process 3 are verified to be the real essential faces of a 3D realizable object.

Because not all of the potential essential faces identified in Process 3 are the real faces of a 3D realizable object, some essential junctions or essential segments are over-identified. By our approach, if there exist some problem lines from the identified essential faces candidates, the second set of rules, rules $E1$, $E2$, and $E3$ generated from the study about the characteristics of 3D realizable objects are also applied in this process to verify such essential faces. The over-identified essential junctions and essential segments can be removed after each rule is orderly applied.

- **Rules for verifying the essential faces.**

  Rule E1: *If any essential segment on the problem line crosses the significant segments, such essential segment on the problem line can be removed.*

  Rule E2: *Closed region that has no significant junction or closed region with only one significant junction cannot be the real face of a 3D realizable object. If such region is formed, it can be removed.*

  Rule E3: *If the ends of essential segments from at least two problem lines are adjacent, such adjacent point and its links formed by the related essential segments can be removed.*

Figure 3.16 shows the detailed steps when the essential faces of an object in Example 1 are verified. After all essential junctions and essential segments are identified in Fig. 3.15(a), new formed regions, R_1, R_2, R_3, R_4, R_5, and R_6, are marked as shown in Fig. 3.16(a). These regions are the candidates of the essential faces of the extracted object. Rules $E1$, $E2$, and $E3$ are orderly applied to remove the over-identified regions.

In Fig. 3.16(a), since there has no essential segment on the problem line crossing the significant segments. Therefore, rule $E1$ cannot be applied. The next rule, $E2$ is considered. Figure 3.16(b) shows the result after rule $E2$ is applied to remove regions R_1 and R_6 in Fig. 3.16(a). Region R_1 consists of just one significant junction which is junction B. Region R_6 also consists of just one significant junction which is junction E. Therefore, by rule $E2$, these two regions can be removed. After regions R_1 and R_6 are removed in Fig. 3.16(b), the problem lines still exist. The next rule, rule $E3$, is applied. In Fig. 3.16(b), the essential segments Ab and Ac are identified on the same line and this two segments are connected. The essential segments Dc and De are also identified on the same line and they are connected. All of this segments are, therefore, identified on the problem lines. We can see that, the ends of the identified essential segments from such two problem lines are adjacent at point c. By rule $E3$, point c and its links formed by the related essential segments can be removed. The result after point c and its links, segments Ac and Dc, are removed as shown in Fig. 3.16(c). Up to this step, there has no problem line exists in the remaining region. Regions R_2, R_4, and R_5 in Fig. 3.16(c) are defined as the real essential faces of an extracted object. They are renamed as faces F_3, F_4, and F_5, respectively in Fig. 3.16(d).

Figure 3.17 shows the detailed steps when the essential faces of an object in Example 2 are verified. After all essential junctions and essential segments are identified in Fig. 3.15(b), new formed regions are R_1, R_2, R_3, R_4, and R_5 as shown in Fig. 3.17(a). Figure 3.17(b) shows the result when rule $E1$ is applied to Fig. 3.17(a). Segments Cd in Fig. 3.17(a) is identified on the problem line and it crosses the significant segment DF of face F_1 at point b. By rule $E1$, therefore, segment Cd can be removed as shown in Fig. 3.17(b). When segment Cd is removed, region R_4 is also automatically removed. In Fig. 3.17(b), region R_5 consists of just one significant junction which is junction G. Therefore, by rule $E2$, region R_5 can also be removed. The result after region R_5 is removed is shown in Fig. 3.17(c). After region R_5 is removed in Fig. 3.17(c), no problem line exists in each region. At this step, all regions can be defined as the real essential faces of the extracted object. Regions R_1, R_2, and R_3 in Fig. 3.17(c) are therefore labeled as faces F_2, F_3, and F_4, respectively in Fig. 3.17(d).

Figure 3.18 shows the detailed steps when the essential faces of an object in Example 3 are verified. After all essential junctions and essential segments are identified in Fig. 3.15(c), new formed regions denoted by solid segments and dashed segments are as shown in Fig. 3.18(a). Figure 3.18(b) shows the result after rule $E1$ is applied to Fig. 3.18(a) to remove segment Ge in Fig. 3.18(a) because this segment crosses the significant segments BE and DE. Figure 3.18(c) shows the result after rule $E2$ is applied to Fig. 3.18(b) to remove regions $abcd$, and cde which have no significant junctions and

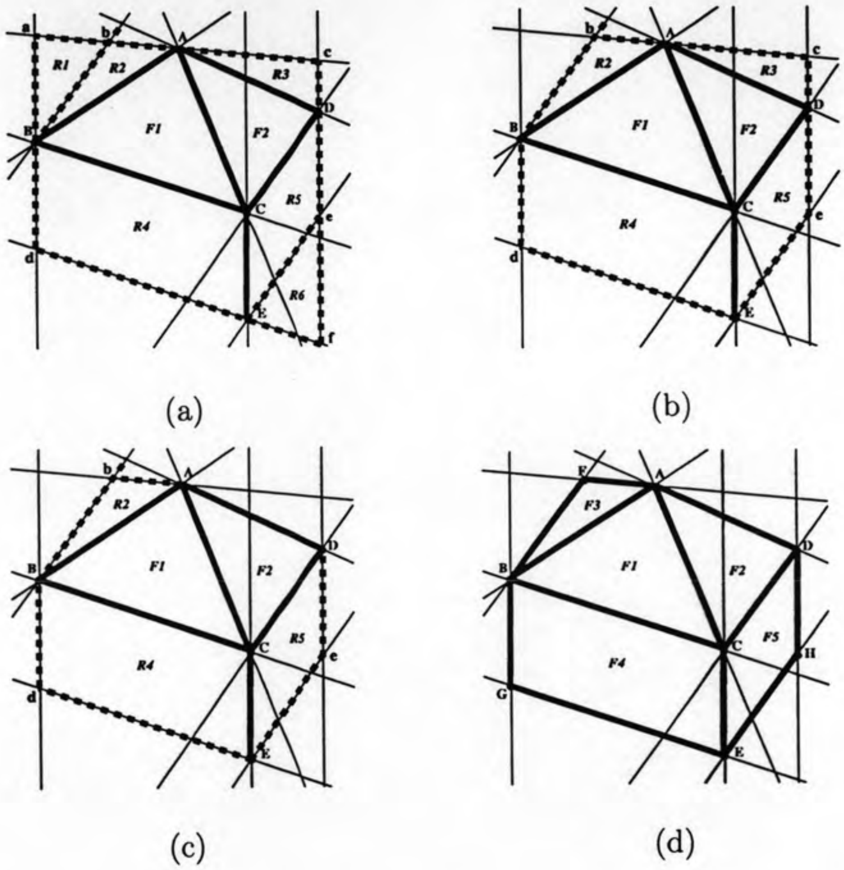


Figure 3.16: Detailed steps when the essential faces of an object in Example 1 are verified. (a) Regions R_1, R_2, R_3, R_4, R_5, and R_6 are formed after the essential segments are identified. (b) Result after rule $E2$ is applied to remove regions R_1 and R_6 in Fig. 3.16(a). (c) Result after point c and its links, segments Ac and Dc, in Fig. 3.16(b) are removed by rule $E3$. (d) When no problem line exists, regions R_2, R_4, and R_5 in Fig. 3.16(c) are then renamed to be faces F_3, F_4, and F_5, respectively.

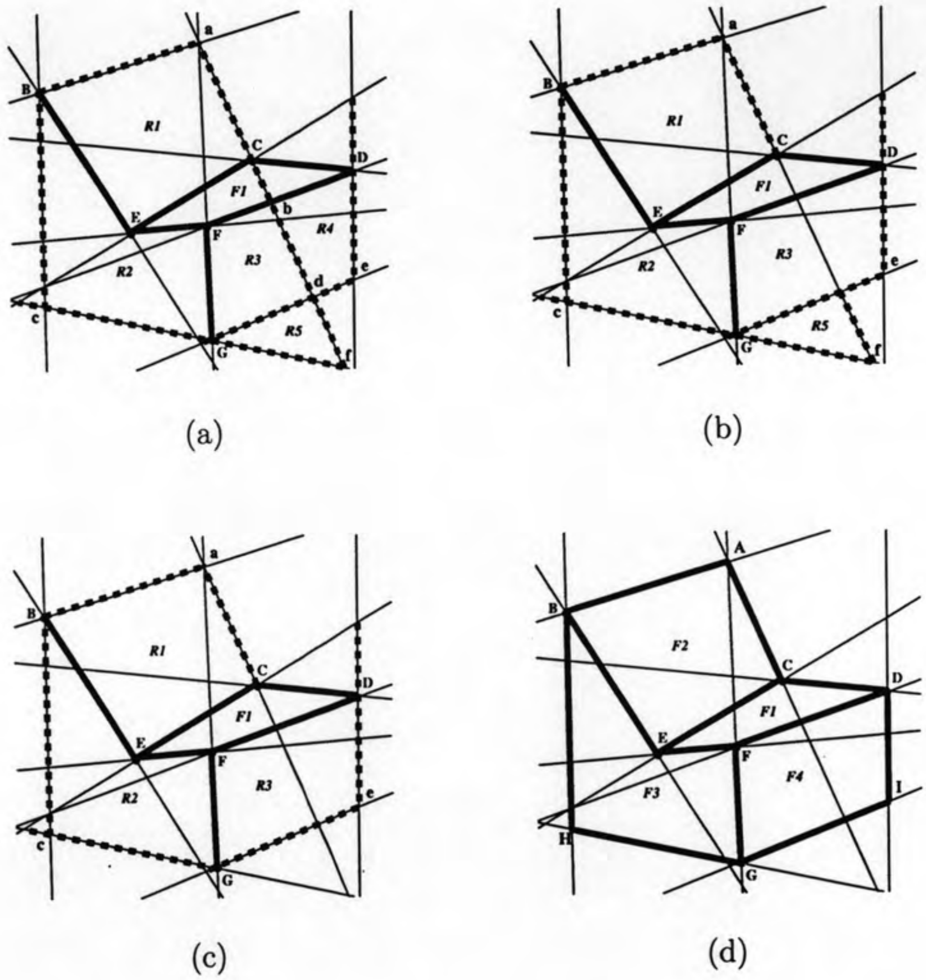


Figure 3.17: Detailed steps when the essential faces of an object in Example 2 are verified. (a) Regions R_1, R_2, R_3, R_4, and R_5 are formed after the essential segments are identified. (b)Result after rule $E1$ is applied to remove segment Cd in Fig. 3.17(a). (c) Result when rule $E2$ is applied to remove region R_5 in Fig. 3.17(b). (d) When no problem line exists, regions R_1, R_2, and R_3 in Fig. 3.17(c) are therefore labeled as faces F_2, F_3, and F_4, respectively.

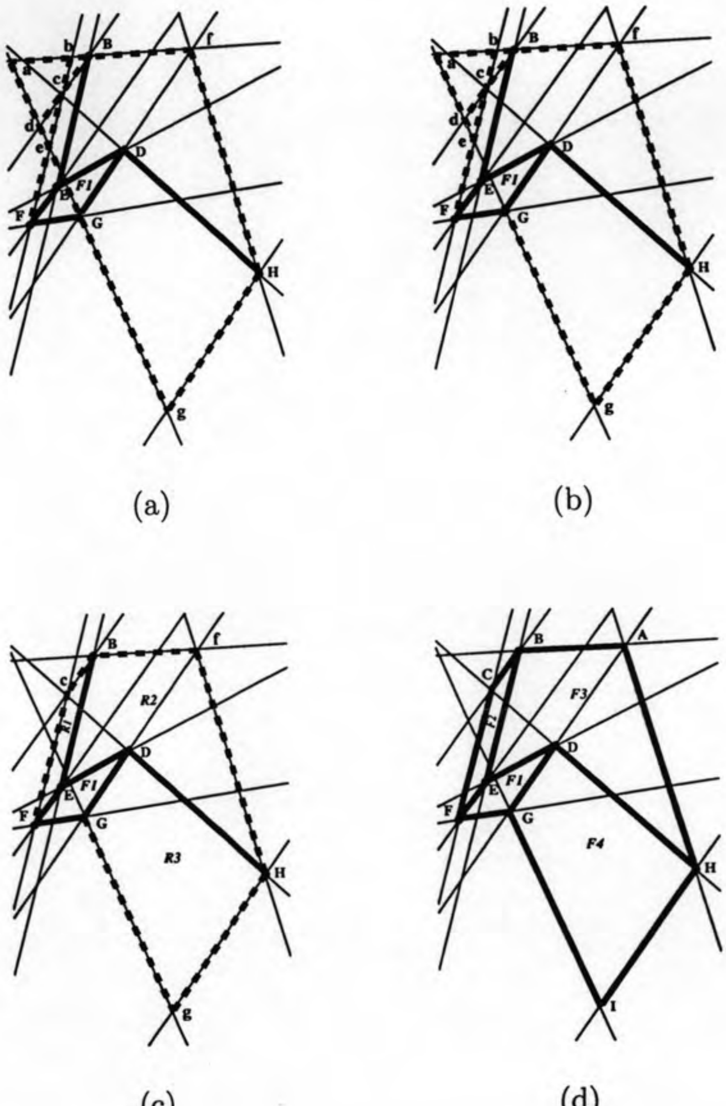


Figure 3.18: Detailed steps when the essential faces of an object in Example 3 are verified. (a) After the essential segments are identified, new regions forming are represented by solid segments and dash segments. (b) Result after rule $E1$ is applied to Fig. 3.18(a) to remove regions *abcd*, and *cde*. (c) Result after rule $E3$ is applied to remove segment Ge in Fig. 3.18(b). (d) When no problem line exists, regions R_1, R_2, and R_3 in Fig. 3.18(c) are then labeled as faces F_2, F_3, and F_4, respectively.

region *Bbc* that has just one significant junction. Figure 3.18(d) shows when all regions in Fig. 3.18(c) are labeled as the real essential faces of the extracted object after there has no problem line exists.

Process 5: Verifying the extracted 3D realizable object.

After all significant faces and all essential faces of an extracted object are already identified and verified as the real faces of a 3D realizable object from the previous four processes, the extracted object are verified before the final 3D realizable object can be extracted.

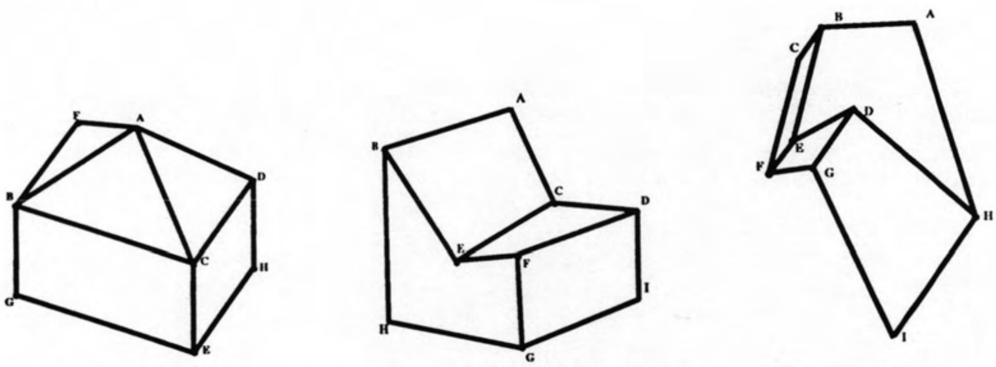
At this process, the existence of the problem lines and the assumption of the extracted object are also checked. Lines that are not used to represent any segments of the extracted object are considered and used before the final object can be extracted. Therefore, when Process 5 is applied, there are three cases as follows.

Case 1: *when no problem line exists and all lines are used.*

Case 2: *when no problem line exists but some lines are not used.*

Case 3: *when there still exist some problem lines.*

After Process 5 is applied to each of the previous three examples, all of them are in Case 1. Process 5 is applied to Fig. 3.16(d) of Example 1, Fig. 3.17(d) of Example 2, and Fig. 3.18(d) of Example 3. In each example, no problem line exists and each line is used to represent at least one segment of the extract object according to the assumption. Therefore, such identified objects in Fig. 3.16(d) of Example 1, Fig. 3.17(d) of Example 2, and Fig. 3.18(d) of Example 3 are the final extracted objects. Figure 3.19 shows the final objects extracted of such three examples. The extracted objects are the same as the realizable objects shown in Fig. 3.8. For the details of Case 2 and Case 3, we will explain in Chapter 4.



(a) (b) (c)

Figure 3.19: Final extracted objects. (a) Example 1. (b) Example 2. (c) Example 3.