CHAPTER IV

PARTICULAR CASES

This chapter will explain the detail of each particular case. There are three cases: (1) when no problem line exists but some lines are not used (Case 2 in Process 5), (2) when there still exist some problem lines (Case 3 in Process 5) after Process 5 of the algorithm is applied, and (3) when rule $S1$ of Process 2 is applied to the complicated example.

4.1 Case 2 in Process 5 of the Algorithm

As mentioned in previous chapter, Case 2 in Process 5 of the algorithm is the case when no problem line exists but some lines are not used. If there exist some lines not representing any identified segment of the extracted object after Process 5 is applied, it means some previously identified segments in an object are incorrectly identified. Such an example shows in Fig. 4.1. Figure 4.1(a) is a realizable object. Figure 4.1(b) is a set of lines extracted from the realizable object in Fig. 4.1(a). Figure 4.1(c) shows when Process 5 of the algorithm checks for the existing of the unused line. In this example, there exists an unused line marked by the dashed line. There is a case like this because some significant segments are incorrectly identified in Process 1 of the algorithm and the generated rules in Process 2 cannot detect such segment. In Fig. 4.1(c), the significant segment AB is incorrectly identified but it cannot detect by the generated rules in Process 2. In Process 5 of the algorithm, therefore, the unused line marked by the dashed line is considered again. In this example, there exists only one essential junction on such line which is junction C. If this line represents some segments of an object, such only one essential junction must be the real junction of an object. Therefore, segment BC in Fig. 4.1(d) can be suddenly identified as a segment of an extracted object. The previously incorrect identified segment, segment AB, must be removed in Fig. 4.1(d). Then, junction A must be checked whether it still has the property of the significant junction. After segment AB is removed, the degree of junction A is four. It still has the property of the significant junction because its degree does not less than three. Therefore, it is possible to remove segment AB. Figure 4.1(d) is the final 3D realizable object extracted by our algorithm.

Figure 4.2 shows another example of the case when some significant segments are miss-identified. Figure 4.2(a) is a realizable object. Figure 4.2(b) shows the object when the significant junctions and the significant segments are identified in Process 1. Figure 4.2(c) shows the result when rules in Process 4 are applied to remove all segments marked by dashed segments. Segments ab and bc are removed by rule $E1$ because they cross the significant segment AB. Segments cd and Dd are removed by rule $E2$ because they form a region that has only one significant junction. Therefore, in this example, there are two lines that do not represent any segments in the extracted object. They are the lines that have the dashed segments passing through in Fig. 4.2(c). Such lines must be considered again in Process 5 of the algorithm. Figure 4.2(d) shows the result when all of the crossing points on both lines are considered. There are two crossing points on the first line which are point d and point e. If point e is identified, segment Ae is also identified. Therefore, segment AC must be removed. But if segment AC is removed, the degree of the significant junction C is only two. As a result, segment AC cannot removed and point e cannot be identified as the next junction. Point d is then considered. If point d is identified, segments cd and Dd are also identified as shown in

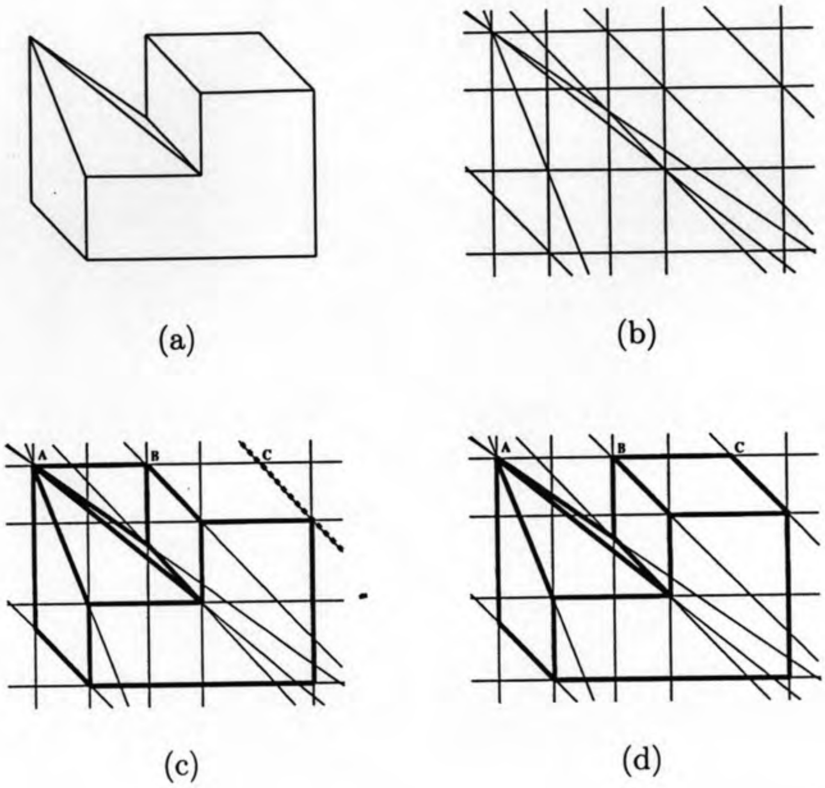


Figure 4.1: Example of the case when some significant segments are incorrectly identified. (a) The realizable object. (b) Set of lines extracted. (c) AB is the incorrect identified segment and line marked by dashed line is an unused line. (d) The final 3D realizable extracted object.

Fig. 4.2(e). Because point c separates the previously identified segment BD to be two connected segments, one of such connected segment must be removed. It is impossible to remove segment Bc because a junction B will have degree of only two and the extracted object is not the closed object. Another segment, segment Dc is then removed as shown in Fig. 4.2(f). After segment Dc is removed, a junction D still has a degree equal to three. Therefore, it is possible to remove segment Dc. Figure 4.2(f) is the final extracted object of this example.

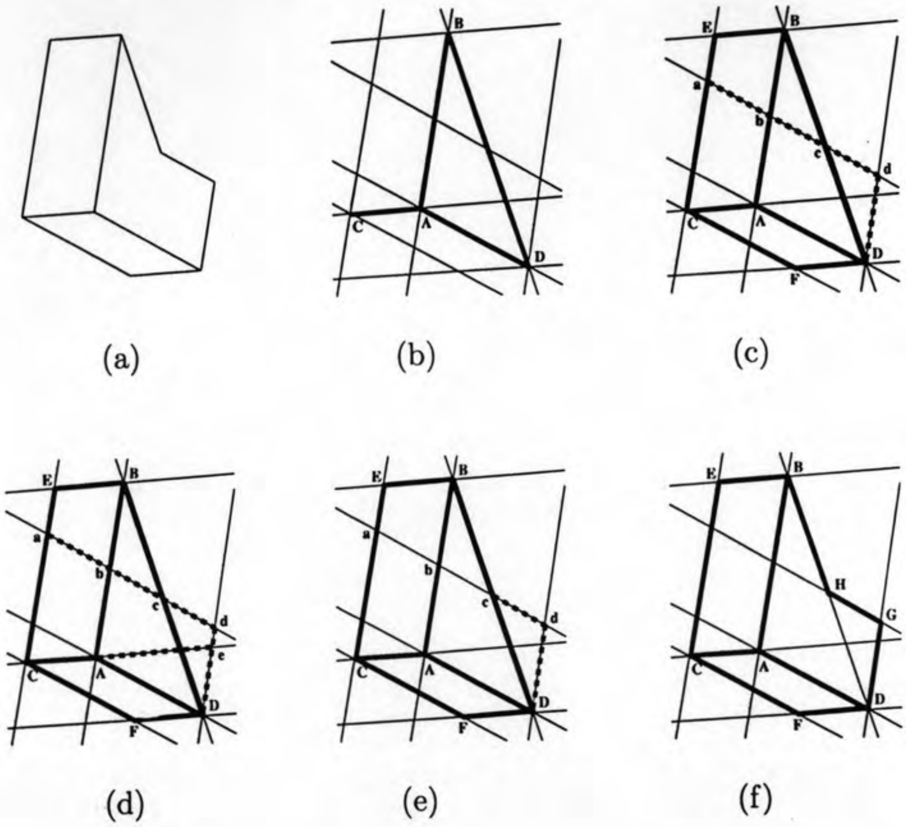
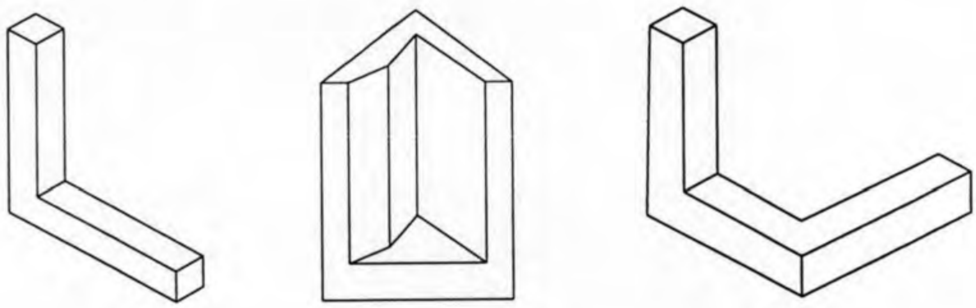


Figure 4.2: Another example of the case when some significant segments are incorrectly identified. (a) The realizable object. (b) Set of lines extracted from the realizable object in (a) and the significant junctions and significant segments are identified. (c) Lines that the dashed segments passing through are the unused lines after rules in Process 4 are applied. (d) All crossing points along the two unused lines are considered again in Process 5. (e) Segments *cd* and *Dd* are considered to be the next identified segments. (f) The final 3D realizable object is extracted by our algorithm.

4.2 Case 3 in Process 5 of the Algorithm

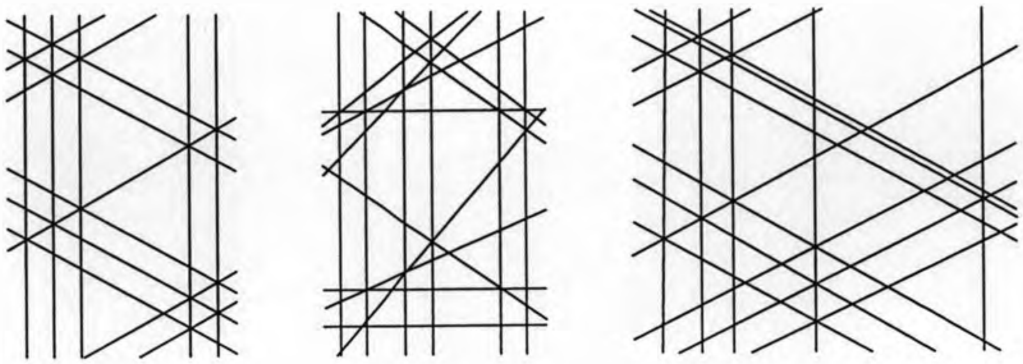
Case 3 in Process 5 of the algorithm is the case when some problem lines still exist after Process 5 of the algorithm is applied. In some cases, problem lines may still exist after all rules in Process 2 are applied to verify the identified the significant faces. By our method, we ignore such problem lines first by further applying Process 3 and Process 4 to identify and verify the essential faces, respectively. In Process 5, the additional rule, rule $SE1$, is applied to verify the problem lines again. Figure 4.3 to Fig. 4.8 show the detailed steps of three examples when rule $SE1$ is applied. Figure 4.4 shows the set of crossing lines extracted from each realizable object in Fig. 4.3.



(a) (b) (c)

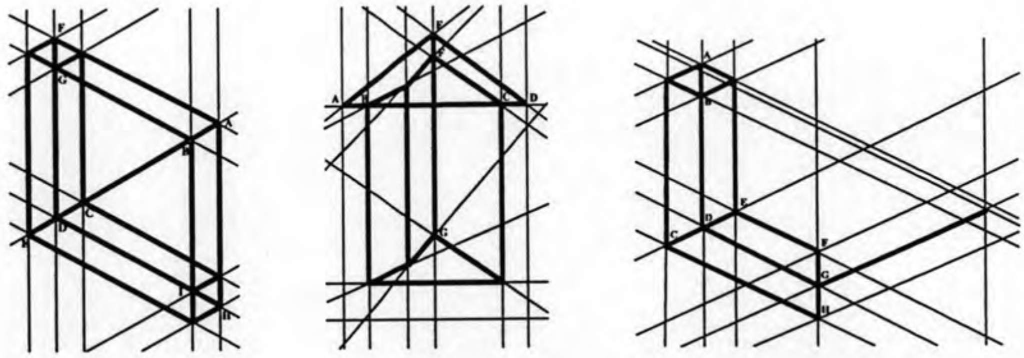
Figure 4.3: Examples of the realizable objects used in explaining when rule $SE1$ is applied. (a) Realizable object 1. (b) Realizable object 2. (c) Realizable object 3.

Figure 4.5 is the result when the significant junctions and significant segments of each object are identified. The results after all rules in Process 2 are applied to remove some over-identified segments are shown in Fig. 4.6. After all rules in Process 2 are already applied, some problem lines still exist in Fig. 4.6. There still exists one problem line in Fig. 4.6(a) containing segments CD and DE. In Fig. 4.6(b), one problem line also exist containing segment EF and FG. In Fig. 4.6(c), two problem lines exist. Segments CD and DE are connected on the same line but segments FG and GH are also connected



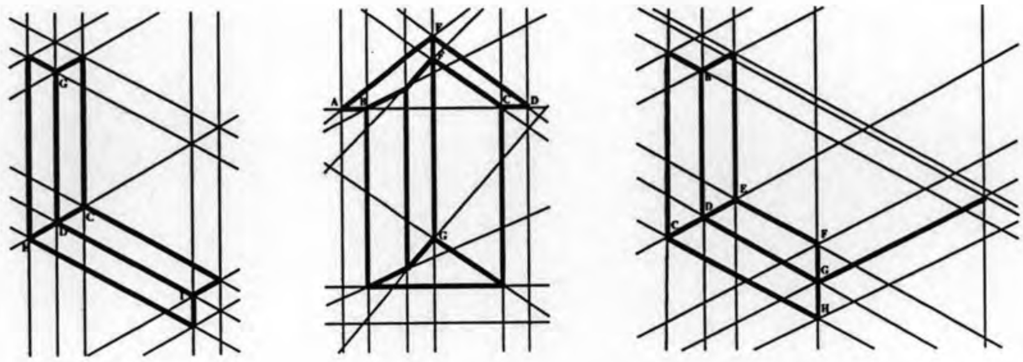
(a) (b) (c)

Figure 4.4: Set of crossing lines extracted from each realizable object in Fig. 4.3. (a) Set of extracted crossing line from the realizable object 1. (b) Set of extracted crossing line from the realizable object 2. (c) Set of extracted crossing line from the realizable object 3.



(a) (b) (c)

Figure 4.5: Results after Process 1 of the algorithm is applied to identify the significant junctions and significant segments of each object. (a) The identified significant junctions and significant segments of object 1. (b) The identified significant junctions and significant segments of object 2. (c) The identified significant junctions and significant segments of object 3.



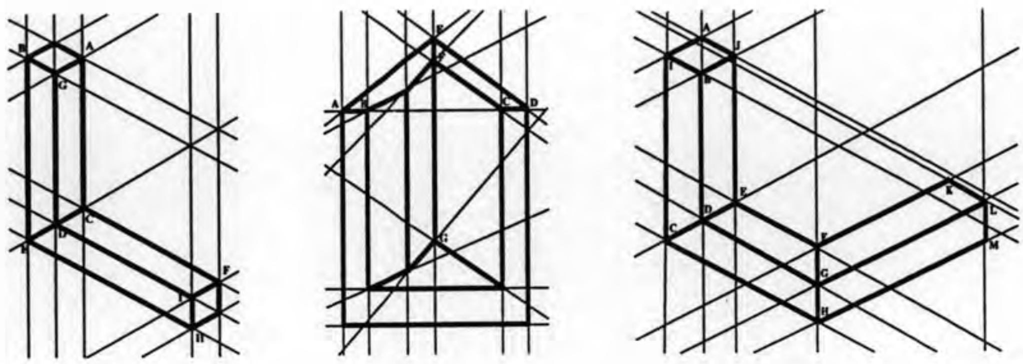
(a) (b) (c)

Figure 4.6: Results after the significant faces are formed by the significant junctions and the significant segments in Fig. 4.6 are verified using rules in Process 2 but some problem lines still exist. (a) Segments CD and DE are still connected and are on the same line. (b) Segments EF and FG are still connected and are on the same line. (c) Segments CD and DE are still connected and are on the same line but segments FG and GH are also connected on another line.

on another line.

By our method, although some problem lines still exist after all rules in Process 2 are already applied, we ignore them first by further applying Process 3 and Process 4 of the algorithm. The results after Process 3 and Process 4 of the algorithm are applied to identify and verify the essential faces of each example are shown in Fig. 4.7. After that Process 5 of the algorithm is further applied by checking whether some problem lines exist. If the problem lines exist, rule $SE1$ is then applied to remove some over-identified segments on such problem lines. Rule $SE1$ is generated as follows.

Rule SE1: *If each of the identified segment on a problem line does not parallel to any other identified segments in the same region, such identified segment can be removed.*

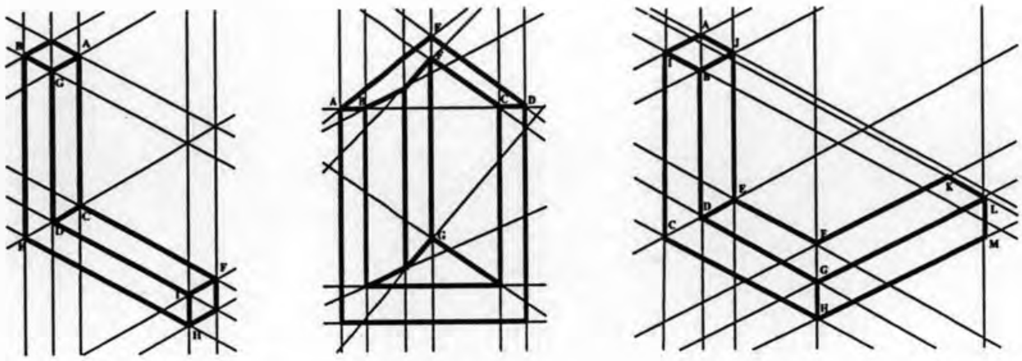


(a) (b) (c)

Figure 4.7: Results after the problem lines in Fig. 4.6 are ignored and then Process 3 and Process 4 of the algorithm are applied to identify and verify the essential faces, respectively. (a) Object 1 after Process 3 and Process 4 are applied. (b) Object 2 after Process 3 and Process 4 are applied. (c) Object 3 after Process 3 and Process 4 are applied.

In Fig. 4.7(a), there still exists one problem line containing segments CD and DE. Rule $SE1$ is applied to verify each of such segment. We can see that, a segment DE is the segment of regions $BGDE$ and $DEIH$. Whereas segment DE is not parallel to any segment in both regions. By rule $SE1$, therefore, a segment DE can be removed. The result after the segment DE in Fig. 4.7(a) is removed is shown in Fig. 4.8(a). In Fig. 4.7(b), there also exists one problem line containing segments EF and FG. A segment EF is the segment of regions $ABFE$ and $CDEF$ but it is not parallel to any segment in both regions. It also can be removed by rule $SE1$. The result after a segment EF in Fig. 4.7(b) is removed is shown in Fig. 4.8(b). In the same way, segments CD and FG in Fig. 4.7(c) can be removed by rule $SE1$. The result after such two segments are removed is shown in Fig. 4.8(c).

After rule $SE1$ is already applied, the features of the extracted 3D realizable object are then verified in the final step of the algorithm. All of three extracted objects in



(a) (b) (c)

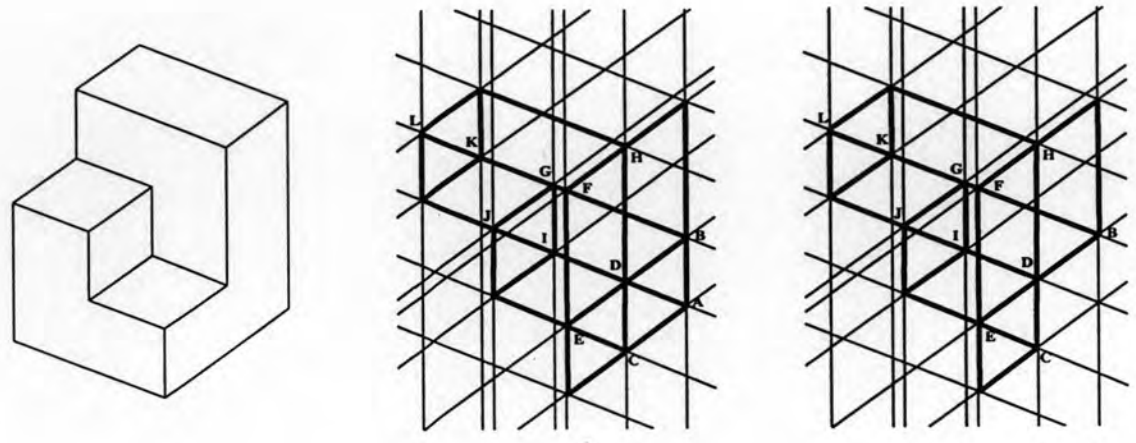
Figure 4.8: Results after rule $SE1$ is applied in Process 5 to remove the remaining over-identified segments. (a) Result after rule $SE1$ is applied to remove segment DE in Fig. 4.7(a). (b) Result after rule $SE1$ is applied to remove segment EF in Fig. 4.7(b). (c) Result after rule $SE1$ is applied to remove segment CD and FG in Fig. 4.7(c).

Fig. 4.8 have the features of the 3D realizable object.

4.3 When Rule *S1* is Applied to Complicated Examples

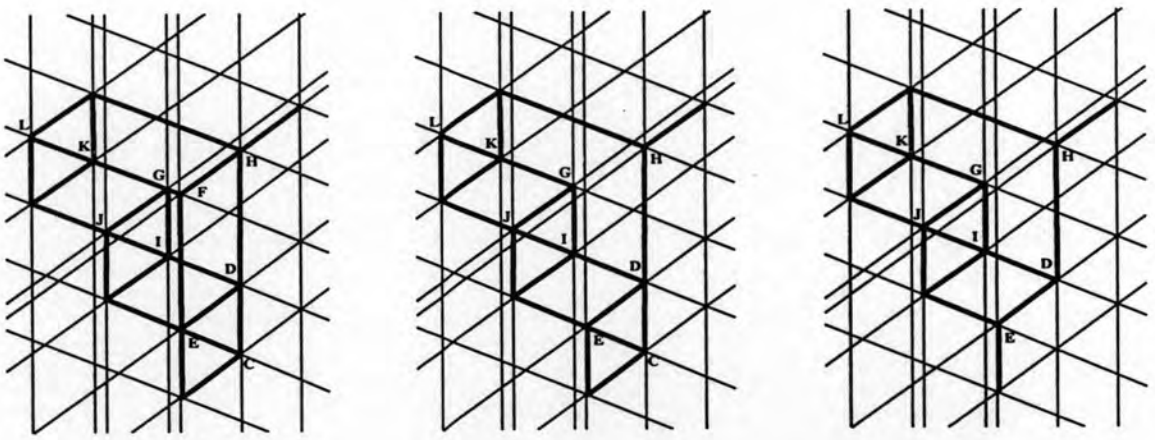
If the end of the identified significant segments from at least two problem lines are adjacent, rule $S1$ is applied in Process 2 of the algorithm. In some cases, there are more than one adjacent point of the problem lines. Some adjacent points are formed by just two problem lines but some are formed by more than two problem lines. By our method, if there are more than one adjacent point, the point formed by the highest number of the problem lines are therefore orderly considered. Figure 4.9 and Fig. 4.10 show when rule $S1$ is applied to the complicated example. The points formed by the highest number of the problem lines and its links are orderly removed.

Figure 4.9(a) is the realizable object that set of lines extracted from this object is used as input. Figure 4.9(b) represents when the significant junctions and the significant



(a) (b) (c)

Figure 4.9: When rule $S1$ is applied to more complicate object. (a) A realizable object. (b) When the significant junctions and the significant segments are identified. (c) A result after rule $S1$ is applied to remove point A and its related links.



(a) (b) (c)

Figure 4.10: Further steps from Fig. 4.9. (a) A result after rule $S1$ is applied to remove point B and its related links. (b) A result after rule $S1$ is applied to remove point F and its related links. (c) A result after rule $S1$ is applied to remove point C and its related links.

segments are identified. Points A, B, C and F in Fig. 4.9(b) are the adjacency points from three, two, two, and two problem lines, respectively. Point A has the highest number, therefore it is removed first. The result after point A and its related links are removed is showed in Fig. 4.9(c). Figure 4.10(a) shows the result when the next adjacency point, point B and its related links in Fig. 4.9(c) are removed. After point B is removed in Fig. 4.10(a), point F is adjacent by three problem lines which is above point C. Point F is the next point to be removed. The result after point F and its related links in Fig. 4.10(a) are removed is shown in Fig. 4.10(b). And the result after point C and its related links in Fig. 4.10(b) are removed is shown in Fig. 4.10(c).

Figure 4.11(a) shows the result after rule $S3$ is used to removed segments KL and IJ in Fig. 4.10(c). This two segments are on the problem lines and they form the triangles in the other significant faces. Figure 4.11(b) shows the result when the candidates of the essential faces are identified. Figure 4.11(c) shows the result after such potential essential faces in Fig. 4.11(b) are verified and the final object is extracted.

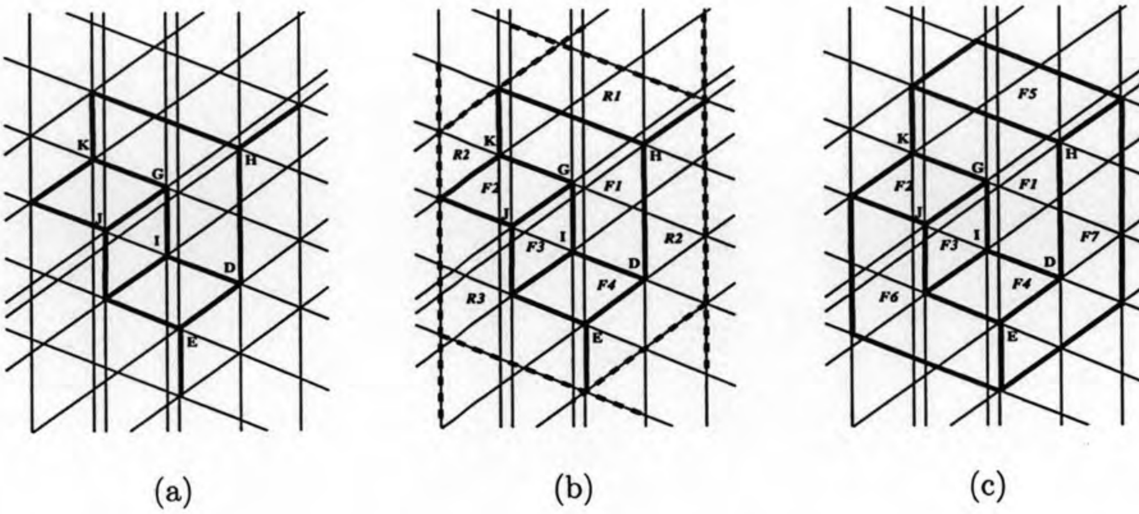


Figure 4.11: Further steps from Fig. 4.10. (a) A result after rule $S3$ is used to removed segments KL and IJ in Fig. 4.10(c). (b) When the candidates of the essential faces are identified. (c) A result after such potential essential faces in Fig. 4.11 are verified and the final object is extracted.

4.4 Time Complexity of the Algorithm

Let $L = \{l_1, l_2, ..., l_m\}$ be a given set of crossing lines and $P = \{p_1, p_2, ..., p_n\}$ a set of crossing points of which $p_i = (c_i, S_i)$, $1 \leq i \leq n$, where c_i is the coordinate in XY plane of the crossing point i and $S_i \subset L$, the crossing lines at the crossing point i.

The time complexity of the algorithm can be roughly analyzed in the worst case as follows.

Process 1: Identifying the significant junctions and significant segments as the significant face candidates of a 3D realizable object.

Worst case = O(mn)

Process 2: Verifying the significant faces candidates whether they are the real significant faces of a 3D realizable object.

(Rules $S1$, $S2$, and $S3$ are orderly applied)

Worst case = O(mn)

Process 3: Identifying the essential junctions and essential segments as the essential faces candidates of a 3D realizable object.

Worst case = O(mn)

Process 4: Verifying the essential face candidates whether they are the real essential faces of a 3D realizable object.

(Rules $E1$, $E2$, and $E3$ are orderly applied)

Worst case = O(mn)

Process 5: Verifying the extracted 3D realizable object.

(Rules $SE1$ is applied and the unused lines are also considered)

Worst case = O(mn)

Total time complexity = $O(mn)$, where m is the number of initial crossing lines and n is the number of initial crossing points.