PERIODIC-REVIEW ORDER POLICY FOR A TWO-ECHELON INVENTORY PROBLEM WITH SEASONAL DEMAND

Mr. Noppadon Sakulsom

A Dissertation Submitted in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy in Industrial Engineering
Department of Industrial Engineering
Faculty of Engineering
Chulalongkorn University
Academic Year 2018

นโยบายการสั่งตามรอบเวลาสำหรับปัญหาสินค้าคงคลังสองระดับโดยมีความต้องการสินค้าแบบมีฤดูกาล

นายนพปฎล สกุลสม

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรดุษฎีบัณฑิต
สาขาวิชาวิศวกรรมอุตสาหการ ภาควิชาวิศวกรรมอุตสาหการ
คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
ปีการศึกษา 2561

Thesis Title	PERIODIC-REVIEW ORDER POLICY FOR A TWO-ECHELON INVENTORY PROBLEM WITH SEASONAL DEMAND
By	Mr. Noppadon Sakulsom
Field of Study	Industrial Engineering
Thesis Advisor	Associate Professor WIPAWEE THARMMAPHORNPHILAS, Ph.D.

Accepted by the Faculty of Engineering, Chulalongkorn University in Partial Fulfillment of the Requirement for the Doctor of Philosophy

.... Dean of the Faculty of Engineering
(Associate Professor SUPOT TEACHAVORASINSKUN, D.Eng.)

DISSERTATION COMMITTEE

.... Chairman
(Associate Professor PAVEENA CHAOVALITWONGSE, Ph.D.)

.... Thesis Advisor
(Associate Professor WIPAWEE THARMMAPHORNPHILAS, Ph.D.)

.... Examiner
(Assistant Professor Daricha Sutivong, Ph.D.)

.... Examiner
(Assistant Professor NARAGAIN PHUMCHUSRI, Ph.D.)

.... External Examiner
(Assistant Professor Nantachai Kantanantha, Ph.D.)

นพปฎล สกุลสม : นโยบายการสั่งตามรอบเวลาสำหรับปัญหาสินค้าคงคลังสองระดับโดยมีความต้องการสินค้าแบบมีฤดูกาล. (PERIODIC-REVIEW ORDER POLICY FOR A TWO-ECHELON INVENTORY PROBLEM WITH SEASONAL DEMAND) อ.ที่ปรึกษาหลัก : รศ. ดร.วิภาวี ธรรมาภรณ์พิลาศ

วิทยานิพนธ์ฉบับนี้นำเสนอกระบวนการในการระบุนโยบายการสั่งสำหรับระบบสินค้าคงคลังสองระดับซึ่งมีวิธีการเติมสินค้าหลายวิธีและมีความต้องการสินค้าแบบมีฤดูกาล ระบบดำเนินการโดยการสั่งตามรอบเวลาโดยใช้จุดสั่งซื้อและระดับสินค้าสูงสุด หรือระบบ (R,s,S) ระบบใช้แนวคิดสินค้าคงคลังตามลำดับ ในการสั่งสินค้าโดยจุดเก็บสินค้าแต่ละแห่งตัดสินใจสั่งโดยอ้างอิงจากระดับสินค้าคงคลังของตนเองและจุดเก็บสินค้าทั้งหมดที่รับสินค้าต่อจากจุดเก็บสินค้านี้ ผู้วิจัยแบ่งปัญหาออกเป็นสองส่วน และเสนอวิธีการซึ่งพัฒนาด้วยแบบจำลองเชิงเส้นผสมจำนวนเต็มเพื่อระบุนโยบายสั่งสำหรับระบบสินค้าคงคลังซึ่งมีวิธีการเติมสินค้าวิธีเดียว จากนั้นจึงสร้างกระบวนการซึ่งพัฒนาด้วยขั้นตอนวิธีเชิงพันธุกรรม และการค้นหาแบบไบนารีขึ้นเพื่อใช้แก้ปัญหาด้วยเวลาที่น้อยลง กระบวนการแบบหลังนี้สามารถแก้ปัญหาได้ด้วยเวลาที่สั้นลงโดยเฉลี่ย 97.49% ในปัญหาส่วนแรกและสั้นลงโดยเฉลี่ย 95.99% ในส่วนที่สอง จากนั้นผู้วิจัยขยายปัญหาจากเดิมที่มีวิธีการเติมสินค้าวิธีเดียวเป็นปัญหาซึ่งมีวิธีการเติมสินค้าปกติเป็นวิธีหลักและมีวิธีเติมสินค้าแบบพิเศษอีกสองวิธีเพื่อป้องกันการขาดสินค้า นอกจากนี้เนื่องจากความต้องการสินค้าเป็นแบบมีฤดูกาล นโยบายการสั่งของวิธีเติมสินค้าแบบพิเศษจึงสามารถกำหนดได้สองแบบได้แก่ นโยบายแบบสถิต ซึ่งใช้นโยบายการสั่งแบบเดียวกันในทุกๆคาบ และนโยบายการสั่งแบบพลวัต ซึ่งใช้นโยบายการสั่งแตกต่างกันในแต่ละคาบในแต่ละรอบของความต้องการสินค้า วิธีเติมสินค้าแบบพิเศษสามารถกำจัดคาบเวลาที่ระดับการให้บริการต่ำกว่าที่ต้องการลงได้ โดยนโยบายการสั่งแบบพลวัตมีแนวโน้มที่จะทำการสั่งน้อยกว่า แต่ในกรณีที่ความต้องการสินค้ามีการกระจายตัวมากและมีรอบสั้น นโยบายการสั่งแบบสถิตสามารถทดแทนนโยบายการสั่งแบบพลวัตได้จากความง่ายในการใช้

สาขาวิชา	วิศวกรรมอุตสาหการ	ลายมือชื่อนิสิต	
ปีการศึกษา	2561	ลายมือชื่อ อ.ที่ปรึกษาหลัก	

5571446821 : MAJOR INDUSTRIAL ENGINEERING
KEYWORD: INVENTORY SYSTEM / MULTI-ECHELON / SEASONAL DEMAND / HEURISTICS

Noppadon Sakulsom : PERIODIC-REVIEW ORDER POLICY FOR A TWO-ECHELON INVENTORY PROBLEM WITH SEASONAL DEMAND. Advisor: Assoc. Prof. WIPAWEE THARMMAPHORNPHILAS, Ph.D.

This dissertation presents methodologies to determine ordering policies for a 2-echelon inventory system with multiple replenishing modes under seasonal demand. The system operates on periodic review basis using reorder point and order-up-to point or *(R,s,S)*. An Echelon stock concept is applied where each location makes decisions on its own inventory information and the information of all locations downstream. We decompose a problem into two phases and propose a methodology based on mixed-integer programming models to determine ordering policies for a system with one replenishing mode. Then, another methodology based on the genetic algorithm and binary search is developed to solve the problem with shorter computational time. The later methodology can solve problem with 97.49% on average less computational time in the first phase and 95.99% on average less time in the second phase. Afterwards, a problem is extended to a system with a regular replenishing mode as a main replenishing mode and two special modes to prevent stockouts. Under seasonal demand, special modes could have either a static policy which applies one policy on every period or a dynamic policy which applies a different policy for each period in the demand cycle. Special replenishing modes can eliminate the periods with unsatisfied service level with slightly higher holding cost. A dynamic policy tends to give smaller number of special orders. However, in case of high deviation demand and short demand cycle, for ease of use, the static policy can substitute the dynamic policy.

Field of Study: Industrial Engineering		Student's Signature ....
Academic Year:	2018	Advisor's Signature ....

ACKNOWLEDGEMENTS

This work was supported by the scholarships from Graduate School, Chulalongkorn University to commemorate the 72nd Anniversary of his Majesty King Bhumibala Aduladeja, and from the 90th Anniversary Chulalongkorn University Fund (Ratchadaphiseksomphot Endowment Fund).

For the completion of this dissertation, I would like to thank the following people for their valuable contribution in making this dissertation what it is. First, I would like to express my appreciation and gratitude to my advisor, Assoc.Prof. Wipawee Tharmmaphornphilas, for all her guidance and continuous support during my master and Ph.D. study. She has provided me not only academic advices but also great encouragement in every step of this dissertation with patience. Thinking about my Ph.D. study, one of my favorite times will be working with my advisor and I am very thankful.

I would like to thank my thesis committee: Assoc.Prof.Paveena Chaovalitwongse, Asst.Prof.Daricha Sutivong, Asst.Prof.Naragain Phumchusri and Asst.Prof.Nantachai Kantanantha. For their valuable questions and comments. I truly appreciate their time and willingness to serve on my committee, as well as their helpful suggestions. My gratitude also goes to Asst.Prof.Seeronk Prichanont, Asst.Prof.Oran Kittithreerapronchai and Asst.Prof.Pisit Jarumaneeroj for their helpful suggestions. Also, I would like to thank all staffs in the Industrial Engineering department for their helps during my study. My gratitude also goes to Siriraj hospital for letting me study their system. I would like to thank Assoc.Prof.Cherdchai Nopmaneejumruslers and Ms.Nonglak Puenchompoo for letting me be a part of their improvement plan. I would like to thank Ms. Panita Jantapatchothe, Mr. Nithat Rahannok, Ms. Araya Puntagoun, Ms. Sunicha Limkobpaiboon, Ms. Narisa Tantai, Ms.Panattharin Lerdkiattikorn and Ms. Roongrat Piyananjaruhsri for their valuable information and suggestions. I am thankful for the opportunity to work with them.

I would like to thank my colleagues in ORC lab and other labs: P.Pe, P.Pat, P.Toy, Bell, New, Khaotu, Tee, P, Fah, Rin, Nett, June, Off and Cake for their friendship and support through the good time and through the challenging times.

My deepest gratitude goes to my family and Lalin who support and accompany me in this Ph.D. journey. I would like to thank them for being my inspiration and for everything.

Noppadon Sakulsom

TABLE OF CONTENTS

Page

LIST OF TABLES

Page

LIST OF FIGURES

Page

CHAPTER I

INTRODUCTION

Inventory management is involved with almost all industries, from items in grocery stores to drugs in hospitals to spare parts in automobile manufacturers. Inventory cost is one of major costs of the business and managing inventory also relates to whether shortage occurs or not which directly affects customer's satisfaction. However, demand is the key factor determining the difficulty of inventory management. When demand is known in advance it is quite easy to decide how to store items. If demand is stochastic it will be more difficult to manage, especially when demand pattern changes over time. Furthermore, an inventory system can be considered as multiple levels of locations or so-called a multi-echelon system such as a system consisting of retailers supplied by the warehouse(s). The more levels the system has, the more decisions it requires. The system with multiple levels is very complicated since a decision any location makes will affect other locations in the system. In this thesis, we develop methodologies to manage a 2-echelon inventory system under seasonal demand with the goal to determine the ordering policies for all locations with minimum cost respecting to expected service level.

In this work, we focus on the system in a service industry. The system has a single warehouse and *N* non-identical retailers under seasonal demand. Each location replenishes inventory in a fixed time interval. Retailers are supplied by a warehouse which is supplied by external suppliers and items can be stored at the warehouse and retailers. Unsatisfied demand is considered lost.

Demand is assumed to be seasonal without trend. However, it is driven by human factor like the stationary items are driven by school calendars. Since the human factor can drive demand in various lengths of cycle such as a month, a week or even a day. The system considered in this thesis has seasonal demand within a cycle of one week. For example, the demand could be high on Monday, Wednesday, and Thursday. On the other hand, demand could be very low on weekend and this pattern repeats every week. This demand pattern can be found at a drug dispensing system in a hospital. The number of patients diagnosed depends on the number of physicians. Physicians' working days are pre-scheduled and the number of physicians is different on each day. The physician's schedule repeats every week. Therefore, demand of each drug depends on the physicians' schedule.

Dealing with seasonal demand as if it is stationary demand can lead to shortage or high holding cost. Therefore, many papers (Graves and Willems, 2008; Reddy and Rajendran, 2005; Kim, Wu, and Huang, 2015, Grewal, Enns, and Rogers, 2015) tackled a system with trend and seasonal demand by varying ordering policy along with the changing phases of demand. However, their demands changed slowly compared to lead time such as the length of demand phase is 2,000 hours where lead time is 16 hours (Grewal, Enns, and Rogers, 2015) or 100-period demand phase with 10 period lead time (Graves and Willems, 2008). This thesis considers 1-day demand phase and 1-day lead time; therefore, changing ordering policy everyday as demand changes would not be practical.

1.1 System overview

A single warehouse, multi-retailer inventory system provides items to customers. Customer demand occurs at retailers which are supplied by the central warehouse and the warehouse is supplied by external suppliers as shown in Figure 1. In the system, items are stored at both warehouse and retailers which are assumed to have unlimited storage space. When the warehouse or any retailer orders items, they are replenished with known lead time. The demand which is not satisfied by on-hand inventory is considered lost.

Demand is assumed to be seasonal fluctuating in a cycle of a certain span of periods. The demand pattern repeats every cycle and the total demand per cycle is assumed to be stationary.

Normally, items are regularly replenished from the warehouse to each retailer. This regular replenishing mode operates with a deterministic lead time. However, in some cases, when a retailer faces risk of stockouts, this lead time may be too long to satisfy customer demand.

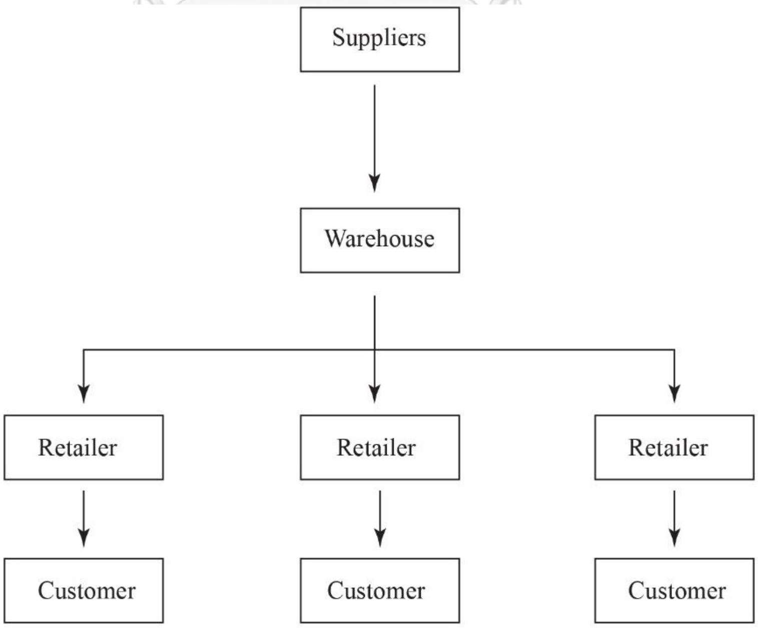

Figure 1 Overview of the inventory system

To deal with the risk of stockouts, there are two other special replenishing modes: an emergency replenishing mode and a transshipment mode as shown in Figure 2. The first one is a special delivery with shorter lead time from warehouse to a requesting retailer. The second one is a delivery, also with shorter lead time, from another retailer with excessive items, to the requesting one. Both special modes have shorter fixed lead time than the regular mode.

The system operates under periodic review basis using (R,s,S) or a periodic base stock policy. The system controls inventory with echelon stock basis. Therefore, each location makes decision based on inventory information of its own and of all locations downstream.

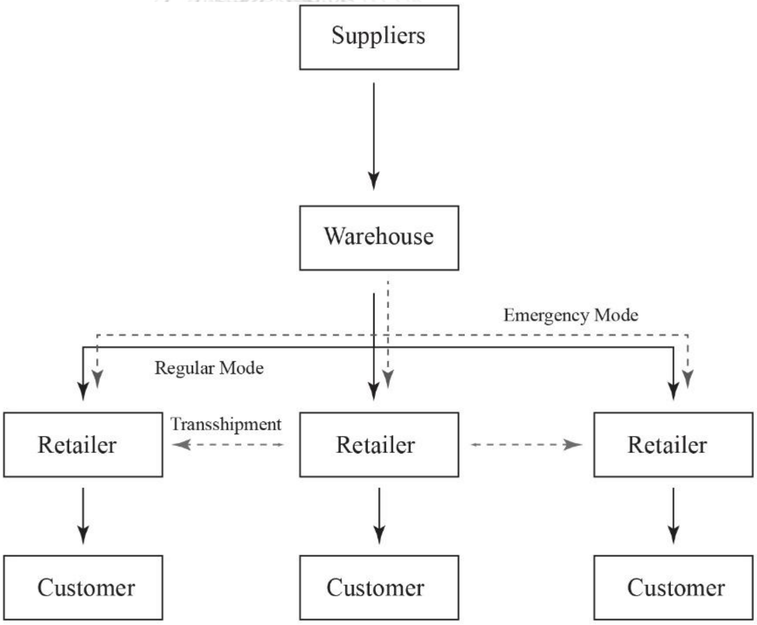

Figure 2 Replenishing modes in the system

1.2 Dissertation objective

The objective of this dissertation is to study a divergent 2-echelon inventory problem with multiple replenishing modes under seasonal demand. The objective is to develop a method to determine ordering policies for items of which demand is seasonal without trend. The method determines (R, s, S) ordering policy for each item to minimize total cost owing to ordering costs and holding cost respecting to expected service level.

1.3 Dissertation scope and assumption

1.3.1 Scope

1. This research focuses only on a divergent 2-echelon inventory system.
2. This research mainly studies items with seasonal demands without trend and total cycle demand is stationary.
3. This research studies only system with (R, s, S) ordering policy.
4. There are 2 additional replenishing modes to be considered as an option.
5. An ordering policy is determined for only one item at a time.

1.3.2 Assumption

1. Warehouse and retailers have unlimited storage spaces.
2. The entire ordered lot is delivered at the same time.
3. Suppliers are always sufficient.
4. There is no quantity discount from suppliers.
5. Lead time is deterministic.
6. Unsatisfied demand is considered lost.
7. Review period, R, is given as 1 period.

1.4 Expected dissertation contribution

Multi-echelon problem is introduced for decades and stay interesting to many researchers. There are papers about multi-echelon inventor problem in various aspects. These papers cover a variety of system structures, assumptions and solution approaches. Unfortunately, to our extent, no paper is found dealing with multi-echelon under seasonal demand or multi-echelon with more than 2 replenishing modes.

To analyze the system, demand is often assumed stationary with some distribution function. This assumption helps simplify the problem's complexity. This assumption is relaxed in this research to reflect the real-life situation. Additionally, as the system has single warehouse and multiple retailers, these retailers can also be assumed to be identical or non-identical. The developed heuristic algorithm is expected to be able to solve both the system with identical and non-identical retailers.

There are some papers dealing with system with multiple replenishing modes, but they are limited to only 2 types of replenishing mode: a regular mode with an emergency mode or a regular mode with a transshipment mode. In this study, the number of replenishing modes is extended to 3 modes. Additional replenishing mode makes the problem more complicated. The main contribution of this dissertation is to develop a method to determine proper (R, s, S) ordering policy for a divergent 2-echelon system with 3 replenishing modes under seasonal demand.

The remainder of this thesis is organized as follows. Chapter 2 reviews the literature related to the problem. Chapter 3 presents the first part of the thesis where a methodology based on MIP models are proposed. Chapter 4 presents the second part of the thesis where algorithms to improve computational time are proposed. Then, the third part of thesis in which a system with multiple replenishing modes are studied is presented in Chapter 5. Finally, Chapter 6 concludes and suggests future research extensions.

CHAPTER II

LITERATURE REVIEW

2.1 Inventory system

The fundamental decisions of inventory management are simple two questions: when to order and how much to order. Based on these two questions, the complexity of the inventory models depending on assumptions and constraints about the systems considered. There are many types of inventories in context of manufacturing and distribution. However, one of the most natural ways is to classify by value added (Nahmias, 2009). With this way of classification, there are 4 types of inventories: 1) Raw materials 2) Components 3) Work-in-process and 4) Finished goods. All those types of inventories are items that should be kept as few as possible, since they cause unnecessary costs. There are many reasons to hold inventories.

1) Economies of scale. Since machine needs setting up to produce a certain type of item, it should be economical to produce a number of items in each production setup and store them for future use.
2) Uncertainties. Uncertainties are often a major reason to hold inventories and uncertain demand is the most important. When items are unavailable, unsatisfied customers may go elsewhere and never return. Inventory provides a buffer to prevent these circumstances.
3) Speculation. Inventory could help reducing cost by storing item or resource of which values are expected to increase.
4) Transportation. When transportation takes long time, in-transit or pipeline inventories exist.

5) Smoothing. Anticipating changes in demand and storing inventory for peak demand can help level production rates and workforce levels.
6) System constraints. There are some constraints about logistics leading to hold inventory. One example is that some items require buying in minimum quantities or buying in multiple of batch size.
7) Control costs. Different inventory control model leads to different costs of inventory control. Controlling inventories in detail to keep them at minimum is more costly than just keeping them in large number to be sufficient to use. The suitability of control model depends on many factors such as item cost or difficulty to store.

Inventory systems are different in details. There are 5 main characteristics of inventory systems.

1) Demand. The most significant factors affecting complexity of the model are pattern and characteristics of the demand. There are two aspects about demand: it is constant or variable and it is known or random.
2) Lead time. Lead time is the amount of time taken from placing order until receiving items. Lead time can be both deterministic and stochastic.
3) Review time. The systems that inventory level is known all times are referred as continuous review. On the contrary, in the systems with periodic review, inventory levels are known only at discrete points of time.
4) Excess demand. Excess demand is demand that cannot immediately satisfied by on-hand inventory. When excess demand exists, two most common assumptions to deal with it, are either demand is back-ordered (or demand can wait to be satisfied in the future) or demand is lost (or demand

will be served by outside). With these two assumptions, there are many mixed assumptions between these two such as partial back-ordering (only part of demand is backordered, and others are lost) or customer impatience (customers can wait only within fixed amount of time or order will be cancelled).

5) Changing inventory. In some context, inventory experiences changes over time. Some items deteriorate or get obsolete such as food or fashion and electronic products.

2.2 Relevant costs

To optimize the inventory system, the performance criterion of the system must be determined. Generally, inventory models use cost minimization as its criterion. Although different systems have different characteristics, all costs in the system can be classified as one these three categories: holding cost, order cost or penalty cost.

2.2.1 Holding cost

The holding cost, as known as the carrying cost or the inventory cost, is the sum of all costs that are proportional to the amount of items physically on hand at any point of time. The components of the holding cost include a variety of items. These components are as follows

- cost of space to store items
- taxes and insurance
- breakage, spoilage, deterioration and obsolescence
- opportunity cost of alternative investment

The opportunity cost is often the most significant to determine holding costs. Inventory means cash. To hold inventory means to invest capital. The higher inventory level, the more capital invested. Since this capital could be invested in other operation or project and other components are generally proportional to the value of inventory, the interest rate is often used as components of holding cost. Let c be the value of a unit of inventory, I be the annual interest rate, and h be the holding cost in term of money per unit per year. Then

$$h = Ic$$

2.2.2 Ordering cost

The order cost depends on the amount of inventory that is ordered or produced. In most applications, order cost consists of two components: the fixed cost, K and the variable cost, c. The variable cost is referred to as product cost. The fixed cost is independent of the size of the order as long as the amount ordered is not zero. The fixed cost would be referred to as costs of order generation and receiving and handling the items.

2.2.3 Penalty cost

Penalty cost is also known as shortage cost or stock-out cost. It is the cost of not having sufficient stock on hand to satisfy demand. The penalty costs, p, are different between systems depending on the systems characteristics. If the system assumes demand is back-ordered, the penalty cost includes whatever delay costs might be involved. If demand lost is assumed, the penalty cost includes the lost profit that would have been made. In both cases, a measure of customer satisfaction could be included as the loss-of-goodwill cost. This loss-of-goodwill can be very difficult to estimate in practice.

2.2.4 Service level

It is often difficult to determine the penalty cost since in many cases the shortage cost involving intangible components such as the loss-of-goodwill. A common substitute for the shortage cost is a service level. The service level refers to the probability that a demand is met. The applications of service level in continuous and periodic review systems are different but, generally, there are two types of service level. Type 1 service level is the probability of not stocking out in the lead time or the percentage of periods that all the demand is satisfied. Type 2 service level is the proportion of demand that are met from stock.

2.3 Economic order quantity

Economic order quantity model or EOQ model is the most fundamental inventory model. This model describes the trade-off between fixed order costs and holding costs. It is the basis for analysis of more complex inventory systems.

An EOQ model is based on an assumption that the demand rate is constant at D units per unit time. (The unit of time may be day, week, month, etc.) In the basic model, shortages are not permitted so the costs include fixed order cost or setup cost at K per order placed proportional order cost at c per unit ordered and holding cost at h per unit held per unit time. Let Q be the order size. With no lead time, an order size of Q placed at $t = 0$ instantaneously raises the on-hand inventory level from 0 to Q. As the demand rate D, the order of Q units will be consumed in cycle length of $T = Q/D$. To obtain the cost per unit time, total cost is divided by the cycle length of T. In each cycle, the order cost is $K + cQ$. For the holding cost, the inventory level begins at Q and decreases to 0 at the end of cycle so the average inventory level is $Q/2$. Therefore, holding cost is $\frac{hQ}{2}$. The total cost per cycle, $G(Q)$, is

$$G(Q) = \frac{K + cQ}{T} + \frac{hQ}{2}$$

From cost function, finding the optimal size of order, Q, is as follows (Nahmias, 2009).

$$G(Q) = \frac{K + cQ}{\frac{Q}{D}} + \frac{hQ}{2} = \frac{KD}{Q} + cD + \frac{hQ}{2}$$

$$G(Q) = \frac{KD}{Q} + cD + \frac{hQ}{2}$$

The cost function, $G(Q)$, is composed of three terms of cycle setup cost, cycle product cost and cycle holding cost, respectively. Then, the shape of the curve $G(Q)$ is considered to find Q to minimize $G(Q)$.

$$G'(Q) = -\frac{KD}{Q^2} + h/2$$

and

$G''(Q) = 2KD/Q^3 > 0$ for $Q > 0$

Since $G''(Q) > 0$, the cost function $G(Q)$ is a convex function of Q. Moreover, $G'(0) = -\infty$ and $G'(\infty) = h/2$. The optimal value of Q occurs where $G'(Q) = 0$ which is true when $Q^2 = 2KD/h$. Therefore, the optimal size of order is

$$Q^* = \sqrt{\frac{2KD}{h}}$$

In the basic model, it is assumed that lead time is zero so the order size of Q is placed when the inventory level is zero. In the system with positive lead time, another decision variable needs to be defined. With positive lead time, an order must be placed before on-hand inventory is completely consumed. The point that an order should be placed in advance is called a reorder point. To calculate the reorder point, two factors are used: the first one is demand rate, D, and the other is lead time, l. The reorder point, s, is the product of demand rate and lead time.

$$s = Dl$$

An order must be placed when on-hand inventory level reaches the level of $s = Dl$.

Dealing with inventory system in real life, demand is not always known in advance. The demand rate with uncertainty composed of two components which are deterministic component and random one. As there is random component of demand, the goal of the system is to minimize *expected* costs. However, an EOQ is still useful in the uncertain environment. EOQ can also be used as the size of order in the system but what is difficult to determine is reorder point. Reorder point must be high enough to prevent shortage but not too high that incurs unnecessary holding cost. The exact value for the stock-out cost is often difficult to determine. As mentioned before, a common substitute is a service level. For example, to use the service level (type 1) to determine reorder point is to specify the probability of *not stocking out in the lead time*. The symbol α represents that probability. First the demand distribution must be known. Then probability, α, is specified and reorder point, s is determined to satisfy α. For example, if α is set to 0.95, s must be set to 95% covering demand in lead time period. Finally, size of order, Q, is set to EOQ. This system is known as (s, Q) system which is a continuous review inventory system. When inventory level reaches reorder point, s, order size of Q is placed. Another type of review system is periodic system. A system generally used in periodic review is (R, s, S) policy. In this system, inventory level is only known in a discrete point of time or every R review interval. It is difficult to implement (s, Q) policy in periodic review since, when the inventory level is reviewed, it might be above or below s making it hardly to place an order when inventory level reaches s. With (R, s, S) policy, when inventory level is reviewed, in every R interval, if inventory level, u, is higher than s, there is no order placed. On the other hand, if inventory level, u, is equal or less than s, an order size of $S - u$ is placed. An example for base stock policy, (R, s, S), shown as Figure 3.

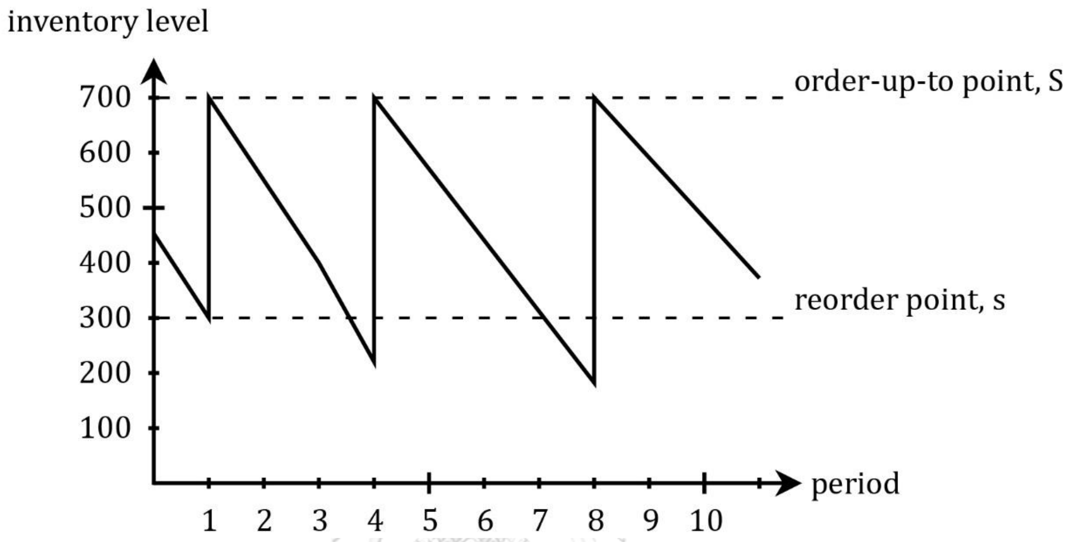

Figure 3 Example for base stock policy, (R, s, S)

In this study the system uses (R, s, S) policy with 1-day review interval and deterministic lead time for warehouse and retailers.

2.4 2-echelon inventory system

Multi-echelon system is the case of an item being stock at more than one location with resupply being made between at least some of the locations (Silver, Pyke, and Peterson, 1998). Diks, De Kok, and Lagodimos (1996) classified multi-echelon system into two types: convergent and divergent structure. The convergent structure is the system that each location is supplied by many higher echelons and has one lower echelon as shown in Figure 4. This structure is also called assembly structure.

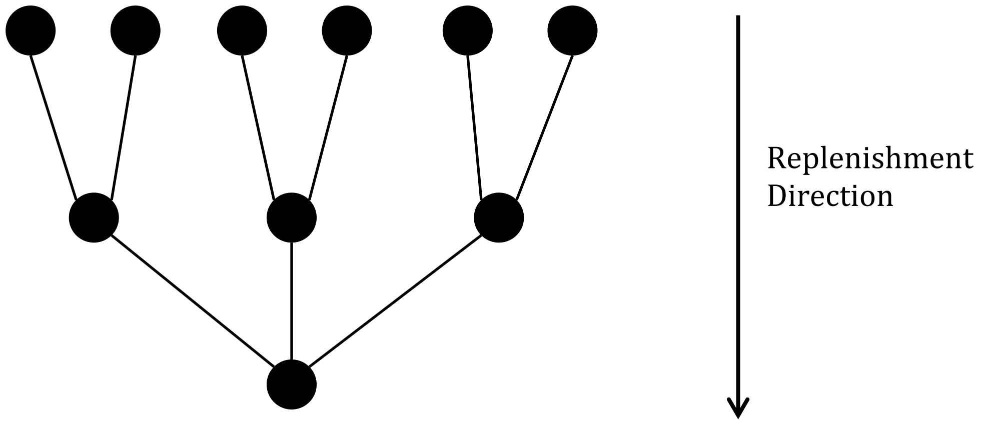

Figure 4 Convergent multi-echelon system

The other system, the divergent structure, is the system that each location has one higher echelon and supplies many lower echelons as shown in Figure 5. This structure is also called distribution structure.

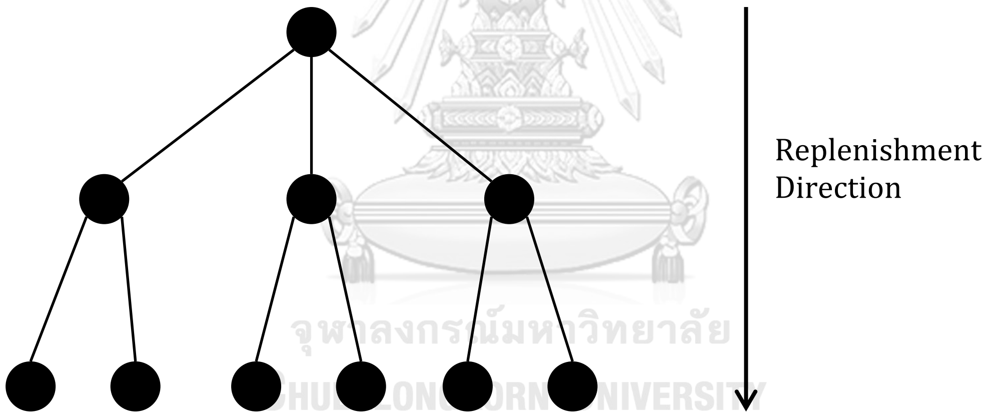

Figure 5 Divergent multi-echelon system

The multi-echelon system is more complex than the single-echelon system because demand at the higher echelon, i.e. the warehouse, is dependent on the demand at the lower echelons. The demand at the warehouse is not the demand directly required by the customers, but it is the demand required by retailers to serve customers. When the warehouse managers make their decision, they should not consider only the demand and inventory level at the warehouse, but they should also consider demands and inventory levels at retailers. This makes the multi-echelon system much more complicated. However, there are many policies to manage the inventory in multi-echelon system. What policy is applicable for the system is dependent on two factors which are information and decision process. If the information at every stock point is shared, this is called centralized information. Otherwise, if the stock point has only information of its own and its lower echelon, it is decentralized information. If every stock point makes the decision together (or someone is authorized to make the decision) with objective for the whole group benefit, this is called global decision. If each stock point makes decision for its own benefit, it is local decision. With these two factors, there are 3 reasonable policies as shown in Figure 6.

Control / Information	Global	Local
Centralized	VMI	Echelon stock
Decentralized	Doesn't make sense!	Installation stock

Figure 6 Multi-echelon system management policies
(Silver, Pyke, and Peterson, 1998)

The VMI or Vendor managed inventory is the policy that the vendor or supplier makes all the decisions that what products, how much and when to refill the inventory in each retailer. An echelon stock policy is each stock point makes its ordering decision based on the sum of the installation inventory positions at the location and all its downstream locations. An installation stock policy is each stock point makes its decision based on its individual inventory position (Axater & Rosling 1993).

Multi-echelon model was first introduced by Clark and Scarf (1960), who studied a serial multi-echelon system. The system operated with uncertain demand, under periodic review ordering policies, and on multi-period time horizon. They introduced the concept of “echelon stock” to find an optimal policy for N-echelon serial system. The echelon stock of a stock point is the stock at the location plus all those items in-transit or on-hand but not yet committed to customer at its downstream stock points. Therefore, the echelon inventory position of a stock point is its echelon stock plus items in-transit to it. Federgruen and Zipkin (1984a) extended a serial N-echelon model proposed by Clark and Scarf (1960) from multi-period horizon to infinite time horizon with stationary uncertain demand. They proposed a new computational approach. However, in their study, the system still operated under an order-up-to-level periodic ordering policy. De Bodt and Graves (1985) continued analyzing the serial N-echelon by applying a (s, Q) continuous review policy. To apply continuous review policy, they considered some more assumptions. One of the key assumptions made was the nested policies which assumed that the order quantity at level must be an integral multiple of the order quantity at an immediately successive stage. With nested policies applied, whenever one stage reorders, all downstream stages also reorder. It was also assumed that an order quantity at higher stage was a multiple of order quantity at lower stage. The model was later modified proposed to apply to fast moving items by Mitra and Chatterjee (2004).

From a serial multi-echelon system, there were studies extending the problem to a divergent multi-echelon system. Bessler and Veinott (1965) extended the pure N-echelon serial inventory system to general arborescent structure: a divergent system that a stock point supplied multiple downstream stock points. The system was analyzed with periodic ordering review and under multi-period time horizon. In this divergent system, when shortage occurred, items could be transshipped from another outlet in the same echelon to the outlet with shortage via their common supply stock point. Then, Eppen and Schrage (1981) analyzed divergent 2-echelon system, which had one depot and multiple end stock points, on infinite time horizon. The problem is known as depot-warehouse problem. To deal with the system on infinite time horizon, the problem was restricted to some more assumptions. At end stock points, there were stationary uncertain demand, which was normally distributed, and the central depot did not hold any stock. Holding and penalty cost and lead times at end stock points were identical. They also held an allocation assumption which assumed that, in each period, the depot received enough material from the supplier, so that each end stock point could be allocated sufficient items to ensure its probability of stock-out equally. Under these assumptions, with no set-up costs, optimal order-up-to-policy at the depot was derived. On the other hand, with fixed set-up costs, an approximately optimal policy was derived. This depot-warehouse problem was further studied in many ways. Federgruen and Zipkin (1984b) extended the model proposed by Eppen and Schrage (1981) by relaxed some assumptions. In their model, holding and penalty costs were non-identical and period demand at end stock points did not have to be normally distributed. The demand was independent in the successive period. Erkip, Hausman, and Nahmias (1990) extended the model considered by Eppen and Schrage (1981) in a different way from that by Federgruen and Zipkin (1984b). They ignored fixed costs and focused on demand correlation over time and analyzed its effect on the safety stock. Bollapragada, Akella, and Srinivasan (1998) also extended depot-warehouse problem in another way. They allowed non-identical warehouses. Various parameters were different across warehouses. These parameters, for instance, were lead times, holding cost and penalty cost. Although Federgruen and Zipkin (1984b) also analyzed the problem with non-identical warehouses, Bollapragada, Akella, and Srinivasan (1998) derived an ordering policy in a different way. Diks and De Kok (1998) extended the model with non-identical warehouses to a higher-than-two-echelon problem with no fixed cost considered. In their analysis, they assumed a balance assumption which assumed that the rationing policy always allocated non-negative stock quantities. De Kok et al. (2018) classified multi-echelon inventory research systematically with various dimensions such as system structure, resource, demand, performance indicator, and research goal. They also identified research gap and potential future research based on recent technology development.

2.5 Fluctuating demand

One of important aspects of the system is demand. Demand pattern can be classified into 4 categories: stationary, trend, seasonal and random (Hanke and Wichern, 2005). These patterns are classified by autocorrelation.

Autocorrelation is the correlation a variable lagged one or more periods and itself. If a series is random, the autocorrelations between Y_t and Y_{t-k} for any lag k are close to zero. The successive values of a time series are not related to each other. If a series has a trend, successive observations are highly correlated, and the autocorrelation coefficient are typically significantly different from zero for the first several time lags and then gradually drop toward zero as the number of lags increase.

If a series has a seasonal pattern, a significant autocorrelation coefficient will occur at the seasonal time lag or multiples of the seasonal lag. The seasonal lag is 4 for quarterly data and 12 monthly data.

The seasonal variation can be divided into 2 kinds: those resulting from natural conditions and those resulting from human decisions (Silver, Pyke, and Peterson, 1998). Therefore, case-study demand is seasonal pattern based on human factors since it is induced by the care unit time pattern and number of physicians.

The demand of the case-study system is seasonal repeatedly fluctuating in 1-week cycle. Demand fluctuates in the same pattern every 7 days. In each cycle, demand fluctuates in the same pattern and total demand of each cycle is stationary. Although the total cycle demand is stationary, periodic seasonal demand makes the system much more complicated. Unlike the system with stationary demand, dealing with inventory in a system with seasonal demand has to take demand pattern into account. If inventory policy is determined as if demand is stationary, the more demand fluctuates, the more shortage is likely to occurs.

With fluctuating demand, there are also researches considering multi-echelon inventory system with deterministic dynamic demand. Zangwill (1969) applied an echelon concept to a dynamic lot sizing problem on multi-echelon system. The multi-echelon dynamic lot sizing problem was extended with various additional constraints and it was mostly solved with mixed integer programming model or algorithms based on the model such as Lagrangian relaxation or decomposition strategy. Diaby and Martel (1993) studied a system with transportation and product price discount based on order quantity. They developed mixed integer linear programming model and Lagrangian relaxation-based procedure to solve the problem. Jaruphongsa, Cetinkaya, and Lee (2004) formulated mixed integer programming model for a problem with demand time window which there were costs occurred both when items shipped earlier or later than definite time ranges. They decomposed the problem into a sequence of smaller problems and developed an algorithm based on dynamic programming to solve them. Afzalabadi, Haiji, and Haiji (2016) proposed heuristics for deterministic dynamic demand in infinite time horizon for a 2-echelon system. The heuristics determined the optimal length of finite time horizon and the optimum ordering pattern which led to the minimum cost within infinite horizon. The proposed heuristics gave better result than Silver-Meal algorithm and EOQ model proposed by Kovalev and Ng (2008) which was developed for a discrete time inventory problem. Besides deterministic demand, some heuristics were developed based on mathematical models to solve uncertain demand. Tarim and Kingsman (2004) developed an algorithm based on a mixed integer programming model to solve lot-sizing problem with service-level constraints for single-item single location on multi-period. Their algorithm was improved from a strategy proposed by Bookbinder and Tan (1988). The algorithm decomposed a problem into two stages: (1) determine timing to replenish orders using expected demand of all periods and (2) adjust actual order size at the time of ordering when actual demand is realized. Tarim and Kingsman (2006), then, applied the algorithm to calculate the (R, S) policies for a single location with non-stationary demand system and Tarim and Smith (2008) improved the algorithm to solve within shorter time by using a constraint programming model.

With non-stationary uncertain demand, there are studies both in single-echelon and multi-echelon systems. Although, there are various methods to deal with non-stationary demand, many methods are based on the same concept. One of the concepts that is widely used is dividing the non-stationary demand into many phases of stationary demand. In a multi-echelon system with non-stationary demand, Graves and Willems (2008) proposed a model to determine locations to hold safety stock and size of safety stock at each location. The model was based on Grave and Willems (2000) which was developed for a system with stationary demand. The model divided a planning horizon into many phases with different stationary demands. Then, safety stock for each phase was determined. The model also determined how safety stock levels changed from phase to phase. Reddy and Rajendran (2005) developed heuristics to determine order-up-to policy for a 5-level serial supply chain with non-stationary demand at the lowest level. They proposed a dynamic order-up-to policy which the policy changed periodically. A simulation study was conducted to evaluate the heuristics in different settings. Kim, Wu, and Huang (2015) applied a multi-period newsvendor model to a perishable product with non-stationary demand in a 2-echelon system. The model gave better solutions compared to those from single-period newsvendor and EOQ model. Grewal, Enns, and Rogers (2015) applied simulation-optimization procedure to solve a single-echelon system with seasonal demand of two products. As demand had seasonal pattern which repeated cycle after cycle, each cycle could be divided into many phases with the same demand's character as other cycles. To correspond with demand in each phase, there were as many ordering policies as number of phases in a demand cycle. Therefore, reorder points and lot sizes varied along demand pattern regions. Ordering policy parameters were iteratively improved via process between simulation and optimization models.

2.6 Multiple replenishing modes

To reduce stock out problem, a company could have more than one replenishing channels. Besides regular mode, other replenishing modes with shorter lead time could be applied. For example, a company, which normally use seafreight, can also use airfreight as an emergency mode with shorter lead time but more expensive.

Emergency replenishing mode is a special replenishment with shorter lead time, but higher cost used in case of imminent shortage from the higher-echelon location (Tagaras and Vlachos, 2001). Generally, a system with more than one source is considered as dual supplies where items are replenished by two sources or one source with 2 modes (Minner, 2003 and Yao and Minner, 2017). The dual-supply problem is studied in various aspects. Two main policies: a continuous review and a periodic review are applied to this problem. Moinzadeh and Nahmias (1988) developed a heuristic algorithm for a system with 2 supply modes under continuous review applying (*Q1*, *Q2*, *R1*, *R2*) policy where an order of *Q1* is placed when on-hand reached *R1* reorder point and an order of *Q2* with shorter lead time is placed when on-hand reached *R2* reorder point. They used a simulation to validate the heuristics and the difference in operation cost between system with and without a special supply mode was studied. Zhou and Yang (2016) proposed heuristics to find policy for 2 replenishing modes under continuous review where both modes must order in batches. For a group of periodic review, various aspects of constraints such as time to place emergency orders or size of orders were studied. Chiang and Gutierrez (1996) proposed a model with 2 replenishing modes under periodic review where, at each review period, either a regular order or an emergency order was placed to raise the inventory position to an expected level. Chiang (2003) extended the model with different variable costs between a regular mode and an emergency mode. Chand, Li and Xu (2016) proposed model similar to Chiang (2003) but they allowed the buyer to choose between two delivery modes at the beginning of the period. They assumed that the unmet demand was backordered and charged a backlogging cost varying with the length of backlogging time. Therefore, the buyer must trade off delivery cost and backlogging cost. Chiang and Gutierrez (1998) allowed multiple emergency orders within a review period. Regular orders and emergency orders are placed periodically but emergency orders have smaller review interval. Chiang (2001) analyzed a special case of the same problem with one-period difference between lead times of a regular mode and an emergency mode. Bylka (2005) proposed a model similar to Chiang and Gutierrez (1998) and the model was extended with an inventory capacity constraint and a limited backlogging constraint. Tagaras and Vlachos (2001) proposed a model for emergency mode where an emergency order would be ordered as late as possible to make the items arrive right before the end of the period. The emergency order is placed to raise on-hand level up to the threshold level. When the on-hand level is less than the threshold level, an emergency order is placed to raise on-hand up to the threshold level and no emergency order is placed otherwise. Huang, Zeng and Xu (2018) proposed a system where regular and emergency orders were supplied by the capacitated suppliers. Regular orders were triggered before the demand is realized but emergency orders were triggered after demand realization. The quantity of emergency order depended on remaining capacity of suppliers. Johansen and Thorstenson (2014) proposed a Markov decision model for a system where regular orders were controlled with reorder point and fixed order quantity and emergency orders were controlled with reorder and order-up-to points. Both regular and emergency orders have constant lead time. Then Johansen (2018) extended the model by assuming stochastic lead time for regular orders.

All those papers studying inventory systems only considered the systems as an arborescent distribution system. (An arborescent system is a tree-like system which each location obtains items only from only one higher location.) However, in this chapter, lateral transshipment is also considered. Lateral transshipments relaxed a system to be more flexible and also more complicated. To allow lateral transshipments, locations of the same level have to pool their inventories (Paterson et al., 2011). There are 2 types of pool policies which are complete pooling and partial pooling. With complete pooling, items can always be transshipped with no condition. On the other hand, with partial pooling, items are reserved for local future demand and will be transshipped when they are excessive items. Another classification of transshipment orders is when they take place. If transshipments take place before demand is observed as predetermined events used to redistribute inventories, they are proactive transshipments. If transshipments take place to respond to stockouts or potential stockouts, they are reactive transshipments. The studies of transshipment orders have both single echelon and multi-echelon structures. Robinson (1990) developed a heuristic technique for multi-location, multi-period problems with transshipments. Optimal ordering policies were determined under two special cases: two non-identical locations and any number of identical locations. Olsson (2015) studied a single echelon, 2 identical locations with positive transshipment lead times. Ordering policy was developed with a heuristic algorithm which separated the whole system into 2 sub-systems, each with one retailer. The positive lead time was treated by keeping track of residual lead time to decide whether to wait for oncoming regular order or request transshipment. Tlili, Moalla, and Campagne (2012) studied 2-echelon system and 2 identical retailers with transshipments. Demand was independent identical normal distribution. They developed initial solution with heuristics based on simulation optimization and, then, used simulation to fine tune to the optimal solution. Tai and Ching (2014) also studied 2-echelon with a number of identical retailers. Ordering policy was developed by using a Markovian model.

Through all those literatures there is no study researches the system of 2-echelon with seasonal demand and emergency replenishment or transshipment are allowed. Summary of literatures are shown in table 1.

Table 1 Summary of literatures.

System structure	Demand	Replenishing mode	Researchers
Multi-echelon (serial)	stationary	1	Clark and Scarf (1960), Federgruen and Zipkin (1984a), Bodt and Graves (1985), Mitra and Chatterjee (2004)
Multi-echelon (divergent)	stationary	1	Bessler and Veinott (1965), Eppen and Schrage (1981), Federgruen and Zipkin (1984b), Erkip et al (1990), Dellaert and Poel (1996), Bollapragada et al (1998), Diks and de Kok (1998), Rivard-Royer et al (2002), Nicholson et al (2004), Meijiboom and Obel (2007), Kumar et al (2008), Kelle et al (2012), Guerrero et al (2013), Uthayakumar and Priyan (2013)
Multi-echelon (divergent)	Deterministic	1	Zangwill (1969), Diaby and Martel (1993), Jaruphongsa et al (2004)
Single-echelon	stationary	2	Robinson (1990), Tagaras and Vlachos (2001), Olsson (2015)
Multi-echelon	stationary	2	Paterson et al (2011), Tlili et al (2012), Tai and Ching (2014)
Multi-echelon (serial)	non-stationary (trend)	1	Reddy and Rajendran (2005)
Single-echelon	non-stationary	1	Grewal, Enns, and Roger (2015), Graves and Willems (2008)
Multi-echelon (divergent)	non-stationary (seasonal)	3	This research

CHAPTER III

PERIODIC-REVIEW POLICY FOR A 2-ECHELON INVENTORY PROBLEM WITH SEASONAL DEMAND

3.1 Introduction

Managing inventory in a multi-echelon system is a very complex problem as it leads to a lot of decisions on many activities and constraints, i.e. what, how much, and when items should be stored at each location, i.e. retailers and warehouse or transported from warehouse to each retailer.

This chapter focuses on an inventory system with a single warehouse and two retailers under seasonal demand. This system is a 2-echelon inventory system whose demand only occurs at retailers. Retailers are supplied by the warehouse and the warehouse is supplied by external suppliers. All locations are replenished with known lead time. The system is illustrated in Figure 7. In the system, items are stored at both warehouse and retailers. The demand which is not satisfied by on-hand inventory is considered as demand loss.

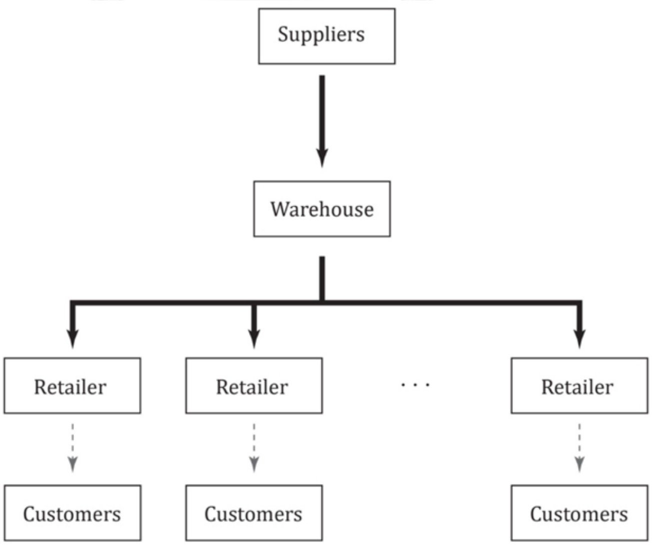

Figure 7 An overview of the inventory system

Demand is assumed to be seasonal without trend and fluctuating within a cycle of a certain span of periods. The demand pattern repeats itself cycle after cycle and, as there is no trend, the total demand per cycle is assumed to be stationary. Seasonal demands can occur as results from natural force or human decisions (Silver, Pyke, and Peterson, 1998). For example, the demand of skiing equipment is driven by weather conditions. On the other hand, department store sales are influenced by holidays and school calendars, which are based on human decision (Hanke and Wichern, 2005). Dealing with seasonal demand as if it is stationary demand can lead to shortage or high holding cost. For example, in Table 2, system A calculates reorder point and order-up-to point by treating demand stationary with an average of 300 units/period. As an order is placed at the end of period and it will arrive at the end of next period, items are backlogged in periods 3, 4 and 5. System B raises reorder and order-up-to points by 300 units to avoid shortage so there is no backlog, but the total holding item is increased to 4,164 units. On the other hand, system C calculates ordering policy by considering seasonal demand. This policy leads to no backlog with the total holding item of 3,408 which is lower than system B. When demand pattern is clearly seasonal. If the ordering policy is developed without taking care of this seasonal pattern, it can lead either to shortage as system A or high holding cost as system B.

This thesis considers the system operated under periodic review basis using (R, s, S) or periodic review base-stock policy in which inventory level is reviewed every R periods and when the level reaches s or lower, an order must be placed to raise the inventory level back to equal or higher than S. The system controls inventory with echelon stock concept where each location makes its own decision.

Each retailer knows its own inventory information while the warehouse can access information of every location. The objective of this chapter is to develop an approach for determining a proper inventory policy for each location to minimize the total inventory cost.

Table 2 The difference between treating demand as stationary and seasonal patterns

System A	**Reorder**	300							
	Order-up-to	600							
	Period	0	1	2	3	4	5	6	Total
	Demand		264	144	360	432	264	144	
	on-hand inventory	600	336	192	240	168	336	192	2064
	order		0	408	360	432	0	408	
	backlogged			0	-168	-192	-96	0	
System B	**Reorder**	600							
	Order-up-to	900							
	Period	0	1	2	3	4	5	6	Total
	Demand		264	144	360	432	264	144	
	on-hand inventory	900	636	492	540	468	636	492	4164
	order		0	408	360	432	0	408	
	backlogged			0	0	0	0	0	
System C	**Reorder**	432							
	Order-up-to	792							
	Period	0	1	2	3	4	5	6	Total
	Demand		264	144	360	432	264	144	
	on-hand inventory	792	528	384	432	360	528	384	3408
	order		0	408	360	432	0	408	
	backlogged			0	0	0	0	0	

The remainder of this chapter is organized as follows. Section 3.2 reviews the literature related to multi-echelon system. Section 3.3 presents a problem description.

Section 3.4 describes the methodology to determine ordering policies. Section 3.5 presents results and discussions. Finally, section 3.6 concludes and suggests future research extensions.

3.2 Literature review

A multi-echelon system is more complex than a single-echelon system because demand at higher echelon, i.e. warehouse, is dependent on demand at lower echelons, i.e. retailers. Demand at warehouse is not directly required by customers but it is required by retailers to serve customers. When a warehouse manager makes decisions, he should consider not only demand and inventory levels at the warehouse, but also consider demand and inventory levels at retailers.

A multi-echelon model was first introduced as a serial multi-echelon system with stationary uncertain demand. Clark and Scarf (1960) analyzed the system on multi-period time horizon by determining the optimal policy for each echelon separately. The system was extended from multi-period horizon to infinite time horizon using periodic base stock policies (Federgruen and Zipkin, 1984a). Besides the periodic review policies, De Bodt and Graves (1985) applied a continuous review policy using reorder point and order quantity or so called (s, Q). The model was later modified and proposed to apply to fast moving items by Mitra and Chatterjee (2004). From a serial inventory system, Bessler and Veinott (1965) generalized the problem by including an arborescent structure - a divergent system that a stock point supplied multiple downstream stock points. The system was analyzed with periodic review ordering policies in multi-period time horizon. Then, the problem was extended to a divergent 2-echelon system with infinite horizon. The problem was studied in many different ways. For example, lower echelon locations were identical, and the demand was stationary and normally distributed (Eppen and Scharage, 1981). Lower echelon locations were non-identical (Bollapragada, Akella, and Srinivasan, 1998). Period demands at end stock points were not normally distributed (Federgruen and Zipkin, 1984b). Demands had correlation over both location and time (Erkip, Hausman, and Nahmias, 1990). However, these studies assume stationary demand while ours assumes seasonal demand. Furthermore, there are some assumptions required in these studies such as nested policy which forces the lower echelon to order when higher echelon orders or no inventory at the higher echelon or no fixed ordering cost.

There are papers considering multi-echelon inventory system with fluctuating deterministic demand or so called a deterministic dynamic demand. Studies in this group were called the multi-echelon dynamic lot sizing problem which was extended with various additional constraints. They were mostly solved by mixed integer programming models or algorithms such as Lagrangian relaxation or decomposition strategy. Zangwill (1969) applied an echelon concept to a multi-echelon dynamic lot sizing problem. Diaby and Martel (1993) studied a system with transportation and product price discount based on order quantity. They developed a mixed integer programming model and used Lagrangian relaxation-based procedure to solve the problem. Jaruphongsa, Cetinkaya, and Lee (2004) formulated a mixed integer programming model for a problem with time window constraint for demand delivery. They decomposed the problem into a sequence of smaller problems and developed an algorithm based on dynamic programming to solve them. Afzalabadi, Haji, and Haji (2016) proposed heuristics for deterministic dynamic demand in infinite time horizon for a 2-echelon system. The heuristics determined the optimal length of finite time horizon and the optimum ordering pattern which led to the minimum cost within infinite horizon. The proposed heuristics gave better result than Silver-Meal algorithm and EOQ model proposed by Kovalev and Ng (2008) which was developed for a discrete time inventory problem. Besides deterministic demand, some heuristics were developed based on mathematical models to solve uncertain demand. Tarim and Kingsman (2004) developed an algorithm based on a mixed integer programming model to solve lot-sizing problem with service-level constraints for single-item single location on multi-period. Their algorithm was improved from a strategy proposed by Bookbinder and Tan (1988). The algorithm decomposed a problem into two stages: (1) determine timing to replenish orders using expected demand of all periods and (2) adjust actual order size at the time of ordering when actual demand is realized. Tarim and Kingsman (2006), then, applied the algorithm to calculate the (R, S) policies for a single location with non-stationary demand system and Tarim and Smith (2008) improved the algorithm to solve within shorter time by using a constraint programming model.

With non-stationary uncertain demand, there are studies both in single-echelon and multi-echelon systems. Although, there are various methods to deal with non-stationary demand, many methods are based on the same concept. One of the concepts that is widely used is dividing the non-stationary demand into many phases of stationary demand. In a multi-echelon system with non-stationary demand, Graves and Willems (2008) proposed a model to determine locations to hold safety stock and size of safety stock at each location. The model was based on Grave and Willems (2000) which was developed for a system with stationary demand. The model divided a planning horizon into many phases with different stationary demands. Then, safety stock for each phase was determined. The model also determined how safety stock levels changed from phase to phase. Reddy and Rajendran (2005) developed heuristics to determine order-up-to policy for a 5-level serial supply chain with non-stationary demand at the lowest level. They proposed a dynamic order-up-to policy which the policy changed periodically. A simulation study was conducted to evaluate the heuristics in different settings. Kim, Wu, and Huang (2015) applied a multi-period newsvendor model to a perishable product with non-stationary demand in a 2-echelon system. The model gave better solutions compared to those from single-period newsvendor and EOQ model. Grewal, Enns, and Rogers (2015) applied simulation-optimization procedure to solve a single-echelon system with seasonal demand of two products. As demand had seasonal pattern which repeated cycle after cycle, each cycle could be divided into many phases with the same demand's character as other cycles. To correspond with demand in each phase, there were as many ordering policies as number of phases in a demand cycle. Therefore, reorder points and lot sizes varied along demand pattern regions. Ordering policy parameters were iteratively improved via process between simulation and optimization models.

Although many researchers chose to apply multiple ordering policies on a system with non-stationary demand, the number of decision conditions will grow rapidly if the system deals with many products and their demands change frequently. Due to complexity, multiple policies for non-stationary demands are not usually practical in real-life situations. The concept to apply different ordering policies to each phase of demand is proper when each demand phase is longer than review period and lead time. For example, when demand phase is 2,000 hours and lead time is 16 hours (Grewal, Enns, and Rogers, 2015). Tunc et al. (2011) investigated that when demands followed a stable seasonal pattern with high uncertainty, stationary policies could reasonably substitute the optimal non-stationary policies. Stationary policies would be efficient in the system with high uncertain demand, high setup cost and low penalty cost. For more classification in multi-echelon inventory system, please refer to De Kok et al. (2018). They classified multi-echelon inventory research systematically with various dimensions such as system structure, resource, demand, performance indicator, and research goal. They also identified research gap and potential future research based on recent technology development.

This chapter studies a divergent 2-echelon system with seasonal demand. Since demand phase in our problem is one period or a length of each season is one period, which is shorter than a review period. Multiple policies as many papers used for seasonal demand may not be proper. Therefore, we choose to apply a single policy to our problem. The objective of this chapter is to develop a methodology to determine ordering policies to minimize the total cost respected to expected service level.

3.3 Problem Statement

This section describes the problem and shows the model formulation to determine ordering policies. The problem considered is a 2-echelon inventory system having one warehouse and N retailers with seasonal stochastic demand. Retailers are supplied by the warehouse and the warehouse is supplied by external suppliers. All locations are replenished with known lead time. Demand that is not satisfied with on-hand inventory will be considered as demand loss. The amount of loss must not exceed expected service level or, in this case, fill rate - the proportion of demand served from on-hand inventory (Nahmias, 2009).

Demand is assumed to be seasonal without trend and fluctuating within a cycle of a certain span of periods. The demand pattern repeats itself cycle after cycle as shown in Figure 8. Period demands are assumed to be normally distributed. For example, in Figure 8, each cycle consists of 4 periods. Average demand of periods 1, 5, 9 and 13 are normally distributed with the same parameters and so are periods 2, 6, 10 and 14.

The system operates on periodic review basis using reorder point and order-up-to point or (R, s, S). The system controls inventory with echelon stock basis which means each location makes decision on its own inventory information and the information of all locations downstream. This chapter proposes a methodology to find optimal inventory policy based on (R, s, S) system to minimize ordering and holding cost respected to expected service level.

Since we consider stochastic demand, it is difficult to find the optimal solution by using a mathematical model. However, a mixed integer programming model is developed to clarify the problem.

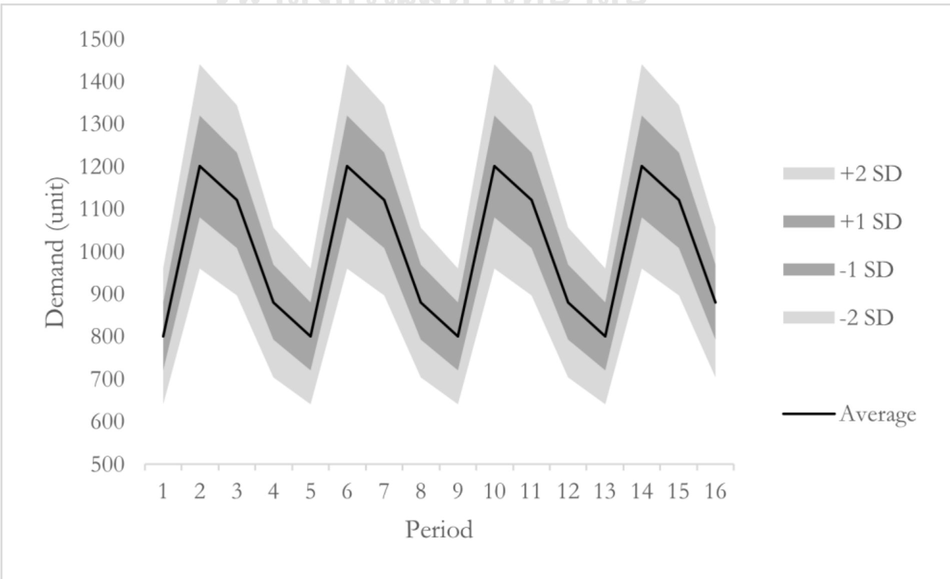

Figure 8 Demand pattern.

The model objective is to minimize the total cost due to ordering and holding costs. A mixed-integer programming model is as follows.

Indices

Ir is a set of retailers $\{1,2,..,n\}$

Irw is a set of stock points including a warehouse and retailers $\{0,1,..,n\}$

(the warehouse is referred as $i = 0$)

J is a set of periods $\{1,2,..,m\}$

Parameters

$demand_{ij}$ = Demand of retailer i in period j (units)

$costorder_i$ = Ordering cost of stock point i ($)

$costholding_i$ = Holding cost of stock point i ($/unit/period)

t_i = Lead time of stock point i (periods)

$servlevel_i$ = Expected service level of retailer i

r_{ij} = 1 if stock point i reviews its inventory in period j;

0 otherwise

M_i = A positive number that is greater than the total demand in planning horizon

of stock point i

Decision variables

I_{ij} = On-hand inventory level at stock point i at the end of period j (units)

O_{ij} = Ordering amount of stock point i at the end of period j (units)

$Lost_{ij}$ = Demand loss of retailer i in period j (units)

L_{ij} = 1 if on-hand inventory of retailer i in period j is not sufficient to cover period's demand;

0 otherwise

Z_{ij} = 1 if an order at stock point i in period j is placed;

0 otherwise

$ReOrder_i$ = reorder point of stock point i (units)

$OrderUpTo_i$ = order-up-to point of stock point i (units)

Objective function

Minimize $$\sum_{i=0}^{n}\sum_{j=1}^{m} Z_{ij} \times costorder_i + \sum_{i=0}^{n}\sum_{j=1}^{m} I_{ij} \times costholding_i \quad (1)$$

Subject to

$$I_{ij-1} + O_{ij-t_i} + Lost_{ij} = demand_{ij} + I_{ij} \quad \forall i \in Ir, \forall j \in J \quad (2)$$

$$I_{0j-1} + O_{0j-t_0} = \sum_{i=1}^{n} O_{ij} + I_{0j} \quad \forall j \in J \quad (3)$$

$$demand_{ij} - \left(I_{ij-1} + O_{ij-t_i}\right) \leq L_{ij} \times M_i \quad \forall i \in Ir, \forall j \in J \quad (4)$$

$$\left(I_{ij-1} + O_{ij-t_i}\right) - demand_{ij} \leq \left(1 - L_{ij}\right) \times M_i \quad \forall i \in Ir, \forall j \in J \quad (5)$$

$$Lost_{ij} \leq L_{ij} \times M_i \quad \forall i \in Ir, \forall j \in J \quad (6)$$

$$I_{ij} \leq \left(1 - L_{ij}\right) \times M_i \quad \forall i \in Ir, \forall j \in J \quad (7)$$

$$Z_{ij} \times r_{ij} \times M_i \geq O_{ij} \quad \forall i \in Irw, \forall j \in J \quad (8)$$

$$I_{ij-1} + \sum_{l=j-t_i}^{j-1} O_{il} + \left(Z_{ij} + \left(1 - r_{ij}\right)\right) \times M_i - 0.5 \geq ReOrder_i$$

$$\forall i \in Ir, \forall j \in J \quad (9)$$

$$I_{ij-1} + \sum_{l=j-t_i}^{j-1} O_{il} \leq ReOrder_i + \left(1 - Z_{ij}\right) \times M_i \quad \forall i \in Ir, \forall j \in J \quad (10)$$

$$I_{ij-1} + \sum_{l=j-t_i}^{j} O_{il} + \left(1 - Z_{ij}\right) \times M_i \geq OrderUpTo_i \qquad \forall i \in Ir, \forall j \in J \qquad (11)$$

$$I_{ij-1} + \sum_{l=j-t_i}^{j} O_{il} \leq OrderUpTo_i + \left(1 - Z_{ij}\right) \times M_i \qquad \forall i \in Ir, \forall j \in J \qquad (12)$$

$$\sum_{i=0}^{n} I_{ij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{il} + \left(Z_{0j} + \left(1 - r_{0j}\right)\right) \times M_i - 0.5 \geq ReOrder_0 \qquad \forall j \in J \qquad (13)$$

$$\sum_{i=0}^{n} I_{ij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{il} \leq ReOrder_0 + \left(1 - Z_{0j}\right) \times M_i \qquad \forall j \in J \qquad (14)$$

$$\sum_{i=0}^{n} I_{ij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{il} + O_{0j} + (1 - Z_{0j}) \times M_i \geq OrderUpTo_0 \qquad \forall j \in J \qquad (15)$$

$$\sum_{i=0}^{n} I_{ij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{il} + O_{0j} \leq OrderUpTo_0 + \left(1 - Z_{0j}\right) \times M_i \qquad \forall j \in J \qquad (16)$$

$$1 - \frac{Lost_{ij}}{demand_{ij}} \geq \ servlevel_i \qquad \forall i \in Ir, \forall j \in J \qquad (17)$$

$$I_{ij}, O_{ij}, ReOrder_i, OrderUpTo_i \geq 0 \qquad \forall i \in Irw, \forall j \in J \qquad (18)$$

$$Z_{ij} \in \{0,1\} \qquad \forall i \in Irw, \forall j \in J \qquad (19)$$

The objective function (1) is to minimize the total cost of a system due to ordering and holding costs. Constraints (2) are inventory levels and product flows in and out (and also loss) at each retailer in each period and constraints (3) are inventory levels and product flows in and out at the warehouse. It is assumed that the warehouse always has sufficient items for retailers' orders so there is no demand loss at the warehouse. Constraints (4) to (7) force decision variables $Lost_{ij}$ and I_{ij}. If available inventory at the retailer is sufficient to serve period's demand, L_{ij} will be zero and $Lost_{ij}$will be zero; otherwise, L_{ij} will be 1 and I_{ij} will be zero. Constraints

(8) define that, on review period ($r_{ij} = 1$), if retailer(s) or warehouse place order, fixed ordering costs will occur. If it is not on review period ($r_{ij} = 0$), order amount, O_{ij}, will be zero. In constraints (9) through (16), reorder points and order-up-to points for the stock points are defined. Constraints (9) to (12) are applied to retailers, while constraints (13) to (16) are applied to the warehouse. Constraints (9) and (10) force the retailers to place orders when their inventory positions (the total level of items on-hand and on-order) are less than or equal to the reorder points and there must be no order placed when inventory positions are higher than the reorder points. In constraints (9), if the inventory positions are equal to or lower than reorder points and $r_{ij} = 1$, Z_{ij} will be 1. If $r_{ij} = 0$, Z_{ij} will be either 1 or 0 where it tends towards 0 due to the objective function. There is a -0.5 term on the left-hand side because, without this term, when the inventory position is equal to reorder points, Z_{ij} can be either 0 or 1 which means that it might be no order placed. In constraints (10), on the other hand, if the inventory positions are higher than reorder points, Z_{ij} will be 0. Constraints (11) and (12) force the inventory positions after placing orders to be equal to the order-up-to points. In these two constraints, if $Z_{ij} = 0$, the constraints will always be true. In constraints (11), when an order is placed or $Z_{ij} = 1$, the inventory level plus order must not less than the order-up-to point. Besides, in constraints (12), the inventory level plus order must not exceed the order-up-to point. Constraints (13) through (16) are similar to constraints (9) to (12) but they are applied to the warehouse. The major difference between the warehouse and retailers is that, at the warehouse, an echelon stock concept is applied so the inventory level is the summation of inventory on-hand and on-order in the system. The echelon stock

concept is applied to the warehouse since demands occur only at retailers. Without information of inventory level at retailers, the warehouse must hold stock sufficient to fill retailers' orders all the time which leads to higher holding cost. By applying the echelon stock concept, the warehouse can predict when retailers are about to place orders and can manage to fill its inventory just before those orders are placed. Constraints (17) guarantee service level for every retailer. Constraints (18) and (19) force all decision variables to be either positive values or binary.

3.4 Methodology

Since the system has stochastic seasonal demand, it cannot be directly solved by a mixed integer programming model. Like an approach proposed by Bookbinder and Tan (1988), they proposed methodology composed of 2 phases – (1) determine timing of replenishment and number of periods to cover (2) determine safety stocks. We propose a 2-phase methodology. The first phase calculated the deterministic policies by using average period demand. These deterministic policies are used to determine when to order and the number of period's demands which the order quantity covers. The second phase is finding appropriate safety stock levels based on the deterministic policies from the first phase to absorb variability of stochastic demand. Safety stock levels can be determined by solving various demand scenarios.

3.4.1 Determining policy for deterministic demand component

This phase is used to determine when to order and the number of periods which the order quantity covers their demand. Since demand is assumed deterministic, policies are determined based on average demand.

3.4.1.1 Concepts for determining initial ordering policy

Since a mixed integer programming model in Section 3 is used to determine deterministic policies. The problem is solved within finite-period horizon while real-life system lies within infinite period. A conflict emerges when an infinite horizon problem is solved using finite horizon. While there is no demand after the planning time horizon, on-hand inventory level at the last period tends to be zero which gives lower holding cost. Applying this type of solutions to an infinite-horizon problem may lead to shortage at the period beyond the considered horizon. Therefore, constraints (20) and (21) are added to the MIP model to force on-hand and on-order inventory at the beginning to be equal to those at the end of horizon. Noted that, in case of stationary demand, the planning horizon can be any periods since every period has the same average demand. However, since we consider seasonal demand, each period has different average demand so on-hand and on-order inventory at the end of horizon must come from the same period of a cycle as the period at the beginning of the horizon. Due to this concept, the planning horizon must be the multiple of cycles.

$$I_{i0} = I_{im} \qquad \forall i \in Irw \qquad (20)$$

$$O_{i0} = O_{im} \qquad \forall i \in Irw \qquad (21)$$

3.4.1.2 Alternatives for determining initial ordering policy

As the system operates on periodic review basis or (R, s, S) policy, there can be alternative solutions for each problem which provide the same minimum total cost. For example, the problem with 4-period demand cycle shown in Table 3 has reorder point of 480 and order-up-to point of 4480. Since an order will be placed whenever the inventory position reaches the reorder point or below to raise the level up to the order-up-to point, the inventory position will raise to the same level no matter how many on-hand items at the time. Therefore, the reorder points between 480 to 1,359 with the same order-up-to point of 4,480 give the same result as shown in Table 3. Although, both 480 and 1,359 reorder points give the same results in this deterministic phase, they can lead to different safety stocks in the second phase which means the different total costs.

Table 3 Example of alternative solutions.

Period	0	1	2	3	4	5	6	7	8
Demand		880	480	1200	1440	880	480	1200	1440
on-hand inventory	1360	480	4000	2800	1360	480	4000	2800	1360
on-order inventory	0	4000	0	0	0	4000	0	0	0
inventory position	1360	4480	4000	2800	1360	4480	4000	2800	1360

With these alternative solutions, there are 2 policies obtained from the model. We name them lower policy and upper policy. The lower one is the policy with the lowest value of reorder point or 480 units in this case and the upper one is the policy with highest value or 1,359 units. The lower and upper policies will apply to all stock points in the system. To obtain lower-alternative policy, the objective function is modified as follow. The objective function (22) is the original function modified as a goal programming model of which the main objective is minimizing the total cost weighted by W_{cost} and the secondary objective is minimizing reorder point. The value of W_{cost} should be high enough to dominate sum of the reorder points.

$$\text{Minimize} \quad W_{cost} \times \begin{pmatrix} \sum_{i=0}^{n} \sum_{j=1}^{m} Z_{ij} \times costorder_i + \\ \sum_{i=0}^{n} \sum_{j=1}^{m} I_{ij} \times costholding_i \end{pmatrix} + \sum_{i=0}^{n} ReOrder_i \tag{22}$$

On the other hand, upper-alternative policy can be obtained via another modified objective function (23).

$$\text{Minimize} \quad W_{cost} \times \begin{pmatrix} \sum_{i=0}^{n} \sum_{j=1}^{m} Z_{ij} \times costorder_i + \\ \sum_{i=0}^{n} \sum_{j=1}^{m} I_{ij} \times costholding_i \end{pmatrix} - \sum_{i=0}^{n} ReOrder_i \tag{23}$$

To compare the quality of solutions, one more policy is developed based on EOQ concept. A solution with EOQ concept is determined by forcing the difference between reorder point and order-up-to point at each retailer equal to EOQ. Constraints (24) is added to the model to force the difference between order-up-to and reorder points close to EOQ value. The $diff_i^+$ is positive when the difference between order-up-to point and reorder point is greater than EOQ. An EOQ-alternative policy is determined with the objective function (25). In this case, the different between order-up-to point and reorder point equals to EOQ is the main objective. Therefore, W_{EOQ} must be high enough to dominate another objective.

$$(OrderUpto_i - ReOrder_i) - eoq_i = diff_i^+ - diff_i^- \qquad \forall i \in Ir \tag{24}$$

$$\text{Minimize} \quad \sum_{i=0}^{n} \sum_{j=1}^{m} Z_{ij} \times costorder_i + \sum_{i=0}^{n} \sum_{j=1}^{m} I_{ij} \times costholding_i + W_{EOQ} \times (diff_i^+ - diff_i^-) \tag{25}$$

Therefore, there are 3 alternative policies for each instance from the first phase. After determining initial policies for each instance, each policy will be used as input to determine safety stock levels to deal with uncertain component of demand in the next phase.

3.4.2 Determining safety stock for random demand component

Since demand is normally distributed, solution from the first phase which based on average demand may not achieve expected service level in various scenarios. In this phase, safety stock levels are calculated for all stock points to absorb variability of demand.

Various scenarios of demand are generated from the normal distribution and a set of scenarios is solved simultaneously with a MIP model to find safety stock level by using ordering policy from the previous model as input parameters. Those input parameters are $determReorder_i$, $determOrderUpTo_i$, $initialI_i$ and $initialO_{il}$. Reorder points and order-up-to points of final ordering policy are reorder points and order-up-to points from initial policy plus safety stock.

The model in this phase is developed based on the model in the previous section with some adjustment to deal with multiple scenarios and demand loss. Additional indices, parameters and decision variables are as follows.

Additional indices

S is a set of scenarios $\{1,2,..,t\}$

L is a set of periods $\{-\max\{leadTime_i\}+1,..,0\}$

Additional parameters

$demand_{sij}$ = Demand of retailer i in period j of scenario s (units)

$initialI_i$ = Initial on-hand inventory level of stock point i (units)

$initialO_{il}$ = Initial on-order of stock point i on period l (units)

$determReorder_i$ = Deterministic reorder point of stock point i (units)

$determOrderUpTo_i$ = Deterministic Order-Up-To point of stock point i (units)

Additional decision variables

I_{sij} = Inventory level at stock point i at the end of period j of scenario s (units)

O_{sij} = Ordering amount of stock point i at the end of period j of scenario s (units)

$Lost_{sij}$ = Demand loss of retailer i in period j of scenario s (units)

L_{sij} = 1 if on-hand inventory of retailer i in period j of scenario s is not sufficient to cover period's demand;
0 otherwise

Z_{sij} = 1 if regular order at stock point i in period j of scenario s is placed;
0 otherwise

SS_i = Safety stock of stock point i (units)

Objective function

Minimize

$$\sum_{s=1}^{t}\sum_{i=0}^{n}\sum_{j=1}^{m} Z_{sij} \times costorder_i + \sum_{s=1}^{t}\sum_{i=0}^{n}\sum_{j=1}^{m} I_{sij} \times costholding_i \quad (26)$$

Subject to

$$I_{sij-1} + O_{sij-t_i} + Lost_{sij} = demand_{sij} + I_{sij} \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \quad (27)$$

$$I_{s0j-1} + O_{s0j-t_0} = \sum_{i=1}^{n} O_{sij} + I_{s0j} \qquad \forall j \in J, \forall s \in S \quad (28)$$

$$demand_{sij} - \left(I_{sij-1} + O_{sij-t_i}\right) \le L_{sij} \times M_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \quad (29)$$

$$\left(I_{sij-1} + O_{sij-t_i}\right) - demand_{sij} \leq \left(1 - L_{sij}\right) \times M_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (30)$$

$$Lost_{sij} \leq L_{sij} \times M_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (31)$$

$$I_{sij} \leq \left(1 - L_{sij}\right) \times M_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (32)$$

$$Z_{sij} \times r_{ij} \times M_i \geq O_{sij} \qquad \forall i \in Irw, \forall j \in J, \forall s \in S \qquad (33)$$

$$I_{sij-1} + \sum_{l=j-t_i}^{j-1} O_{sil} + \left(Z_{sij} + \left(1 - r_{ij}\right)\right) \times M_i - 0.5 \geq ReOrder_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (34)$$

$$I_{sij-1} + \sum_{l=j-t_i}^{j-1} O_{sil} \leq ReOrder_i + \left(1 - Z_{sij}\right) \times M_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (35)$$

$$I_{sij-1} + \sum_{l=j-t_i}^{j} O_{sil} + \left(1 - Z_{sij}\right) \times M_i \geq OrderUpTo_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (36)$$

$$I_{sij-1} + \sum_{l=j-t_i}^{j} O_{sil} \leq OrderUpTo_i + \left(1 - Z_{sij}\right) \times M_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (37)$$

$$\sum_{i=0}^{n} I_{sij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{sil} + \left(Z_{s0j} + \left(1 - r_{0j}\right)\right) \times M_i - 0.5 \geq ReOrder_0 \qquad \forall j \in J, \forall s \in S \qquad (38)$$

$$\sum_{i=0}^{n} I_{ij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{sil} \leq ReOrder_0 + \left(1 - Z_{s0j}\right) \times M_i \qquad \forall j \in J, \forall s \in S \qquad (39)$$

$$\sum_{i=0}^{n} I_{sij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{sil} + O_{s0j} + \left(1 - Z_{0j}\right) \times M_i \geq OrderUpTo_0 \qquad \forall j \in J, \forall s \in S \qquad (40)$$

$$\sum_{i=0}^{n} I_{ij-1} + \sum_{i=0}^{n} \sum_{l=j-t_i}^{j-1} O_{sil} + O_{s0j} \leq OrderUpTo_0 + \left(1 - Z_{s0j}\right) \times M_i \qquad \forall j \in J, \forall s \in S \qquad (41)$$

$$1 - \frac{Lost_{sij}}{demand_{sij}} \geq servlevel_i \qquad \forall i \in Ir, \forall j \in J, \forall s \in S \qquad (42)$$

$$ReOrder_i = determReorder_i + SS_i \qquad \forall i \in Ir, \forall s \in S \qquad (43)$$

$$OrderUpTo_i = determOrderUpTo_i + SS_i$$

$$\forall i \in Ir, \forall s \in S \qquad (44)$$

$$I_{si0} = initialI_i + SS_i \qquad \forall i \in Ir, \forall s \in S \qquad (45)$$

$$O_{sil} = initialO_{il} \qquad \forall i \in Ir, \forall l \in L, \forall s \in S \qquad (46)$$

$$I_{sij}, O_{sij}, ReOrder_i, OrderUpTo_i \geq 0 \qquad \forall i \in Irw, \forall j \in J, \forall s \in S \qquad (47)$$

$$Z_{sij} \in \{0,1\} \qquad \forall i \in Irw, \forall j \in J, \forall s \in S \qquad (48)$$

The objective function (26) is to minimize the total cost of the system in all scenarios due to ordering cost and holding cost. Constraints (27) through (42) force variables in the same way as constraints (2) to (17). Constraints (43) and (44) calculate reorder points and order-up-to points of the final policy. Constraints (45) force initial on-hand inventory, I_{si0}, equal to an initial amount obtained from the previous model plus safety stock. Constraints (46) force initial on-order before the first period, O_{sil}, equal to an initial amount obtained from the previous model, $initialO_{il}$. Constraints (47) and (48) force all decision variables to be either positive values or binary.

3.5 Experimental Results and Discussion

The experiment summary is shown in Figure 9. To test the proposed methodology, in the first phase, six instances are developed. Three instances have 4-period cycle and the others have 7-period cycle. Each group of three instances has the same period-cycle demand pattern and holding cost but different ordering costs: high, low and zero. High value is ordering costs of which EOQ values close to the retailers' cycle demand and low value is costs of which EOQ values smaller than the cycle demand. Certainly, the zero value is an ordering cost with value of zero. In Figure 9, the ordering cost and holding cost ratios are shown for each location as (WH, R1, R2) which means the warehouse, retailer1 and retailer2. In all instances, holding cost is 1 ($/unit/period). Demand pattern for each retailer is also shown as average demand in each period. These six instances are solved using an MIP model in phase I to find the best reorder and order-up-to points for average demand under three initial policies. These three initial policies are lower, upper and EOQ discussed in section 3.4.1.2. Since the horizon must be multiples of cycles, 6-cycle planning horizon is used in this phase. Therefore, the 4-period cycle instances have 24-period horizon and the 7-period cycle instances have 42-period horizon. The MIP models were solved by using CPLEX. All experiments ran on a computer with 2.00 GHz Intel Core i7 processor and 4 GB of RAM.

In the second phase, 3 different standard deviations are used to generate various scenarios of the problem, i.e. 10%, 20% and 25% of average demand. We assumed that demands are normally distributed. Demand in each period is randomly generated based on average and standard deviation of that period. Demand of the 1^{st} period in every cycle has the same average and standard deviation. This also applies to the 2^{nd}, 3^{rd} and so on. A set of 4 scenarios is randomly generated based on the same parameters and solved for safety stock with the second MIP model. Since they are randomly generated, actual demand in each scenario is different. Each scenario has 24-cycle horizon which is 96-period horizon for 4-period instance and 168-period horizon for 7-period instance. The instance with 4-period cycle under 99% is used as the base case. Then a 4-period cycle under 95% service level is used to measure the differences between service levels. Furthermore, the instances with 7-period cycle under 99% service level is used to measure the differences between the number of periods in cycle.

Policy Determination

Period/cycle	Average demand in each period: Retailer 1	Average demand in each period: Retailer 2	Ordering cost/ Holding cost (WH, R1, R2)
4	880, 480, 1200, 1440	880, 1840, 2400, 2880	High (12,000, 12,000, 12,000)
4	880, 480, 1200, 1440	880, 1840, 2400, 2880	Low (750, 750, 750)
4	880, 480, 1200, 1440	880, 1840, 2400, 2880	Zero (0, 0, 0)
7	105, 99, 109, 121, 88, 140, 38	226, 228, 220, 209, 159, 287, 71	High (2,450, 2,450, 4,900)
7	105, 99, 109, 121, 88, 140, 38	226, 228, 220, 209, 159, 287, 71	Low (612.5, 612.5, 1225)
7	105, 99, 109, 121, 88, 140, 38	226, 228, 220, 209, 159, 287, 71	Zero (0, 0, 0)

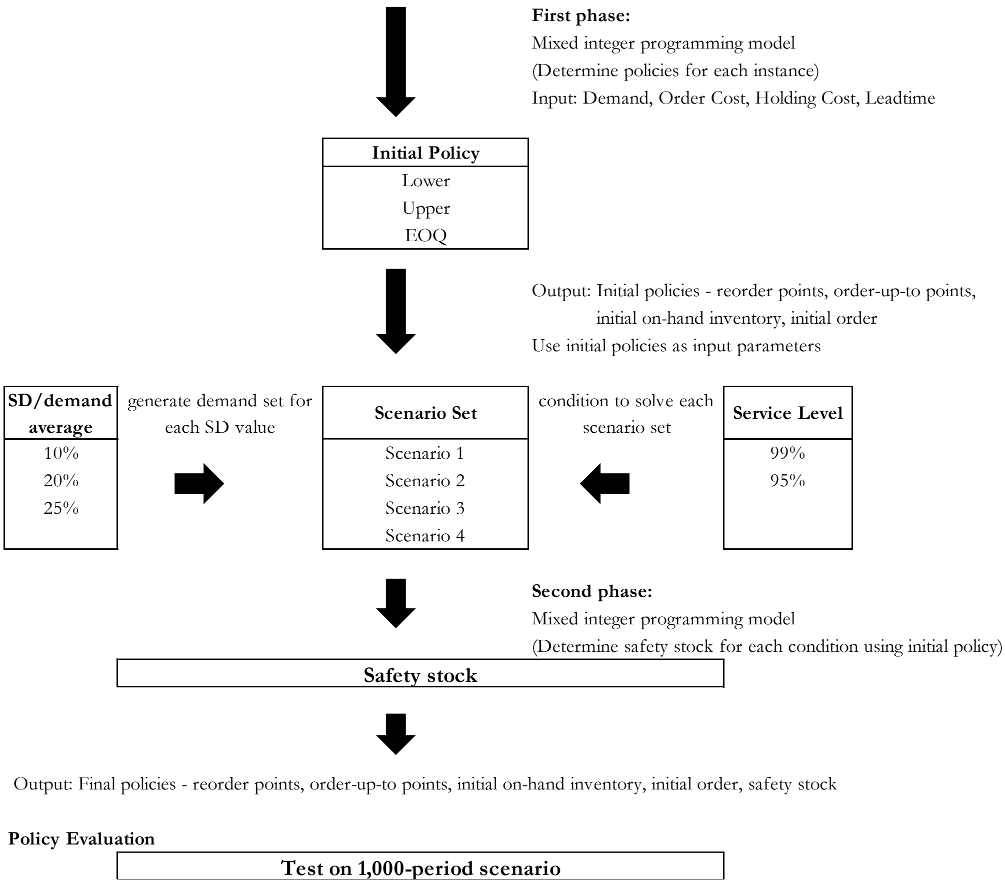

Figure 9 Experimental summary

After final policy is developed from the second phase, the performance of the policy is tested with additional 1,000-period scenario to investigate whether demand loss is within expected service level.

3.5.1 Comparison among the lower, upper and EOQ alternatives.

Under both 99% and 95% service level, in 4-period cycle instances, all three alternatives perform the same way. Comparing upper and lower policies, upper policy gives the lower total cost in every scenario. The upper policy could give lower total costs since it has higher reorder points which lead to lower safety stock in the second model. Since the upper and lower policies have different reorder points but the same order-up-to points. Although reorder points are raised by safety stock to the same levels, the upper policies will have lower final order-up-to points which lead to smaller sizes of orders and lower holding costs. In case of zero ordering cost, EOQ policy gives the lowest total cost. In this case, EOQ is zero which means reorder points and order-up-to points of retailers in EOQ policy are the same points. It means that the policy of retailers are (R, S) instead of (R, s, S) and they have inventory position filled at order-up-to point in every period, which leads to low holding cost and low total cost since ordering cost is zero. The average differences in costs are shown in Table 4 and Table 5. The average difference from the best solution is calculated from 4 scenarios, so alternatives with 0.00% difference are the alternatives that perform better than the others in all 4 scenarios. However, in some case, one alternative may outperform other alternatives in some scenarios when another alternative outperforms it in other scenarios. For example, in Table 4, at high ordering cost and SD to demand average ratio of 25%, upper policy gives better results than EOQ policy in 3 scenarios and the EOQ policy performs better in 1 scenario. Therefore, upper-alternative difference is 0.01% and EOQ-alternative difference is 0.61%.

Generally, in Table 4 and Table 5, most instances can be solved within 5 minutes but, in some cases, it can take almost an hour. It takes longer time to solve an instance with low service level than an instance with high service level. Since lower service level means higher demand loss allowed, the search space is larger than those problems with higher service level. As there are more options to be chosen, it takes longer time to find the optimal solution.

Furthermore, in 7-period cycle instances, upper policy also gives lower total cost than lower policy in all scenarios as shown in Table 6. However, in case of zero ordering cost, all three alternatives receive the same policy with the same total cost in phase 1 but the upper policy has the highest reorder points at the warehouse. Therefore, when their reorder points are raised by safety stock to the same values, the upper policy has the lowest order-up-to points in phase 2 which leads to the lowest holding costs.

Most instances in Table 6 can be solved within 15 minutes but, in some cases, it can take up to 3 hours. Comparing computational time, the problems with longer planning horizon require longer computational time. For example, the instances with 168-period planning horizon require 1527.89 seconds on average which is longer than 224.24 s required by the instances with 96-period horizon. Details of all scenarios' costs and computational time are provided at http://pioneer.netserv.chula.ac.th/~twipawee/RsS_MIP.zip

Table 4 Average difference and computational time for 4-period cycle instances at 99% service level.

			Average % Difference from Best Solution			**Computational Time (s)**		
Service Level	**Ordering Cost/ Holding Cost**	**SD/ Average**	**Upper**	**Lower**	**EOQ**	**Upper**	**Lower**	**EOQ**
0.99	High	10%	0.00%	14.64%	10.39%	148.52	110.03	94.14
		20%	0.00%	3.61%	1.18%	70.73	93.31	70.86
		25%	0.01%	4.46%	0.61%	158.42	91.94	113.72
	Low	10%	0.00%	26.84%	12.72%	2641.00	111.80	286.22
		20%	0.00%	19.54%	11.09%	124.95	95.75	234.00
		25%	0.00%	18.29%	11.01%	126.12	138.34	110.75
	Zero	10%	6.82%	35.97%	0.00%	125.69	195.44	106.02
		20%	9.60%	30.18%	0.00%	96.38	104.39	60.38
		25%	8.07%	27.48%	0.00%	61.55	411.69	72.31

Table 5 Average difference and computational time for 4-period cycle instances at 95% service level.

			Average % Difference from Best Solution			**Computational Time (s)**		
Service Level	**Ordering Cost/ Holding Cost**	**SD/ Average**	**Upper**	**Lower**	**EOQ**	**Upper**	**Lower**	**EOQ**
0.95	High	10%	0.00%	14.74%	10.44%	1576.31	242.22	143.74
		20%	0.00%	3.55%	1.30%	108.73	172.39	74.34
		25%	0.10%	4.27%	0.39%	112.86	77.00	1625.42
	Low	10%	0.00%	27.02%	12.77%	687.84	144.97	1191.17
		20%	0.00%	19.75%	11.39%	127.03	117.03	271.11
		25%	0.00%	18.49%	11.20%	81.33	88.27	150.52
	Zero	10%	7.09%	36.41%	0.00%	727.36	104.42	113.27
		20%	9.25%	30.17%	0.00%	121.42	151.84	97.67
		25%	7.88%	27.58%	0.00%	107.27	87.50	92.12

Table 6 Average difference and computational time for 7-period cycle instances at 99% service level.

			Average % Difference from Best Solution			Computational Time (s)		
Service Level	**Ordering Cost/ Holding Cost**	**SD/ Average**	**Upper**	**Lower**	**EOQ**	**Upper**	**Lower**	**EOQ**
0.99	High	10%	0.00%	5.84%	5.84%	486.16	827.66	844.09
		20%	0.00%	2.32%	2.32%	1437.59	719.77	745.90
		25%	0.00%	3.74%	3.74%	11458.00	1893.64	1732.43
	Low	10%	26.65%	45.16%	0.00%	654.94	672.63	358.24
		20%	30.52%	43.01%	0.00%	695.42	1359.44	653.16
		25%	29.57%	43.74%	0.00%	8220.59	1389.48	1394.17
	Zero	10%	0.00%	70.26%	50.34%	697.93	409.28	189.54
		20%	0.00%	45.41%	31.98%	787.83	378.00	1491.00
		25%	0.00%	37.80%	26.37%	617.67	469.65	668.90

3.5.2 Efficiency of each alternative on another set of demands

Initial policies from the first phase and safety stock from the second phase are combined as final ordering policy. These combinations of deterministic policy and safety stock are tested on another scenario with 1,000 periods to compare its robustness with total cost and service level.

Focusing on the robustness of policies, on average for 4-period cycle, average loss is around 0.00% to 0.19% for 99% service level and 0.00% to 0.22% for 95% service level as shown in Figure 10 and Figure 11. In a certain period, applying upper and EOQ policies can lead to the maximum loss (not the average loss shown in Figure 10 and Figure 11) as high as 39.75% for 99% service level and 42.58% for 95% service level while the maximum loss of lower policies is no higher than 25% and 30% respectively. However, the number of periods that loss is higher than the expected service level is smaller than 2% in a span of 1,000 periods. For 7-period cycle, average loss is around 0.00% to 0.05% for 99% service level as shown in Figure 12. In a certain period, applying upper and EOQ policies can lead to the maximum loss as high as 23.84% for 99% service level while the maximum loss of lower policies is no higher than 18.90%. In a span of 1,000 periods, the number of periods that loss is higher than the expected service level is smaller than 1%. Therefore, the policies obtained from proposed approach are robust for the problem and are practical to use in real life.

When standard deviation increases, the total cost and loss tends to increase. Lower policies give the highest total cost but the lowest loss. Normally, the upper policies tend to give the lowest total cost and highest loss. Lower policies have the highest cost and lowest loss since they have bigger size of orders and hold more inventory than other alternatives. The policies have the bigger size of orders because they have bigger differences between reorder points and order-up-to points. On the contrary, upper and EOQ policies give lower total costs due to their smaller orders leading to lower holding costs. In summary, there is trade-off between total cost and loss. For cost-concern, upper or EOQ policies are the preferred. For loss-concern, lower policies are the best choices.

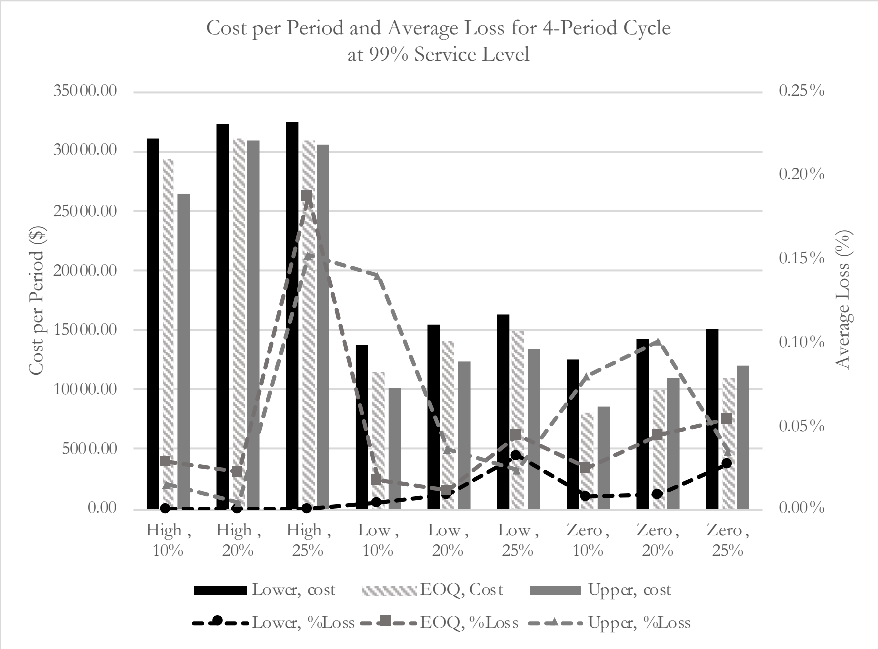

Figure 10 Total cost and average loss for 4-period cycle instances at 99% service level on 1,000-period horizon.

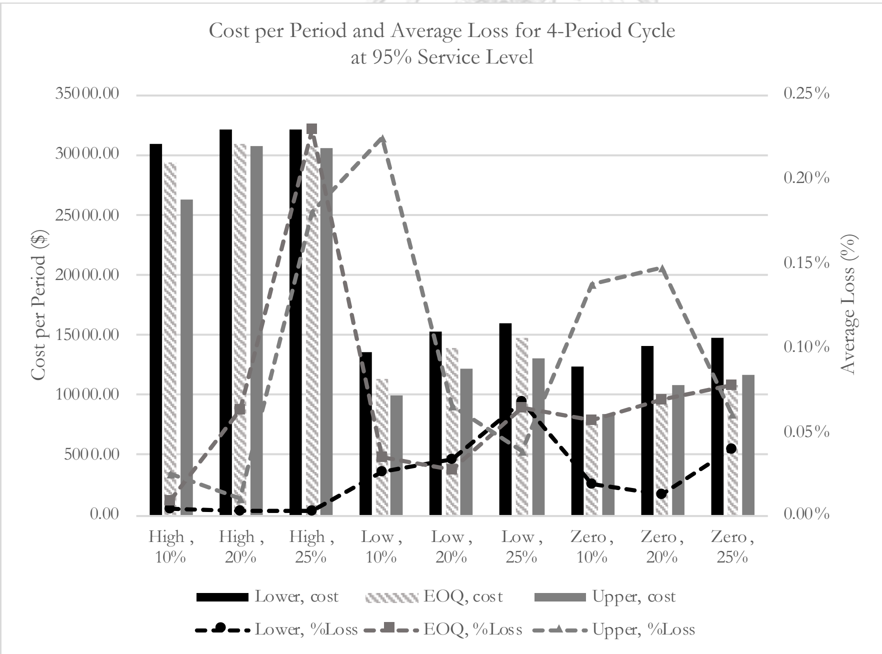

Figure 11 Total cost and average loss for 4-period cycle instances at 95% service level on 1,000-period horizon.

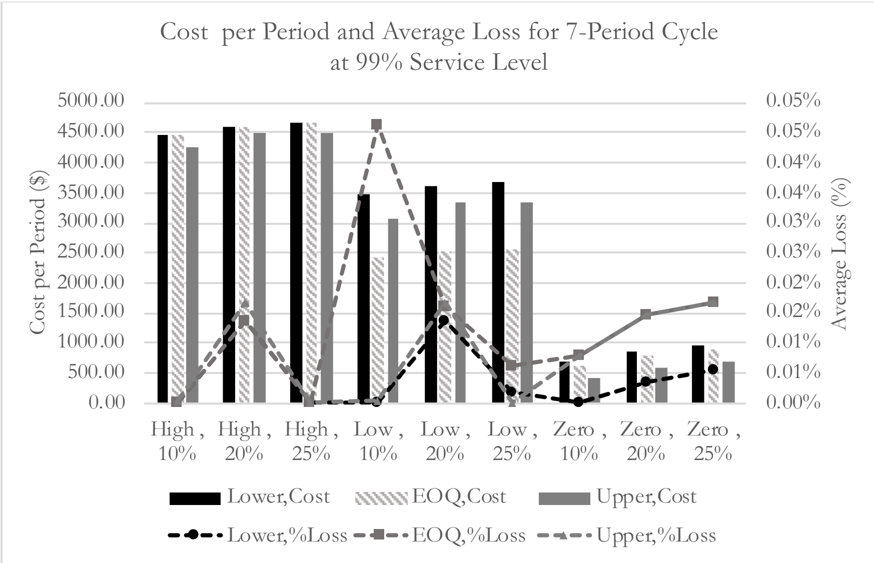

Figure 12 Total cost and average loss for 7-period cycle instances at 99% service level on 1,000-period horizon.

3.5.3 Effect of on-hand and on-order inventory through inventory policy

Since system operates under periodic review and seasonal demand, some values of initial on-hand inventory may lead to shortage while other values may not, even though they are applied to the same ordering policy. The example in Table 7 illustrates that an ordering policy cannot be applied to every value of initial on-hand inventory. In Table 7, an order is placed at the end of each period. The system has 1-period lead time so an order placed at the end of this period will be ready to use at the end of the next period. The system A with 1,360 initial inventory faces no shortage. On the contrary, the system B with 4,000 initial on-hand inventory faces shortage at period 5. Therefore, under seasonal demand, it is better to find the proper initial on-hand level by letting it be a decision variable than an input parameter. This can help the system avoid shortage.

To implement the (R, s, S) policies with predetermined initial inventory obtained from the algorithm, inventory managers may start using the (R, S) policies first. When the inventory position of any retailer reaches the required level, that retailer can start using (R, s, S) policy. After all retailers use (R, s, S), the warehouse can use its (R, s, S) policy when the echelon inventory position reaches the required level.

Table 7 Example of effect of initial on-hand inventory on shortage.

	Reorder	480						
	Order-up-to	4480						
	Period	0	1	2	3	4	5	6
System A	**Demand**		880	480	1200	1440	880	480
	on-hand inventory	1360	480	0	2800	1360	480	0
	order		4000	0	0	0	4000	0
System B	**Demand**		880	480	1200	1440	880	480
	on-hand inventory	4000	3120	2640	1440	0	-880	3120
	order		0	0	0	4480	0	0

3.6 Conclusion

A methodology to deal with multi-echelon inventory system with seasonal demand is proposed. Since the seasonal stochastic demand cannot be directly solved with a MIP model, the 2-phase methodology is proposed. The first phase determines deterministic policy by solving a deterministic problem based on average demand and the second phase calculates safety stock by solving multiple scenarios of the problem generated from demand distribution. The MIP model for the first phase is used to find reorder and order-up-to points. We found that there are alternative optimal solutions where multiple reorder points provide the same total cost. However, these alternatives may lead to different cost in the second phase. Therefore, we explore 3 different alternatives from the first phase including lower, upper and EOQ. In the second phase, another MIP model is developed. It uses reorder and order-up-to points from phase 1 as inputs and solves multiple randomly-generated scenarios simultaneously for safety stock that leads to the minimum total cost respected to required service level.

The proposed methodology can find solutions within a reasonable amount of time (generally within 15 minutes). The policy with the highest reorder point tends to get the lowest cost in most scenarios but they face more demand loss comparing to other policies. On average, when the upper-alternative policies is the best policy, it gives the total cost that is around 17% better than other policies. On the other hand, the policy with the lowest reorder point tends to get the highest cost but the lowest demand loss. The highest average loss from the lower-alternative policies is only 0.07% while the highest average loss from other policies is 0.22%.

As the problem that we considered in this chapter assumed that unsatisfied demand was lost and the problem with lost-sales assumption received more attention recently (De Kok et al., 2018), to make the problem be more interesting, it can be extended in various aspects such as considering shortage cost, considering ordering batch sizes, exploring other replenishment policies or exploring other replenishment modes. The methodology can be improved as well. The proposed methodology based upon MIP models can solve only limited problem size. While the problem sizes increase such as increasing in demand volume or increasing in the number of periods per cycle, the optimal solution may not be able to obtain using the MIP models. Heuristic approach should be developed to solve the problem.

CHAPTER IV

HEURISTICS FOR PERIODIC-REVIEW POLICY IN A 2-ECHELON INVENTORY PROBLEM WITH SEASONAL DEMAND

4.1 Introduction

A distribution system with one-warehouse and multi-retailer is a very complex system since it is involved with a lot of decisions from many activities and constraints such as how many and at what time items should be transported from the warehouse to retailers or stored at the warehouse or each retailer.

This chapter focuses on an inventory system with a single warehouse and N non-identical retailers under seasonal demand. This system is a 2-echelon inventory system in which each location replenishes inventory in a fixed time interval. Retailers are supplied by a warehouse which is supplied by external suppliers and items can be stored at the warehouse and retailers. Unfilled demand is considered as demand loss.

Demand is assumed to be seasonal without trend. In this case, demand is fluctuating within a certain span of periods called cycle and demand pattern repeats itself cycle after cycle. However, as there is no trend, the total demand per cycle is assumed to be stationary. When it comes to the term seasonal demand, both natural force and human decisions can be factors of seasonal pattern (Silver, Pyke, and Peterson, 1998). For example, the demand of umbrella or raincoat is driven by weather conditions. On the other hand, stationary sales are influenced by school calendars, which are based on human decision (Hanke and Wichern, 2005). Human factor can drive demand in the same way as natural force does and it can drive demand in various lengths of cycle such as a month, a week or even a day. The system considered in this chapter has seasonal demand within a cycle of one week as it is driven by human factor. For example, the demand could be high on Monday, Wednesday, and Thursday. On the other hand, demand could be very low on weekend and this pattern repeats every week. This demand pattern can be found at a drug dispensing system in a hospital. The number of patients diagnosed depends on the number of physicians. Physicians' working days are pre-scheduled and the number of physicians is different on each day. The physician's schedule repeats every week. Therefore, demand of each drug depends on the physicians' schedule.

Dealing with seasonal demand as if it is stationary demand can lead to shortage or high holding cost. Therefore, many papers (Graves and Willems, 2008; Reddy and Rajendran, 2005; Kim, Wu, and Huang, 2015, Grewal, Enns, and Rogers, 2015) tackled a system with trend and seasonal demand by varying ordering policy along with the changing phases of demand. However, their demands changed slowly compared to lead time such as the length of demand phase is 2,000 hours where lead time is 16 hours (Grewal, Enns, and Rogers, 2015) or 100-period demand phase with 10 period lead time (Graves and Willems, 2008). This thesis considers 1-day demand phase and 1-day lead time; therefore, changing ordering policy everyday as demand changes would not be practical.

As we developed a 2-phase mixed-integer programming model for the problem in Chapter 3. The proposed methodology has some limitations. For example, the number of potential solutions increases along with the number of retailers and the computational time grows exponentially as the number of retailers increases. In a preliminary study, the average computational time for instances with 2, 3, and 5 retailers are 24.70s, 69.87s, and 7,571.30s, respectively. We use the modeling framework from Chapter 3 to develop heuristics to determine proper ordering policy to minimize the total inventory cost.

The remainder of this chapter is organized as follows. Section 4.2 reviews the literature related to the multi-echelon system. Section 4.3 presents a problem description. Section 4.4 describes heuristics to determine ordering policies. Section 4.5 presents results and discussions. Finally, section 4.6 concludes and suggests future research extensions.

4.2 Literature review

A multi-echelon model was first introduced as a serial multi-echelon system with stationary demand. Papers in this group include Clark and Scarf (1960), De Bodt and Graves (1985) and Ji et al. (2016). Besides a serial multi-echelon system, a divergent and general structure inventory systems were also studied including Bessler and Veinott (1965), Erkip, Hausman, and Nahmias (1990), Chu and Shen (2010), Erugaz et al. (2014) and Shang, Tao, and Zhou (2015). These studies explored various aspects of multi-echelon inventory system, but they assumed stationary demand which made them different from our system with seasonal demand.

Many researchers studied a multi-echelon system with deterministic demand called the multi-echelon dynamic lot sizing problem. Examples of techniques used to solve this problem are mixed integer programming models, Lagrangian relaxation and decomposition strategy. Papers in this group include Zangwill (1969), Bookbinder and Tan (1988), Diaby and Martel (1993), and Tarim and Smith (2008), Afzalabadi, Haji, and Haji (2016).

Various methods were used to deal with non-stationary demand in both trend and seasonal patterns. These methods were based on the same concept. The concept that was widely used was dividing non-stationary demand into many phases of stationary demand. Then, an ordering policy was developed for each phase. Graves and Willems (2008) proposed a model to determine the size of safety stock at each location in a multi-echelon system. As the model divided a planning horizon into phases of different stationary demand, it determined size of safety stock in each phase and also how safety stock changed from phase to phase. Reddy and Rajendran (2005) proposed a dynamic order-up-to policy which a policy changed periodically and developed heuristics to determine a policy for a 5-level serial supply chain where non-stationary demand occurred at the lowest level. Kim, Wu, and Huang (2015) applied a multi-period newsvendor model in a 2-echelon system to a perishable product with non-stationary demand. Grewal, Enns, and Rogers (2015) applied simulation-optimization procedure to solve a single-echelon system of two products with seasonal demand.

Despite efficiency of the method that divides demand into many phases and applies different ordering policy to each phase, the number of decision conditions grows rapidly when demand changes frequently. The concept to divide demand into phases and develop policy for each phase is proper when lengths of phases are longer than review period and replenishing lead time which means there could be a number of orders placed within one phase. For example, a length of each demand phase is a quarter and lengths of review period and lead time are days or weeks. In our problem, review period and replenishing lead time are both one period which is equal to the length of each demand phase. It could be a situation in which an order arrives when the policy has already changed, and it would be confusing for users to take care of the inventory level. In this problem, using one ordering policy on each location is more practical than using many policies. However, using one ordering policy on each location instead of using many policies leads to higher total cost. It is a trade-off between ease of use and the total cost. Tunc et al. (2011) investigated that using only one policy for each location could reasonably substitute the method of using many policies when demands followed a stable seasonal pattern with high uncertainty, high setup cost and low penalty cost.

4.3 Problem statement

This chapter considers a 2-echelon inventory system having one warehouse and N non-identical retailers with seasonal demand. Each location replenishes inventory in a fixed time interval. Retailers are supplied by the warehouse which is supplied by external suppliers and items can be stored at the warehouse and retailers. Demand that is not satisfied with on-hand inventory are considered as demand loss. This demand loss must not exceed expected service level. In this problem, the service level is fill rate - the proportion of demand served from on-hand inventory (Nahmias, 2009).

Demand is assumed to be seasonal without trend and fluctuating within a cycle. The demand pattern repeats cycle after cycle. Period demand is assumed to be normally distributed. For example, if each cycle consists of 4 periods, average demand of periods 1, 5, 9 and 13 will be normally distributed with the same parameters and so will periods 2, 6, 10 and 14. This results in the same expected total demand in every cycle.

The system operates on periodic review basis using reorder point and order-up-to point or (R, s, S). The system controls inventory with echelon stock basis where each location makes decision based on inventory information of its own and of all locations downstream. This thesis proposes algorithms to find ordering policy based on (R, s, S) system to minimize ordering and holding costs respected to expected service level.

With (R, s, S) policy, location i reviews its inventory position every R_i periods. If its inventory position (on-hand plus on-order inventory) is equal to or lower than its reorder point, s_i, an order will be placed to raise inventory position to be equal to or higher than its order-up-to point, S_i. For the warehouse, as echelon concept is applied, its inventory position is the total inventory position of its own and all retailers.

An example of the system with one warehouse and 2 retailers is shown in Figure 13. Retailer 1 and retailer 2 show their inventory positions and on-hand inventories. Warehouse (Installation Stock) shows its local inventory position and on-hand inventory. Warehouse (Echelon Stock) shows its echelon inventory position and on-hand inventory.

Each location has 1-period lead time and reviews its inventory at the end of every period. Retailers have a 4-period cycle or retailers' average demands repeat every 4 periods. In Figure 13, the slope in each period represents an average demand of that period. With 4-period cycle, the slopes of the 4 periods are different but the slopes of any consecutive 4 periods repeat the same pattern. This is the main difference between seasonal demand and stationary or trend demand as stationary demand is assumed to have the same slope on every period and trend demand is assumed to have either steeper or flatter slopes along the planning horizon.

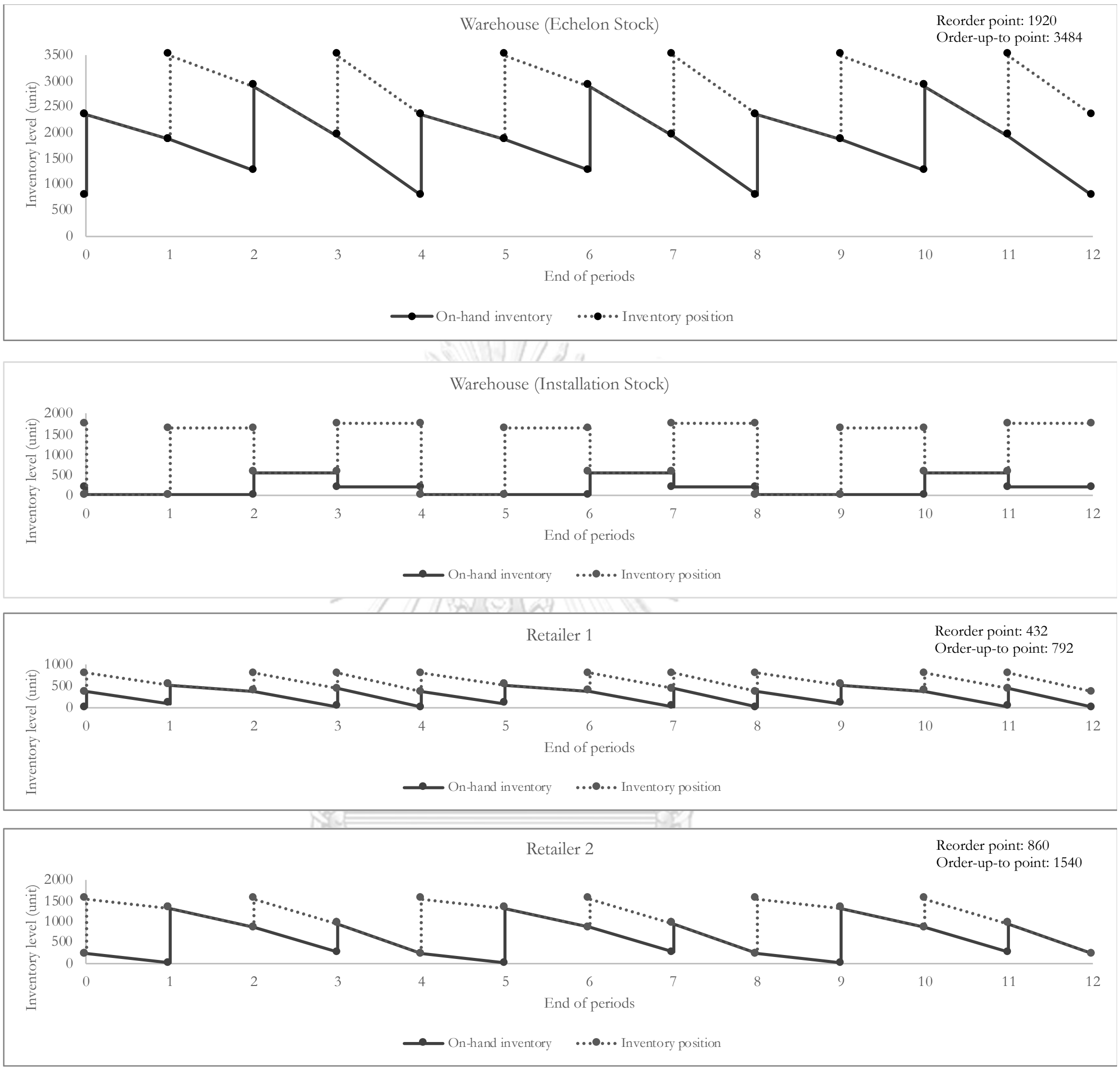

Figure 13 Inventory levels at each location in the system.

Retailers 1 and 2 operate with reorder points of 432 and 860, and order-up-to points of 792 and 1,540, respectively. When their inventory positions are at or below reorder points, orders are placed to raise inventory positions up to order-up-to points and the on-hand levels increase when orders arrive in the next period. Orders from retailers are demand for the warehouse. The warehouse's reorder and order-up-to points are 1,920 and 3,484. Retailer 1 orders at the end of periods 2, 3, and 4 of each cycle and retailer 2 orders at the end of periods 2 and 4 of each cycle. The warehouse orders at the end of periods 1 and 3 of each cycle. With 1-period lead time, an order placed at the end of period j will arrives at the end of period $j+1$ and will be available to use from the beginning of period $j+2$. The total order from all retailers is demand of the warehouse as the warehouse's inventory position or on-hand inventory (installation stock) drops at the end of periods 2, 3, and 4. In warehouse (echelon stock), inventory position and on-hand inventory do not drop when retailers order because items are only moved from the warehouse to retailers, not out of the system.

The system shown in Figure 13 assumes deterministic demand by using average demand of each period in a cycle to determine initial ordering policy. As demand is normally distributed, actual demand can be either higher or lower than the average. Therefore, using average demand to determine ordering policy will have 50% probability to serve actual demand. To raise probability of serving actual demand, the system must have safety stock. Safety stock level at each location is mainly determined by service level and variability of demand.

4.4 Methodology

To solve the problem, we develop heuristic algorithms based on a 2-phase methodology proposed in Chapter 3. A methodology composed of 2 phases like an approach proposed by Bookbinder and Tan (1988). The first phase determines timing of replenishment and number of periods that an order serves based on average demand. The second phase determines safety stock levels based on the initial policies from the first phase to absorb variability of stochastic demand.

4.4.1 Heuristics to determine policy for average demand

An algorithm in this phase is used to determine when to order and the number of periods which an order quantity serves their demand. Policies are determined based on an average demand of each period.

4.4.1.1 Heuristics to find potential policies for retailers

In this system, customers' demand occur only at retailers and demand at a warehouse is the total order placed by retailers. Consequently, an ordering policy of the warehouse depends on ordering policies of retailers. The algorithm finds all potential policies for retailers and use combinations of retailers' policies to find the optimal policy for the warehouse and retailers.

Since determining optimal (R, s, S) is very difficult, an approximation relationship which is widely used to determine order quantity, Q, reorder point, s, and, order-up-to point, S, is that S is equal to $s + Q$ (Silver, Pyke, and Peterson, 1998; Nahmias, 2009). The order quantity, Q, determines timing of replenishment. The smaller Q requires more frequent replenishment while the bigger Q requires less frequent replenishment. With average demand 200 units per period and Q of 600 units, an order would be placed every 3 periods. However, if average demands of periods are different, such as, in a cycle of 4 periods are 100, 200, 300, and 200, an order of 600 can serve a group of periods 1, 2, and 3 but it cannot serve a group of periods 2, 3, and 4. Furthermore, if a location has on-hand of 800 at the beginning of period 1, the on-hand level at the end of each period will be 700, 500, 200, and 0. If it has the on-hand of 800 at the beginning of period 3, the on-hand levels at the next 4 periods will be 500, 300, 200, and 0. Both cases result in zero on-hand at the end of the fourth period but the total on-hand levels of them are 1,400 and 1,000, respectively. This means that the first case costs more holding cost. Therefore, choosing the period on which an order is placed can reduce the total cost. Therefore, with seasonal demand, proper timing of replenishment must be determined. Different replenishment periods result in different outcomes.

The proposed algorithm starts by determining period(s) in a cycle to place order(s) and calculating Q. Q is determined by the number of periods that each order should cover or the span of periods, sp. Since average demand of each period within a cycle is different, same sp may come up with different Q. Therefore, the period that an order starts covering demand, ap, is used to determine different ordering policy. Then, with sp and ap, order-up-to point can be calculated based upon equation (50).

$$OrderUpTo_i = \sum_{j=ap}^{j=ap+leadtime_i+sp-1} Demand_{ij} \qquad (50)$$

The order-up-to point for location i, $OrderUpTo_i$, is the summation of demand during $sp + leadtime_i$ starting from period ap so, after serving demand sp periods, it has enough on-hand inventory until the next order arrives. Therefore, reorder point, $Reorder_i$, is the inventory level that the location should place an order or the inventory level after it serves demand in sp periods as in equation (51). We generate different reorder points by choosing different starting points to calculate sp-period demand. The starting point is referred as sh.

$$Reorder_i = OrderUpTo_i - \sum_{j=ap+sh}^{j=ap+sh+sp-1} Demand_{ij} \qquad (51)$$

Then, an initial on-hand inventory, I_{i0}, and an initial on-order inventory, O_{i0}, are determined. For the first iteration, O_{i0} is set to be zero and I_{i0} is a sufficient amount of inventory to cover demand until the first order arrives. Equation (52) is used to calculate an initial on-hand inventory.

$$I_{i0} = Reorder_i + \sum_{j=1}^{j=ap-leadtime_i} Demand_{ij} \quad (52)$$

The algorithm varies sp, ap and sh to find the potential policies. Parameters ap and sh vary from 1 to the number of periods in a cycle and parameter sp can vary in any number of periods. Therefore, there should be a method to limit search range for sp.

As the average demand of each cycle is stationary, economic order quantity, EOQ could be used to find preliminary order quantity. The EOQ of any location can be calculated as $\sqrt{\frac{2KD}{h}}$ where K is ordering cost, D is cycle demand and h is holding cost per unit per cycle. It is proved that, in stationary demand, the farther Q is away from EOQ, the higher total cost occurs (Nahmias, 2009). Therefore, even though, our problem has seasonal demand pattern, we would like to search sp within a range close to EOQ.

From retailer's point of view, there are 2 ways to consider the warehouse's ordering cost. 1) The warehouse orders items to serve any specific retailer only. In this case, the warehouse's ordering cost should be considered as a part of retailer's ordering cost. 2) The warehouse serves any retailer with its on-hand inventory. In this case, there is no cost from warehouse added to retailer's ordering cost. Hence, there would be two EOQ values for each retailer - i.e., one that uses only retailer ordering cost, $EOQlower_i$, and one that uses sum of warehouse and retailer ordering cost, $EOQupper_i$.

With seasonal demand, the policy with Q equals to fraction of cycle demand would give higher cost although it is closer to EOQ. We observed that sp which are multiple of cycles can always give policies that on-hand level reaches zero when an order arrives. On the other hand, sp equal to fraction of cycle might give policies where there is on-hand inventory left on order's arrival. Figure 14 shows an example of 4-period cycle demand. Two systems operate with different ordering policies while they have the same parameters. Their average demand is 300 units per period, ordering cost is \$1,350 and holding cost is 1 \$ per unit per period. Therefore, the EOQ is 900 units. The system A is expected to order every 4 periods starting on period 1 of a cycle while the system B is expected to order every 3 periods starting on period 1. System A uses up all its inventory when next order arrives as in periods 2, 6, and 10. On the other hand, system B has redundant inventory in some periods such as periods 2 and 10. In a horizon of 12 periods, the system B has higher total on-hand inventory and also orders more frequently than system A which leads to higher total holding cost and higher total ordering cost. Noted that system B has Q closer to EOQ than system A.

System A

Reorder	144													
Order-up	1344													
Period	0	1	2	3	4	5	6	7	8	9	10	11	12	Total
Demand		264	144	360	432	264	144	360	432	264	144	360	432	
on-hand	408	144	1200	840	408	144	1200	840	408	144	1200	840	408	5184
order		1200	0	0	0	1200	0	0	0	1200	0	0	0	

System B

Reorder	504													
Order-up	1200													
Period	0	1	2	3	4	5	6	7	8	9	10	11	12	Total
Demand		264	144	360	432	264	144	360	432	264	144	360	432	
on-hand	768	504	1056	696	264	936	792	432	768	504	1056	696	264	5448
order		696	0	0	936	0	0	768	0	696	0	0	936	

Figure 14 Example for search range.

Therefore, Q should range within the numbers of cycles close to $EOQlower_i$ and $EOQupper_i$, for example, at any location, if the cycle demand is 1,000 and

$EOQlower_i$ is 2,100 and $EOQupper_i$ is 3,200, the sp is between 2 and 4 cycles. The range for sp is determined as equations (53) and (54).

$$lowerRange = min\left\{\left\lfloor\frac{EOQlower_i}{cycleDemand}\right\rfloor \times cyclePeriod, \forall i \in I\right\} \quad (53)$$

$$upperRange = max\left\{\left\lceil\frac{EOQupper_i}{cycleDemand}\right\rceil \times cyclePeriod, \forall i \in I\right\} \quad (54)$$

The algorithm to determine ordering policy for each retailer can be summarized as shown in Figure 15.

procedure $FindLocationPolicy(lowerRange, upperRange)$
F_i: a set of potential policies of location i
P_i: a potential policy of location i
$cyclePeriod$: the number of periods per cycle
$lowerRange$: lower search range
$upperRange$: upper search range
for sp in $[lowerRange..upperRange]$ **do**
    for ap in $[1..cyclePeriod]$ **do**
        for sh in $[1..cyclePeriod]$ **do**
            Compute $OrderUpTo_i$, $Reorder_i$, I_{i0}, and O_{i0}
            Compute demand loss for period j, $lost_{ij}$
            if $\max\{lost_{ij}\} = 0$ **do**
                Reassign I_{i0} and O_{i0}
                Compute policy total cost, $Cost_i$
                $P_i \leftarrow \{OrderUpTo_i, Reorder_i, I_{i0}, O_{i0}, Cost_i\}$
                Collect policy $F_i \leftarrow F_i \cup \{P_i\}$
return F_i

Figure 15 The algorithm to find potential policies of each location.

The algorithm will find all potential policies for each retailer by changing sp, ap and sh. If the policy gives no demand loss, which means it is an acceptable policy, the algorithm will reassign I_{i0} and O_{i0} to be on-hand and on-order levels at the end of a cycle after sp to make ordering pattern repeat. Then, the policy will be collected as one of potential solutions.

4.4.1.2 Heuristics to find potential policies for the warehouse

From the previous section, potential ordering policies for each retailer are found. Since warehouse demand comes from retailers' orders, warehouse policy must be obtained based upon combinations of retailer policies. Due to a large number of retailer policy combinations, a genetic algorithm (GA) is used to find the best set of warehouse and retailer policies.

A GA starts by ranking policies of each retailer ascending by the total retailer cost, breaks tie arbitrarily. Then, a set of chromosomes representing combinations of retailer policies are created. A chromosome consists of N members where N is a number of retailers. Each member, r_i, represents the policy of retailer i. For example, a chromosome of 2 retailers, $r_1|r_2 = 10|15$, represents the combination of the 10th and 15th policies of retailers 1 and 2, respectively. Chromosomes are randomly generated and added to a set called chromosome pool until size of chromosome pool is equal to population size, P. In our experiment, P is 200.

With a set of P chromosomes, two genetic operators, crossover and mutation, are used. Crossover operator creates new chromosomes by randomly picking up two chromosomes as parents and swapping members between parents to create two new chromosomes as offspring. We use k-point crossover where k or the number of crossover sites can be any number between 1 to $N-1$ (Sastry, Goldberg, and

Kendall, 2005). For example, two parents, $1|3$ and $12|15$, will give two offspring, $1|15$ and $12|3$. When a chromosome has more than 2 members, the algorithm randomly selects k and crossover sites. For example, two parents, $1|3|7$ and $12|15|11$, might give offspring as $1|3|11$ and $12|15|7$ or $1|15|7$ and $12|3|11$. In our experiment, crossover probability is 0.7 which means parents chromosomes are randomly selected from the chromosome pool and give offspring until the number of offspring is equal to $0.7P$. Another operator, mutation, creates chromosomes by randomly assigning a new value to a randomly selected chromosome's member. For example, a second member of a chromosome, $10|15$, is randomly selected and it is assigned a new value as $10|23$. In our experiment, the mutation probability is 0.2 and only one member in any chromosome can be mutated in each iteration. Therefore, the number of new chromosomes from mutation operator is $0.2P$. After crossover and mutation, the chromosome pool is an enlarged set with new chromosomes added from these operators and its size will become, P' which is $1.9P$ in our experiment.

The policy for the warehouse is determined based on chromosomes of retailer policies. Each solution evaluated by its total cost (including warehouse cost). Each chromosome p will be given its value, $DiffCost_p$, as its difference between its total cost and the maximum cost among chromosomes. Therefore, the probability to be selected for the next iteration of each chromosome is calculated as equation (55). A solution with lower total cost will have higher probability of selection.

$$Prob_p = \frac{DiffCost_p}{\sum_{p=1}^{p=P'} DiffCost_p} \tag{55}$$

Then, P chromosomes will be selected for the next iteration. The best solution is always included in the next iteration while another $P-1$ solutions are selected based upon their probabilities.

The algorithm continues until there is no improvement for C generations where C is called stopping criteria. In our experiment, C is set to be 20.

A GA is used to find proper combination of retailer policies. To evaluate each combination, we need to find warehouse policy based on that combination and calculate total inventory cost (warehouse + retailer). With echelon stock concept a methodology to generate warehouse policy is slightly different from retailer policy.

With each combination, information from each retailer is used to determine policies for the warehouse. Range of warehouse's sp are the minimum and maximum sp of retailers. Other parameters can be calculated as equation (56) – (59).

$$OrderUpTo_0 = \sum_{i \in I} OnHand_{i,ap-leadtime_0-1} + \sum_{i \in I} \sum_{j=ap-leadtime_0-leadtime_i}^{j=ap+sp-1} Order_{ij} \qquad (56)$$

$$Reorder_0 = OrderUpTo_0 - \sum_{j=ap-leadtime_0+sh}^{j=ap+sh+sp-leadtime_0-1} \sum_{i \in I} demand_{ij} \qquad (57)$$

$$I_{0,0} = \sum_{j=1}^{j=ap-1} \sum_{i \in I} Order_{ij} \qquad (58)$$

$$O_{0,0} = 0 \qquad (59)$$

With echelon stock concept, when the warehouse orders, it has to consider retailers' inventory position which is the on-hand and on-order levels at any given period. $OrderUpTo_0$ is the summation of retailers' inventory position at the warehouse's ordering period, $ap - leadtime_0 - 1$, and the total demand from retailers within a span of periods, $sp + leadtime_0$, starting from the ordering period. The retailers' inventory position level is $\sum_{i \in I} OnHand_{i,ap-leadtime_0-1} +$

$\sum_{i \in I} \sum_{j=ap-leadtime_0-leadtime_i}^{j=ap-leadtime_0-1} Order_{ij}$ and the total demand from retailers is $\sum_{j=ap-leadtime_0}^{j=ap+sp-1} \sum_{i \in I} Order_{ij}$ which lead to the summation in equation (56). $Reorder_0$ is the inventory level that the warehouse should place an order or the level after the system serves customer’s demand in sp periods. Other potential $Reorder_0$ are also calculated by changing group of periods with sh. $\sum_{j=ap-leadtime_0+sh}^{j=ap+sh+sp-leadtime_0-1} \sum_{i \in I} demand_{ij}$ is considered to determine $Reorder_0$ because it is the number of items flow out of the system and decreases the echelon inventory position. Initial $I_{0,0}$ is a sufficient amount of inventory to cover demand until the first order arrives and initial $O_{0,0}$ is zero.

Like retailers, if the policy gives no demand loss, the algorithm will reassign values to $I_{0,0}$ and $O_{0,0}$ and collect the policy.

4.4.2 Heuristics to determine safety stock levels

Since the solutions found in the first phase are based on average demand, they may not achieve expected service level under demand which is normally distributed. To absorb demand’s variability, in this phase, safety stock levels are determined for all locations.

Instances of demand with 400-period horizon are generated from the normal distribution and solved to find safety stock level by using an ordering policy from the previous phase as input parameters - $Reorder_i$, $OrderUpTo_i$, I_{i0} and O_{i0}. Once safety stock is found, the final on-hand inventory at period 0, reorder points and order-up-to points are these values from the initial policy plus safety stock.

To find safety stock of the system; first, the algorithm finds safety stock level for each retailer independently. Then, the safety stock level of the warehouse is determined based on retailers' safety stock levels. The algorithm can be described as follows.

The algorithm begins with determining search range. With expected service level, ESL, the z value respected to ESL is determined. The algorithm sets the search range for safety stock levels from $low = 0$ to $high = \lceil z \times SD \rceil$ where SD is the standard deviation of demand. For the warehouse, SD is the standard deviation of the total orders from retailers. Then the safety stock level is set to be $high$ and its actual service level is evaluated using simulation based on actual demand. If actual service level for $high$, ASL_{high}, is lower than expected service level, ESL, the search range will change to $low = high$, $z = z + 1$ and $high = \lceil z \times SD \rceil$ and ASL_{high} is re-calculated. This process will repeat until ASL_{high} is greater than ESL. Then, search range is between low and $high$. After determining search range, mid is set to be $\left\lceil \frac{low+high}{2} \right\rceil$. Then, the algorithm starts a binary search.

A binary search is then applied to find optimal safety stock. If $ASL_{mid} \geq ESL$, search range will change to $high = mid$ and $mid = \left\lceil \frac{low+high}{2} \right\rceil$. Otherwise, if $ASL_{mid} < ESL$, search range will change to $low = mid$ and $mid = \left\lceil \frac{low+high}{2} \right\rceil$. The algorithm iterates as long as $high > low + 1$. Then, the safety stock for the location is $high$. Consequently, with $high$ as the safety stock, the final policy for the location can be calculated.

The algorithm finds the safety stock level for each location. For the warehouse, demand is total order from retailers in each period. Safety stock of the warehouse can serve all demand within training instances.

4.5 Experimental Results and Discussion

In this section, the proposed algorithms are compared with MIP models on their solution quality and computational time. The MIP models are solved with CPLEX Studio 12.6 and the proposed algorithms ran on Python. All experiments ran on a computer with 2.00 GHz Intel Core i7 processor and 4 GB of RAM. Then, the proper lengths of planning horizon to determine safety stock levels are investigated.

4.5.1 Comparison of the proposed algorithms and the MIP models

Recall that the problem is divided into 2 phases which are solved independently. The heuristic solutions in phase 1 are compared with the optimal solutions from the MIP model for phase 1 and the heuristic solutions in phase 2 are compared with those optimal solutions from the MIP model of the second phase.

4.5.1.1 Comparison on the first phase

In the first phase, 9 instances are used to test algorithm's performance: 3 instances with 2 retailers, 3 instances with 3 retailers and the other 3 instances with 5 retailers. These 3 instances for each size have the same demand pattern and holding cost but different ordering cost i.e. high, low and zero as shown in table 8. All retailers and warehouse have the same ordering cost. High, low and zero values are determined by relationship between retailers' EOQ and retailer's cycle demand. High value is an ordering cost of which EOQ is equal to the cycle demand, low value is a cost leading to EOQ equal to a quarter of cycle demand and zero is the one with zero ordering cost.

Table 8 Summary of instance parameters.

		Average Demand				Ordering Cost		
Size	**Retailer**	**Period 1**	**Period 2**	**Period 3**	**Period 4**	**High**	**Low**	**Zero**
2 Retailer	1	880	480	1,200	1,440	12,000	750	0
	2	880	1,840	2,400	2,880			
3 Retailer	1	264	144	360	432	3,200	200	0
	2	176	368	480	576			
	3	800	100	900	200			
5 Retailer	1	80	160	60	100	800	50	0
	2	160	60	80	100			
	3	120	80	100	100			
	4	88	48	120	144			
	5	48	120	144	88			

The algorithm can find the optimal solution for every instance. With 2 and 3 retailers, the algorithm spends slightly longer time to find the optimal solutions but, with 5 retailers, the algorithm's computational time is 97.49% on average less than the MIP model's as shown in Figure 16.

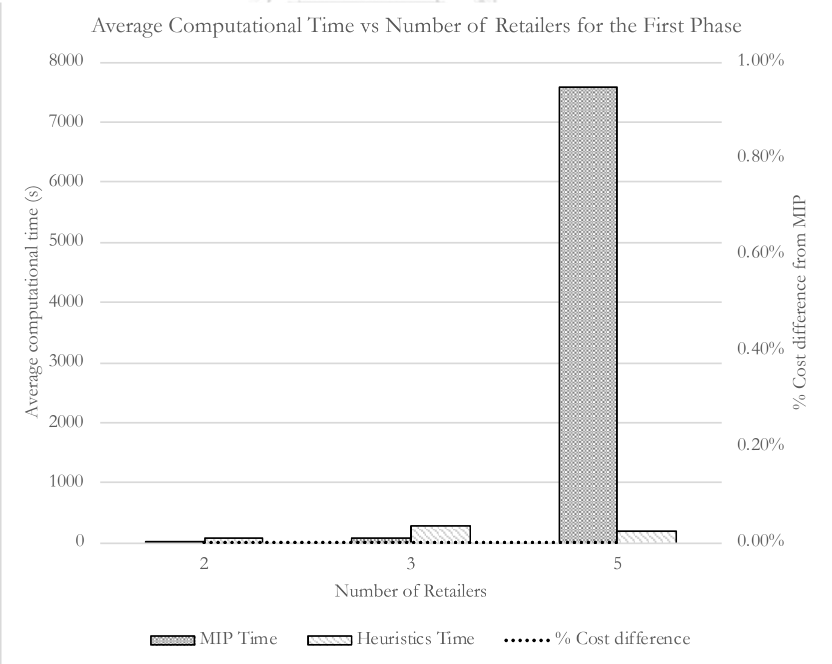

Figure 16 Comparisons of average computational time and cost difference of MIP model and heuristics.

4.5.1.2 Comparison on the second phase

In the second phase, to evaluate performance of the algorithm for safety stock, 3 instances having 2 retailers from the first phase are used to compare the results. The instances for 3 and 5 retailers cannot used in this phase because the MIP model cannot find the optimal solutions within 4 hours. Actual demand of each instance is generated based on 3 different values of standard deviation i.e. 10%, 20% and 25% of average demand. Therefore, with 3 instances from the first phase and 3 standard deviations, 9 instances are used in the second phase.

Demand of 400 periods are generated for each instance to find safety stock. Every instance is solved with 95% service level. The proposed algorithm can find the optimal solutions for all instances. The computational time for the second phase of the MIP model is between 77.00 s to 242.22 s while the computational time of the proposed algorithm is between 5.01 s to 5.70 s which is 95.99% less on average. The computational time and cost difference are shown in Figure 17.

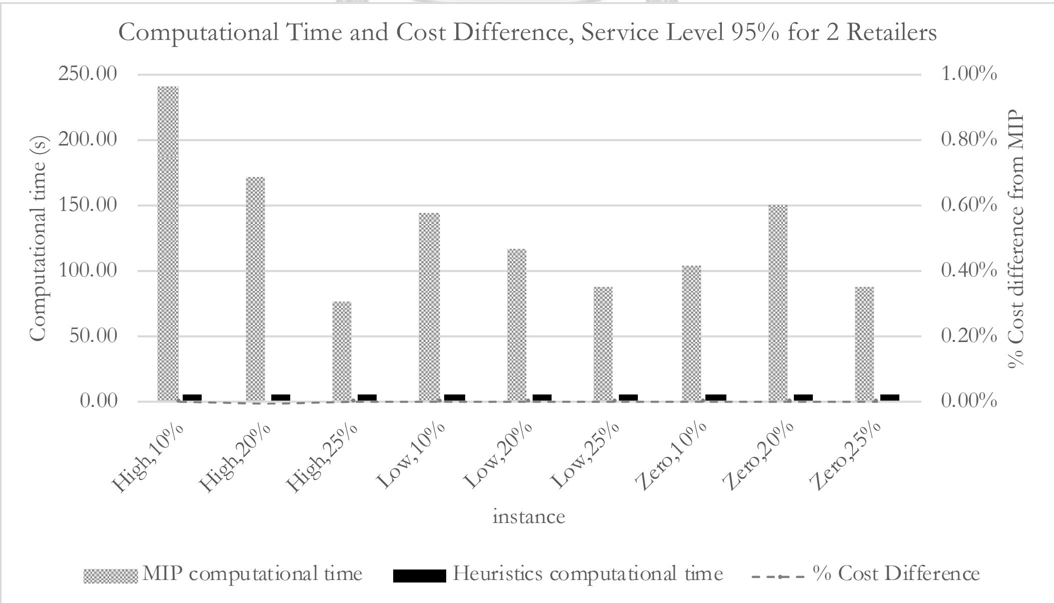

Figure 17 Comparisons of computational time and cost difference of MIP model and heuristics for the second phase.

4.5.2 Length of planning horizon to determine safety stock

After the final policy is obtained, another question is how it performs on other instances or how robust it is. To test the robustness of the solution, each final policy is applied to another 10,000-period instance to investigate demand loss compared to expected service level. The final policies can provide less than 1% average demand loss in 10,000 periods. On average, 32.78 out of 10,000 periods or 0.33% have service level lower than 95%. Therefore, the final policy from the second phase is efficient enough for the real-life situation.

However, if we want to improve the number of periods with unsatisfied expected service level (NUS), longer horizon must be used in the second phase of algorithm. In the experiment, different lengths of horizon, which are 1,000, 10,000, 20,000, 30,000, 40,000 and 50,000 periods, are generated to determine safety stock. Then, the final policy is applied to instances with 10,000-period horizon to investigate the NUS. It is shown in Figure 18 that the longer horizon used, the smaller NUS becomes. However, the safety stock level increases as the NUS decreases. Hence, there is a trade-off between the NUS and the holding cost due to safety stock level. Noted that the computational time increases along with the length of horizon but a 50,000-period instance can be solved within 20 minutes.

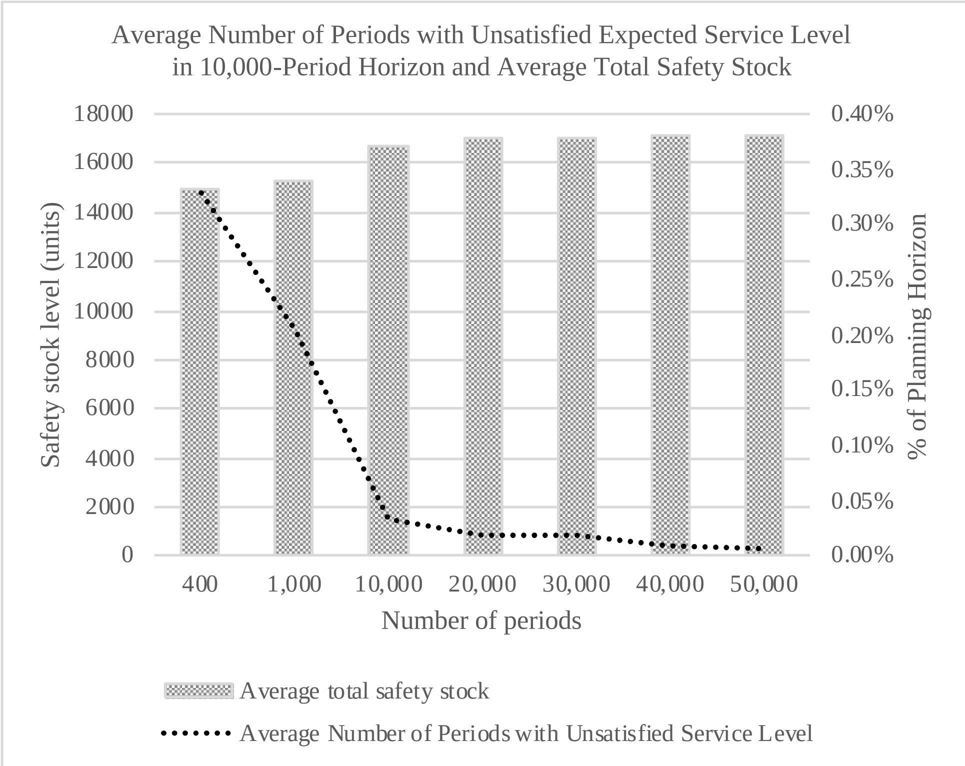

Figure 18 Average number of periods with unsatisfied expected service level (NUS) and average total safety stock with different training planning horizon lengths.

4.6 Conclusion

A 2-phase algorithm to determine ordering policy for a 2-echelon inventory system is proposed. The first phase is used to determine initial ordering policy for each location. The second phase is used to determine safety stock. The algorithm can find the solutions which are as good as the optimal solutions from the MIP models. With 2 and 3 retailers, the first-phase algorithm requires computational time slightly higher than the MIP model but, with 5 retailers, the algorithm requires much smaller amount of time. With 2 retailers, the second-phase algorithm requires 95.99% less time on average. The final policies are applied to the problems with longer horizon. The results show that the policies are robust since they give acceptable average demand loss. However, the longer horizon should be used in the second phase to determine service level if the lower number of periods with unsatisfied expected service level is required. Noted that longer horizon requires longer computational time and leads to higher safety stock.

This chapter considers only one replenishing mode which is from upstream warehouse. However, in reality, other replenishing modes can be explored such as emergency replenishing from the warehouse or transshipment between retailers. Another way to extend the algorithm is considering a batch-size constraint. The batch size could even be different for the warehouse and retailers. This constraint would be a useful research extension in real-life situation.

CHAPTER V

MULTIPLE REPLENISHING MODES FOR PERIODIC-REVIEW IN A 2-ECHELON INVENTORY PROBLEM WITH SEASONAL DEMAND

5.1. Introduction

An inventory system with one-warehouse and multi-retailer is a complex system which receives attention from many researchers. The system is involved with a lot of decisions such as how many items should be stored at each location and at what time they should be transported from the warehouse to retailers. Determining ordering policy for such system is a very difficult task. Even though proper inventory policy is applied. Under stochastic demand, a retailer could run out of stock and the demand will be considered lost. In this situation, a special replenishing mode with shorter lead time may be used to prevent stockouts. These special modes can be an emergency replenishing mode which is a replenishing mode with shorter lead time supplied by the warehouse (Minner, 2003) or a transshipment which is a mode where items are requested from other retailers with excessive on-hand stock (Paterson et al., 2011).

This chapter focuses on a 2-echelon inventory system with a single warehouse and N non-identical retailers under seasonal demand. Generally, retailers are supplied by a warehouse which is supplied by external suppliers and items can be stored at the warehouse and retailers. This regular replenishment has a fixed lead time. When a retailer faces the risk of stockout, items could be transported via one of special modes with fixed shorter lead times i.e. emergency replenishment from warehouse and transshipment from another retailer. Unfilled demand is considered as demand loss.

Demand is assumed to be seasonal without trend. It fluctuates within a certain span of periods called cycle and its pattern repeats cycle after cycle. The system considered in this thesis has seasonal demand driven by human factor and the demand fluctuates within a cycle of one week. Under this seasonal demand, the ordering policies for the regular replenishment in this chapter is obtained from an algorithm proposed in Chapter 4. However, although the policies from the algorithm give low total demand loss in the test problem, there are some periods with unsatisfied service level. Therefore, to reduce the number of these periods, special replenishing modes are applied. For ease of use, the ordering policy for both special modes could be static policy which applies one policy on every period. However, since demand is seasonal, dynamic policy which applies a different policy for each period in the demand cycle is introduced. Since the dynamic policy varies policies depending on demand on each period, it tends to give better result such as lower number of special orders and also lower ordering cost. Therefore, there is a trade-off between static and dynamic policies. Then, we investigate the differences between using static and dynamic policies for special modes and find situations which are appropriate for static and dynamic policies.

The remainder of this chapter is organized as follows. Section 5.2 reviews the literature related to the system with multiple replenishing modes. Section 5.3 presents a problem description. Section 5.4 describes a methodology to determine ordering policies for special modes. Section 5.5 presents results and discussions. Finally, Section 5.6 concludes and suggests future research extensions.

5.2. Literature Review

To reduce stock out problem, a company could have more than one replenishing channels. Besides regular mode, other replenishing modes with shorter lead time could be applied. For example, a company, which normally use seafreight, can also use airfreight as an emergency mode with shorter lead time but more expensive.

Emergency replenishing mode is a special replenishment with shorter lead time, but higher cost used in case of imminent shortage from the higher-echelon location (Tagaras and Vlachos, 2001). Generally, a system with more than one source is considered as dual supplies where items are replenished by two sources or one source with 2 modes (Minner, 2003 and Yao and Minner, 2017). The dual-supply problem is studied in various aspects. Two main policies: a continuous review and a periodic review are applied to this problem. Moinzadeh and Nahmias (1988) developed a heuristic algorithm for a system with 2 supply modes under continuous review applying (Q1, Q2, R1, R2) policy where an order of Q1 is placed when on-hand reached R1 reorder point and an order of Q2 with shorter lead time is placed when on-hand reached R2 reorder point. They used a simulation to validate the heuristics and the difference in operation cost between system with and without a special supply mode was studied. Zhou and Yang (2016) proposed heuristics to find policy for 2 replenishing modes under continuous review where both modes must order in batches. For a group of periodic review, various aspects of constraints such as time to place emergency orders or size of orders were studied. Chiang and Gutierrez (1996) proposed a model with 2 replenishing modes under periodic review where, at each review period, either a regular order or an emergency order was placed to raise the inventory position to an expected level. Chiang (2003) extended the model with different variable costs between a regular mode and an emergency mode. Chand, Li and Xu (2016) proposed model similar to Chiang (2003) but they allowed the buyer to choose between two delivery modes at the beginning of the period. They assumed that the unmet demand was backordered and charged a backlogging cost varying with the length of backlogging time. Therefore, the buyer must trade off delivery cost and backlogging cost. Chiang and Gutierrez (1998) allowed multiple emergency orders within a review period. Regular orders and emergency orders are placed periodically but emergency orders have smaller review interval. Chiang (2001) analyzed a special case of the same problem with one-period difference between lead times of a regular mode and an emergency mode. Bylka (2005) proposed a model similar to Chiang and Gutierrez (1998) and the model was extended with an inventory capacity constraint and a limited backlogging constraint. Tagaras and Vlachos (2001) proposed a model for emergency mode where an emergency order would be ordered as late as possible to make the items arrive right before the end of the period. The emergency order is placed to raise on-hand level up to the threshold level. When the on-hand level is less than the threshold level, an emergency order is placed to raise on-hand up to the threshold level and no emergency order is placed otherwise. Huang, Zeng and Xu (2018) proposed a system where regular and emergency orders were supplied by the capacitated suppliers. Regular orders were triggered before the demand is realized but emergency orders were triggered after demand realization. The quantity of emergency order depended on remaining capacity of suppliers. Johansen and Thorstenson (2014) proposed a Markov decision model for a system where regular orders were controlled with reorder point and fixed order quantity and emergency orders were controlled with reorder and order-up-to points. Both regular and emergency orders have constant lead time. Then Johansen (2018) extended the model by assuming stochastic lead time for regular orders.

All those papers studying inventory systems only considered the systems as an arborescent distribution system. (An arborescent system is a tree-like system which each location obtains items only from only one higher location.) However, in this chapter, lateral transshipment is also considered. Lateral transshipments relaxed a system to be more flexible and also more complicated. To allow lateral transshipments, locations of the same level have to pool their inventories (Paterson et al., 2011). There are 2 types of pool policies which are complete pooling and partial pooling. With complete pooling, items can always be transshipped with no condition. On the other hand, with partial pooling, items are reserved for local future demand and will be transshipped when they are excessive items. Another classification of transshipment orders is when they take place. If transshipments take place before demand is observed as predetermined events used to redistribute inventories, they are proactive transshipments. If transshipments take place to respond to stockouts or potential stockouts, they are reactive transshipments. The studies of transshipment orders have both single echelon and multi-echelon structures. Robinson (1990) developed a heuristic technique for multi-location, multi-period problems with transshipments. Optimal ordering policies were determined under two special cases: two non-identical locations and any number of identical locations. Olsson (2015) studied a single echelon, 2 identical locations with positive transshipment lead times. Ordering policy was developed with a heuristic algorithm which separated the whole system into 2 sub-systems, each with one retailer. The positive lead time was treated by keeping track of residual lead time to decide whether to wait for oncoming regular order or request transshipment. Tlili, Moalla, and Campagne (2012) studied 2-echelon system and 2 identical retailers with transshipments. Demand was independent identical normal distribution. They developed initial solution with heuristics based on simulation optimization and, then, used simulation to fine tune to the optimal solution. Tai and Ching (2014) also studied 2-echelon with a number of identical retailers. Ordering policy was developed by using a Markovian model.

In this chapter we studied 2 special modes. The emergency mode places orders in a similar way to Gutierrez (1998) and Chiang (2001) where the inventory position is reviewed periodically, and there can be both regular and emergency orders in each period. An emergency order has smaller review interval. Then, if the warehouse cannot fulfill an emergency order, the retailer will request transshipment from another retailer. The transshipment applies partial pooling concept where items will be transshipped only when they are excessive items.

5.3. Problem Statement

This thesis considers a 2-echelon inventory system having one warehouse and N non-identical retailers with seasonal demand. Demand in each period is normally distributed with different means and standard deviations. Each location replenishes inventory in a fixed lead time. Normally, retailers are supplied by the warehouse which is supplied by external suppliers and items can be stored at the warehouse and retailers. Demand that is not satisfied with on-hand inventory are considered as demand loss. In this problem, the service level is fill rate - the proportion of demand served from on-hand inventory (Nahmias, 2009). Therefore, in each period, the ratio of demand served from on-hand inventory to period's demand must not be lower than expected service level.

Despite safety stock level, with uncertain demand, there could be some periods which satisfied demand is below the expected service level. To reduce these periods with unsatisfied service level, an emergency replenishing mode is applied. With this special mode, a retailer can order items from the warehouse with shorter fixed lead time but higher ordering cost than regular replenishing mode. However, since the system is a 2-echelon system and the warehouse has limited items, the warehouse might not be able to fulfill some emergency orders due to its on-hand items. When the warehouse cannot serve an emergency order, there should be another special mode to help retailer prevent shortage. A transshipment mode is another mode where a retailer requests items from another retailer on the same echelon which has excessive on-hand items. A transshipment has shorter fixed lead time than an emergency mode because the warehouse locates far from customers and retailers. However, since the transshipment disturbs a retailer which gives items to the location facing stockouts, the system prefers an emergency mode to a transshipment and assume that transshipment has higher ordering cost. Hence, when a retailer faces stockouts it will consider requesting emergency order, then, if the warehouse cannot serve the order, a transshipment will be considered. Therefore, two special replenishing modes respond to situations where one of the retailers faces the risk of stockouts. One is an emergency mode from the warehouse and the other is a transshipment from another retailer with sufficient stock on hand. All replenishing modes operate on periodic review basis using reorder point and order-up-to point or (R, s, S). The special modes have shorter review interval and lead time than the regular mode.

When there are 3 replenishing modes, an example for the inventory movement in the system can be shown as Figure 19. In Figure 19, there are one warehouse and 2 retailers. To make it easy, every location has 1-period lead time and 1-period review interval for a regular replenishing mode and zero lead time and continuous review for other special modes. With 1-period review interval, a regular mode reviews location's inventory position at the end of every period. If an inventory position reaches a reorder point during any period, an order will be placed at the end of that period. On the other hand, with continuous review, whenever an inventory position reaches a reorder point, an order is immediately placed. The reorder point and order-up-to point for a regular mode of each location is shown in Figure 19.

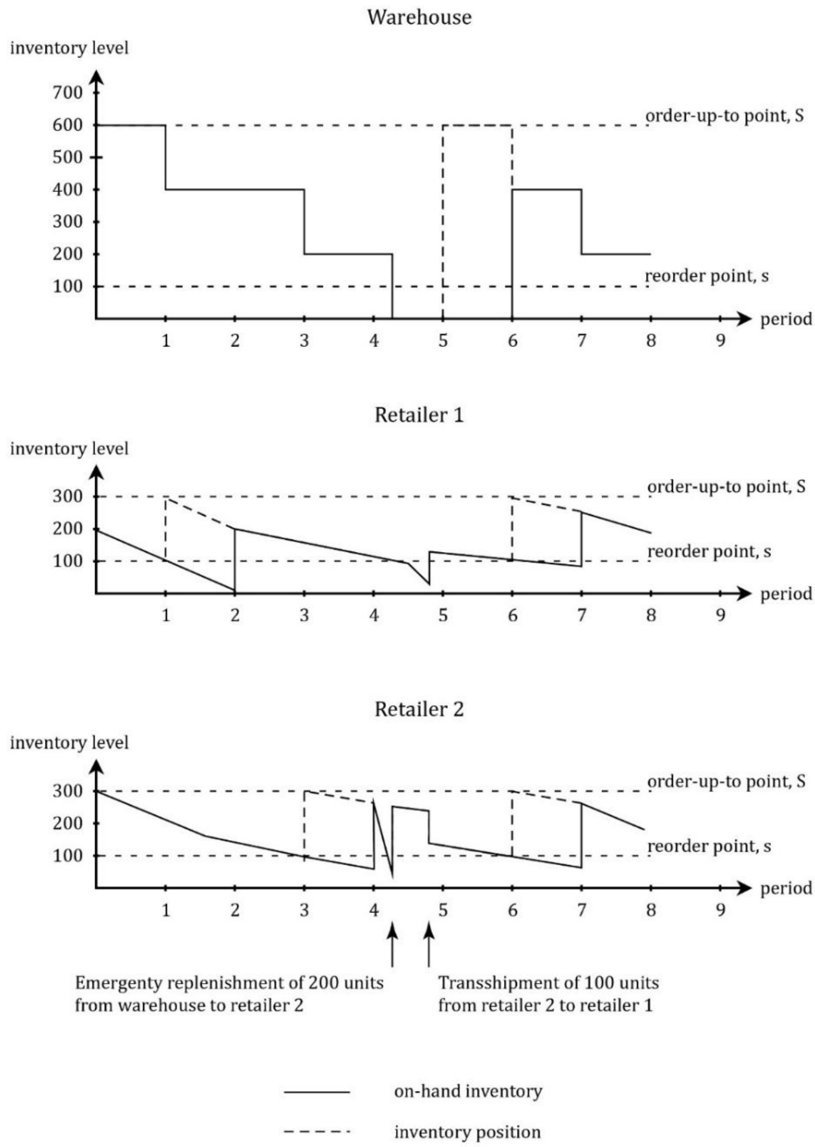

Figure 19 Inventory movement in a system with 3 replenishing modes.

An order is placed as the inventory level reaches reorder point. At the beginning of period 1, retailer 1 reaches the reorder point and places an order of 200 units. The inventory position immediately rises to 300 units and the inventory level at the warehouse drops from 600 to 400 units. Afterwards, the order arrives at the beginning of period 2.

In period 4, demand spike consumes items stored at retailer 2. The retailer is at risk of stockouts, so it requests an emergency replenishing from the warehouse. The order depletes the inventory at the warehouse. Therefore, in the same period, when retailer 1 requests an emergency order afterward, the warehouse cannot satisfy an order. Consequently, retailer 1 has to request a transshipment order from retailer 2.

5.4. Methodology

The policy of the regular mode is determined with the methodology proposed in Chapter 4. Initial reorder point and order-up-to point for each location are determined with heuristic algorithm and safety stock levels are determined with simulation on training instances to find the final policy which satisfies the expected service level. However, under stochastic demand, a retailer could run out of stock which leads to demand loss. Then, special replenishing modes with shorter lead time would be used to prevent shortage.

Since the demand is seasonal, a concept that is widely used is to apply different policies on different periods of demand (Tunc et al., 2011). However, applying multiple policies would be a difficult task, especially when the demand changes frequently. A trade-off between ease of use and a lower cost is taken into account. Therefore, two types of ordering policies for special modes are determined i.e. static and dynamic policies. A static policy is a policy which every period applies the same policy. A dynamic policy is a policy which each period in a cycle has its own policy. Both dynamic and static policies for special modes operate based on (*R, s, S*) policy in the same way as the regular mode. When an inventory position reaches a reorder point during any period, an order is placed to raise an inventory position to an order-up-to point of that period.

Special modes have shorter lead time and review interval than a regular mode. To consider special modes, each period is divided into a group of sub-period. Demand in each sub-period is assumed to be fraction of period's demand and the demand of every sub-period in the same period is distributed with the same mean and standard deviation. Each special mode reviews inventory position at the end of every sub-period. If an order is placed at the end of sub-period *j*, items will be delivered at the end of sub-period *j+lead time*. Therefore, items which arrive at the end sub-period *j+lead time* can serve customer demand in that sub-period. This is different from the regular mode where an order arriving at the end of period *j* will be available to serve customer from the next period. As special orders are supposed to prevent stockouts on any period, any special order which will arrive on next period cannot be placed.

First, an order-up-to point for an emergency mode on each period is determined as follow. As it is found in Chapter 3 that, under seasonal demand, the same size of order could lead to different holding cost when it was ordered in a different period. If an emergency order is too large, it will delay a regular order or if it is too low it will rush regular order. From a regular replenishing mode, we can calculate expected on-hand levels of each period. In Table 9, for a 4-period demand cycle, a regular order is expected in period 1 of each cycle. The on-hand levels shown are the levels at the end of periods. These levels are determined by an ordering policy of a regular mode. These numbers repeat cycle after cycle. Therefore, expected on-hand levels at the end of periods 1, 2, 3 and 4 are 1480, 5000, 3800 and 2,360 respectively. Therefore, for a dynamic policy, if retailer's on-hand level reaches emergency reorder point in any sub-period, an emergency order is placed to raise on-hand level to the expected on-hand level of that period. For a static policy, an expected on-hand level is the lowest level which is higher than the emergency reorder point. Please note that the expected on-hand levels are calculated based on the expected demand without considering variation of demand.

Table 9 An example of regular-mode ordering policy

Reorder	1,480
Order-up-to	5,480

		Cycle 1				Cycle 2			
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Demand		880	480	1,200	1,440	880	480	1,200	1,440
On hand	2,360	1,480	5,000	3,800	2,360	1,480	5,000	3,800	2,360
Order		4,000				4,000			

Then, since an emergency mode is triggered when a location faces risk of stockouts under the same demand distribution, for a dynamic policy, a reorder point for each period can be calculated with equation (60) as follows. For a static policy, the reorder point is the highest reorder point of all periods in a cycle.

$$s = \hat{x}_{R+L} + z\sigma_{R+L} \qquad (60)$$

where,

R = review interval (sub-period)

L = lead time (sub-period)

$\hat{x}_t$= expected demand in a period of duration t (unit)

σ_t= standard deviation of demand over a period of duration t

z = the z-value corresponding to the expected service level

When an emergency order is requested but the warehouse does not have sufficient items, a transshipment is considered. In contrast to emergency mode, a transshipment mode is one retailer requests items from another retailer. As a transshipment decreases on-hand level on another retailer, this mode will only request an amount to prevent stockouts. Therefore, an order-up-to point for a transshipment mode, S_T, is calculated with equation (60) which is used to calculate a reorder point for an emergency mode. To prevent stockouts, a location should hold items at least equal to S_T. Therefore, a reorder point is $S_T - 1$. Please note that although the S_T is calculated with equation (60), it is not the same value of a reorder point of an emergency mode due to different lead time between two modes.

An example of ordering policies for emergency and transshipment modes is shown in Table 10 and Table 11. In the example, standard deviation of demand is assumed to be 5% of period's demand. Each period is divided into 4 sub-periods. A lead time for an emergency mode is 2 sub-period and a lead time for a transshipment is 1 sub-period. A review interval for special modes is 1 sub-period. The ordering policy for a regular mode, demand pattern and expected on-hand are the same as in Table 9. In each period, average demand is the same for each sub-period so sub-period's demand in period 1 is $\frac{880}{4} = 220$ and its standard deviation is $\sqrt{\frac{1}{4}}(880 \times 5\%) = 22$.

Therefore, for an emergency mode, order-up-to points for the dynamic policy are determined. Reorder points for the dynamic policy are determined with 2 sub-period lead time and 1 sub-period review interval. For period 1, average demand during lead time and review period is $\hat{x}_{R+L} = \hat{x}_{1+2} = 660$ and standard deviation is $\sigma_{R+L} = \sqrt{1+2}(22) = 38.11$ and, then, a reorder point is $s = \hat{x}_{R+L} + z\sigma_{R+L} = 660 + 1.64 \times 38.11 = 722.54$. Then, a reorder point is rounded up to 723. A static policy is determined as the highest reorder point and lowest order-up-to point. For a transshipment mode, order-up-to points for the dynamic policy are determined with equation (1) with 1 sub-period lead time and 1 sub-period review interval. Therefore, its order-up-to points are lower than the reorder points of an emergency mode. Then, reorder points are determined by order-up-to minus 1. Afterwards, a static policy is determined as the highest order-up-to and reorder points.

Too many items transshipped would lead to stockouts at the retailer that provides items. We apply a partial pooling concept that items are reserved for local future demand (Paterson et al., 2011). The items will be transshipped from a retailer when they are excessive items only.

Table 10 An example of ordering policies for special replenishing modes.

Number of Sub-periods	4	**SD/Average Demand**	5%
Lead Time (Emergency)	2	**Expected Service Level**	95%
Lead Time (Transshipment)	1	**z (Expected Service Level)**	1.64
Review Interval	1		

Periods	1	2	3	4
Demand (Period)	880	480	1,200	1,440
On-Hand (unit)	1,480	5,000	3,800	2,360
Demand (Sub-period)	220	120	300	360
SD (sub-period)	22	12	30	36
SD (Emergency)	38.11	20.78	51.96	62.35
SD (Transshipment)	31.11	16.97	42.43	50.91

Table 11 An example of dynamic and static ordering policies

	Dynamic Policy				**Static Policy**
Periods	1	2	3	4	
Order-up-to (Emergency)	1,480	5,000	3,800	2,360	1,480
Reorder (Emergency)	723	395	986	1,183	1,183
Order-up-to (Transshipment)	492	268	670	804	804
Reorder (Transshipment)	491	267	669	803	803

Using the transshipment mode, we also need to decide which retailer should be requested order. These potential retailers are the location with on-hand items more than its reorder point after sending a considered order. We use a following ratio to find potential retailers. A potential retailer is the one with ratio higher than 1. Then, the transshipping retailer is the potential retailer with the highest ratio.

$$r_i = \frac{I_i - T}{s_i} \qquad (61)$$

where,

T= a considered transshipment order (unit)

r_i= ratio of retailer i

I_i= on-hand items of retailer i (unit)

s_i= reorder point of retailer i (unit)

On each sub-period, special modes place order only when the items arrive within that period. Therefore, special modes consider only on-hand inventory of a retailer and on-order of emergency and transshipment modes. When the inventory position gets below both emergency and transshipment reorder points, an emergency will always be considered first. When an inventory position reaches an emergency reorder point, an order is placed to raise the on-hand to an emergency order-up-to point. If an emergency order is placed on that sub-period, an inventory position is raised to be higher than both emergency and transshipment reorder points and there will be no transshipment order placed. If the warehouse cannot deliver the whole order, no items is delivered. Then, a transshipment is considered when there is no emergency order on that sub-period.

5.5. Result and Discussion

To prevent periods with unsatisfied service level, safety stock would be very high. Therefore, we expect special replenishing modes to eliminate the number of periods with unsatisfied service level (NUS). The special modes should help reduce the NUS without increasing holding cost. Since it transfers items from one location to another in the system to serve demand.

There are 3 settings with 4-period cycle and 3 settings with 7-period cycle and these 6 settings are solved on demand standard deviation of 5%, 10%, 15% and 20% of demand average. Therefore, 24 instances are used to explore benefit of using special replenishing modes. Example of parameter settings are shown in Table 12. In Table 11, each retailer has different demand pattern. Average demand on each period is shown for every retailer. All settings for the same length of cycle have different ordering costs for a regular replenishing mode. With high ordering cost, a retailer would order less frequent than with low ordering cost. As Chapter 3 studied the problem with regular mode, ordering policies would control the locations to place orders according to the periods in demand cycle. Therefore, with high ordering cost, a retailer would order only once in a cycle but with low ordering cost, it would order every period. On different demand deviation, the policies for a regular mode would be different due to different safety stock but, as they have the same demand pattern, they would be expected to place orders on the same period in a cycle. Each location has its own number of expected orders per cycle. For example, if a retailer under 7-period cycle has this number of 1, it places an order every 7 periods. If a retailer under 4-period cycle the number of expected orders of 3, it may place orders on periods 1, 2, 3 or 1, 3, 4 of the cycle or any other group of 3 periods.

Every instance is tested under 95% service level. The regular replenishment has 1-period lead time and reviews inventory levels every period. For the special replenishing modes, each period is divided into 4 sub-period and special modes review inventory level every sub-period. The emergency mode has 2-sub-period lead time and the transshipment has 1-sub-period lead time. Every policy for special modes is tested on a 10,000-period instance.

Table 12 Parameters of each instance.

Setting	No. Retailer	Period/ Cycle	Average Demand/Period			Number of Expected Orders/Cycle			
			Retailer 1	Retailer 2	Retailer 3	Warehouse	Retailer 1	Retailer 2	Retailer 3
1	2	4	880, 480,1200, 1440	880, 1840, 2400, 2880		1	1	1	
2			880, 480,1200, 1440	880, 1840, 2400, 2880		2	3	2	
3			88,48, 120, 144	88, 184, 240, 288		2	3	2	
1	3	7	107, 101, 111, 109, 76, 142, 54	242, 269, 263, 281, 184, 106, 55	458, 344, 396, 452, 295, 611,244	1	0.5	7	7
2			107, 101, 111, 109, 76, 142, 54	242, 269, 263, 281, 184, 106, 55	458, 344, 396, 452, 295, 611,244	2	1	1	7
3			107, 101, 111, 109, 76, 142, 54	242, 269, 263, 281, 184, 106, 55	458, 344, 396, 452, 295, 611,244	7	7	1	7

5.5.1 The difference between emergency and transshipment modes

The number of emergency and transshipment orders decreases as the deviation of demand increases as shown in Figure 20. The number of orders tends to decrease because, under higher deviation demand, the safety stock on each location is higher where retailers face the risk of stockouts less frequently. However, instances under the same demand deviation have different number of special orders. Another factor affecting the number of orders is the number of expected orders per cycle shown in Table 11. The instances with higher number of expected orders tend to have more special orders. With 4-period cycle, instances with setting 1 have the lowest Number of expected orders and they also have the lowest number of special orders. Instances with 7-period behave in a similar way. Instances with setting 2 also have the lowest number of expected orders and number of special orders. However, the instances with setting 1 have more special orders than instances with setting 3. The reason could be that the setting 1 orders frequent on a retailer 2 which has demand which is larger and fluctuates more than retailer 1. A retailer faces the risk of stockouts when its on-hand is low. This situation would be in the period where a retailer waits for an order to arrive. Since an on-hand level is at its highest in the period that an order arrives and at its lowest in the period right before an order's arrival. With a policy that leads to order frequently, a retailer tends to face the risk of stockouts more frequently than with a policy that rarely orders.

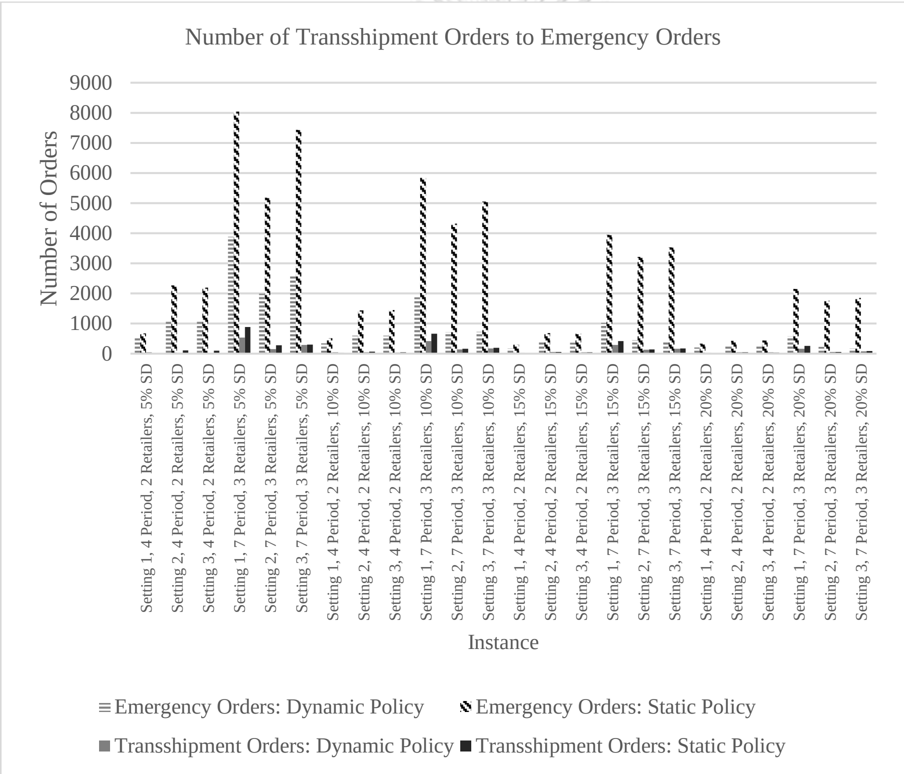

Figure 20 Number of transshipment orders to emergency orders

Between an emergency mode and transshipment mode, the number of transshipment orders is around 12% of the number of emergency orders. Since the system always considers emergency mode before a transshipment mode, a transshipment order can occur when an emergency order cannot be placed such as when the warehouse cannot fulfill the order or at the last sub-period. To consider this relationship, the ratio of number of transshipment orders to number of emergency orders is calculated as $R_{T/E} = \frac{Number\ of\ transshipment\ orders}{Number\ of emergency\ orders} \times 100$. The ratio for each instance is shown on Figure 21. On average, $R_{T/E}$ is 18% under dynamic policies and 6% under static policies. Due to more emergency orders, $R_{T/E}$ under static policies is much lower than $R_{T/E}$ under dynamic policies. The ratio tends to increase as the deviation of demand increases for dynamic policies because, with high deviation demand, the number of emergency and transshipment orders decreases but the number of emergency orders decreases more rapidly.

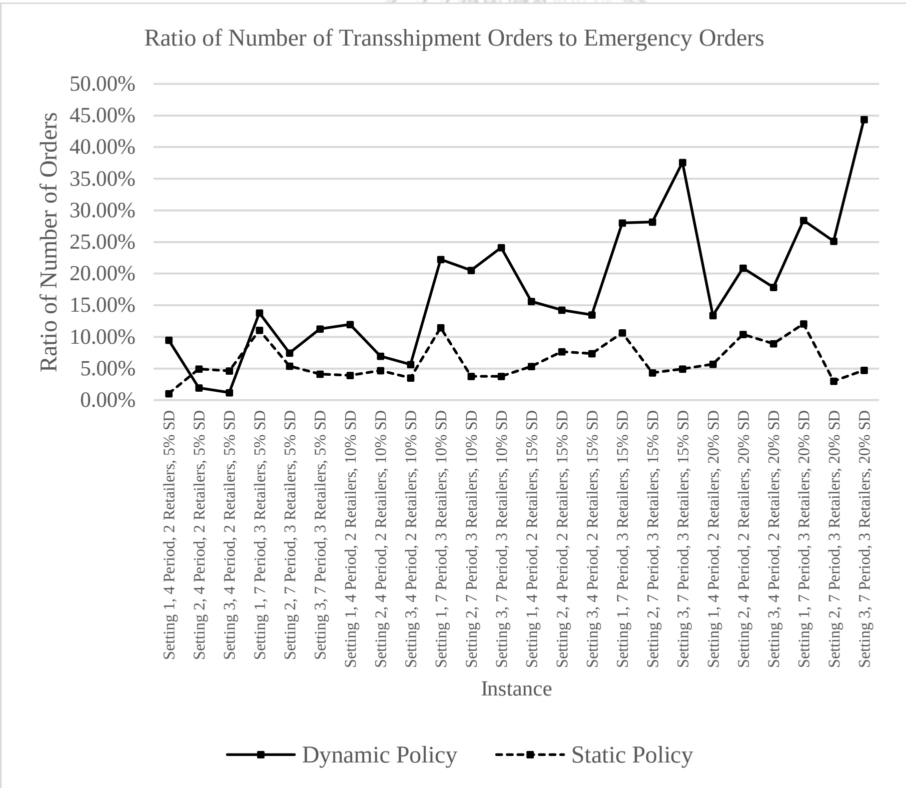

Figure 21 Ratio of number of transshipment orders to emergency orders

5.5.2 The difference between dynamic and static policies

With special replenishing modes, the number of periods with unsatisfied service level, NUS, is eliminated while the holding cost for each instance slightly increases. On average, static and dynamic policies give holding costs 0.85% and 0.72% higher than those of instances without any special mode. However, as the static policy uses the highest reorder point of the demand cycle, it is expected to place more special orders than the dynamic policy. The number of special orders of a static and dynamic policies are shown in Figure 22. The Figure 22 also illustrates the NUS of the system without special modes and with special modes under static and dynamic policies

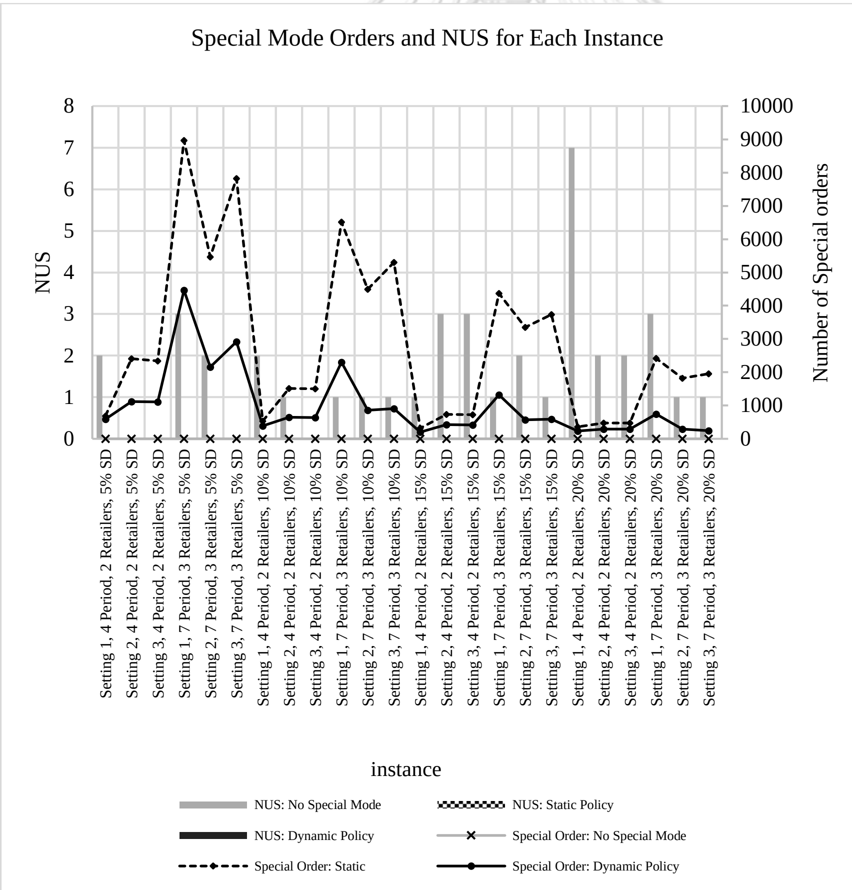

Figure 22 Number of periods with unsatisfied service level and number of special orders on each instance

However, the number of special orders from a dynamic policy gets closer to those from a static policy under higher standard deviation of demand and shorter demand cycle as shown in Figure 23. We calculate a ratio of number of special orders from dynamic policy to static policy as $R_{D/S} = \frac{Number\ of\ special\ orders_{dynamic}}{Number\ of\ special\ orders_{static}} \times 100$. As the number of special orders for 7-period instances is higher than those for 4-period instances, $R_{D/S}$ is lower for 7-period instances. On average, 4-period instances give $R_{D/S}$ of 53% while 7-period instances have $R_{D/S}$ only 31%. To consider whether to choose a static policy or a dynamic policy, one factor to take into account is the frequency of special orders. The ratio of number of special orders to regular orders is calculated as $R_{S/R} = \frac{Number\ of\ special\ orders}{Number\ of\ regular\ orders} \times 100$. In Figure 23, $R_{S/R}$ is around 1% to 25% under dynamic policies and 4% to 55% under static policies. Similar to the ratio of number of special orders from dynamic policy to static policy, $R_{D/S}$, 7-period instances give higher $R_{S/R}$ than 4-period instances. For 7-period instances, $R_{S/R}$ is around 10% to 55% under static policies and 1% to 25% under dynamic policies. For 4-period instances, $R_{S/R}$ is around 4% to 23% and 3% to 14% under static and dynamic policies, respectively. The $R_{D/S}$ is higher with shorter demand cycle because the difference between dynamic and static policies is directly affected by the number of policies for special modes. While a static policy has only one policy for each special mode, a dynamic policy has policies as many as the number of periods in a cycle. Therefore, the longer cycle is, the bigger difference between dynamic and static policies becomes. Furthermore, $R_{S/R}$ of dynamic and static policies get closer when the length of cycle is shorter. It is due to the same reason that the difference between two policies is affected by the length of cycle or the number of dynamic policies.

Therefore, from $R_{D/S}$ and $R_{S/R}$ when the deviation of demand is high and demand cycle is short, a static policy can substitute the dynamic policy.

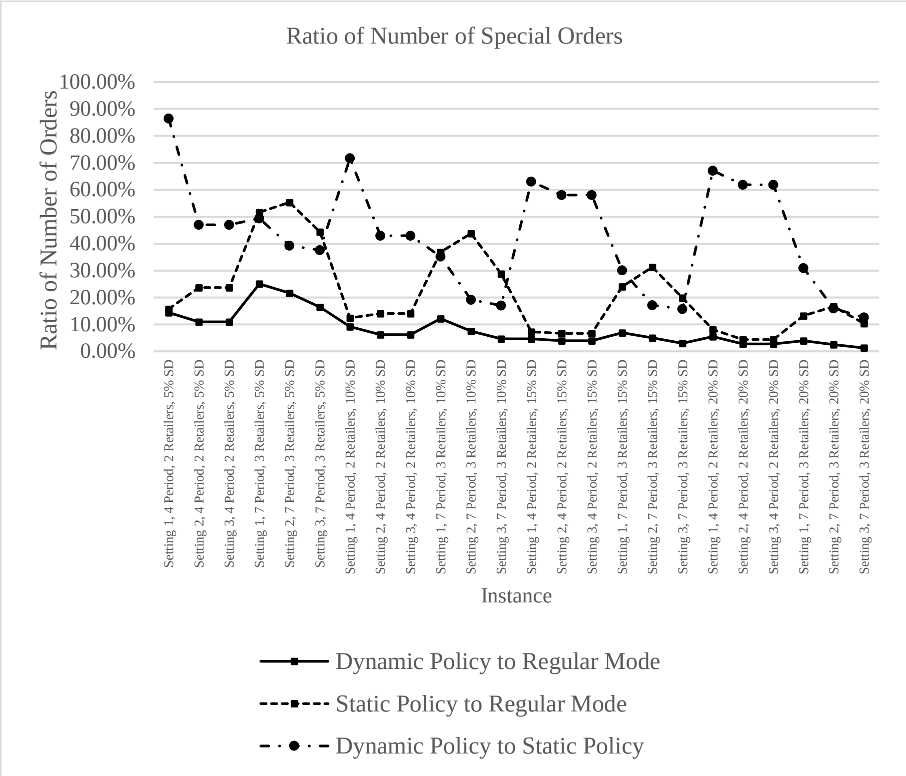

Figure 23 Ratio of number of special orders

5.5.3 Safety stock levels and number of special orders

It is obvious that when the safety stock is higher the ratio, $R_{S/R}$, as in section 5.5.2, will be lower and vice versa. We investigate how different safety stock affects number of special orders. In Figure 24, $R_{S/R}$ decreases when the safety stock increases. The relationship goes in the same way for both static and dynamic policies. Therefore, if holding cost of the system is high compared to ordering cost of special modes, decrease safety stock level and let the special modes respond to stockouts would be a good choice.

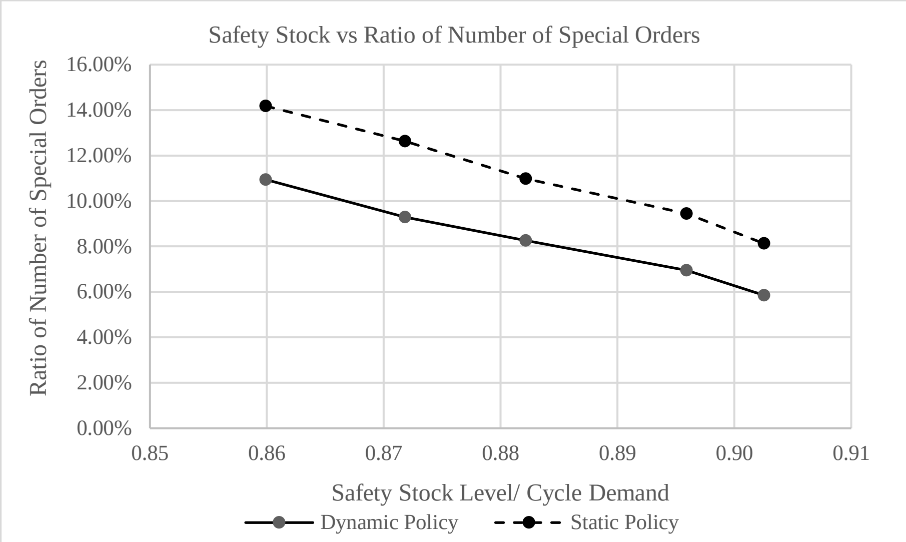

Figure 24 Safety stock levels vs number of special orders

5.6 Conclusion

An emergency and transshipment modes are considered as special modes to prevent stockouts. In our experiment, the number of special orders tends to decrease in instances with high deviation demand because the safety stock on each location under these instances is high. Therefore, the retailers face the risk of stockouts less frequently. With a policy for a regular mode that leads to order frequently, a retailer tends to face the risk of stockouts more frequently which leads to more special orders. Since we let the system considers emergency mode before a transshipment mode, the number of transshipment orders is around 12% of the number of emergency orders. The ratio tends to increase under the higher deviation of demand. With high deviation demand, the number of emergency and transshipment orders decreases but the number of emergency orders decreases more rapidly.

Two types of policies for special replenishing modes are considered. The first one is a static policy which applies one policy on every period and the other is a dynamic policy which varies policy for each period in the demand cycle. The static policy would be easier to implement while the dynamic policy would give a better result. We investigate these policies to find how they affect the system in different situations. With special replenishing modes, the number of periods with unsatisfied service level, NUS, is eliminated while the holding cost for each instance is slightly higher. A dynamic policy tends to give smaller number of special orders. However, the number of special orders from a dynamic policy gets closer to those from a static policy with higher demand deviation and shorter demand cycle. Therefore, in case of high deviation demand and short demand cycle, for ease of use the static policy can substitute the dynamic policy.

Moreover, the number of special orders is in inverse proportion to the size of safety stock levels. Therefore, one way to reduce holding cost is to let special modes respond to stockouts and decrease safety stock.

As this chapter focuses on the 2-echelon system under seasonal demand, there are various aspects to extend the problem. This chapter considers only replenishing policies with a unit size of items. However, in reality, a batch-size constraint would be applied. The batch size could even be different for the warehouse and retailers. Furthermore, the system would be explored how it performs when each location has stochastic lead time. Another constraint which reflects real-life is how to determine ordering policies for the system with capacitated space. These constraints would be a useful research extension in real-life situation.

CHAPTER VI

CONCLUDING REMARKS AND FUTURE WORKS

In this thesis, we propose methodologies to determine ordering policies for a 2-echelon inventory system under seasonal demand. Our objective is to determine the ordering policies for all locations with minimum cost respected to expected service level. The first part of the dissertation decomposes a problem into 2 phases and develop a methodology based on mixed-integer programming models (MIP models). The first phase determines deterministic policy by solving a deterministic problem based on average demand. The MIP model in this phase is used to find reorder and order-up-to points. Then, the second phase calculates safety stock by solving multiple scenarios of the problem generated from demand distribution. We found that there are alternative optimal solutions where multiple reorder points give the same total cost, but these alternatives lead to different cost in the second phase. In the second phase, another MIP model uses reorder and order-up-to points from the first phase as inputs and solve multiple scenarios with randomly generated demand simultaneously for safety stock that leads to the minimum total cost respected to required service level. The policy with the highest reorder point tends to give the lowest cost but they face more demand loss than other policies. On the other hand, the policy with the lowest reorder point tends to give the highest cost but the lowest demand loss.

The methodology considers the average demand of each period in the first phase, so it absorbs the fluctuation of demand's mean in this phase. Therefore, we believe that the model could deal with demand with any seasonal pattern as the policy tends to place an order on the period with small demand size to minimize holding cost. However, as we assume that the standard deviation of demand is a proportion of demand size, a period with smaller size of demand also has smaller deviation. Relaxing this assumption would lead to different results which should be studied further.

The proposed methodology in Chapter 3 can find solutions within a reasonable amount of time. However, it can be improved. Since the methodology is based upon MIP modes, it can solve limited problem size. While the problem sizes increase i.e. larger demand volume or greater number of retailers, the optimal solution may not be able to obtain with MIP models.

In the second part of the thesis (Chapter 4), we develop algorithms to solve the problem with smaller computational time. The problem is still decomposed into 2 phases. In the first phase to determine deterministic policies, a heuristic algorithm is developed partially based on the concept of the genetic algorithm. Then, in the second phase to determine safety stock levels, another algorithm is developed using binary search. The algorithms can find the solutions which are as good as the optimal solutions from the MIP models. Solving for the solutions in 2 phases, the total computational time from proposed algorithms is much smaller than those from the MIP models. Furthermore, we also find that the policies are more robust when the longer planning horizon is used in the second phase.

The genetic algorithm is chosen in this thesis since it is an efficient metaheuristic used in various research domains. The algorithm is also appropriate for this problem since the solutions can be encoded into chromosomes which stay feasible after crossover and mutation. Furthermore, the objective to find the solution with lowest cost is easy to define a fitness function as shown in Chapter 4. It is also easy to adjust the search by tuning crossover rate and mutation rate. With too high crossover rate, the search would converge too soon while, with too high mutation rate, the search would lose the good solutions. In our experiment, we investigate the results of using crossover rate of 0.6, 0.7, 0.8, and 0.9, and mutation rate of 0.1, 0.2, and 0.3. Crossover rates of 0.8 and 0.9 lead to premature convergence with mutation rate of 0.1. On the other hand, with crossover rate of 0.6, the search finds the optimal solution with mutation rate of 0.1 but it cannot find the optimal solution with mutation rate of 0.2 and 0.3. With crossover rate of 0.7, the search finds the optimal solution with all mutation rates. Therefore, we choose the crossover rate of 0.7 and mutation rate of 0.2.

From Figure 16, the first phase of the proposed methodology requires more computational time when the system has more retailers since the more retailers means the longer chromosomes to calculate. Consequently, the second phase also requires more computational time with more retailers as it has more periods to simulate. However, as the computational time grows linearly, we believe that the computational time will be reasonable to use this methodology until the size of 20 retailers. Therefore, the algorithm is appropriate for the problem since the size of case study is around 10 retailers.

Several directions are possible for future work related to Chapter 3 and 4. It can be extended in various aspects such as considering shortage cost or considering batch-size constraint. Considering multiple items is also an interesting way since it could lead to difficult decisions such as joint ordering or shared storage space. Another way to extend the problem is to consider multiple replenishing modes.

In the third part of the thesis (Chapter 5), beside the regular replenishing, two special replenishing modes with shorter fixed lead time are considered. These special modes are an emergency mode which is a mode supplied by the warehouse and a transshipment which is a mode where items are requested from other retailers with excessive on-hand stock. Normally, retailers are supplied by the warehouse via a regular replenishing mode but when any retailer faces the risk of stockout, items could be transported via one of special modes. An emergency mode is always considered first, while a transshipment mode is considered when the warehouse cannot fulfill an emergency order. As the demand is seasonal, two types of policies for special modes are considered. The first one is a static policy which applies one policy on every period and the other is a dynamic policy which varies policy for each period in the demand cycle. We investigate these policies to find how they affect the system in different situations. A dynamic policy tends to give smaller number of special orders. However, the number of special orders from a dynamic policy gets closer to those from a static policy with higher demand deviation and shorter demand cycle. Therefore, since a static policy is easier to implement, in case of high deviation demand and short demand cycle, the static policy can substitute the dynamic policy. Moreover, the number of special orders is in inverse proportion to the size of safety stock levels. Therefore, one way to reduce holding cost is to let special modes respond to stockouts and decrease safety stock. In Chapter 5, the ratio of number of special orders to regular orders and the ratio of number of transshipment orders to emergency orders are investigated in various deviation. With these ratios, if ordering cost of each special mode is realized, an approximate cost of using special modes could be calculated. For example, in a system with 5% deviation of demand average and 4-period cycle, Figure 21 and 23 show that, for a static policy, the number of special orders is around 25% of number of regular orders and the number of transshipment orders around 5% of emergency orders. Therefore, if the number of regular orders is 1,000 and ordering costs an emergency mode and a transshipment mode are \$10 and \$20, the approximate cost of applying special modes is around $(1{,}000 \times 25\% \times 95\% \times 10) + (1{,}000 \times 25\% \times 5\% \times 20) = \$2{,}625.$

There are several possible extensions of Chapter 5. First, like in Chapter 3 and 4, a batch-size constraint can be applied to reflect real-life situations. With multiple replenishing modes, a batch-size constraint could be different for each echelon and each replenishing mode. For example, a regular mode must be placed as multiple of large packages, but an emergency mode can be placed as multiple of small packages and a transshipment can be place in unit. Another interesting constraint is capacitated space constraint. Since each location stores many types of items to serve customers, storage space must be shared among these different items. Therefore, the space used to store each item would be limited. Therefore, ordering policy should take storage capacity into account. Furthermore, one might explore how the system performs when each mode has stochastic lead time. These additional constraints would be useful research directions.

Overall, we develop methodologies to determine ordering policies for a 2-echelon inventory system under seasonal demand. We decompose a problem into 2 phases. Then, we propose a methodology based on MIP models and we improve the methodology by developing algorithms based on the genetic algorithm and binary search to solve the problem with smaller computational time. Beside a regular replenishing mode in the first and second parts, a problem is extended with two special replenishing modes which used to prevent stockouts. Our work is motivated by a case-study problem in a healthcare sector, but we believe that the methodology is useful for any other inventory system with similar characteristics.

REFERENCES

Afzalabadi, M., Haji, A., & Haji, R. (2016). Vendor's optimal inventory policy with dynamic and discrete demands in an infinite time horizon. *Computers and Industrial Engineering*, 102, 368-373.

Axater, S., & Rosling, K. (1993). Notes: Installation vs. echelon stock policies for multilevel inventory control. *Management Science*, 39, 1274-1280.

Bessler, S.A., & Veinott, A.F. (1965). Optimal policy for a dynamic multi-echelon inventory model. *Naval Research Logistics Quarterly*, 13(4), 355-389.

Bollapragada, S., Akella, R., & Srinivasan, R. (1998). Centralized ordering and allocation policies in a two-echelon system with non-identical warehouses. *European Journal of Operational Research*, 106, 74-81.

Bookbinder, J.H., & Tan, J.-Y. (1988). Strategies for the probabilistic lot-sizing problem with service-level constraints. *Management Science*, 34(9), 1096-1109.

Bylka, S. (2005). Turnpike policies for periodic review inventory model with emergency orders. *International Journal of Production Economics*, 93-94, 357-373.

Bylka, S., & Krupa, A. (2018). A note on the optimal periodic pattern (OPP) algorithm for the system in which buyers periodically order from a vendor. *Computers and Industrial Engineering*, 115, 37-40

Chand, S., Li, J., & Xu, Y. (2016). A periodic review inventory model with two delivery modes, fractional lead-times, and age-and-period-dependent backlogging costs. *International Journal of Production Economics*, 173, 199-206.

Chiang, C. (2001). A note on optimal policies for a periodic inventory system with emergency orders. *Computers and Operations Research*, 28, 93-103.

Chiang, C. (2003). Optimal replenishment for a periodic review inventory system with two supply modes. *European Journal of Operational Research*, 149, 229-244.

Chiang, C., & Gutierrez, G.J. (1996). A periodic review inventory system with two supply modes. *European Journal of Operational Research*, 28, 93-103.

Chiang, C., & Gutierrez, G.J. (1998). Optimal control policies for a periodic review inventory system with emergency orders. *Naval Research Logistics*, 45, 187-204.

Chu, L.Y. & Shen, Z.-J.M. (2010). A power-of-two ordering policy for one-warehouse multiretailer systems with stochastic demand. *Operations Research*, 59(2), 492-502.

Clark, A.J., & Scarf, H. (1960). Optimal policies for a multi-echelon inventory problem. *Management Science*, 2, 475-490.

De Bodt, M.A., & Graves, S.C. (1985). Continuous-review for a multi-echelon inventory problem with stochastic demand. *Management Science*, 31(10), 1286-1299.

De Kok, T., Grob, C., Laumanns, M., Minner, S., & Rambau, J. (2018). A typology and literature review on stochastic multi-echelon inventory models. *European Journal of Operational Research*, 269, 955-983.

Diaby, M., & Martel, A. (1993). Dynamic lot sizing for multi-echelon distribution systems with purchasing and transportation price discounts. *Operations Research*, 41(1), 48-59.

Diks, E.B., de Kok, A.G., & Lagodimos, A.G. (1996). Multi-echelon systems: a service measure perspective. *European Journal of Operational Research*, 95, 241-263.

Eppen, G., & Scharage, L. (1981). Centralized ordering policies in a multi-warehouse system with lead times and random demand. *TIMS Studies in the Management Sciences*, 16, 51-67.

Erkip, N., Hausman, W.H., & Nahmias, S. (1990). Optimal centralized ordering policies in multi-echelon inventory systems with correlated demands. *Management Science*, 36(3), 381-392.

Eruguz, A.S., Jemai, Z., Sahin, E., & Dallery, Y. (2014). Optimising reorder intervals and order-up-to levels in guaranteed service supply chains. *International Journal of Production Research*, 52(1), 149-164.

Federgruen, A., & Zipkin, P. (1984a). Computational issues in an infinite-horizon, multiechelon inventory model. *Operations Research*, 32(4), 818-836.

Federgruen, A., & Zipkin, P. (1984b). Approximations of dynamic multilocation production and inventory problems. *Management Science*, 30, 69-84.

Graves, S.C., & Willems, S.P. (2000). Optimizing strategic safety stock placement in supply chains. *Manufacturing and Service Operation Management*, 2, 68-83.

Graves, S.C., & Willems, S.P. (2008). Strategic inventory placement in supply chain: non-stationary demand. *Manufacturing and Service Operation Management*, 10(2), 278-287.

Grewal, C.S., Enns, S.T., & Rogers, P. (2015). Dynamic reorder point replenishment strategies for a capacitated supply chain with seasonal demand. *Computers and Industrial Engineering*, 80, 97-110.

Hanke, J.E., & Wichern, D.W. (2005). *Business forecasting*. New Jersey: Prentice Hall.

Huang, H., Zeng, N., & Xu, H. (2018). Procurement mechanism for dual sourcing and emergency production under capacity constraint. *Computers and Industrial Engineering*, 119, 204-218.

Jaruphongsa, W., Cetinkaya, S., & Lee, C-.Y. (2004). A two-echelon inventory optimization model with demand time window considerations. *Journal of Global Optimization*, 30, 347-366.

Ji, Q., Sun, L., Hu, X., & Hou, J. (2016). Optimal policies of a two-echelon serial inventory system with general limited capacities. *International Journal of Production Research*, 54(20), 6142-6155.

Johansen, S.G. (2018). Emergency orders in the periodic-review inventory system with fixed ordering costs and stochastic lead times for normal orders. *International Journal of Production Economics*, in Press.

Johansen, S.G., & Thorstenson, A. (2014). Emergency orders in the periodic-review inventory system with fixed ordering costs and compound poisson demand. *International Journal of Production Economics*, 157, 147-157.

Kim, G., Wu, K., & Huang, E. (2015). Optimal inventory control in a multi-period newsvendor problem with non-stationary demand. *Advanced Engineering Informatics*, 29, 139-145.

Kovalev, A., & Ng, C.T. (2008). A discret EOQ problem is solvable in O(log n) time. *European Journal of Operational Research*, 189, 914-919.

Minner, S. (2003). Multiple-supplier inventory models in supply chain management: a review. *International Journal of Production Economics*, 81-82, 265-279.

Mitra, S., & Chatterjee, A.K. (2004). Echelon stock based continuous review (R,Q) policy for fast moving items. *Omega*, 32, 161-166.

Moinzadeh, K., & Nahmias, S. (1988). A continuous review model for an inventory system with two supply modes. *Management Science*, 34, 761-773.

Nahmias, S. (2009). *Production and operations analysis*. New York: McGraw-Hill

Olsson, F. (2015). Emergency lateral transshipments in a two-location inventory system with positive transshipment leadtimes. *European Journal of Operational Research*, 242, 424-433.

Pacheco, E.O., Cannella, S., Luders, R., & Barbosa-Povoa, A.P. (2017). Order-up-to-level policy update procedure for a supply chain subject to market demand uncertainty. *Computers and Industrial Engineering*, 113, 347-355.

Paterson, C., Kiesmuller, G., Teunter, R. & Glazebrook, K. (2011). Inventory models with lateral transshipment: a review. *European Journal of Operational Research*, 210, 125-136.

Reddy, A.M., & Rajendran, C. (2005). A simulation study of dynamic order-up-to policies in a supply chain with non-stationary customer demand and information sharing. *International Journal of Advance Manufacturing Technology*, 25, 1029-1045.

Robinson, L.W. (1990). Optimal and approximate policies in multiperiod, multilocation inventory models with transshipments. *Operations Research*, 38(2), 278-295.

Sastry, K., Goldberg, D., & Kendall, G. (2005). *Search methodologies*. Boston: Springer.

Shang, K.H., Tao, Z., & Zhou, S.X. (2015). Optimizing reorder intervals for two-echelon distribution systems with stochastic demand. *Operations Research*, 63(2), 458-475.

Silver, E.A., Pyke, D., & Peterson, R.F. (1998). *Inventory management and production planning and scheduling*, New York: John Wiley and Sons.

Tagaras, G. & Vlachos, D. (2001). A periodic review inventory system with emergency replenishments. *Management Science*, 47 (3), 415-429.

Tai, A.H., & Ching, W-.K. (2014). Optimal inventory policy for a Markovian two-echelon system with returns and lateral transshipment. *International Journal of Production Economics*, 151, 48-55.

Tarim, S.A., & Kingsman, B.G. (2004). The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. *International Journal of Production Economics*, 88, 105-119.

Tarim, S.A., & Kingsman, B.G. (2006). Modelling and Computing (R^n, S^n) policies for inventory systems with non-stationary stochastic demand. *European Journal of Operational Research*, 174, 581-599.

Tarim, S.A., & Smith, B.M. (2008). Constraint programming for computing non-stationary (R, S) inventory policies. *European Journal of Operational Research*, 189, 1004-1021.

Tlili, M., Moalla, M., & Campagne, J.-P. (2012). The trans-shipment problem in a two-echelon, multi-location inventory system with lost sales. *International Journal of Production Research*, 50(13), 3547-3559.

Tunc, H., Kilic, O.A., Tarim, S.A., & Eksioglu, B. (2011). The cost of using stationary inventory policies when demand is non-stationary. *Omega*, 39, 410-415.

Yao, M., & Minner, S. (2017). Review of multi-supplier inventory models in supply chain management: an update. Working paper.

Zangwill, W. (1969). A backlogging model and a multi-echelon model of a dynamic economic lot size production system – a network approach. *Management Science*, 15, 506-527.

Zhou, S.X., & Yang, C. (2016). Continuous-review (R, nQ) policies for inventory systems with dual delivery modes. *Operations Research*, 64(6), 1302-1319.

REFERENCES

VITA

NAME	NOPPADON SAKULSOM
DATE OF BIRTH	5 May 1984
PLACE OF BIRTH	Saraburi
INSTITUTIONS ATTENDED	Chulalongkorn University Kasetsart University
HOME ADDRESS	448/299 Pracha-U-Thit Rd. Huaikwang Huaikwang Bangkok 10310
PUBLICATION	A Multi-Objective Music Rehearsal Scheduling Problem (2011) Scheduling a music rehearsal problem with unequal music piece length (2014)